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Algebra: Introductory and Intermediate

Aufmann/Barker/Lockwood Third Edition

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by PAUL NOLTING

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Preface

The third edition of *Algebra: Introductory and Intermediate* examines the fundamental ideas of algebra. Recognizing that the basic principles of geometry are a necessary part of mathematics, we have also included a separate chapter on geometry (Chapter 3) and have integrated geometry topics, where appropriate, throughout the text. The text has been designed not only to meet the needs of the traditional college student but also to serve the needs of returning students whose mathematical proficiency may have declined during years away from formal education.

In this new edition of *Algebra: Introductory and Intermediate*, we have continued to integrate the approaches suggested by AMATYC. Each chapter opens with a photo and a reference to a mathematical application within the chapter. At the end of each section, there are "Applying the Concepts" exercises that include writing, synthesis, critical thinking, and challenge problems. At the end of each chapter, there is a "Focus on Problem Solving" that introduces students to various problem-solving strategies. This is followed by "Projects and Group Activities" that can be used for cooperative-learning activities.

NEW! Changes to This Edition

New to this edition is the topic of graphing scatter diagrams. See Objective 4.1C in Chapter 4, *Linear Functions and Inequalities in Two Variables*. This objective reinforces the skill of graphing points in a rectangular coordinate system while providing interesting applications of the skill.

In response to user requests, we have expanded the material on functions to include the mapping of functions and illustrations of function machines. See, for example, Section 2 of Chapter 4, *Linear Functions and Inequalities in Two Variables*. This chapter also now includes graphing exercises that require critical thinking. See the exercises in Section 4.4.

In Chapter 5, *Systems of Linear Equations and Inequalities*, the discussion of solutions of systems of equations has been expanded and improved. This development should lead to greater student understanding of dependent, inconsistent, and independent systems of equations.

In Chapter 9, *Rational Exponents and Radicals*, nth roots are simplified by using perfect powers rather than by using prime factorization. Therefore, rather than rewriting the square root of 20 as the square root of 2^2 times 5, 20 is written as the product of 4 and 5; the cube root of 16 is rewritten as the cube root of 8 times 2, not 2^4 and then 2^3 times 2. This approach better mirrors what the student is thinking mentally when simplifying radical expressions.

A new chapter, Chapter R, *Review of Introductory Algebra Topics*, follows Chapter 12 and was written specifically for this edition of *Algebra: Introductory and Intermediate*. This chapter provides a review of the main topics of first-year algebra. It can be used at any point in the course. For students who need more in-depth review of a topic, there are references within the chapter that refer them back to the expanded coverage within the text.

Concept-based writing exercises have been added at the beginning of many of the objective-specific exercise sets. These exercises require a student to verbalize basic concepts presented in the lesson. These are concepts that the student must

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have an understanding of before attempting any exercises. For example, in Chapter 12, *Exponential and Logarithmic Functions*, the first exercise for Objective 12.2A asks, "What is a common logarithm? How is the common logarithm of $4z$ written?"

Many of the exercise sets now include developmental exercises, the intent of which is to reinforce the concepts underlying the skills presented in the lesson. For example, in Chapter 10, *Quadratic Equations*, the exercises for the first section require students to write quadratic equations in standard form and then to name the values of a, b, and c prior to having them solve any quadratic equations.

Throughout the text, data problems have been updated to reflect current data and trends. These application problems will demonstrate to students the variety of problems that require mathematical analysis. Instructors will find that many of these problems may lead to interesting class discussions. Besides the Index of Applications on the inside front cover, a chapter-by-chapter index of the variety of application problems can be found in the Table of Contents. This additional index highlights, in an easily accessible location, the importance and scope of the applications of mathematics.

Another new feature of this edition is *AIM for Success*, which explains what is required of a student to be successful and how this text has been designed to foster student success. *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills. There are suggestions for teaching this lesson in the *Instructor's Resource Manual*.

Related to *AIM for Success* are *Prep Tests*, which occur at the beginning of each chapter. These tests focus on the particular prerequisite skills that will be used in the upcoming chapter. The answers to these questions can be found in the Answer Section along with a reference (except for Chapter 1) to the objective from which the question was taken. Students who miss a question are encouraged to review the objective from which the question was taken.

The *Go Figure* problem that follows the *Prep Test* is a puzzle problem designed to engage students in problem solving.

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Chapter Opening Features

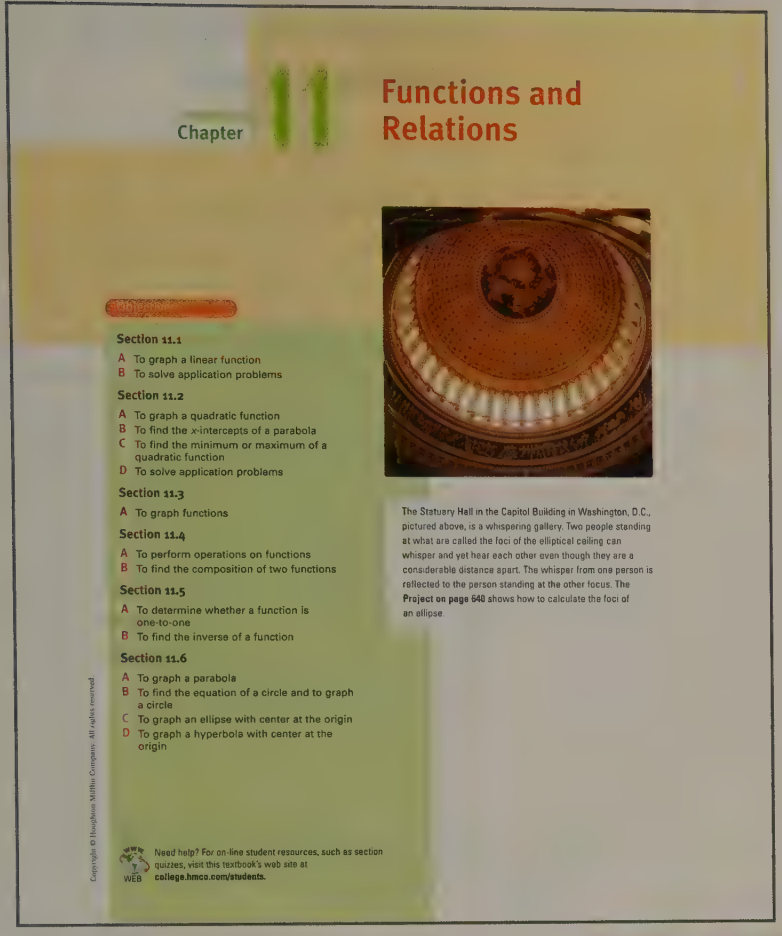
NEW! Chapter Opener

New, motivating chapter opener photos and captions have been added, illustrating and referencing specific applications from the chapter.

The WEB at the bottom of the page lets students know of additional on-line resources at **math.college.hmco.com/students.**

Objective-Specific Approach

Each chapter begins with a list of learning objectives which form the framework for a complete learning system. The objectives are woven throughout the text (i.e., Exercises, Prep Tests, Chapter Reviews, Chapter Tests, and Cumulative Reviews) as well as through the print and multimedia ancillaries. This results in a seamless learning system delivered in one consistent voice.



Chapter 11

Functions and Relations

Section 11.1
A To graph a linear function
B To solve application problems

Section 11.2
A To graph a quadratic function
B To find the x-intercepts of a parabola
C To find the minimum or maximum of a quadratic function
D To solve application problems

Section 11.3
A To graph functions

Section 11.4
A To perform operations on functions
B To find the composition of two functions

Section 11.5
A To determine whether a function is one-to-one
B To find the inverse of a function

Section 11.6
A To graph a parabola
B To find the equation of a circle and to graph a circle
C To graph an ellipse with center at the origin
D To graph a hyperbola with center at the origin

The Statuary Hall in the Capitol Building in Washington, D.C., pictured above, is a whispering gallery. Two people standing at what are called the foci of the elliptical ceiling can whisper and yet hear each other even though they are a considerable distance apart. The whisper from one person is reflected to the person standing at the other focus. The **Project on page 640** shows how to calculate the foci of an ellipse.

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

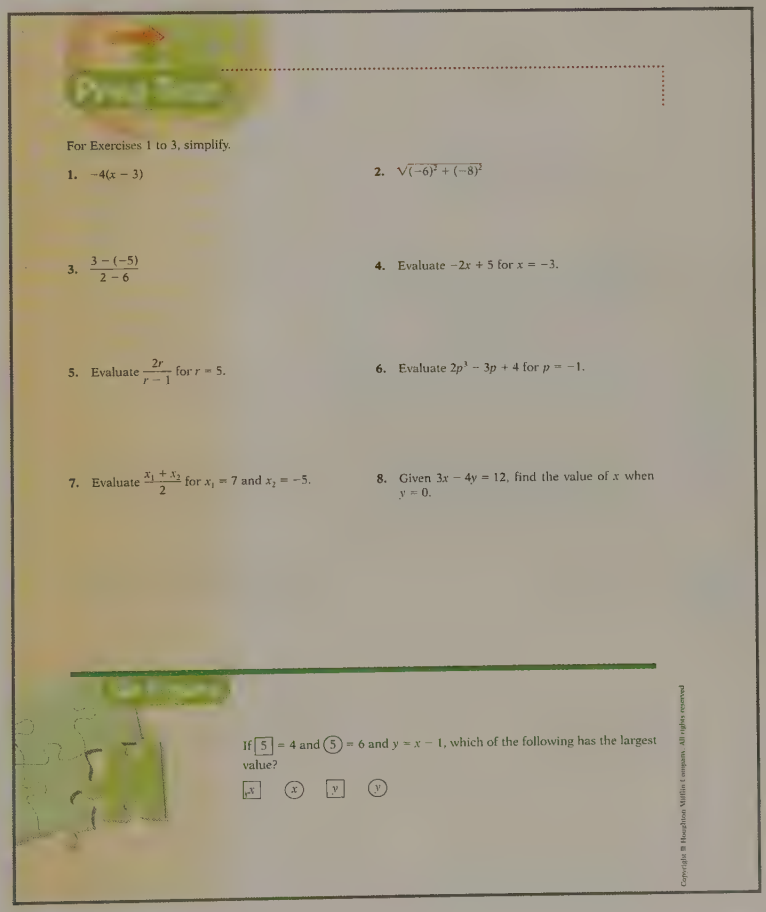
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NEW! Prep Test and Go Figure

Prep Tests occur at the beginning of each chapter and test students on *previously covered* concepts that are required in the upcoming chapter. Answers are provided in the Answer Section. Objective references are also provided if a student needs to review specific concepts.

The **Go Figure** problem that follows the *Prep Test* is a playful puzzle problem designed to engage students in problem solving.



Prep Test

For Exercises 1 to 3, simplify.

1. $-4(x-3)$
2. $\sqrt{(-6)^2+(-8)^2}$
3. $\frac{3-(-5)}{2-6}$
4. Evaluate $-2x+5$ for $x=-3$.
5. Evaluate $\frac{2r}{r-1}$ for $r=5$.
6. Evaluate $2p^3-3p+4$ for $p=-1$.
7. Evaluate $\frac{x_1+x_2}{2}$ for $x_1=7$ and $x_2=-5$.
8. Given $3x-4y=12$, find the value of x when $y=0$.

Go Figure

If $\boxed{5}=4$ and $(5)=6$ and $y=x-1$, which of the following has the largest value?

$\boxed{x}$ (x) $\boxed{y}$ (y)

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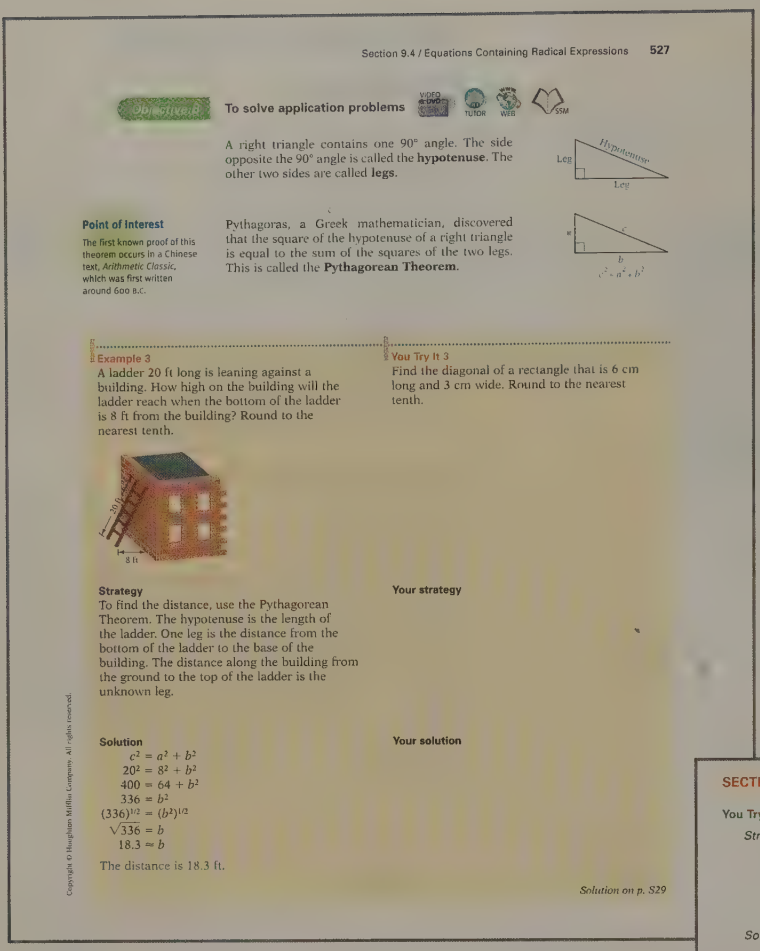
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Aufmann Interactive Method (AIM)

An Interactive Approach

Algebra: Introductory and Intermediate uses an interactive style that provides a student with an opportunity to try a skill as it is presented. Each section is divided into objectives, and every objective contains one or more sets of matched-pair examples. The first example in each set is worked out; the second example, called "You Try It," is for the student to work. By solving this You Try It, the student actively practices concepts as they are presented in the text.

There are *complete, worked-out* solutions to these examples in an appendix. By comparing their solutions to the solution in the appendix, students obtain immediate feedback on, and reinforcement of, the concept.



Section 9.4 / Equations Containing Radical Expressions 527

Objective B: To solve application problems

A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse**. The other two sides are called **legs**.

Hypotenuse, Leg, Leg

Point of Interest
The first known proof of this theorem occurs in a Chinese text, *Arithmetic Classic*, which was first written around 600 B.C.

Pythagoras, a Greek mathematician, discovered that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. This is called the **Pythagorean Theorem**.

$c^2 = a^2 + b^2$

Example 3
A ladder 20 ft long is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 8 ft from the building? Round to the nearest tenth.

20 ft, 8 ft

Strategy
To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$c^2 = a^2 + b^2$
$20^2 = 8^2 + b^2$
$400 = 64 + b^2$
$336 = b^2$
$(336)^{1/2} = (b^2)^{1/2}$
$\sqrt{336} = b$
$18.3 \approx b$

The distance is 18.3 ft.

Solution on p. S29

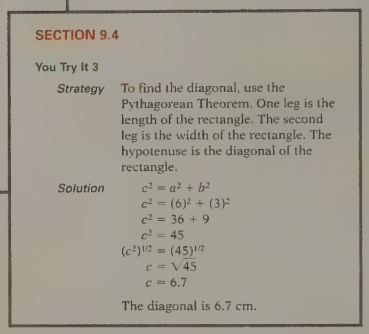
You Try It 3
Find the diagonal of a rectangle that is 6 cm long and 3 cm wide. Round to the nearest tenth.

Your strategy

Your solution

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SECTION 9.4

You Try It 3

Strategy To find the diagonal, use the Pythagorean Theorem. One leg is the length of the rectangle. The second leg is the width of the rectangle. The hypotenuse is the diagonal of the rectangle.

Solution

$c^2 = a^2 + b^2$
$c^2 = (6)^2 + (3)^2$
$c^2 = 36 + 9$
$c^2 = 45$
$(c^2)^{1/2} = (45)^{1/2}$
$c = \sqrt{45}$
$c \approx 6.7$

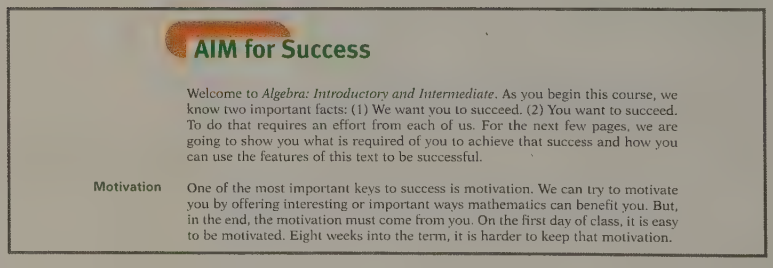
The diagonal is 6.7 cm.

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NEW! AIM for Success Student Preface

This new student "how to use this book" preface explains what is required of a student to be successful and how this text has been designed to foster student success, including the Aufmann Interactive Method (AIM). *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills. There are suggestions for teaching this lesson in the *Instructor's Resource Manual.*

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AIM for Success

Welcome to *Algebra: Introductory and Intermediate*. As you begin this course, we know two important facts: (1) We want you to succeed. (2) You want to succeed. To do that requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways mathematics can benefit you. But, in the end, the motivation must come from you. On the first day of class, it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

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Problem Solving

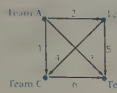
Focus on Problem Solving

At the end of each chapter is a **Focus on Problem Solving** feature, which introduces the student to various successful problem-solving strategies. Strategies such as drawing a diagram, applying solutions to other problems, working backwards, inductive reasoning, and trial and error are some of the techniques that are demonstrated.

Focus on Problem Solving

Solve an Easier Problem

Suppose you are in charge of your softball league, which consists of 15 teams. You must devise a schedule in which each team plays every other team once. How many games must be scheduled?



To solve this problem, we will attempt an easier problem first. Suppose that your league contains only a small number of teams. For instance, if there were only 1 team, you would schedule 0 games. If there were 2 teams, you would schedule 1 game. If there were 3 teams, you would schedule 3 games. The diagram at the left shows that 6 games must be scheduled when there are 4 teams in the league.

Here is a table of our results so far. (Remember that making a table is a strategy to be used in problem solving.)

Number of Teams	*Number of Games*	*Possible Pattern*
1	0	0
2	1	1
3	3	1 + 2
4	6	1 + 2 + 3

1. Draw a diagram with 5 dots to represent the teams. Draw lines from each dot to a second dot, and determine the number of games required.
2. What is the apparent pattern for the number of games required?
3. Assuming that the pattern continues, how many games must be scheduled for the 15 teams of the original problem?

After solving a problem, good problem solvers ask whether it is possible to solve the problem in a different manner. Here is a possible alternative method of solving the scheduling problem.

Begin with one of the 15 teams (say team A) and ask, "How many games must this team play?" Because there are 14 teams left to play, you must schedule 14 games. Now move to team B. It is already scheduled to play team A, and it does not play itself, so there are 13 teams left for it to play. Consequently, you must schedule 14 + 13 games.

4. Continue this reasoning for the remaining teams and determine the number of games that must be scheduled. Does this answer correspond to the answer you obtained using the first method?
5. Visit www.sports.com to get lists of professional sports teams, divisions, etc. Find the strategy used to create the schedule of your favorite team.

Projects and Group Activities

Using a Graphing Calculator to Solve a System of Equations

A graphing calculator can be used to solve a system of equations. For this procedure to work on most calculators, it is necessary that the point of intersection be on the screen. This means that you may have to experiment with Xmin, Xmax, Ymin, and Ymax values until the graphs intersect on the screen.

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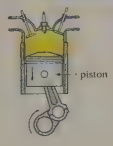
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Problem-Solving Strategies

The text features a carefully developed approach to problem solving that emphasizes the importance of *strategy* when solving problems. Students are encouraged to develop their own strategies—to draw diagrams, to write out the solution steps in words—as part of their solution to a problem. In each case, model strategies are presented as guides for students to follow as they attempt the "You Try It" problem. Having students provide strategies is a natural way to incorporate writing into the math curriculum.

Objective To solve application problems



The **tolerance** of a component, or part, is the acceptable amount by which the component may vary from a given measurement. For example, the diameter of a piston may vary from the given measurement of 9 cm by 0.001 cm. This is written 9 cm ± 0.001 cm and is read "9 centimeters plus or minus 0.001 centimeter." The maximum diameter, or **upper limit**, of the piston is 9 cm + 0.001 cm = 9.001 cm. The minimum diameter, or **lower limit**, is 9 cm − 0.001 cm = 8.999 cm.

The lower and upper limits of the diameter of the piston could also be found by solving the absolute value inequality $|d - 9| \le 0.001$, where d is the diameter of the piston.

$$|d - 9| \le 0.001$$
$$-0.001 \le d - 9 \le 0.001$$
$$-0.001 + 9 \le d - 9 + 9 \le 0.001 + 9$$
$$8.999 \le d \le 9.001$$

The lower and upper limits of the diameter of the piston are 8.999 cm and 9.001 cm.

Example 5

A doctor has prescribed 2 cc of medication for a patient. The tolerance is 0.03 cc. (In the medical field, cubic centimeter is usually abbreviated cc.) Find the lower and upper limits of the amount of medication to be given.

Strategy

Let p represent the prescribed amount of medication, T the tolerance, and m the given amount of medication. Solve the absolute value inequality $|m - p| \le T$ for m.

Solution

$|m - p| \le T$
$|m - 2| \le 0.03$

$$-0.03 \le m - 2 \le 0.03$$
$$-0.03 + 2 \le m - 2 + 2 \le 0.03 + 2$$
$$1.97 \le m \le 2.03$$

The lower and upper limits of the amount of medication to be given are 1.97 cc and 2.03 cc.

You Try It 5

A machinist must make a bushing that has a diameter of 2.55 in. The tolerance of the bushing is 0.003 in. Find the lower and upper limits of the diameter of the bushing.

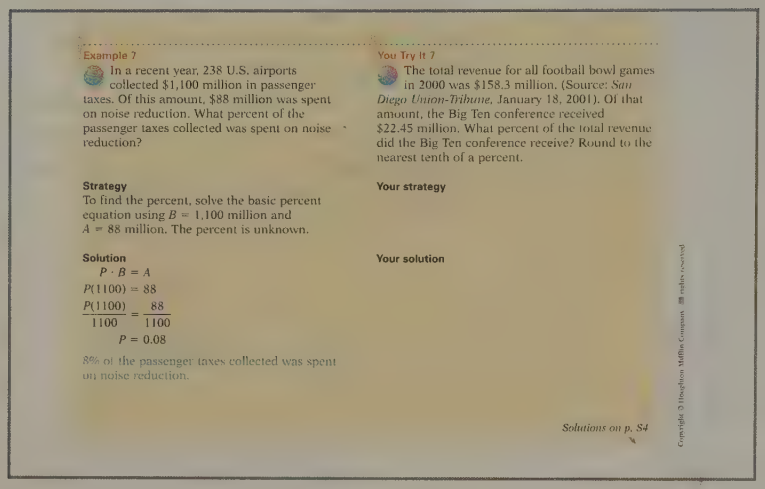
Your strategy

Your solution

Solution on p. S7

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Real Data and Applications



Example 7

In a recent year, 238 U.S. airports collected $1,100 million in passenger taxes. Of this amount, $88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$
$$P(1100) = 88$$
$$\frac{P(1100)}{1100} = \frac{88}{1100}$$
$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was $158.3 million. (Source: *San Diego Union-Tribune*, January 18, 2001). Of that amount, the Big Ten conference received $22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent.

Your strategy

Your solution

Solutions on p. S4

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Applications

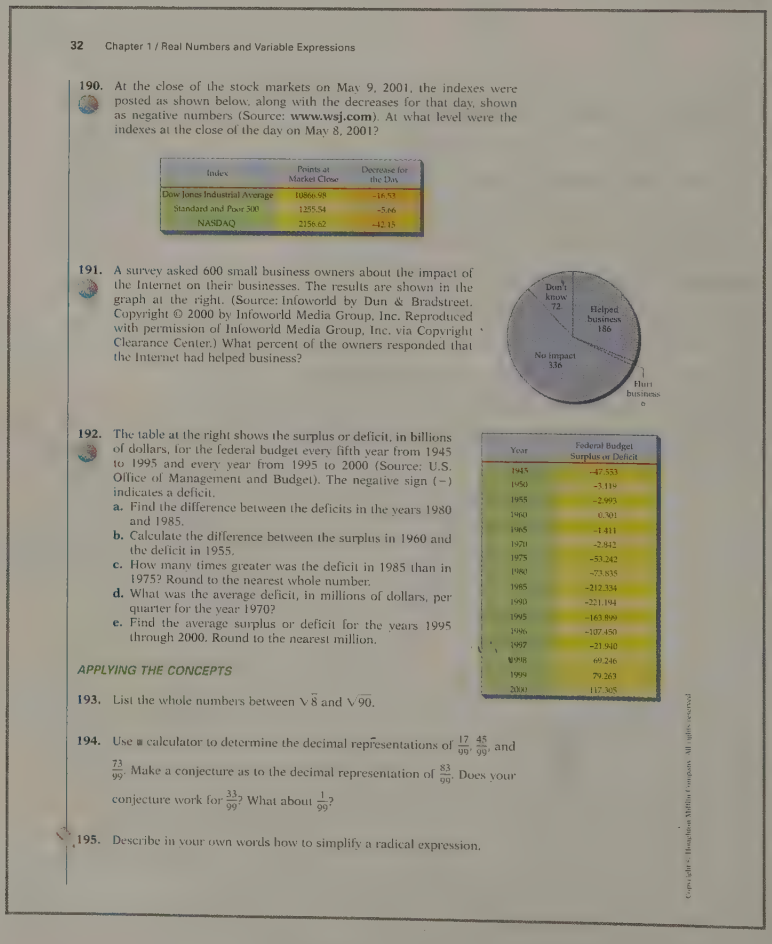
One way to motivate an interest in mathematics is through applications. Wherever appropriate, the last objective of a section presents applications that require the student to use problem-solving strategies, along with the skills covered in that section, to solve practical problems. This carefully integrated applied approach generates student awareness of the value of algebra as a real-life tool.

Applications are taken from many disciplines, including agriculture, business, carpentry, chemistry, construction, Earth science, education, manufacturing, nutrition, real estate, and sociology.

Real Data

Real data examples and exercises, identified by , ask students to analyze and solve problems taken from actual situations. Students are often required to work with tables, graphs, and charts drawn from a variety of disciplines.

Page 32



32 Chapter 1 / Real Numbers and Variable Expressions

190. At the close of the stock markets on May 9, 2001, the indexes were posted as shown below, along with the decreases for that day, shown as negative numbers (Source: **www.wsj.com**). At what level were the indexes at the close of the day on May 8, 2001?

Index	Points at Market Close	Decrease for the Day
Dow Jones Industrial Average	10866.98	−16.53
Standard and Poor 500	1255.54	−5.66
NASDAQ	2156.62	−42.15

191. A survey asked 600 small business owners about the impact of the Internet on their businesses. The results are shown in the graph at the right. (Source: Infoworld by Dun & Bradstreet. Copyright © 2000 by Infoworld Media Group, Inc. Reproduced with permission of Infoworld Media Group, Inc. via Copyright Clearance Center.) What percent of the owners responded that the Internet had helped business?

192. The table at the right shows the surplus or deficit, in billions of dollars, for the federal budget every fifth year from 1945 to 1995 and every year from 1995 to 2000 (Source: U.S. Office of Management and Budget). The negative sign (−) indicates a deficit.

a. Find the difference between the deficits in the years 1980 and 1985.

b. Calculate the difference between the surplus in 1960 and the deficit in 1955.

c. How many times greater was the deficit in 1985 than in 1975? Round to the nearest whole number.

d. What was the average deficit, in millions of dollars, per quarter for the year 1970?

e. Find the average surplus or deficit for the years 1995 through 2000. Round to the nearest million.

Year	Federal Budget Surplus or Deficit
1945	−47.553
1950	−3.119
1955	−2.993
1960	0.301
1965	−1.411
1970	−2.842
1975	−53.242
1980	−73.835
1985	−212.334
1990	−221.194
1995	−163.899
1996	−107.450
1997	−21.940
1998	69.246
1999	79.263
2000	117.305

APPLYING THE CONCEPTS

193. List the whole numbers between $\sqrt{8}$ and $\sqrt{90}$.

194. Use a calculator to determine the decimal representations of $\frac{17}{99}$, $\frac{45}{99}$, and $\frac{73}{99}$. Make a conjecture as to the decimal representation of $\frac{83}{99}$. Does your conjecture work for $\frac{33}{99}$? What about $\frac{1}{99}$?

195. Describe in your own words how to simplify a radical expression.

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Student Pedagogy

Icons

The VIDEO & DVD, CD TUTOR, WWW WEB, SSM at each objective head remind students of the many and varied additional resources available for each objective.

Key Terms and Concepts

Key terms, in bold, emphasize important terms. The key terms are also provided in a **Glossary** at the back of the text.

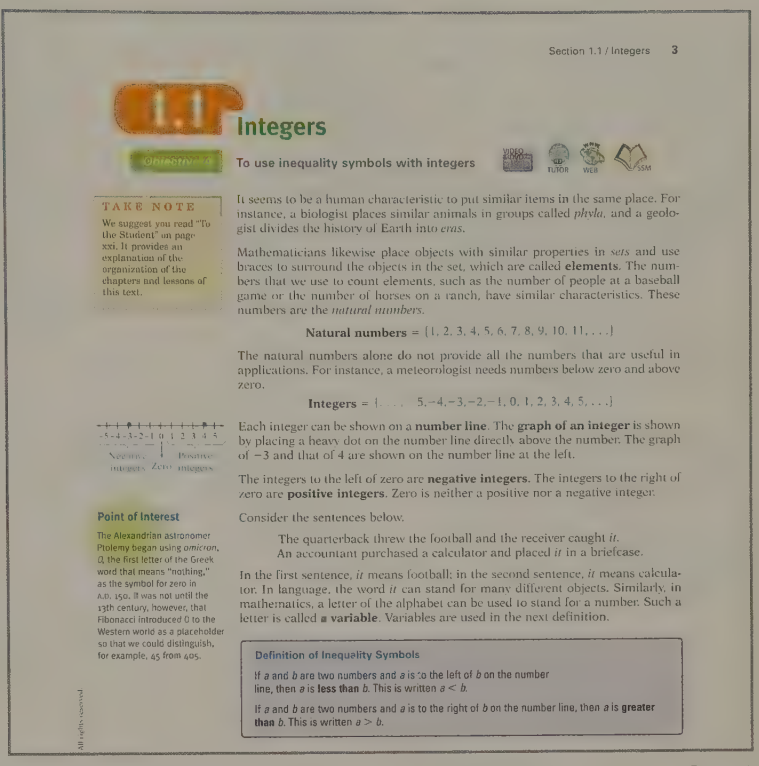
Key concepts are presented in purple boxes in order to highlight these important concepts and to provide for easy reference.

Point of Interest

These margin notes contain interesting sidelights about mathematics, its history, or its application.

Take Note

These margin notes alert students to a point requiring special attention or are used to amplify the concepts under discussion.



Section 1.1 / Integers 3

1.1 Integers

Objective To use inequality symbols with integers

TAKE NOTE
We suggest you read "To the Student" on page xxi. It provides an explanation of the organization of the chapters and lessons of this text.

It seems to be a human characteristic to put similar items in the same place. For instance, a biologist places similar animals in groups called *phyla*, and a geologist divides the history of Earth into *eras*.

Mathematicians likewise place objects with similar properties in *sets* and use braces to surround the objects in the set, which are called **elements**. The numbers that we use to count elements, such as the number of people at a baseball game or the number of horses on a ranch, have similar characteristics. These numbers are the *natural numbers*.

Natural numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .}

The natural numbers alone do not provide all the numbers that are useful in applications. For instance, a meteorologist needs numbers below zero and above zero.

Integers = {. . . −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, . . .}

Each integer can be shown on a **number line**. The **graph of an integer** is shown by placing a heavy dot on the number line directly above the number. The graph of −3 and that of 4 are shown on the number line at the left.

The integers to the left of zero are **negative integers**. The integers to the right of zero are **positive integers**. Zero is neither a positive nor a negative integer.

Point of Interest
The Alexandrian astronomer Ptolemy began using *omicron*, O, the first letter of the Greek word that means "nothing," as the symbol for zero in A.D. 150. It was not until the 13th century, however, that Fibonacci introduced 0 to the Western world as a placeholder so that we could distinguish, for example, 45 from 405.

Consider the sentences below.

The quarterback threw the football and the receiver caught *it*.
An accountant purchased a calculator and placed *it* in a briefcase.

In the first sentence, *it* means football; in the second sentence, *it* means calculator. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable**. Variables are used in the next definition.

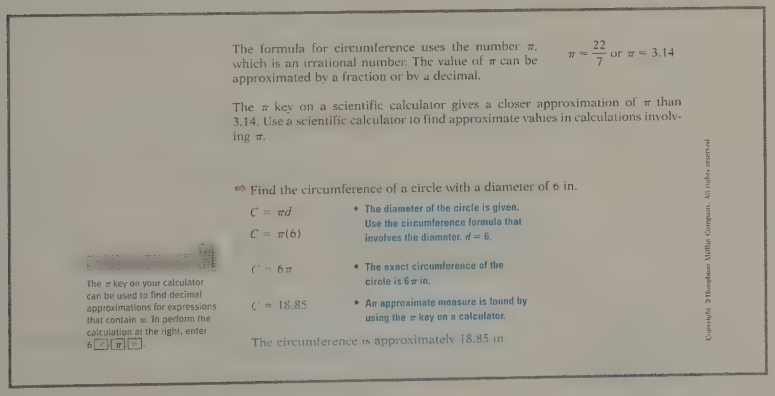
Definition of Inequality Symbols

If a and b are two numbers and a is to the left of b on the number line, then a is **less than** b. This is written $a < b$.

If a and b are two numbers and a is to the right of b on the number line, then a is **greater than** b. This is written $a > b$.

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Page 3



The formula for circumference uses the number π, which is an irrational number. The value of π can be approximated by a fraction or by a decimal. $\pi \approx \frac{22}{7}$ or $\pi \approx 3.14$

The π key on a scientific calculator gives a closer approximation of π than 3.14. Use a scientific calculator to find approximate values in calculations involving π.

Find the circumference of a circle with a diameter of 6 in.

$C = \pi d$ • The diameter of the circle is given. Use the circumference formula that involves the diameter. $d = 6$.

$C = \pi(6)$

$C = 6\pi$ • The exact circumference of the circle is 6π in.

$C \approx 18.85$ • An approximate measure is found by using the π key on a calculator.

The circumference is approximately 18.85 in.

The π key on your calculator can be used to find decimal approximations for expressions that contain π. To perform the calculation at the right, enter 6 × π =.

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Annotated Examples

Examples indicated by use annotations in blue to explain what is happening in key steps of the complete, worked-out solutions.

Calculator Note

These margin notes provide suggestions for using a calculator in certain situations.

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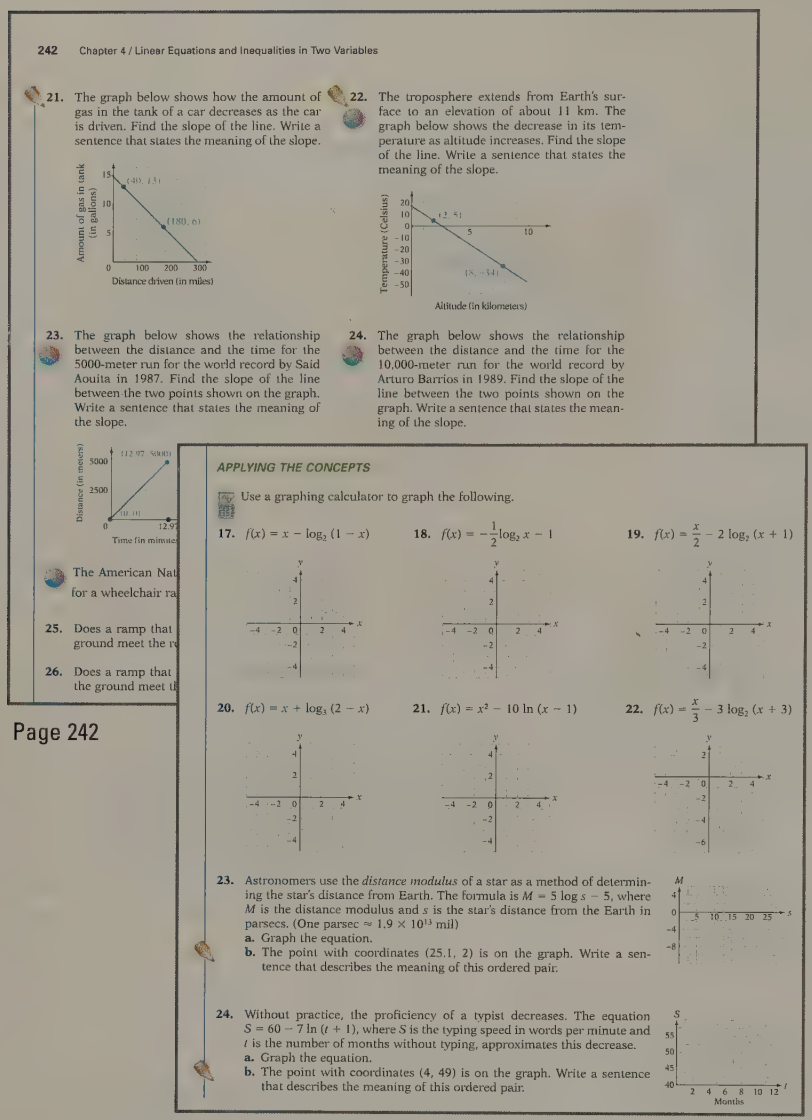
Exercises and Projects

Exercises

The exercise sets of *Algebra: Introductory and Intermediate* emphasize skill building, skill maintenance, and applications.

Concept-based writing or developmental exercises have been integrated with the exercise sets. Icons identify appropriate writing, data analysis, and calculator exercises.

Included in each exercise set is a section called **Applying the Concepts,** which presents extensions of topics, requires analysis, or offers challenge problems. The writing exercises ask students to explain answers, write about a topic in the section, or research and report on a related topic.



242 Chapter 4 / Linear Equations and Inequalities in Two Variables

21. The graph below shows how the amount of gas in the tank of a car decreases as the car is driven. Find the slope of the line. Write a sentence that states the meaning of the slope.

Amount of gas in tank (in gallons)
Distance driven (in miles)

22. The troposphere extends from Earth's surface to an elevation of about 11 km. The graph below shows the decrease in its temperature as altitude increases. Find the slope of the line. Write a sentence that states the meaning of the slope.

Temperature (Celsius)
Altitude (in kilometers)

23. The graph below shows the relationship between the distance and the time for the 5000-meter run for the world record by Said Aouita in 1987. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

Distance (in meters)
Time (in minutes)

24. The graph below shows the relationship between the distance and the time for the 10,000-meter run for the world record by Arturo Barrios in 1989. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

25. Does a ramp that
ground meet the r

26. Does a ramp that
the ground meet t

The American Nat
for a wheelchair ra

Page 242

APPLYING THE CONCEPTS

Use a graphing calculator to graph the following.

17. $f(x) = x - \log_2 (1 - x)$

18. $f(x) = -\frac{1}{2}\log_2 x - 1$

19. $f(x) = \frac{x}{2} - 2 \log_2 (x + 1)$

20. $f(x) = x + \log_3 (2 - x)$

21. $f(x) = x^2 - 10 \ln (x - 1)$

22. $f(x) = \frac{x}{3} - 3 \log_2 (x + 3)$

23. Astronomers use the *distance modulus* of a star as a method of determining the star's distance from Earth. The formula is $M = 5 \log s - 5$, where M is the distance modulus and s is the star's distance from the Earth in parsecs. (One parsec $\approx 1.9 \times 10^{13}$ mil)
a. Graph the equation.
b. The point with coordinates (25.1, 2) is on the graph. Write a sentence that describes the meaning of this ordered pair.

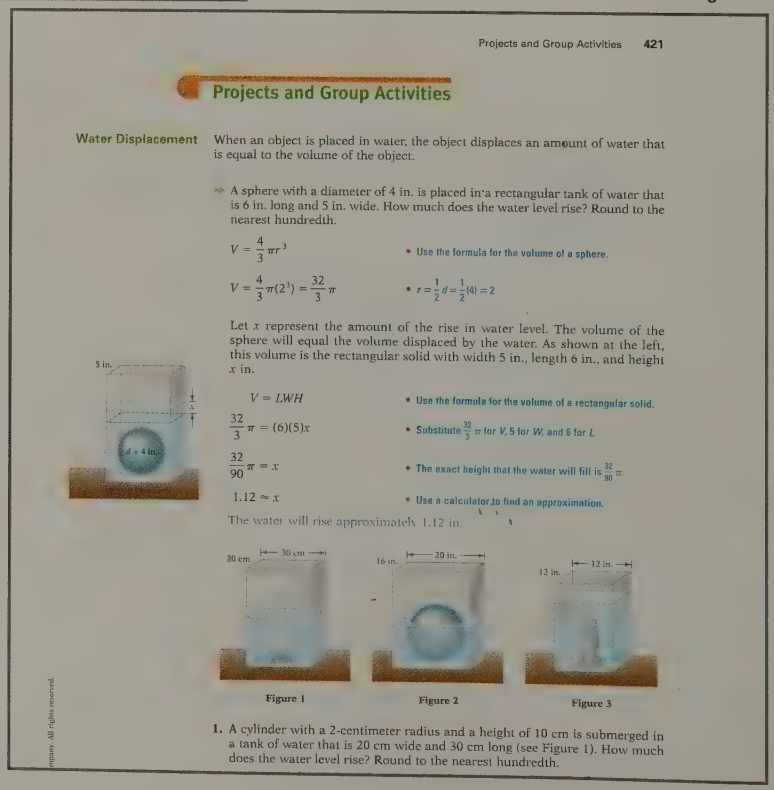
24. Without practice, the proficiency of a typist decreases. The equation $S = 60 - 7 \ln (t + 1)$, where S is the typing speed in words per minute and t is the number of months without typing, approximates this decrease.
a. Graph the equation.
b. The point with coordinates (4, 49) is on the graph. Write a sentence that describes the meaning of this ordered pair.

Months

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Projects and Group Activities

The **Projects and Group Activities** feature at the end of each chapter can be used as extra credit or for cooperative learning activities. The projects cover various aspects of mathematics, including the use of calculators, collecting data from the Internet, data analysis, and extended applications.



Projects and Group Activities 421

Projects and Group Activities

Water Displacement

When an object is placed in water, the object displaces an amount of water that is equal to the volume of the object.

A sphere with a diameter of 4 in. is placed in a rectangular tank of water that is 6 in. long and 5 in. wide. How much does the water level rise? Round to the nearest hundredth.

$V = \frac{4}{3}\pi r^3$ • Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi(2^3) = \frac{32}{3}\pi$ • $r = \frac{1}{2}d = \frac{1}{2}(4) = 2$

Let x represent the amount of the rise in water level. The volume of the sphere will equal the volume displaced by the water. As shown at the left, this volume is the rectangular solid with width 5 in., length 6 in., and height x in.

5 in.
6 in.
$d = 4$ in.

$V = LWH$ • Use the formula for the volume of a rectangular solid.

$\frac{32}{3}\pi = (6)(5)x$ • Substitute $\frac{32}{3}\pi$ for V, 5 for W, and 6 for L.

$\frac{32}{90}\pi = x$ • The exact height that the water will fill is $\frac{32}{90}\pi$.

$1.12 \approx x$ • Use a calculator to find an approximation.

The water will rise approximately 1.12 in.

20 cm, 30 cm
16 in., 20 in.
12 in., 12 in.

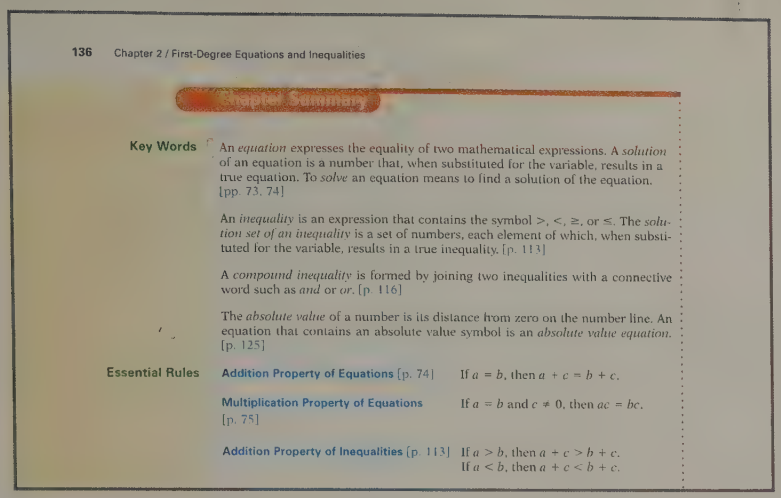
Figure 1 Figure 2 Figure 3

1. A cylinder with a 2-centimeter radius and a height of 10 cm is submerged in a tank of water that is 20 cm wide and 30 cm long (see Figure 1). How much does the water level rise? Round to the nearest hundredth.

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End of Chapter



136 Chapter 2 / First-Degree Equations and Inequalities

Chapter Summary

Key Words

An *equation* expresses the equality of two mathematical expressions. A *solution* of an equation is a number that, when substituted for the variable, results in a true equation. To *solve* an equation means to find a solution of the equation. [pp. 73, 74]

An *inequality* is an expression that contains the symbol >, <, ≥, or ≤. The *solution set of an inequality* is a set of numbers, each element of which, when substituted for the variable, results in a true inequality. [p. 113]

A *compound inequality* is formed by joining two inequalities with a connective word such as *and* or *or*. [p. 116]

The *absolute value* of a number is its distance from zero on the number line. An equation that contains an absolute value symbol is an *absolute value equation*. [p. 125]

Essential Rules

Addition Property of Equations [p. 74] If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations [p. 75] If $a = b$ and $c \neq 0$, then $ac = bc$.

Addition Property of Inequalities [p. 113] If $a > b$, then $a + c > b + c$. If $a < b$, then $a + c < b + c$.

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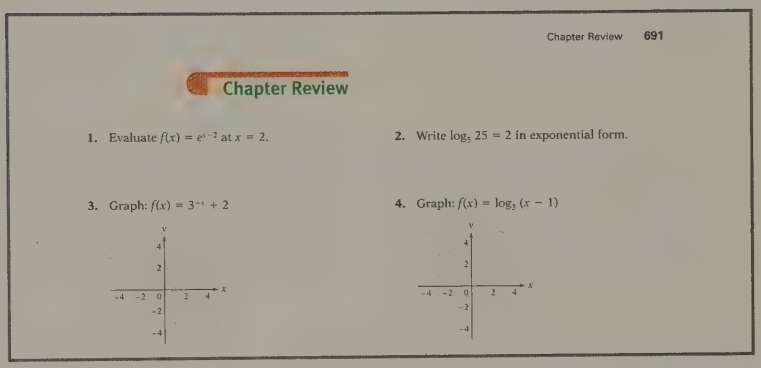
Chapter Summary

At the end of each chapter there is a **Chapter Summary** that includes *Key Words* and *Essential Rules* that were covered in the chapter. These chapter summaries provide a single point of reference as the student prepares for a test. Each concept references the page number from the lesson where the concept is introduced.

Chapter Review

Review exercises are found at the end of each chapter. These exercises are selected to help the student integrate all of the topics presented in the chapter.

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Chapter Review 691

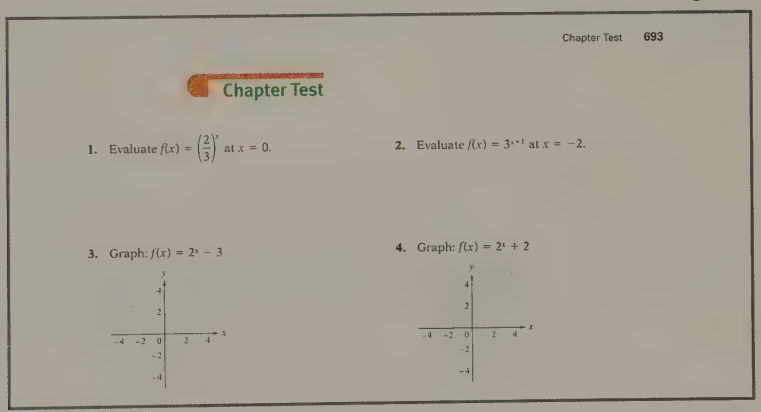
Chapter Review

1. Evaluate $f(x) = e^{x-2}$ at $x = 2$.
2. Write $\log_5 25 = 2$ in exponential form.
3. Graph: $f(x) = 3^{x-1} + 2$
4. Graph: $f(x) = \log_3 (x - 1)$

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Chapter Test

The Chapter Test exercises are designed to simulate a possible test of the material in the chapter.



Chapter Test 693

Chapter Test

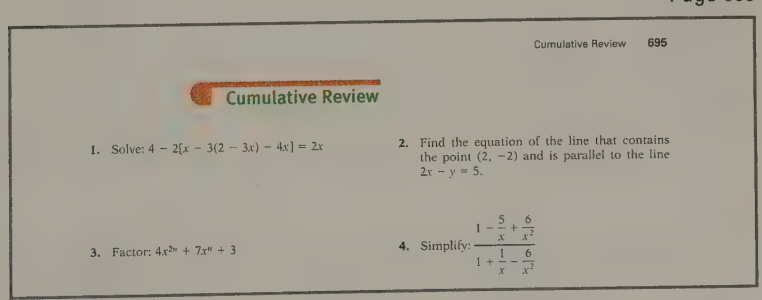
1. Evaluate $f(x) = \left(\frac{2}{3}\right)^x$ at $x = 0$.
2. Evaluate $f(x) = 3^{x+1}$ at $x = -2$.
3. Graph: $f(x) = 2^x - 3$
4. Graph: $f(x) = 2^x + 2$

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Cumulative Review

Cumulative Review exercises, which appear at the end of each chapter (beginning with Chapter 2), help students maintain skills learned in previous chapters.

The answers to all Chapter Review exercises, all Chapter Test exercises, and all Cumulative Review exercises are given in the Answer Section. Along with the answer, there is a reference to the objective that pertains to each exercise.



Cumulative Review 695

Cumulative Review

1. Solve: $4 - 2[x - 3(2 - 3x) - 4x] = 2x$
2. Find the equation of the line that contains the point $(2, -2)$ and is parallel to the line $2x - y = 5$.
3. Factor: $4x^{2n} + 7x^n + 3$
4. Simplify: $\dfrac{1 - \frac{5}{x} + \frac{6}{x^2}}{1 + \frac{1}{x} - \frac{6}{x^2}}$

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Instructor Resources

Algebra: Introductory and Intermediate has a complete set of teaching aids for the instructor.

Instructor's Annotated Edition This edition contains a replica of the student text and additional items just for the instructor. These include: *Instructor Notes, Transparency Master icons, In-Class Examples, Concept Checks, Discuss the Concepts, New Vocabulary/Symbols, etc., Vocabulary/Symbols, etc. to Review, Optional Student Activities, Quick Quizzes, Answers to Writing Exercises,* and *Suggested Assignments*. Answers to all exercises are also provided.

Instructor's Resource Manual with Solutions Manual The *Instructor's Resource Manual* includes a lesson plan for the *AIM for Success* as well as suggested course sequences. The *Instructor's Solutions Manual* contains worked-out solutions for all exercises in the text.

Printed Test Bank The *Printed Test Bank* provides a printout of one example of each of the algorithmic items in *HM Testing*.

HM³ Tutorial (Instructor version) This tutorial CD software package for Microsoft Windows® was written and developed by the authors specifically for use with this text. For each objective, exercises and quizzes are algorithmically generated, solution steps are animated, lessons and exercises are presented in a colorful, lively manner, and an integrated classroom management system tracks student performance.

NEW! ***WebCT ePacks*** *WebCT ePacks* provide instructors with a flexible, Internet-based education platform providing multiple ways to present learning materials. The *WebCT ePacks* come with a full array of features to enrich the on-line learning experience.

NEW! ***Blackboard Cartridges*** The *Houghton Mifflin Blackboard Cartridge* allows flexible, efficient, and creative ways to present learning materials and opportunities. In addition to course management benefits, instructors may make use of an electronic grade book, receive papers from students enrolled in the course via the Internet, and track student use of the communication and collaboration functions.

NEW! ***HM ClassPrep with HM Testing CD-ROM*** *HM ClassPrep* contains a multitude of text-specific resources for instructors to use to enhance the classroom experience. These resources can be easily accessed by chapter or resource type and can also link you to the text's web site. *HM Testing* is our computerized test generator and contains a database of algorithmic test items as well as providing **on-line testing** and **gradebook** functions.

NEW! ***Instructor Text-Specific Web Site*** The resources available on the *ClassPrep CD* are also available on the instructor web site at math.college.hmco.com/instructors. Appropriate items are password protected. Instructors also have access to the student part of the text's web site.

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Student Resources

Student Solutions Manual The *Student Solutions Manual* contains complete solutions to all odd-numbered exercises in the text.

Math Study Skills Workbook by Paul D. Nolting This workbook is designed to reinforce skills and minimize frustration for students in any math class, lab, or study skills course. It offers a wealth of study tips and sound advice on note taking, time management, and reducing math anxiety. In addition, numerous opportunities for self assessment enable students to track their own progress.

NEW! ***HM eduSpace*** *eduSpace* is a new content delivery system, combining an algorithmic tutorial program, on-line delivery of course materials, and classroom management functions. The interactive on-line content correlates directly to this text and can be accessed 24 hours a day.

HM³ Tutorial (Student version) This tutorial CD software package for Microsoft Windows® was written and developed by the authors specifically for use with this text. HM³ is an interactive tutorial containing lessons, exercises, and quizzes for every section of the text. Lessons are presented in a colorful, lively manner, and solution steps are animated. Exercises and quizzes are algorithmically generated. Next to every objective head, the  serves as a reminder that there is an HM³ tutorial lesson corresponding to that objective.

NEW! ***SMARTHINKING™ Live, On-line Tutoring*** Houghton Mifflin has partnered with SMARTHINKING to provide an easy-to-use and effective on-line tutorial service. **Whiteboard Simulations** and **Practice Area** promote real-time visual interaction.

Three levels of service are offered.

- **Text-Specific Tutoring** provides real-time, one-on-one instruction with a specially qualified "e-structor."
- **Questions Any Time** allows students to submit questions to the tutor outside the scheduled hours and receive a reply within 24 hours.
- **Independent Study Resources** connect students with around-the-clock access to additional educational services, including interactive web sites, diagnostic tests, and Frequently Asked Questions posed to SMARTHINKING e-structors.

NEW! ***Videos and DVDs*** This edition offers brand new text-specific videos and DVDs, hosted by Dana Mosely, covering all sections of the text and providing a valuable resource for further instruction and review. Next to every objective head, the 

serves as a reminder that the objective is covered in a video/DVD lesson.

NEW! ***Student Text-Specific Web Site*** On-line student resources, such as section quizzes, can be found at this text's web site at math.college.hmco.com/students.

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AIM for Success

Welcome to *Algebra: Introductory and Intermediate*. As you begin this course, we know two important facts: (1) We want you to succeed. (2) You want to succeed. To do that requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways mathematics can benefit you. But, in the end, the motivation must come from you. On the first day of class, it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

TAKE NOTE

Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out.

_____

_____

_____

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. Although you may not agree that this course is necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better.

ACTIVITY	TIME SPENT	TIME WISHED SPENT

Thinking about these activities, put the number of hours that you spend each week practicing these activities next to the activity. Next to that number, indicate the number of hours per week you would like to spend on these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend doing it. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, such as lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first

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learned the activities you listed above, you probably could not do them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful atheletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "Can Do" attitude. Develop this attitude toward math.

Strategies for Success

Textbook Reconnaissance Right now, do a 15-minute "textbook reconnaissance" of this book. Here's how:

First, read the table of contents. Do it in three minutes or less. Next, look through the entire book, page by page. Move quickly. Scan titles, look at pictures, notice diagrams.

A textbook reconnaissance shows you where a course is going. It gives you the big picture. That's useful because brains work best when going from the general to the specific. Getting the big picture before you start makes details easier to recall and understand later on.

Your textbook reconnaissance will work even better if, as you scan, you look for ideas or topics that are interesting to you. List three facts, topics, or problems that you found interesting during your textbook reconnaissance.

_____

_____

_____

The idea behind this technique is simple: It's easier to work at learning material if you know it's going to be useful to you.

Not all the topics in this book will be "interesting" to you. But that is true of any subject. Surfers find that on some days the waves are better than others, musicians find some music more appealing than other music, computer gamers find some computer games more interesting than others, car enthusiasts find some cars more exciting than others. Some car enthusiasts would rather have a completely restored 1957 Chevrolet than a new Ferrari.

Know the Course Requirements To do your best in this course, you must know exactly what your instructor requires. Course requirements may be stated in a *syllabus*, which is a printed outline of the main topics of the course, or they may be presented orally. When they are listed in a syllabus or on other printed pages, keep them in a safe place. When they are presented orally, make sure to take complete notes. In either case, it is important that you understand them completely and follow them exactly. Be sure you know the answer to each of the following questions.

1. What is your instructor's name?
2. Where is your instructor's office?
3. At what times does your instructor hold office hours?
4. Besides the textbook, what other materials does your instructor require?
5. What is your instructor's attendance policy?
6. If you must be absent from a class meeting, what should you do before returning to class? What should you do when you return to class?

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7. What is the instructor's policy regarding collection or grading of homework assignments?
8. What options are available if you are having difficulty with an assignment? Is there a math tutoring center?
9. If there is a math lab at your school, where is it located? What hours is it open?
10. What is the instructor's policy if you miss a quiz?
11. What is the instructor's policy if you miss an exam?
12. Where can you get help when studying for an exam?

Remember: Your instructor wants to see you succeed. If you need help, ask! Do not fall behind. If you are running a race and fall behind by 100 yards, you may be able to catch up but it will require more effort than had you not fallen behind.

TAKE NOTE

Besides time management, there must be realistic ideas of how much time is available. There are very few people who can *successfully* work full-time and go to school full-time. If you work 40 hours a week, take 15 units, spend the recommended study time given at the right, and sleep 8 hours a day, you will use over 80% of the available hours in a week. That leaves less than 20% of the hours in a week for family, friends, eating, recreation, and other activities.

Time Management We know that there are demands on your time. Family, work, friends, and entertainment all compete for your time. We do not want to see you receive poor job evaluations because you are studying math. However, it is also true that we do not want to see you receive poor math test scores because you devoted too much time to work. When several competing and important tasks require your time and energy, the only way to manage the stress of being successful at both is to manage your time efficiently.

Instructors often advise students to spend twice the amount of time outside of class studying as they spend in the classroom. Time management is important if you are to accomplish this goal and succeed in school. The following activity is intended to help you structure your time more efficiently.

List the name of each course you are taking this term, the number of class hours each course meets, and the number of hours you should spend studying each subject outside of class. Then fill in a weekly schedule like the one printed below. Begin by writing in the hours spent in your classes, the hours spent at work (if you have a job), and any other commitments that are not flexible with respect to the time that you do them. Then begin to write down commitments that are more flexible, including hours spent studying. Remember to reserve time for activities such as meals and exercise. You should also schedule free time.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7–8 a.m.							
8–9 a.m.							
9–10 a.m.							
10–11 a.m.							
11–12 p.m.							
12–1 p.m.							
1–2 p.m.							
2–3 p.m.							
3–4 p.m.							
4–5 p.m.							
5–6 p.m.							
6–7 p.m.							
7–8 p.m.							
8–9 p.m.							
9–10 p.m.							
10–11 p.m.							
11–12 a.m.							

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We know that many of you must work. If that is the case, realize that working 10 hours a week at a part-time job is equivalent to taking a three-unit class. If you must work, consider letting your education progress at a slower rate to allow you to be successful at both work and school. There is no rule that says you must finish school in a certain time frame.

Schedule Study Time As we encouraged you to do by filling out the time management form above, schedule a certain time to study. You should think of this time the way you would the time for work or class—that is, reasons for missing study time should be as compelling as reasons for missing work or class. "I just didn't feel like it" is not a good reason to miss your scheduled study time.

Although this may seem like an obvious exercise, list a few reasons you might want to study.

_____

_____

_____

Of course we have no way of knowing the reasons you listed, but from our experience one reason given quite frequently is "To pass the course." There is nothing wrong with that reason. If that is the most important reason for you to study, then use it to stay focused.

One method of keeping to a study schedule is to form a ***study group***. Look for people who are committed to learning, who pay attention in class, and who are punctual. Ask them to join your group. Choose people with similar educational goals but different methods of learning. You can gain insight from seeing the material from a new perspective. Limit groups to four or five people; larger groups are unwieldy.

There are many ways to conduct a study group. Begin with the following suggestions and see what works best for your group.

1. Test each other by asking questions. Each group member might bring two or three sample test questions to each meeting.
2. Practice teaching each other. Many of us who are teachers learned a lot about our subject when we had to explain it to someone else.
3. Compare class notes. You might ask other students about material in your notes that is difficult for you to understand.
4. Brainstorm test questions.
5. Set an agenda for each meeting. Set approximate time limits for each agenda item and determine a quitting time.

And finally, probably the most important aspect of studying is that it should be done in relatively small chunks. If you can only study three hours a week for this course (probably not enough for most people), do it in blocks of one hour on three separate days, preferably after class. Three hours of studying on a Sunday is not as productive as three hours of paced study.

Text Features That Promote Success

There are 12 chapters in this text. Each chapter is divided into sections, and each section is subdivided into learning objectives. Each learning objective is labeled with a letter from A to G.

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Preparing for a Chapter Before you begin a new chapter, you should take some time to review previously learned skills. There are two ways to do this. The first is to complete the ***Cumulative Review***, which occurs after every chapter (except Chapter 1). For instance, turn to page 325. The questions in this review are taken from the previous chapters. The answers for all these exercises can be found on page A16. Turn to that page now and locate the answers for the Chapter 5 Cumulative Review. After the answer to the first exercise, which is $-6\sqrt{10}$, you will see the objective reference [1.2F]. This means that this question was taken from Chapter 1, Section 2, Objective F. If you missed this question, you should return to that objective and restudy the material.

A second way of preparing for a new chapter is to complete the ***Prep Test***. This test focuses on the particular skills that will be required for the new chapter. Turn to page 274 to see a Prep Test. The answers for the Prep Test are the first set of answers in the answer section for a chapter. Turn to page A13 to see the answers for the Chapter 5 Prep Test. Note that an objective reference is given for each question. If you answer a question incorrectly, restudy the objective from which the question was taken.

Before the class meeting in which your professor begins a new section, you should read each objective statement for that section. Next, browse through the objective material, being sure to note each word in bold type. These words indicate important concepts that you must know in order to learn the material. Do not worry about trying to understand all the material. Your professor is there to assist you with that endeavor. The purpose of browsing through the material is so that your brain will be prepared to accept and organize the new information when it is presented to you.

Turn to page 3. Write down the title of the first objective in Section 1.1. Under the title of the objective, write down the words in the objective that are in bold print. It is not necessary for you to understand the meaning of these words. You are in this class to learn their meaning.

_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____

Math Is not a Spectator Sport To learn mathematics you must be an active participant. Listening and watching your professor do mathematics is not enough. Mathematics requires that you interact with the lesson you are studying. If you filled in the blanks above, you were being interactive. There are other ways this textbook has been designed to help you be an active learner.

Annotated Examples An orange arrow indicates an example with explanatory remarks to the right of the work. Using paper and pencil, you should work along as you go through the example.

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$3x + 2 < -4$
$3x < -6$
$\frac{3x}{3} < \frac{-6}{3}$
$x < -2$
The solution set is $\{x|x < -2\}$.

➡ Solve: $3x + 2 < -4$

$3x + 2 < -4$

$3x < -6$ • Subtract 2 from each side of the inequality.

$\frac{3x}{3} < \frac{-6}{3}$ • Divide each side of the inequality by the coefficient 3.

$x < -2$

The solution set is $\{x \mid x < -2\}$.

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When you complete the example, get a clean sheet of paper. Write down the problem and then try to complete the solution without referring to your notes or the book. When you can do that, move on to the next part of the objective.

Leaf through the book now and write down the page numbers of two other occurrences of an arrowed example.

You Try Its One of the key instructional features of this text is the paired examples. Notice that in each example box, the example on the left is completely worked out and the "You Try It" example on the right is not. Study the worked-out example carefully by working through each step. Then work the You Try It. If you get stuck, refer to the page number at the end of the example, which directs you to the place where the You Try It is solved—a complete worked-out solution is provided. Try to use the given solution to get a hint for the step you are stuck on. Then try to complete your solution.

Example 2 Solve:
$3x - 5 \le 3 - 2(3x + 1)$

Solution

$3x - 5 \le 3 - 2(3x + 1)$
$3x - 5 \le 3 - 6x - 2$
$3x - 5 \le 1 - 6x$
$9x - 5 \le 1$
$9x \le 6$
$\frac{9x}{9} \le \frac{6}{9}$
$x \le \frac{2}{3}$

$\left\{x \mid x \le \frac{2}{3}\right\}$

You Try It 2 Solve:
$5x - 2 \le 4 - 3(x - 2)$

Your solution

$5x - 2 \le 4 - 3x + 6$
$5x - 2 \le 10 - 3x$
$8x - 2 \le 10$
$8x \le 12$
$\frac{8x}{8} \le \frac{12}{8}$
$x \le \frac{3}{2}$

$\left\{x|x \le \frac{3}{2}\right\}$

Solution on p. S6

Page 116

When you have completed your solution, check your work against the solution we provided. (Turn to page S6 to see the solution of You Try It 2.) Be aware that frequently there is more than one way to solve a problem. Your answer, however, should be the same as the given answer. If you have any question as to whether your method will "always work," check with your instructor or with someone in the math center.

Browse through the textbook and write down the page numbers where two other paired example features occur.

Remember: Be an active participant in your learning process. When you are sitting in class watching and listening to an explanation, you may think that you understand. However, until you actually try to do it, you will have no confirmation of the new knowledge or skill. Most of us have had the experience of sitting in class thinking we knew how to do something only to get home and realize that we didn't.

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TAKE NOTE

There is a strong connection between reading and being a successful student in math or any other subject. If you have difficulty reading, consider taking a reading course. Reading is much like other skills. There are certain things you can learn that will make you a better reader.

Word Problems Word problems are difficult because we must read the problem, determine the quantity we must find, think of a method to do that, and then actually solve the problem. In short, we must formulate a *strategy* to solve the problem and then devise a *solution*.

Note in the paired example below that part of every word problem is a strategy and part is a solution. The strategy is a written description of how we will solve the problem. In the corresponding You Try It, you are asked to formulate a strategy. Do not skip this step, and be sure to write it out.

Example 7

In a recent year, 238 U.S. airports collected $1,100 million in passenger taxes. Of this amount, $88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$
$$P(1100) = 88$$
$$\frac{P(1100)}{1100} = \frac{88}{1100}$$
$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was $158.3 million. (Source: *San Diego Union-Tribune*, January 18, 2001). Of that amount, the Big Ten conference received $22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent.

Your strategy

Use the basic percent equation, $P \cdot B = A$, with $B = 158.3$ million and $A = 22.45$ million. Solve for P.

Your solution

$$P \cdot B = A$$
$$P(158.3) = 22.45$$
$$\frac{P(158.3)}{158.3} = \frac{22.56}{158.3}$$
$$P \approx 0.142$$

The Big Ten conference received approximately 14.2% of the total revenue.

Solutions on p. S4

Page 78

TAKE NOTE

If a rule has more than one part, be sure to make a notation to that effect.

Rule Boxes Pay special attention to rules placed in boxes. These rules give you the reasons certain types of problems are solved the way they are. When you see a rule, try to rewrite the rule in your own words.

Multiplying each side of an equation by the same number (NOT 0) will not change the solution.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

Page 75

Chapter Exercises When you have completed studying an objective, do the exercises in the exercise set that correspond with that objective. The exercises are labeled with the same letter as the objective. Math is a subject that needs to be learned in small sections and practiced continually in order to be mastered. Doing all of the exercises in each exercise set will help you master the problem-solving techniques necessary for success. As you work through the exercises for an objective, check your answers to the odd-numbered exercises with those in the back of the book.

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Preparing for a Test There are important features of this text that can be used to prepare for a test.

- Chapter Summary
- Chapter Review
- Chapter Test

After completing a chapter, read the Chapter Summary. (See page 320 for the Chapter 5 Summary.) This summary highlights the important topics covered in the chapter. The page number following each topic refers you to the page in the text on which you can find more information about the concept.

Following the Chapter Summary are a Chapter Review (see page 321) and a Chapter Test (see page 323). Doing the review exercises is an important way of testing your understanding of the chapter. The answer to each review exercise is given at the back of the book, along with its objective reference. After checking your answers, restudy any objective from which a question you missed was taken. It may be helpful to retry some of the exercises for that objective to reinforce your problem-solving techniques.

The Chapter Test should be used to prepare for an exam. We suggest that you try the Chapter Test a few days before your actual exam. Take the test in a quiet place and try to complete the test in the same amount of time you will be allowed for your exam. When taking the Chapter Test, practice the strategies of successful test takers: (1) scan the entire test to get a feel for the questions; (2) read the directions carefully; (3) work the problems that are easiest for you first; and perhaps most importantly, (4) try to stay calm.

When you have completed the Chapter Test, check your answers. If you missed a question, review the material in that objective and rework some of the exercises from that objective. This will strengthen your ability to perform the skills in that objective.

Your career goal goes here

Is it difficult to be successful? YES! Successful music groups, artists, professional athletes, chefs, and _____ have to work very hard to achieve their goals. They focus on their goals and ignore distractions. The things we ask you to do to achieve success take time and commitment. We are confident that if you follow our suggestions, you will succeed.

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Chapter 1

Real Numbers and Variable Expressions



Containers full of products to be shipped and sold in other countries are piled high onto huge cargo ships at ports around the United States. The depot at this port, near Newark, New Jersey, sees activity every day from trucks dropping off containers. The more goods a country exports, the better the likelihood of a favorable balance of trade. In order to avoid a trade deficit, a country needs to export more than it imports. The **Project on pages 63–64** shows how to calculate a country's balance of trade and determine whether it is favorable or unfavorable.

Section 1.1

- A To use inequality symbols with integers
- B To find the additive inverse and absolute value of a number
- C To add or subtract integers
- D To multiply or divide integers
- E To solve application problems

Section 1.2

- A To write a rational number as a decimal
- B To convert among percents, fractions, and decimals
- C To add or subtract rational numbers
- D To multiply or divide rational numbers
- E To evaluate exponential expressions
- F To simplify numerical radical expressions
- G To solve application problems

Section 1.3

- A To use the Order of Operations Agreement to simplify expressions

Section 1.4

- A To evaluate a variable expression
- B To simplify a variable expression using the Properties of Addition
- C To simplify a variable expression using the Properties of Multiplication
- D To simplify a variable expression using the Distributive Property
- E To translate a verbal expression into a variable expression

Section 1.5

- A To write a set using the roster method
- B To write a set using set-builder notation
- C To graph an inequality on the number line



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

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Prep Test

1. What is 127.1649 rounded to the nearest hundredth?

2. Add: 49.147 + 5.96

3. Subtract: 5004 − 487

4. Multiply: 407 × 28

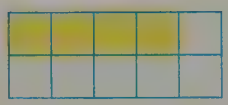
5. Divide: 456 ÷ 19

6. What is the smallest number that both 8 and 12 divide evenly into?

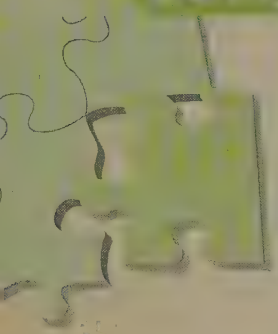
7. What is the greatest number that divides evenly into both 16 and 20?

8. Without using 1, write 21 as a product of two whole numbers.

9. Represent the shaded portion of the figure as a fraction.



Do Figure



If you multiply the first 20 natural numbers

$$(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots \cdot 17 \cdot 18 \cdot 19 \cdot 20),$$

how many zeros will be at the end of the number?

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1.1 Integers

Objective A To use inequality symbols with integers

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TAKE NOTE
We suggest you read "To the Student" on page xxi. It provides an explanation of the organization of the chapters and lessons of this text.

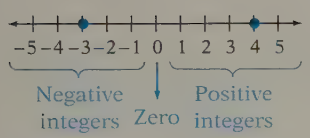
It seems to be a human characteristic to put similar items in the same place. For instance, a biologist places similar animals in groups called *phyla,* and a geologist divides the history of Earth into *eras.*

Mathematicians likewise place objects with similar properties in *sets* and use braces to surround the objects in the set, which are called **elements**. The numbers that we use to count elements, such as the number of people at a baseball game or the number of horses on a ranch, have similar characteristics. These numbers are the *natural numbers.*

Natural numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .}

The natural numbers alone do not provide all the numbers that are useful in applications. For instance, a meteorologist needs numbers below zero and above zero.

Integers = {. . . , −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, . . .}



Each integer can be shown on a **number line**. The **graph of an integer** is shown by placing a heavy dot on the number line directly above the number. The graph of −3 and that of 4 are shown on the number line at the left.

The integers to the left of zero are **negative integers**. The integers to the right of zero are **positive integers**. Zero is neither a positive nor a negative integer.

Point of Interest
The Alexandrian astronomer Ptolemy began using *omicron, O,* the first letter of the Greek word that means "nothing," as the symbol for zero in A.D. 150. It was not until the 13th century, however, that Fibonacci introduced 0 to the Western world as a placeholder so that we could distinguish, for example, 45 from 405.

Consider the sentences below.

The quarterback threw the football and the receiver caught *it*.
An accountant purchased a calculator and placed *it* in a briefcase.

In the first sentence, *it* means football; in the second sentence, *it* means calculator. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable**. Variables are used in the next definition.

Definition of Inequality Symbols

If a and b are two numbers and a is to the left of b on the number line, then a is **less than** b. This is written $a < b$.

If a and b are two numbers and a is to the right of b on the number line, then a is **greater than** b. This is written $a > b$.

There are also inequality symbols for **less than or equal to** ($\leq$) and **greater than or equal to** ($\geq$). For instance,

$6 \leq 6$ because $6 = 6$. $7 \leq 15$ because $7 < 15$.

It is convenient to use a variable to represent, or stand for, any one of the elements of a set. For instance, the statement "x is an element of the set {0, 2, 4, 6}" means that x can be replaced by 0, 2, 4, or 6. The symbol for "is an element of" is $\in$; the symbol for "is not an element of" is $\notin$. For example,

$2 \in \{0, 2, 4, 6\}$ $6 \in \{0, 2, 4, 6\}$ $7 \notin \{0, 2, 4, 6\}$

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Example 1

Let $x \in \{-6, -2, 0\}$. For which values of x is the inequality $x \leq -2$ a true statement?

Solution

Replace x by each element of the set and determine whether the inequality is true.

$x \leq -2$

$-6 \leq -2$ True. $-6 < -2$

$-2 \leq -2$ True. $-2 = -2$

$0 \leq -2$ False.

The inequality is true for -6 and -2.

You Try It 1

Let $y \in \{-5, -1, 5\}$. For which values of y is the inequality $y > -1$ a true statement?

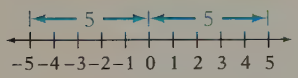
Your solution

Solution on p. S1

Objective B

To find the additive inverse and absolute value of a number

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On the number line, the numbers 5 and -5 are the same distance from zero but on opposite sides of zero. The numbers 5 and -5 are called **opposites** or **additive inverses** of each other. (See the number line at the left.)

TAKE NOTE

The distance from 0 to 5 is 5; $|5| = 5$. The distance from 0 to -5 is 5; $|-5| = 5$.

The opposite (or additive inverse) of 5 is -5. The opposite of -5 is 5. The symbol for opposite is $-$.

$-(5)$ means the opposite of *positive* 5. $-(5) = -5$

$-(-5)$ means the opposite of *negative* 5. $-(-5) = 5$

The **absolute value** of a number is its distance from zero on the number line. The symbol for absolute value is two vertical bars, $|\ |$.

Point of Interest

The definition of absolute value that we have given in the box is written in what is called "rhetorical style." That is, it is written without the use of variables. This is how *all* mathematics was written prior to the Renaissance. During that period, from the 14th to the 16th century, the idea of expressing a variable symbolically was developed.

Absolute Value

The absolute value of a positive number is the number itself. The absolute value of zero is zero. The absolute value of a negative number is the opposite of the negative number.

$|9| = 9$
$|0| = 0$
$|-7| = 7$

Evaluate: $-|-12|$

$-|-12| = -12$

- The absolute value sign does not affect the negative sign in front of the absolute value sign.

Example 2

Let $a \in \{-12, 0, 4\}$. Find the additive inverse of a and the absolute value of a for each element of the set.

Solution

Replace a by each element of the set.

$-a$	$\lvert a \rvert$
$-(-12) = 12$	$\lvert -12 \rvert = 12$
$-(0) = 0$	$\lvert 0 \rvert = 0$
$-(4) = -4$	$\lvert 4 \rvert = 4$

You Try It 2

Let $z \in \{-11, 0, 8\}$. Find the additive inverse of z and the absolute value of z for each element of the set.

Your solution

Solution on p. S1

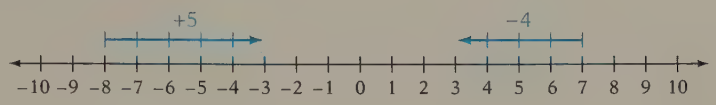
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Objective C

To add or subtract integers

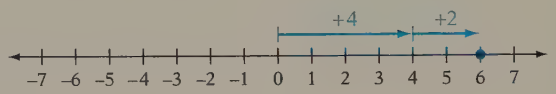
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A number can be represented anywhere along the number line by an arrow. A positive number is represented by an arrow pointing to the right, and a negative number is represented by an arrow pointing to the left. The size of the number is represented by the length of the arrow.

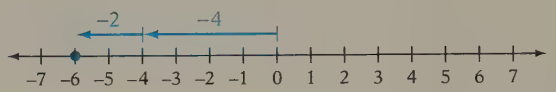


Addition of integers can be shown on the number line. To add integers, start at zero and draw an arrow representing the first number. At the tip of the first arrow, draw a second arrow representing the second number. The sum is below the tip of the second arrow.

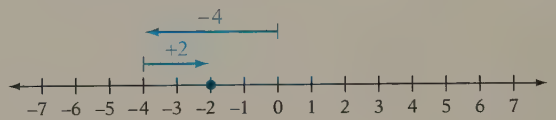
$4 + 2 = 6$



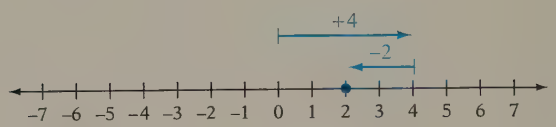
$-4 + (-2) = -6$



$-4 + 2 = -2$



$4 + (-2) = 2$



The pattern for the addition of integers shown on the number line can be summarized in the following rule.

Addition of Integers

- *Numbers with the same sign*
  To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.
- *Numbers with different signs*
  To add two numbers with different signs, find the absolute value of each number. Then subtract the smaller of these numbers from the larger one. Attach the sign of the number with the larger absolute value.

Add: $(-9) + 8$

$|-9| = 9 \quad |8| = 8$

$9 - 8 = 1$

$(-9) + 8 = -1$

- The signs are different. Find the absolute value of each number.
- Subtract the smaller number from the larger.
- Attach the sign of the number with the larger absolute value. Because $|-9| > |8|$, use the sign of -9.

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Add: $(-23) + 47 + (-18) + 5$

To add more than two numbers, add the first two numbers. Then add the sum to the third number. Continue until all the numbers are added.

$$\begin{aligned}(-23) + 47 + (-18) + 5 &= 24 + (-18) + 5\\ &= 6 + 5\\ &= 11\end{aligned}$$

Look at the two expressions below and note that each expression equals the same number.

$$\begin{aligned}8 - 3 &= 5 \quad && 8 \text{ minus } 3 \text{ is } 5.\\ 8 + (-3) &= 5 \quad && 8 \text{ plus the opposite of } 3 \text{ is } 5.\end{aligned}$$

This example suggests that to subtract two numbers, we add the opposite of the second number to the first number.

first number	$-$	second number	$=$	first number	$+$	the opposite of the second number	
40	$-$	60	$=$	40	$+$	(-60)	$= -20$
-40	$-$	60	$=$	-40	$+$	(-60)	$= -100$
-40	$-$	(-60)	$=$	-40	$+$	60	$= 20$
40	$-$	(-60)	$=$	40	$+$	60	$= 100$

Subtract: $-21 - (-40)$

Change this sign to plus.

$$-21 - (-40) = -21 + 40 = 19$$

Change -40 to the opposite of -40.

- **Rewrite each subtraction as addition of the opposite. Then add.**

Subtract: $15 - 51$

Change this sign to plus.

$$15 - 51 = 15 + (-51) = -36$$

Change 51 to the opposite of 51.

- **Rewrite each subtraction as addition of the opposite. Then add.**

Subtract: $-12 - (-21) - 15$

$$\begin{aligned}-12 - (-21) - 15 &= -12 + 21 + (-15)\\ &= 9 + (-15)\\ &= -6\end{aligned}$$

- **Rewrite each subtraction as addition of the opposite. Then add.**

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Example 3 Add: $(-52) + (-39)$

Solution The signs are the same. Add the absolute values of the numbers:
$52 + 39 = 91$

Attach the sign of the addends:
$(-52) + (-39) = -91$

You Try It 3 Add: $100 + (-43)$

Your solution

Example 4 Add:
$37 + (-52) + (-21) + (-7)$

Solution
$$\begin{aligned} &37 + (-52) + (-21) + (-7) \\ &= -15 + (-21) + (-7) \\ &= -36 + (-7) \\ &= -43 \end{aligned}$$

You Try It 4 Add:
$(-51) + 42 + 17 + (-102)$

Your solution

Example 5 Subtract: $-11 - 15$

Solution
$$\begin{aligned} -11 - 15 &= -11 + (-15) \\ &= -26 \end{aligned}$$

You Try It 5 Subtract: $19 - (-32)$

Your solution

Example 6 Subtract:
$-14 - 18 - (-21) - 4$

Solution
$$\begin{aligned} &-14 - 18 - (-21) - 4 \\ &= -14 + (-18) + 21 + (-4) \\ &= -32 + 21 + (-4) \\ &= -11 + (-4) \\ &= -15 \end{aligned}$$

You Try It 6 Subtract:
$-9 - (-12) - 17 - 4$

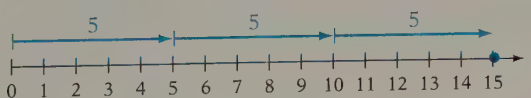
Your solution

Solutions on p. S1

Objective D To multiply or divide integers



Multiplication is the repeated addition of the same number. The product 3×5 is shown on the number line below.



5 is added 3 times.

$3 \times 5 = 5 + 5 + 5 = 15$

Point of Interest

The cross $\times$ was first used as a symbol for multiplication in 1631 in a book titled *The Key to Mathematics*. Also in that year, another book, *Practice of the Analytical Art*, advocated the use of a dot to indicate multiplication.

To indicate multiplication, several different symbols are used.

$3 \times 5 = 15$ $\quad 3 \cdot 5 = 15$ $\quad (3)(5) = 15$ $\quad 3(5) = 15$ $\quad (3)5 = 15$

Note that when parentheses are used and there is no arithmetic operation symbol, the operation is multiplication. Each number in a product is called a **factor**. For instance, 3 and 5 are factors of the product $3 \cdot 5 = 15$.

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Now consider the product of a positive and a negative number.

TAKE NOTE
3(−5) is 3 times −5.

$$3(-5) = \underbrace{(-5) + (-5) + (-5)}_{-5 \text{ is added 3 times.}} = -15$$

• **Multiplication is repeated addition.**

This suggests that the product of a positive number and a negative number is negative. Here are a few more examples.

$$4(-7) = -28 \qquad (-6)5 = -30 \qquad (-5) \cdot 7 = -35$$

To find the product of two negative numbers, look at the pattern at the right. As −5 multiplies a sequence of decreasing integers, the products increase by 5.

The pattern can be continued by requiring that the product of two negative numbers be positive.

These numbers decrease by 1. ↓ ↓ These numbers increase by 5.

$$\begin{aligned} -5(3) &= -15 \\ -5(2) &= -10 \\ -5(1) &= -5 \\ -5(0) &= 0 \\ -5(-1) &= 5 \\ -5(-2) &= 10 \\ -5(-3) &= 15 \end{aligned}$$

Multiplication of Integers

- *Numbers with the same sign*
  To multiply two numbers with the same sign, multiply the absolute values of the numbers. The product is positive.
- *Numbers with different signs*
  To multiply two numbers with different signs, multiply the absolute values of the numbers. The product is negative.

➡ Multiply: −2(5)(−7)(−4)

$$\begin{aligned} -2(5)(-7)(-4) &= -10(-7)(-4) \\ &= 70(-4) \\ &= -280 \end{aligned}$$

• **To multiply more than two numbers, multiply the first two. Then multiply the product by the third number. Continue until all the numbers are multiplied.**

For every division problem there is a related multiplication problem.

$$\underbrace{\frac{8}{2} = 4}_{\text{Division}} \quad \text{because} \quad \underbrace{4 \cdot 2 = 8}_{\text{Related multiplication}}$$

This fact and the rules for multiplying integers can be used to illustrate the rules for dividing integers.

Note in the following examples that the quotient of two numbers with the same sign is positive.

$$\frac{12}{3} = 4 \text{ because } 4 \cdot 3 = 12 \qquad \frac{-12}{-3} = 4 \text{ because } 4 \cdot (-3) = -12$$

The next two examples illustrate that the quotient of two numbers with different signs is negative.

$$\frac{12}{-3} = -4 \text{ because } (-4)(-3) = 12 \qquad \frac{-12}{3} = -4 \text{ because } (-4) \cdot 3 = -12$$

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Division of Integers

- *Numbers with the same sign*
  To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive.
- *Numbers with different signs*
  To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative.

Simplify: $-\frac{-56}{7}$

$$-\frac{-56}{7} = -\left(\frac{-56}{7}\right) = -(-8) = 8$$

Note that $\frac{-12}{3} = -4$, $\frac{12}{-3} = -4$, and $-\frac{12}{3} = -4$. This suggests the following rule.

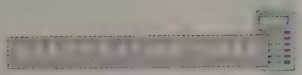
TAKE NOTE
The symbol $\neq$ is read "is not equal to."

If a and b are integers, and $b \neq 0$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

PROPERTIES OF ZERO AND ONE IN DIVISION

- Zero divided by any number other than zero is zero.

$\frac{0}{a} = 0$ because $0 \cdot a = 0$ For example, $\frac{0}{7} = 0$ because $0 \cdot 7 = 0$.



Enter 4 ÷ 0 = on your calculator. You will get an error message.

- Division by zero is not defined.

To understand that division by zero is not permitted, suppose that $\frac{4}{0}$ were equal to n, where n is some number. Because each division problem has a related multiplication problem, $\frac{4}{0} = n$ means $n \cdot 0 = 4$. But $n \cdot 0 = 4$ is impossible because any number times 0 is 0. Therefore, division by 0 is not defined.

- Any number other than zero divided by itself is 1.

$\frac{a}{a} = 1$, $a \neq 0$ For example, $\frac{-8}{-8} = 1$.

TAKE NOTE
"To the Student" on page xi explains how best to use the boxed examples in this text, such as the one below.

- Any number divided by one is the number.

$\frac{a}{1} = a$ For example, $\frac{9}{1} = 9$.

Example 7 Multiply: $(-3)4(-5)$

Solution $(-3)4(-5) = (-12)(-5) = 60$

You Try It 7 Multiply: $8(-9)10$

Your solution

Solution on p. S1

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Example 8 Multiply: $12(-4)(-3)(-5)$

Solution
$$12(-4)(-3)(-5)$$
$$= (-48)(-3)(-5)$$
$$= 144(-5) = -720$$

You Try It 8 Multiply: $(-2)3(-8)7$

Your solution

Example 9 Divide: $(-120) \div (-8)$

Solution $(-120) \div (-8) = 15$

You Try It 9 Divide: $(-135) \div (-9)$

Your solution

Example 10

Divide: $\frac{95}{-5}$

Solution $\frac{95}{-5} = -19$

You Try It 10

Divide: $\frac{-72}{4}$

Your solution

Example 11

Divide: $-\frac{-81}{3}$

Solution $-\frac{-81}{3} = -(-27) = 27$

You Try It 11

Divide: $-\frac{36}{-12}$

Your solution

Solutions on p. S1

Objective E To solve application problems



To solve an application problem, first read the problem carefully. The Strategy involves identifying the quantity to be found and planning the steps that are necessary to find that quantity. The Solution involves performing each operation stated in the Strategy and writing the answer.

Example 12

The average temperature on Mercury's sunlit side is 950°F. The average temperature on Mercury's dark side is −346°F. Find the difference between these two average temperatures.

Strategy

To find the difference, subtract the average temperature on the dark side (−346) from the average temperature on the sunlit side (950).

Solution

$950 - (-346) = 950 + 346$
$= 1296$

The difference between these average temperatures is 1296°F.

You Try It 12

The daily low temperatures (in degrees Celsius) during one week were recorded as follows: −6°, −7°, 0°, −5°, −8°, −1°, −1°. Find the average daily low temperature.

Your strategy

Your solution

Solution on p. S1

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1.1 Exercises

> **TAKE NOTE**
> "To the Student" on page xxi discusses the exercise sets in this textbook.

Objective A

Place the correct symbol, $<$ or $>$, between the two numbers.

1. 8 $\quad -6$ **2.** $-14 \quad 16$ **3.** $-12 \quad 1$ **4.** $35 \quad 28$ **5.** $42 \quad 19$

6. $-42 \quad 27$ **7.** $0 \quad -31$ **8.** $-17 \quad 0$ **9.** $53 \quad -46$ **10.** $-27 \quad -39$

Answer true or false.

11. $-13 > 0$ **12.** $-20 > 3$ **13.** $12 > -31$ **14.** $9 > 7$ **15.** $-5 > -2$

16. $-44 > -21$ **17.** $-4 > -120$ **18.** $0 > -8$ **19.** $-1 > 0$ **20.** $-10 > -88$

21. Let $x \in \{-23, -18, -8, 0\}$. For which values of x is the inequality $x < -8$ a true statement?

22. Let $w \in \{-33, -24, -10, 0\}$. For which values of w is the inequality $w < -10$ a true statement?

23. Let $a \in \{-33, -15, 21, 37\}$. For which values of a is the inequality $a > -10$ a true statement?

24. Let $v \in \{-27, -14, 14, 27\}$. For which values of v is the inequality $v > -15$ a true statement?

25. Let $n \in \{-23, -1, 0, 4, 29\}$. For which values of n is the inequality $-6 > n$ a true statement?

26. Let $m \in \{-33, -11, 0, 12, 45\}$. For which values of m is the inequality $-15 > m$ a true statement?

Objective B

Find the additive inverse.

27. 4 **28.** 8 **29.** -9 **30.** -12 **31.** -28 **32.** -36

Evaluate.

33. $-(-14)$ **34.** $-(-40)$ **35.** $-(77)$ **36.** $-(39)$ **37.** $-(0)$ **38.** $-(-13)$

39. $|-74|$ **40.** $|-96|$ **41.** $-|-82|$ **42.** $-|-53|$ **43.** $-|81|$ **44.** $-|38|$

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Place the correct symbol, $<$ or $>$, between the values of the two numbers.

45. $|-83|$ $|58|$ **46.** $|22|$ $|-19|$ **47.** $|43|$ $|-52|$ **48.** $|-71|$ $|-92|$

49. $|-68|$ $|-42|$ **50.** $|12|$ $|-31|$ **51.** $|-45|$ $|-61|$ **52.** $|-28|$ $|43|$

53. Let $p \in \{-19, 0, 28\}$. Evaluate $-p$ for each element of the set.

54. Let $q \in \{-34, 0, 31\}$. Evaluate $-q$ for each element of the set.

55. Let $x \in \{-45, 0, 17\}$. Evaluate $-|x|$ for each element of the set.

56. Let $y \in \{-91, 0, 48\}$. Evaluate $-|y|$ for each element of the set.

Objective C

57. **a.** Explain the rule for adding two integers with the same sign.
b. Explain the rule for adding two integers with different signs.

58. Explain how to rewrite the subtraction $8 - (-6)$ as addition of the opposite.

Add or subtract.

59. $-3 + (-8)$ **60.** $-6 + (-9)$ **61.** $-8 + 3$ **62.** $-9 + 2$

63. $-3 + (-80)$ **64.** $-12 + (-1)$ **65.** $-23 + (-23)$ **66.** $-12 + (-12)$

67. $16 + (-16)$ **68.** $-17 + 17$ **69.** $48 + (-53)$ **70.** $19 + (-41)$

71. $-17 + (-3) + 29$ **72.** $13 + 62 + (-38)$ **73.** $-3 + (-8) + 12$ **74.** $-27 + (-42) + (-18)$

75. $16 - 8$ **76.** $12 - 3$ **77.** $7 - 14$ **78.** $6 - 9$

79. $-7 - 2$ **80.** $-9 - 4$ **81.** $7 - (-2)$ **82.** $3 - (-4)$

83. $-6 - (-3)$ **84.** $-4 - (-2)$ **85.** $6 - (-12)$ **86.** $-12 - 16$

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87. $13 + (-22) + 4 + (-5)$

88. $-14 + (-3) + 7 + (-21)$

89. $-16 + (-17) + (-18) + 10$

90. $-25 + (-31) + 24 + 19$

91. $26 + (-15) + (-11) + (-12)$

92. $-32 + 40 + (-8) + (-19)$

93. $-14 + (-15) + (-11) + 40$

94. $28 + (-19) + (-8) + (-1)$

95. $-4 - 3 - 2$

96. $4 - 5 - 12$

97. $12 - (-7) - 8$

98. $-12 - (-3) - (-15)$

99. $-19 - (-19) - 18$

100. $-8 - (-8) - 14$

101. $-17 - (-8) - (-9)$

102. $7 - 8 - (-1)$

103. $-30 - (-65) - 29 - 4$

104. $42 - (-82) - 65 - 7$

105. $-16 - 47 - 63 - 12$

106. $42 - (-30) - 65 - (-11)$

107. $-47 - (-67) - 13 - 15$

108. $-18 - 49 - (-84) - 27$

109. $-19 - 17 - (-36) - 12$

110. $48 - 19 - 29 - 51$

111. $21 - (-14) - 43 - 12$

112. $17 - (-17) - 14 - 21$

Objective D

113. Describe the rules for multiplying two integers.

114. Name the operation in each expression. Justify your answer.
a. $8(-7)$ **b.** $8 - 7$ **c.** $8 - (-7)$ **d.** $-xy$ **e.** $x(-y)$ **f.** $-x - y$

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Multiply or divide.

115. $(14)3$ **116.** $(17)6$ **117.** $-7 \cdot 4$ **118.** $-8 \cdot 7$ **119.** $(-12)(-5)$ **120.** $(-13)(-9)$

121. $-11(23)$ **122.** $-8(21)$ **123.** $(-17)14$ **124.** $(-15)12$ **125.** $6(-19)$ **126.** $17(-13)$

127. $12 \div (-6)$ **128.** $18 \div (-3)$ **129.** $(-72) \div (-9)$ **130.** $(-64) \div (-8)$ **131.** $-42 \div 6$

132. $(-56) \div 8$ **133.** $(-144) \div 12$ **134.** $(-93) \div (-3)$ **135.** $48 \div (-8)$ **136.** $57 \div (-3)$

137. $\frac{-49}{7}$ **138.** $\frac{-45}{5}$ **139.** $\frac{-44}{-4}$ **140.** $\frac{-36}{-9}$ **141.** $\frac{98}{-7}$

142. $\frac{85}{-5}$ **143.** $-\frac{-120}{8}$ **144.** $-\frac{-72}{4}$ **145.** $-\frac{-80}{-5}$ **146.** $-\frac{-114}{-6}$

147. $0 \div (-9)$ **148.** $0 \div (-14)$ **149.** $\frac{-261}{9}$ **150.** $\frac{-128}{4}$ **151.** $9 \div 0$

152. $(-21) \div 0$ **153.** $\frac{132}{-12}$ **154.** $\frac{250}{-25}$ **155.** $\frac{0}{0}$ **156.** $\frac{-58}{0}$

157. $7(5)(-3)$ **158.** $(-3)(-2)8$ **159.** $9(-7)(-4)$ **160.** $(-2)(6)(-4)$

161. $16(-3)5$ **162.** $20(-4)3$ **163.** $-4(-3)8$ **164.** $-5(-9)6$

165. $-3(-8)(-9)$ **166.** $-7(-6)(-5)$ **167.** $(-9)7(5)$ **168.** $(-8)7(10)$

169. $7(-2)(5)(-6)$ **170.** $(-3)7(-2)8$ **171.** $-9(-4)(-8)(-10)$ **172.** $-11(-3)(-5)(-2)$

173. $7(9)(-11)4$ **174.** $-12(-4)7(-2)$ **175.** $(-14)9(-11)0$ **176.** $(-13)(15)(-19)0$

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Objective E Application Problems

The elevation, or height, of places on Earth is measured in relation to sea level, or the average level of the ocean's surface. The table below shows height above sea level as a positive number and depth below sea level as a negative number. Use the table for Exercises 177 to 179.

Continent	Highest Elevation (in meters)		Lowest Elevation (in meters)	
Africa	Mt. Kilimanjaro	5895	Qattara Depression	−133
Asia	Mt. Everest	8848	Dead Sea	−400
Europe	Mt. Elbrus	5634	Caspian Sea	−28
America	Mt. Aconcagua	6960	Death Valley	−86

177. Find the difference in elevation between Mt. Aconcagua and Death Valley.

178. What is the difference in elevation between Mt. Kilimanjaro and the Qattara Depression?

179. For which continent shown is the difference between the highest and lowest elevations greatest?

The table at the right shows the boiling point and the melting point in degrees Celsius of three chemical elements. Use this table for Exercises 180 and 181.

Chemical Element	Boiling Point	Melting Point
Mercury	357	−39
Radon	−62	−71
Xenon	−107	−112

180. Find the difference between the boiling point and the melting point of mercury.

181. Find the difference between the boiling point and the melting point of xenon.

182. To discourage random guessing on a multiple-choice exam, a professor assigns 5 points for a correct answer, −2 points for an incorrect answer, and 0 points for leaving the question blank. What is the score for a student who had 20 correct answers, had 13 incorrect answers, and left 7 questions blank?

183. To discourage random guessing on a multiple-choice exam, a professor assigns 7 points for a correct answer, −3 points for an incorrect answer, and −1 point for leaving the question blank. What is the score for a student who had 17 correct answers, had 8 incorrect answers, and left 2 questions blank?

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The table at the right shows the average temperatures at different cruising altitudes for airplanes. Use the table for Exercises 184 to 186.

Cruising Altitude	Average Temperature
12,000 ft	16°F
20,000 ft	−12°F
30,000 ft	−48°F
40,000 ft	−70°F
50,000 ft	−70°F

184. What is the difference between the average temperatures at 12,000 ft and at 40,000 ft?

185. What is the difference between the average temperatures at 40,000 ft and at 50,000 ft?

186. How much colder is the average temperature at 30,000 ft than at 20,000 ft?

A meteorologist may report a wind-chill temperature. This is the equivalent temperature, including the effects of wind and temperature, that a person would feel in calm air conditions. The table below gives the wind-chill temperature for various wind speeds and temperatures. For instance, when the temperature is 5°F and the wind is blowing at 15 mph, the wind-chill temperature is −25°F. Use this table for Exercises 187 and 188.

Wind-Chill Factors

Wind Speed (mph)	Thermometer Reading (degrees Fahrenheit)																
	35	30	25	20	15	10	5	0	−5	−10	−15	−20	−25	−30	−35	−40	−45
5	33	27	21	19	12	7	0	−5	−10	−15	−21	−26	−31	−36	−42	−47	−52
10	22	16	10	3	−3	−9	−15	−22	−27	−34	−40	−46	−52	−58	−64	−71	−77
15	16	9	2	−5	−11	−18	−25	−31	−38	−45	−51	−58	−65	−72	−78	−85	−92
20	12	4	−3	−10	−17	−24	−31	−39	−46	−53	−60	−67	−74	−81	−88	−95	−103
25	8	1	−7	−15	−22	−29	−36	−44	−51	−59	−66	−74	−81	−88	−96	−103	−110
30	6	−2	−10	−18	−25	−33	−41	−49	−56	−64	−71	−79	−86	−93	−101	−109	−116
35	4	−4	−12	−20	−27	−35	−43	−52	−58	−67	−74	−82	−89	−97	−105	−113	−120
40	3	−5	−13	−21	−29	−37	−45	−53	−60	−69	−76	−84	−92	−100	−107	−115	−123
45	2	−6	−14	−22	−30	−38	−46	−54	−62	−70	−78	−85	−93	−102	−109	−117	125

187. When the thermometer reading is −5°F, what is the difference between the wind-chill factor when the wind is blowing 10 mph and when the wind is blowing 30 mph?

188. When the thermometer reading is −20°F, what is the difference between the wind-chill factor when the wind is blowing 15 mph and when the wind is blowing 25 mph?

APPLYING THE CONCEPTS

189. If $-4x$ equals a positive integer, is x a positive or a negative integer? Explain your answer.

190. Is the difference between two integers always smaller than either of the integers? If not, give an example for which the difference between two integers is greater than either integer.

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1.2 Rational and Irrational Numbers

To write a rational number as a decimal

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Point of Interest

As early as A.D. 630, the Hindu mathematician Brahmagupta wrote a fraction as one number over another separated by a space. The Arab mathematician al Hassar (around A.D. 1050) was the first to show a fraction with the horizontal bar separating the numerator and denominator.

A *rational number* is the quotient of two integers. A rational number written in this way is commonly called a fraction. Here are some examples of rational numbers.

$$\frac{3}{4}, \quad \frac{-4}{9}, \quad \frac{15}{-4}, \quad \frac{8}{1}, \quad -\frac{5}{6}$$

Rational Numbers

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Because an integer can be written as the quotient of the integer and 1, every integer is a rational number. For instance,

$$\frac{6}{1} = 6 \qquad \frac{-8}{1} = -8$$

Point of Interest

Simon Stevin (1548–1620) was the first to name decimal numbers. He wrote the number 2.345 as 2 0 3 1 4 2 5 3. He called the whole number part the *commencement*, the tenths digit was *prime*, the hundredths digit was *second*, the thousandths digit was *third*, and so on.

A number written in **decimal notation** is also a rational number.

three-tenths $0.3 = \frac{3}{10}$ forty-three thousandths $0.043 = \frac{43}{1000}$

A rational number written as a fraction can be written in decimal notation.

Write $\frac{5}{8}$ as a decimal.

The fraction bar can be read "÷".

$$\frac{5}{8} = 5 \div 8$$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

← This is called a **terminating decimal.**

← The remainder is zero.

$$\frac{5}{8} = 0.625$$

Write $\frac{4}{11}$ as a decimal.

$$\begin{array}{r} 0.3636\ldots \\ 11\overline{)4.0000} \\ -33 \\ \hline 70 \\ -66 \\ \hline 40 \\ -33 \\ \hline 70 \\ -66 \\ \hline 4 \end{array}$$

← This is called a **repeating decimal.**

← The remainder is never zero.

$$\frac{4}{11} = 0.\overline{36}$$

← The bar over the digits 3 and 6 is used to show that these digits repeat.

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Example 1

Write $\frac{8}{11}$ as a decimal. Place a bar over the repeating digits of the decimal.

Solution $\frac{8}{11} = 8 \div 11 = 0.7272\ldots = 0.\overline{72}$

You Try It 1

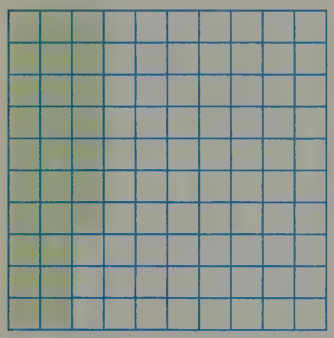
Write $\frac{4}{9}$ as a decimal. Place a bar over the repeating digits of the decimal.

Your solution

Solution on p. S1

Objective B To convert among percents, fractions, and decimals

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27% of the region is shaded.

Percent means "parts of 100." Thus 27% means 27 parts of 100.

In applied problems involving percent, it may be necessary to rewrite a percent as a fraction or decimal or to rewrite a fraction or decimal as a percent.

To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

Write 27% as a fraction.

$$27\% = 27\left(\frac{1}{100}\right) = \frac{27}{100}$$

• Remove the percent sign and multiply by $\frac{1}{100}$.

To write a percent as a decimal, remove the percent sign and multiply by 0.01.

$$33\% = 33(0.01) = 0.33$$

Move the decimal point two places to the left. Then remove the percent sign.

A fraction or decimal can be written as a percent by multiplying by 100%.

$$\frac{5}{8} = \frac{5}{8}(100\%) = \frac{500}{8}\% = 62.5\%, \text{ or } 62\frac{1}{2}\%$$

$$0.82 = 0.82(100\%) = 82\%$$

Move the decimal point two places to the right. Then write the percent sign.

Example 2

Write 130% as a fraction and as a decimal.

Solution

$$130\% = 130\left(\frac{1}{100}\right) = \frac{130}{100} = \frac{13}{10}$$

$$130\% = 130(0.01) = 1.30$$

You Try It 2

Write 125% as a fraction and as a decimal.

Your solution

Solution on p. S1

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Example 3

Write $\frac{5}{6}$ as a percent.

Solution $\frac{5}{6} = \frac{5}{6}(100\%) = \frac{500}{6}\% = 83\frac{1}{3}\%$

You Try It 3

Write $\frac{1}{3}$ as a percent.

Your solution

Example 4 Write 0.092 as a percent.

Solution $0.092 = 0.092(100\%) = 9.2\%$

You Try It 4 Write 0.043 as a percent.

Your solution

Solutions on p. S1

Objective C To add or subtract rational numbers

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Fractions with the same denominator are added by adding the numerators and placing the sum over the common denominator.

Addition of Fractions

To add two fractions with the same denominator, add the numerators and place the sum over the common denominator.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with different denominators, first rewrite the fractions as equivalent fractions with a common denominator. Then add the fractions.

The least common denominator is the **least common multiple** (LCM) of the denominators. This is the smallest number that is a multiple of each of the denominators.

TAKE NOTE

You can find the LCM by multiplying the denominators and then dividing by the *common factor* of the two denominators. In the case of 6 and 10, $6 \cdot 10 = 60$. Now divide by 2, the common factor of 6 and 10.

$60 \div 2 = 30$.

- Add: $-\frac{5}{6} + \frac{3}{10}$

The LCM of 6 and 10 is 30. Rewrite the fractions as equivalent fractions with the denominator 30. Then add the fractions.

$$-\frac{5}{6} + \frac{3}{10} = -\frac{5}{6}\cdot\frac{5}{5} + \frac{3}{10}\cdot\frac{3}{3} = -\frac{25}{30} + \frac{9}{30} = \frac{-25+9}{30} = \frac{-16}{30} = -\frac{8}{15}$$

To subtract fractions with the same denominator, subtract the numerators and place the difference over the common denominator.

TAKE NOTE

The least common multiple of the denominators is frequently called the **least common denominator** (LCD).

- Subtract: $-\frac{4}{9} - \left(-\frac{7}{12}\right)$

The LCM of 9 and 12 is 36. Rewrite the fractions as equivalent fractions with the denominator 36. Then subtract the fractions.

$$-\frac{4}{9} - \left(-\frac{7}{12}\right) = -\frac{16}{36} - \left(-\frac{21}{36}\right) = \frac{-16-(-21)}{36} = \frac{-16+21}{36} = \frac{5}{36}$$

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To add or subtract decimals, write the numbers so that the decimal points are in a vertical line. Then proceed as in the addition or subtraction of integers. Write the decimal point in the answer directly below the decimal points in the problem.

Add: $-114.039 + 84.76$

$|-114.039| = 114.039$

$|84.76| = 84.76$

- The signs are different. Find the absolute value of each number.

$$\begin{array}{r} 114.039 \\ -\ 84.76 \\ \hline 29.279 \end{array}$$

- Subtract the smaller of these numbers from the larger.

$-114.039 + 84.76 = -29.279$

- Attach the sign of the number with the larger absolute value. Because $|-114.039| > |84.76|$, use the sign of -114.039.

Example 5

Simplify: $-\frac{3}{4} + \frac{1}{6} - \frac{5}{8}$

Solution The LCM of 4, 6, and 8 is 24.

$$-\frac{3}{4} + \frac{1}{6} - \frac{5}{8} = -\frac{18}{24} + \frac{4}{24} - \frac{15}{24}$$

$$= \frac{-18 + 4 - 15}{24}$$

$$= \frac{-29}{24} = -\frac{29}{24}$$

You Try It 5

Simplify: $-\frac{7}{8} - \frac{5}{6} + \frac{3}{4}$

Your solution

Example 6 Subtract: $42.987 - 98.61$

Solution

$$42.987 - 98.61$$
$$= 42.987 + (-98.61)$$
$$= -55.623$$

You Try It 6 Subtract: $16.127 - 67.91$

Your solution

Solutions on p. S2

Objective 3 To multiply or divide rational numbers

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The product of two fractions is the product of the numerators divided by the product of the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Multiply: $\frac{3}{8} \cdot \frac{12}{17}$

$$\frac{3}{8} \cdot \frac{12}{17} = \frac{3 \cdot 12}{8 \cdot 17}$$

- Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 17}$$

- Write the prime factorization of each factor. Divide by the common factors.

$$= \frac{9}{34}$$

- Multiply the factors in the numerator and in the denominator.

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TAKE NOTE
To invert the divisor means to write its reciprocal. The reciprocal of $\frac{18}{25}$ is $\frac{25}{18}$.

To divide fractions, invert the divisor. Then multiply the fractions.

➡ Divide: $\frac{3}{10} \div \left(-\frac{18}{25}\right)$

The signs are different. The quotient is negative.

$$\frac{3}{10} \div \left(-\frac{18}{25}\right) = -\left(\frac{3}{10} \div \frac{18}{25}\right) = -\left(\frac{3}{10} \cdot \frac{25}{18}\right) = -\left(\frac{3 \cdot 25}{10 \cdot 18}\right)$$

$$= -\left(\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{2 \cdot \underset{1}{\cancel{5}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3}\right) = -\frac{5}{12}$$

To multiply decimals, multiply as with integers. Write the decimal point in the product so that the number of decimal places in the product equals the sum of the decimal places in the factors.

➡ Multiply: $-6.89(0.00035)$

$$\begin{array}{rl} 6.89 & 2 \text{ decimal places} \\ \times\ 0.00035 & 5 \text{ decimal places} \\ \hline 3445 & \\ 2067 & \\ \hline 0.0024115 & 7 \text{ decimal places} \end{array}$$

• Multiply the absolute values.

$-6.89(0.00035) = -0.0024115$

• The signs are different. The product is negative.

To divide decimals, move the decimal point in the divisor to the right to make it a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly over the decimal point in the dividend. Then divide as with whole numbers.

TAKE NOTE
The symbol $\approx$ is used to indicate that the quotient is an approximate value that has been rounded off.

➡ Divide: $1.32 \div 0.27$. Round to the nearest tenth.

$$\begin{array}{r} 4.88 \approx 4.9 \\ 0.27\overset{\curvearrowright}{.}\overset{\curvearrowright}{)}1.32.00 \\ \underline{-1\ 08} \\ 240 \\ \underline{-216} \\ 240 \\ \underline{-216} \\ 24 \end{array}$$

• Move the decimal point 2 places to the right in the divisor and then in the dividend. Place the decimal point in the quotient.

Example 7

Divide: $-\frac{5}{8} \div \left(-\frac{5}{40}\right)$

Solution The quotient is positive.

$$-\frac{5}{8} \div \left(-\frac{5}{40}\right) = \frac{5}{8} \div \frac{5}{40} = \frac{5}{8} \cdot \frac{40}{5} = \frac{5 \cdot 40}{8 \cdot 5}$$

$$= \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{5}}} = \frac{5}{1} = 5$$

You Try It 7

Divide: $-\frac{3}{8} \div \left(-\frac{5}{12}\right)$

Your solution

Solution on p. S2

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Example 8 Multiply: $-4.29(8.2)$

Solution The product is negative.

$$\begin{array}{r} 4.29 \\ \times\ 8.2 \\ \hline 858 \\ 3432 \\ \hline 35.178 \end{array}$$

$-4.29(8.2) = -35.178$

You Try It 8 Multiply: $-5.44(3.8)$

Your solution

Solution on p. S2

Objective E To evaluate exponential expressions

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Repeated multiplication of the same factor can be written using an exponent.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ ← exponent, ↑ base

$a \cdot a \cdot a \cdot a = a^4$ ← exponent, ↑ base

Point of Interest

René Descartes (1596–1650) was the first mathematician extensively to use exponential notation as it is used today. However, for some unknown reason, he always used xx for x^2.

The **exponent** indicates how many times the factor, called the **base**, occurs in the multiplication. The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in **factored form**. The exponential expression 2^5 is in **exponential form**.

2^1 is read "the first power of 2" or just 2. Usually the exponent 1 is not written.

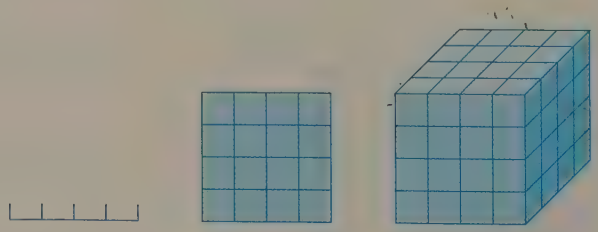
2^2 is read "the second power of 2" or "2 squared."

2^3 is read "the third power of 2" or "2 cubed."

2^4 is read "the fourth power of 2."

a^4 is read "the fourth power of a."

There is a geometric interpretation of the first three natural-number powers.



$4^1 = 4$
Length: 4 ft

$4^2 = 16$
Area: 16 ft^2

$4^3 = 64$
Volume: 64 ft^3

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To evaluate an exponential expression, write each factor as many times as indicated by the exponent. Then multiply.

➡ Evaluate $(-2)^4$.

$(-2)^4 = (-2)(-2)(-2)(-2)$ • Write (-2) as a factor 4 times.
$= 16$ • Multiply.

➡ Evaluate -2^4.

$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$ • Write 2 as a factor 4 times.
$= -16$ • Multiply.

From these last two examples, note the difference between $(-2)^4$ and -2^4.

$(-2)^4 = 16$
$-2^4 = -(2^4) = -16$

Example 9 Evaluate -5^3.

Solution $-5^3 = -(5 \cdot 5 \cdot 5) = -125$

You Try It 9 Evaluate -6^3.

Your solution

Example 10 Evaluate $(-4)^4$.

Solution $(-4)^4 = (-4)(-4)(-4)(-4)$
$= 256$

You Try It 10 Evaluate $(-3)^4$.

Your solution

Example 11 Evaluate $(-3)^2 \cdot 2^3$.

Solution $(-3)^2 \cdot 2^3 = (-3)(-3) \cdot (2)(2)(2)$
$= 9 \cdot 8 = 72$

You Try It 11 Evaluate $(3^3)(-2)^3$.

Your solution

Example 12
Evaluate $\left(-\frac{2}{3}\right)^3$.

Solution $\left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$
$= -\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = -\frac{8}{27}$

You Try It 12
Evaluate $\left(-\frac{2}{5}\right)^2$.

Your solution

Example 13 Evaluate $-4(0.7)^2$.

Solution $-4(0.7)^2 = -4(0.7)(0.7)$
$= -2.8(0.7) = -1.96$

You Try It 13 Evaluate $-3(0.3)^3$.

Your solution

Solutions on p. S2

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Objective F

To simplify numerical radical expressions

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A **square root** of a positive number x is a number whose square is x.

A square root of 16 is 4 because $4^2 = 16$.
A square root of 16 is -4 because $(-4)^2 = 16$.

Every positive number has two square roots, one a positive and one a negative number. The symbol "$\sqrt{\ }$," called a **radical sign**, is used to indicate the positive or **principal square root** of a number. For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$. The number under the radical sign is called the **radicand**.

When the negative square root of a number is to be found, a negative sign is placed in front of the radical. For example, $-\sqrt{16} = -4$ and $-\sqrt{25} = -5$.

Square Roots of Perfect Squares

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$

The square of an integer is a **perfect square**. 49, 81, and 144 are examples of perfect squares.

$7^2 = 49$
$9^2 = 81$
$12^2 = 144$

The principal square root of an integer that is a perfect square is a positive integer.

$\sqrt{49} = 7$
$\sqrt{81} = 9$
$\sqrt{144} = 12$

If a number is not a perfect square, its square root can only be approximated. For example, 2 and 7 are not perfect squares. The square roots of these numbers are **irrational numbers**. Their decimal representations never terminate or repeat.

$$\sqrt{2} \approx 1.4142135 \ldots \qquad \sqrt{7} \approx 2.6457513 \ldots$$

Recall that rational numbers are fractions such as $-\frac{6}{7}$ or $-\frac{10}{3}$, where the numerator and denominator are integers. Rational numbers are also represented by repeating decimals, such as $0.25767676 \ldots$, and by terminating decimals, such as 1.73. An irrational number is neither a repeating nor a terminating decimal. For instance, $2.45445444544445 \ldots$ is an irrational number.

Real Numbers

The rational numbers and the irrational numbers taken together are called the **real numbers**.

TAKE NOTE

Recall that a factor of a number divides the number evenly. For instance, 6 is a factor of 18. The perfect square 9 is also a factor of 18. 9 is a *perfect-square factor* of 18, whereas 6 is not a perfect-square factor of 18.

Radical expressions that contain radicands that are not perfect squares are frequently written in simplest form. A radical expression is in simplest form when the radicand contains no factor greater than 1 that is a perfect square. For instance, $\sqrt{50}$ is not in simplest form because 25 is a perfect square factor of 50. The radical expression $\sqrt{15}$ is in simplest form because there are no perfect-square factors of 15 that are greater than 1.

The Product Property of Square Roots and a knowledge of perfect squares are used to simplify radicands that are not perfect squares.

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The Product Property of Square Roots

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

TAKE NOTE

From the example at the right, $\sqrt{72} = 6\sqrt{2}$. The two expressions are different representations of the same number. Using a calculator, we find that $\sqrt{72} \approx 8.485281$ and $6\sqrt{2} \approx 8.485281$.

Simplify: $\sqrt{72}$

$\sqrt{72} = \sqrt{36 \cdot 2}$ • Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{36}\,\sqrt{2}$ • Use the Product Property of Square Roots to write the expression as a product.

$= 6\sqrt{2}$ • Simplify $\sqrt{36}$.

Note that 72 must be written as the product of a perfect square and *a factor that does not contain a perfect square.* Therefore, it would not be correct to rewrite $\sqrt{72}$ as $\sqrt{9 \cdot 8}$ and simplify the expression as shown at the right. Although 9 is a perfect-square factor of 72, 8 contains a perfect square ($8 = 4 \cdot 2$). Therefore, $\sqrt{8}$ is not in simplest form. Remember to find the *largest* perfect-square factor of the radicand.

$\sqrt{72} = \sqrt{9 \cdot 8}$
$= \sqrt{9}\,\sqrt{8}$
$= 3\sqrt{8}$

Not in simplest form

Simplify: $\sqrt{-16}$

Because the square of any real number is positive, there is no real number whose square is -16. $\sqrt{-16}$ is not a real number.

Example 14 Simplify: $3\sqrt{90}$

Solution

$3\sqrt{90} = 3\sqrt{9 \cdot 10}$
$= 3\sqrt{9}\,\sqrt{10}$
$= 3 \cdot 3\sqrt{10}$
$= 9\sqrt{10}$

You Try It 14 Simplify: $-5\sqrt{32}$

Your solution

Example 15 Simplify: $\sqrt{252}$

Solution

$\sqrt{252} = \sqrt{36 \cdot 7}$
$= \sqrt{36}\,\sqrt{7}$
$= 6\sqrt{7}$

You Try It 15 Simplify: $\sqrt{216}$

Your solution

Solutions on p. S2

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Objective G To solve application problems

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One of the applications of percent is to express a portion of a total as a percent. For instance, a recent survey of 450 mall shoppers found that 270 preferred the mall closest to their home even though it did not have as much store variety as a mall farther from home. The percent of shoppers who preferred the mall closest to home can be found by converting a fraction to a percent.

$$\frac{\text{Portion preferring mall closest to home}}{\text{Total number surveyed}} = \frac{270}{450}$$

$$= 0.60 = 60\%$$

The Congressional Budget Office projected that the total surpluses for 2001 through 2011 would be \$5.6 trillion. The number 5.6 trillion means

$$5.6 \times \underbrace{1{,}000{,}000{,}000{,}000}_{\text{1 trillion}} = 5{,}600{,}000{,}000{,}000$$

Numbers such as 5.6 trillion are used in many instances because they are easy to read and offer an approximation of the actual number.

The table below shows the net incomes, in millions of dollars, for the first quarter of 2001 and the first quarter of 2000 for two U.S. companies. Profits are shown as positive numbers; losses are shown as negative numbers. One quarter of a year is three months. Use this table for Example 16 and You Try It 16.

Company	1st Quarter 2001 Net Income	1st Quarter 2000 Net Income
Cisco Systems, Inc.	−2,693	641
Friendly Ice Cream	−3.203	−18.510

Source: **www.wsj.com**

Example 16

If earnings continued throughout the year at the same level, what would be the 2001 annual net income for Cisco Systems?

Strategy

To find the annual net income, multiply the net income in the first quarter of 2001 (−2,693) by the number of quarters in one year (4).

Solution

$-2{,}693(4) = -10{,}772$

The annual net income for Cisco Systems in 2001 would be −\$10,772 million.

You Try It 16

For the first quarter of 2000, what was the average monthly net income for Friendly Ice Cream?

Your strategy

Your solution

Solution on p. S2

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1.2 Exercises

Objective A

Write as a decimal. Place a bar over the repeating digits of a repeating decimal.

1. $\frac{1}{8}$ **2.** $\frac{7}{8}$ **3.** $\frac{2}{9}$ **4.** $\frac{8}{9}$ **5.** $\frac{1}{6}$

6. $\frac{5}{6}$ **7.** $\frac{9}{16}$ **8.** $\frac{15}{16}$ **9.** $\frac{7}{12}$ **10.** $\frac{11}{12}$

11. $\frac{6}{25}$ **12.** $\frac{14}{25}$ **13.** $\frac{9}{40}$ **14.** $\frac{21}{40}$ **15.** $\frac{5}{11}$

Objective B

16. **a.** Explain how to convert a fraction to a percent.
b. Explain how to convert a percent to a fraction.
c. Explain how to convert a decimal to a percent.
d. Explain how to convert a percent to a decimal.

17. Explain why multiplying a number by 100% does not change the value of the number.

Write as a fraction and as a decimal.

18. 75% **19.** 40% **20.** 64% **21.** 88% **22.** 125%

23. 160% **24.** 19% **25.** 87% **26.** 5% **27.** 450%

Write as a fraction.

28. $11\frac{1}{9}\%$ **29.** $4\frac{2}{7}\%$ **30.** $12\frac{1}{2}\%$ **31.** $37\frac{1}{2}\%$ **32.** $66\frac{2}{3}\%$

33. $\frac{1}{4}\%$ **34.** $\frac{1}{2}\%$ **35.** $6\frac{1}{4}\%$ **36.** $83\frac{1}{3}\%$ **37.** $5\frac{3}{4}\%$

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Write as a decimal.

38. 7.3% **39.** 9.1% **40.** 15.8% **41.** 16.7% **42.** 0.3%

43. 0.9% **44.** 9.9% **45.** 9.15% **46.** 121.2% **47.** 18.23%

Write as a percent.

48. 0.15 **49.** 0.37 **50.** 0.05 **51.** 0.02 **52.** 0.175

53. 0.125 **54.** 1.15 **55.** 1.36 **56.** 0.008 **57.** 0.004

58. $\frac{27}{50}$ **59.** $\frac{83}{100}$ **60.** $\frac{1}{3}$ **61.** $\frac{3}{8}$ **62.** $\frac{5}{11}$

63. $\frac{4}{9}$ **64.** $\frac{7}{8}$ **65.** $\frac{9}{20}$ **66.** $1\frac{2}{3}$ **67.** $2\frac{1}{2}$

Objective C

Add or subtract.

68. $-\frac{5}{6}-\frac{5}{9}$ **69.** $-\frac{6}{13}+\frac{17}{26}$ **70.** $-\frac{7}{12}+\frac{5}{8}$ **71.** $\frac{5}{8}-\left(-\frac{3}{4}\right)$

72. $\frac{3}{5}-\frac{11}{12}$ **73.** $\frac{11}{12}-\frac{5}{6}$ **74.** $-\frac{2}{3}-\left(-\frac{11}{18}\right)$ **75.** $-\frac{5}{8}-\left(-\frac{11}{12}\right)$

76. $\frac{1}{3}+\frac{5}{6}-\frac{2}{9}$ **77.** $\frac{1}{2}-\frac{2}{3}+\frac{1}{6}$ **78.** $-\frac{5}{16}+\frac{3}{4}-\frac{7}{8}$ **79.** $\frac{1}{2}-\frac{3}{8}-\left(-\frac{1}{4}\right)$

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80. $\frac{3}{4} - \left(-\frac{7}{12}\right) - \frac{7}{8}$ **81.** $\frac{1}{3} - \frac{1}{4} - \frac{1}{5}$ **82.** $\frac{2}{3} - \frac{1}{2} + \frac{5}{6}$ **83.** $\frac{5}{16} + \frac{1}{8} - \frac{1}{2}$

84. $-13.092 + 6.9$ **85.** $2.54 - 3.6$ **86.** $5.43 + 7.925$ **87.** $-16.92 - 6.925$

88. $-3.87 + 8.546$ **89.** $6.9027 - 17.692$ **90.** $2.09 - 6.72 - 5.4$

91. $-18.39 + 4.9 - 23.7$ **92.** $19 - (-3.72) - 82.75$ **93.** $-3.07 - (-2.97) - 17.4$

94. $16.4 - (-3.09) - 7.93$ **95.** $-3.09 - 4.6 - (-27.3)$ **96.** $2.66 - (-4.66) - 8.2$

Objective D

Multiply or divide.

97. $\frac{1}{2}\left(-\frac{3}{4}\right)$ **98.** $-\frac{2}{9}\left(-\frac{3}{14}\right)$ **99.** $\left(-\frac{3}{8}\right)\left(-\frac{4}{15}\right)$

100. $\left(-\frac{3}{4}\right)\left(-\frac{8}{27}\right)$ **101.** $-\frac{1}{2}\left(\frac{8}{9}\right)$ **102.** $\frac{5}{12}\left(-\frac{8}{15}\right)$

103. $\frac{5}{8}\left(-\frac{7}{12}\right)\frac{16}{25}$ **104.** $\left(\frac{5}{12}\right)\left(-\frac{8}{15}\right)\left(-\frac{1}{3}\right)$ **105.** $\frac{1}{2}\left(-\frac{3}{4}\right)\left(-\frac{5}{8}\right)$

106. $\frac{3}{8} \div \frac{1}{4}$ **107.** $\frac{5}{6} \div \left(-\frac{3}{4}\right)$ **108.** $-\frac{5}{12} \div \frac{15}{32}$

109. $-\frac{7}{8} \div \frac{4}{21}$ **110.** $\frac{7}{10} \div \frac{2}{5}$ **111.** $-\frac{15}{64} \div \left(-\frac{3}{40}\right)$

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112. $\frac{1}{8} \div \left(-\frac{5}{12}\right)$ **113.** $-\frac{4}{9} \div \left(-\frac{2}{3}\right)$ **114.** $-\frac{6}{11} \div \frac{4}{9}$

115. $1.2(3.47)$ **116.** $(-0.8)6.2$ **117.** $(-1.89)(-2.3)$

118. $(6.9)(-4.2)$ **119.** $1.06(-3.8)$ **120.** $-2.7(-3.5)$

121. $1.2(-0.5)(3.7)$ **122.** $-2.4(6.1)(0.9)$ **123.** $2.3(-0.6)(0.8)$

Divide. Round to the nearest hundredth.

124. $-1.27 \div (-1.7)$ **125.** $9.07 \div (-3.5)$ **126.** $0.0976 \div 0.042$

127. $-6.904 \div 1.35$ **128.** $-7.894 \div (-2.06)$ **129.** $-354.2086 \div 0.1719$

Objective E

Evaluate.

130. 6^2 **131.** 7^4 **132.** -7^2 **133.** -4^3 **134.** $(-3)^2$

135. $(-2)^3$ **136.** $(-3)^4$ **137.** $(-5)^3$ **138.** $\left(\frac{1}{2}\right)^2$ **139.** $\left(-\frac{3}{4}\right)^3$

140. $(0.3)^2$ **141.** $(1.5)^3$ **142.** $\left(\frac{2}{3}\right)^2 \cdot 3^3$ **143.** $\left(-\frac{1}{2}\right)^3 \cdot 8$ **144.** $(0.3)^3 \cdot 2^3$

145. $(-2) \cdot (-2)^2$ **146.** $2^3 \cdot 3^3 \cdot (-4)$ **147.** $(-3)^3 \cdot 5^2 \cdot 10$ **148.** $(-7) \cdot 4^2 \cdot 3^2$

149. $(-2) \cdot 2^3 \cdot (-3)^2$ **150.** $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{4} \cdot 3^3$ **151.** $\left(\frac{3}{4}\right)^2 \cdot (-4) \cdot 2^3$ **152.** $8^2 \cdot (-3)^5 \cdot 5$

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Objective F

Simplify.

153. $\sqrt{16}$ **154.** $\sqrt{64}$ **155.** $\sqrt{49}$ **156.** $\sqrt{144}$ **157.** $\sqrt{32}$ **158.** $\sqrt{50}$

159. $\sqrt{8}$ **160.** $\sqrt{12}$ **161.** $6\sqrt{18}$ **162.** $-3\sqrt{48}$ **163.** $5\sqrt{40}$ **164.** $2\sqrt{28}$

165. $\sqrt{15}$ **166.** $\sqrt{21}$ **167.** $\sqrt{29}$ **168.** $\sqrt{13}$ **169.** $-9\sqrt{72}$ **170.** $11\sqrt{80}$

171. $\sqrt{45}$ **172.** $\sqrt{225}$ **173.** $\sqrt{0}$ **174.** $\sqrt{210}$ **175.** $6\sqrt{128}$ **176.** $9\sqrt{288}$

Find the decimal approximation rounded to the nearest thousandth.

177. $\sqrt{240}$ **178.** $\sqrt{300}$ **179.** $\sqrt{288}$ **180.** $\sqrt{600}$ **181.** $\sqrt{256}$ **182.** $\sqrt{324}$

183. $\sqrt{275}$ **184.** $\sqrt{450}$ **185.** $\sqrt{245}$ **186.** $\sqrt{525}$ **187.** $\sqrt{352}$ **188.** $\sqrt{363}$

Objective G *Application Problems*

189. The table below shows the net incomes for the first quarter of 2001 and the first quarter of 2000 for three companies in the entertainment industry (Source: **www.wsj.com**). Figures are in millions of dollars. Profits are shown as positive numbers; losses are shown as negative numbers.

a. If earnings were to continue throughout the year at the same level, what would the 2001 annual net income be for Midway Games?

b. For the first quarter of 2001, what was the average monthly net income for Six Flags?

c. Find the difference between Fox Entertainment's first-quarter net income for 2000 and Midway Games's first-quarter net income for 2000.

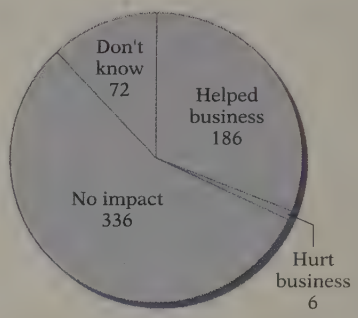
Company	1st Quarter 2001 Net Income	1st Quarter 2000 Net Income
Fox Entertainment	−9.0	19.0
Midway Games, Inc.	−25.852	−11.481
Six Flags, Inc.	−130.752	−113.892

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190. At the close of the stock markets on May 9, 2001, the indexes were posted as shown below, along with the decreases for that day, shown as negative numbers (Source: **www.wsj.com**). At what level were the indexes at the close of the day on May 8, 2001?

Index	Points at Market Close	Decrease for the Day
Dow Jones Industrial Average	10866.98	−16.53
Standard and Poor 500	1255.54	−5.66
NASDAQ	2156.62	−42.15

191. A survey asked 600 small business owners about the impact of the Internet on their businesses. The results are shown in the graph at the right. (Source: Infoworld by Dun & Bradstreet. Copyright © 2000 by Infoworld Media Group, Inc. Reproduced with permission of Infoworld Media Group, Inc. via Copyright Clearance Center.) What percent of the owners responded that the Internet had helped business?



192. The table at the right shows the surplus or deficit, in billions of dollars, for the federal budget every fifth year from 1945 to 1995 and every year from 1995 to 2000 (Source: U.S. Office of Management and Budget). The negative sign (−) indicates a deficit.

a. Find the difference between the deficits in the years 1980 and 1985.

b. Calculate the difference between the surplus in 1960 and the deficit in 1955.

c. How many times greater was the deficit in 1985 than in 1975? Round to the nearest whole number.

d. What was the average deficit, in millions of dollars, per quarter for the year 1970?

e. Find the average surplus or deficit for the years 1995 through 2000. Round to the nearest million.

Year	Federal Budget Surplus or Deficit
1945	−47.553
1950	−3.119
1955	−2.993
1960	0.301
1965	−1.411
1970	−2.842
1975	−53.242
1980	−73.835
1985	−212.334
1990	−221.194
1995	−163.899
1996	−107.450
1997	−21.940
1998	69.246
1999	79.263
2000	117.305

APPLYING THE CONCEPTS

193. List the whole numbers between $\sqrt{8}$ and $\sqrt{90}$.

194. Use a calculator to determine the decimal representations of $\frac{17}{99}$, $\frac{45}{99}$, and $\frac{73}{99}$. Make a conjecture as to the decimal representation of $\frac{83}{99}$. Does your conjecture work for $\frac{33}{99}$? What about $\frac{1}{99}$?

195. Describe in your own words how to simplify a radical expression.

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1.3 The Order of Operations Agreement

Objective A

To use the Order of Operations Agreement to simplify expressions



Let's evaluate $2 + 3 \cdot 5$.

There are two arithmetic operations, addition and multiplication, in this expression. The operations could be performed in different orders.

Multiply first.	$2 + \underbrace{3 \cdot 5}$	Add first.	$\underbrace{2 + 3} \cdot 5$
Then add.	$\underbrace{2 + 15}$	Then multiply.	$\underbrace{5 \cdot 5}$
	17		25

In order to prevent there being more than one answer for a numerical expression, an Order of Operations Agreement has been established.

THE ORDER OF OPERATIONS AGREEMENT

Step 1 Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, absolute value symbols | |, and the fraction bar.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

Evaluate $12 - 24(8 - 5) \div 2^2$.

$$12 - 24(8 - 5) \div 2^2 = 12 - 24(3) \div 2^2$$

- Perform operations inside grouping symbols.

$$= 12 - 24(3) \div 4$$

- Simplify exponential expressions.

$$= 12 - 72 \div 4$$

- Do multiplication and division as they occur from left to right.

$$= 12 - 18$$

$$= -6$$

- Do addition and subtraction as they occur from left to right.

One or more of the above steps may not be needed to evaluate an expression. In that case, proceed to the next step in the Order of Operations Agreement.

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When an expression has grouping symbols inside grouping symbols, perform the operations inside the inner grouping symbols first.

Evaluate $6 \div [4 - (6 - 8)] + 2^2$.

$6 \div [4 - (6 - 8)] + 2^2 = 6 \div [4 - (-2)] + 2^2$ • Perform operations inside grouping symbols.

$= 6 \div 6 + 2^2$

$= 6 \div 6 + 4$ • Simplify exponential expressions.

$= 1 + 4$ • Do multiplication and division as they occur from left to right.

$= 5$ • Do addition and subtraction as they occur from left to right.

Example 1
Evaluate $4 - 3[4 - 2(6 - 3)] \div 2$.

Solution
$4 - 3[4 - 2(6 - 3)] \div 2$
$= 4 - 3[4 - 2 \cdot 3] \div 2$
$= 4 - 3[4 - 6] \div 2$
$= 4 - 3[-2] \div 2$
$= 4 + 6 \div 2$
$= 4 + 3$
$= 7$

You Try It 1
Evaluate $18 - 5[8 - 2(2 - 5)] \div 10$.

Your solution

Example 2
Evaluate $27 \div (5 - 2)^2 + (-3)^2 \cdot 4$.

Solution
$27 \div (5 - 2)^2 + (-3)^2 \cdot 4$
$= 27 \div 3^2 + (-3)^2 \cdot 4$
$= 27 \div 9 + 9 \cdot 4$
$= 3 + 9 \cdot 4$
$= 3 + 36$
$= 39$

You Try It 2
Evaluate $36 \div (8 - 5)^2 - (-3)^2 \cdot 2$.

Your solution

Example 3
Evaluate $(1.75 - 1.3)^2 \div 0.025 + 6.1$.

Solution
$(1.75 - 1.3)^2 \div 0.025 + 6.1$
$= (0.45)^2 \div 0.025 + 6.1$
$= 0.2025 \div 0.025 + 6.1$
$= 8.1 + 6.1$
$= 14.2$

You Try It 3
Evaluate $(6.97 - 4.72)^2 \cdot 4.5 \div 0.05$.

Your solution

Solutions on p. S2

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1.3 Exercises

Objective A

1. Why do we need an Order of Operations Agreement?

2. Describe each step in the Order of Operations Agreement.

Evaluate by using the Order of Operations Agreement.

3. $4 - 8 \div 2$

4. $2^2 \cdot 3 - 3$

5. $2(3 - 4) - (-3)^2$

6. $16 - 32 \div 2^3$

7. $24 - 18 \div 3 + 2$

8. $8 - (-3)^2 - (-2)$

9. $8 - 2(3)^2$

10. $16 - 16 \cdot 2 \div 4$

11. $12 + 16 \div 4 \cdot 2$

12. $16 - 2 \cdot 4^2$

13. $27 - 18 \div (-3^2)$

14. $4 + 12 \div 3 \cdot 2$

15. $16 + 15 \div (-5) - 2$

16. $14 - 2^2 - (4 - 7)$

17. $14 - 2^2 - |4 - 7|$

18. $10 - |5 - 8| + 2^3$

19. $3 - 2[8 - (3 - 2)]$

20. $-2^2 + 4[16 \div (3 - 5)]$

21. $6 + \dfrac{16 - 4}{2^2 + 2} - 2$

22. $24 \div \dfrac{3^2}{8 - 5} - (-5)$

23. $18 \div |9 - 2^3| + (-3)$

24. $96 \div 2[12 + (6 - 2)] - 3^2$

25. $4[16 - (7 - 1)] \div 10$

26. $18 \div 2 - 4^2 - (-3)^2$

27. $20 \div (10 - 2^3) + (-5)$

28. $16 - 3(8 - 3)^2 \div 5$

29. $4(-8) \div [2(7 - 3)^2]$

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30. $\dfrac{(-10)+(-2)}{6^2-30} \div |2-4|$ **31.** $16 - 4 \cdot \dfrac{3^3-7}{2^3+2} - (-2)^2$ **32.** $(0.2)^2 \cdot (-0.5) + 1.72$

33. $0.3(1.7 - 4.8) + (1.2)^2$ **34.** $(1.8)^2 - 2.52 \div 1.8$ **35.** $(1.65 - 1.05)^2 \div 0.4 + 0.8$

APPLYING THE CONCEPTS

36. Find two fractions between $\frac{2}{3}$ and $\frac{3}{4}$. (There is more than one answer to this question.)

37. A **magic square** is one in which the numbers in every row, column, and diagonal sum to the same number. Complete the magic square at the right.

$\frac{2}{3}$		
	$\frac{1}{6}$	$\frac{5}{6}$
		$-\frac{1}{3}$

38. For each part below, find a rational number r that satisfies the condition.
a. $r^2 < r$ **b.** $r^2 = r$ **c.** $r^2 > r$

39. In a survey of consumers, 9% said they would buy an electric vehicle. Approximately 43% said they would be willing to pay between \$1000 and \$2000 more for a new car if the car had an EPA rating of 80 mpg. If your car now gets 28 mpg and you drive approximately 10,000 mi per year, in how many months would your savings on gasoline pay for the increased cost of such a car? Assume the average cost for gasoline is \$1.60 per gallon.



40. Find three different natural numbers a, b, and c such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is a natural number.

41. The following was offered as the simplification of $6 + 2(4 - 9)$.

$$\begin{aligned} 6 + 2(4 - 9) &= 6 + 2(-5) \\ &= 8(-5) \\ &= -40 \end{aligned}$$

Is this a correct simplification? Explain your answer.

42. The following was offered as the simplification of $2 \cdot 3^3$.

$$2 \cdot 3^3 = 6^3 = 216$$

Is this is a correct simplification? Explain your answer.

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1.4 Variable Expressions

To evaluate a variable expression

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Often we discuss a quantity without knowing its exact value—for example, the price of gold next month, the cost of a new automobile next year, or the tuition cost for next semester. Recall that a letter of the alphabet can be used to stand for a quantity that is unknown or that can change, or *vary*. Such a letter is called a variable. An expression that contains one or more variables is called a **variable expression**.

Point of Interest

Historical manuscripts indicate that mathematics is at least 4000 years old. Yet it was only 400 years ago that mathematicians started using variables to stand for numbers. The idea that a letter can stand for some number was a critical turning point in mathematics.

A variable expression is shown at the right. The expression can be rewritten by writing subtraction as the addition of the opposite.

$$3x^2 - 5y + 2xy - x - 7$$

$$3x^2 + (-5y) + 2xy + (-x) + (-7)$$

Note that the expression has 5 addends. The **terms** of a variable expression are the addends of the expression. The expression has 5 terms.

$$\overbrace{\underbrace{3x^2 - 5y + 2xy - x}_{\text{variable terms}} \underbrace{- 7}_{\text{constant term}}}^{5 \text{ terms}}$$

The terms $3x^2$, $-5y$, $2xy$, and $-x$ are **variable terms.**

The term -7 is a **constant term**, or simply a **constant.**

Each variable term is composed of a **numerical coefficient** and a **variable part** (the variable or variables and their exponents).

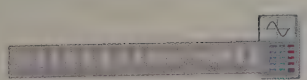
$$\overbrace{3x^2}^{3} - \overbrace{5y}^{-5} + \overbrace{2xy}^{2} - \overbrace{1x}^{1} - 7$$

numerical coefficient

variable part

When the numerical coefficient is 1 or -1, the 1 is usually not written ($x = 1x$ and $-x = -1x$).

Replacing each variable by its value and then simplifying the resulting numerical expression is called **evaluating the variable expression.**



See the appendix "Guidelines for Using Graphing Calculators" for instructions on using a graphing calculator to evaluate variable expressions.

➧ Evaluate $ab - b^2$ when $a = 2$ and $b = -3$.

$ab - b^2$

$2(-3) - (-3)^2$ • Replace each variable in the expression by its value.

$= 2(-3) - 9$ • Use the Order of Operations Agreement to simplify the resulting numerical expression.

$= -6 - 9$

$= -15$

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Example 1

Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

Solution

$$\frac{a^2 - b^2}{a - b}$$

$$\frac{3^2 - (-4)^2}{3 - (-4)} = \frac{9 - 16}{3 - (-4)}$$

$$= \frac{-7}{7} = -1$$

You Try It 1

Evaluate $\frac{a^2 + b^2}{a + b}$ when $a = 5$ and $b = -3$.

Your solution

Example 2 Evaluate $x^2 - 3(x - y) - z^2$ when $x = 2$, $y = -1$, and $z = 3$.

Solution

$$x^2 - 3(x - y) - z^2$$

$$2^2 - 3[2 - (-1)] - 3^2$$

$$= 2^2 - 3(3) - 3^2$$

$$= 4 - 3(3) - 9$$

$$= 4 - 9 - 9$$

$$= -5 - 9$$

$$= -14$$

You Try It 2 Evaluate $x^3 - 2(x + y) + z^2$ when $x = 2$, $y = -4$, and $z = -3$.

Your solution

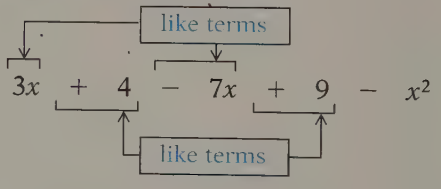
Solutions on p. S2

Objective B

To simplify a variable expression using the Properties of Addition



Like terms of a variable expression are terms with the same variable part. (Because $x^2 = x \cdot x$, x^2 and x are not like terms.)



Constant terms are like terms. 4 and 9 are like terms.

To simplify a variable expression, use the Distributive Property to combine like terms by adding the numerical coefficients. The variable part remains unchanged.

Distributive Property

If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

The Distributive Property can also be written as $ba + ca = (b + c)a$. This form is used to simplify a variable expression.

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Simplify: $2x + 3x$

Use the Distributive Property to add the numerical coefficients of the like variable terms. This is called **combining like terms.**

$$2x + 3x = (2 + 3)x$$
$$= 5x$$

- Use the Distributive Property.

Simplify: $5y - 11y$

$$5y - 11y = (5 - 11)y$$
$$= -6y$$

- Use the Distributive Property. This step is usually done mentally.

TAKE NOTE
Simplifying an expression means combining like terms. A constant term (5) and a variable term ($7p$) are not like terms and therefore cannot be combined.

Simplify: $5 + 7p$

The terms 5 and $7p$ are not like terms.

The expression $5 + 7p$ is in simplest form.

In simplifying variable expressions, the following Properties of Addition are used.

The Associative Property of Addition

If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

When three or more like terms are added, the terms can be grouped (with parentheses, for example) in any order. The sum is the same. For example,

$$(3x + 5x) + 9x = 3x + (5x + 9x)$$
$$8x + 9x = 3x + 14x$$
$$17x = 17x$$

The Commutative Property of Addition

If a and b are real numbers, then $a + b = b + a$.

When two like terms are added, the terms can be added in either order. The sum is the same. For example,

$$2x + (-4x) = -4x + 2x$$
$$-2x = -2x$$

The Addition Property of Zero

If a is a real number, then $a + 0 = 0 + a = a$.

The sum of a term and zero is the term. For example,

$$5x + 0 = 0 + 5x = 5x$$

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The Inverse Property of Addition

If a is a real number, then $a + (-a) = (-a) + a = 0$.

The sum of a term and its opposite is zero. The opposite of a number is called its **additive inverse**.

$$7x + (-7x) = -7x + 7x = 0$$

➡ Simplify: $8x + 4y - 8x + y$

Use the Commutative and Associative Properties of Addition to rearrange and group like terms. Then combine like terms.

$$8x + 4y - 8x + y = \boxed{(8x - 8x) + (4y + y)}$$

• This step is usually done mentally.

$$= 0 + 5y$$
$$= 5y$$

➡ Simplify: $4x^2 + 5x - 6x^2 - 2x + 1$

Use the Commutative and Associative Properties of Addition to rearrange and group like terms. Then combine like terms.

$$4x^2 + 5x - 6x^2 - 2x + 1 = (4x^2 - 6x^2) + (5x - 2x) + 1$$
$$= -2x^2 + 3x + 1$$

Example 3 Simplify: $3x + 4y - 10x + 7y$

Solution $3x + 4y - 10x + 7y = -7x + 11y$

You Try It 3 Simplify: $3a - 2b - 5a + 6b$

Your solution

Example 4 Simplify: $x^2 - 7 + 4x^2 - 16$

Solution $x^2 - 7 + 4x^2 - 16 = 5x^2 - 23$

You Try It 4 Simplify: $-3y^2 + 7 + 8y^2 - 14$

Your solution

Solutions on p. S3

Objective E

To simplify a variable expression using the Properties of Multiplication



In simplifying variable expressions, the following Properties of Multiplication are used.

The Associative Property of Multiplication

If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

When three or more factors are multiplied, the factors can be grouped in any order. The product is the same. For example,

$$2(3x) = (2 \cdot 3)x = 6x$$

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The Commutative Property of Multiplication

If a and b are real numbers, then $a \cdot b = b \cdot a$.

Two factors can be multiplied in either order. The product is the same. For example,

$$(2x) \cdot 3 = 3 \cdot (2x) = 6x$$

The Multiplication Property of One

If a is a real number, then $a \cdot 1 = 1 \cdot a = a$.

The product of a term and one is the term. For example,

$$(8x)(1) = (1)(8x) = 8x$$

The Inverse Property of Multiplication

If a is a real number, and a is not equal to zero, then

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1.$$

$\frac{1}{a}$ is called the **reciprocal** of a. $\frac{1}{a}$ is also called the **multiplicative inverse** of a.

The product of a number and its reciprocal is one. For example,

$$7 \cdot \frac{1}{7} = \frac{1}{7} \cdot 7 = 1$$

The multiplication properties just discussed are used to simplify variable expressions.

➡ Simplify: $2(-x)$

$$\begin{aligned} 2(-x) &= 2(-1 \cdot x) \\ &= [2(-1)]x \\ &= -2x \end{aligned}$$

- Use the Associative Property of Multiplication to group factors.

➡ Simplify: $\frac{3}{2}\left(\frac{2x}{3}\right)$

Use the Associative Property of Multiplication to group factors.

$$\begin{aligned} \frac{3}{2}\left(\frac{2x}{3}\right) &= \frac{3}{2}\left(\frac{2}{3} \cdot x\right) \\ &= \left(\frac{3}{2} \cdot \frac{2}{3}\right)x \\ &= 1 \cdot x \\ &= x \end{aligned}$$

- Note that $\frac{2x}{3} = \frac{2}{3}x$.
- The steps in the dashed box are usually done mentally.

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Simplify: $(16x)2$

Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

$$(16x)2 = \boxed{\begin{aligned} &2(16x) \\ &= (2 \cdot 16)x \end{aligned}}$$
$$= 32x$$

• The steps in the dashed box are usually done mentally.

Example 5 Simplify: $-2(3x^2)$

Solution $-2(3x^2) = -6x^2$

You Try It 5 Simplify: $-5(4y^2)$

Your solution

Example 6 Simplify: $-5(-10x)$

Solution $-5(-10x) = 50x$

You Try It 6 Simplify: $-7(-2a)$

Your solution

Example 7 Simplify: $(6x)(-4)$

Solution $(6x)(-4) = -24x$

You Try It 7 Simplify: $(-5x)(-2)$

Your solution

Solutions on p. S3

Objective D

To simplify a variable expression using the Distributive Property



Recall that the Distributive Property states that if a, b, and c are real numbers, then

$$a(b + c) = ab + ac$$

The Distributive Property is used to remove parentheses from a variable expression.

Simplify: $3(2x + 7)$

$$3(2x + 7) = \boxed{3(2x) + 3(7)}$$
$$= 6x + 21$$

• Use the Distributive Property. Do this step mentally.

Simplify: $-5(4x + 6)$

$$-5(4x + 6) = \boxed{-5(4x) + (-5) \cdot 6}$$
$$= -20x - 30$$

• Use the Distributive Property. Do this step mentally.

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Simplify: $-(2x - 4)$

$$-(2x - 4) = \boxed{\begin{array}{l} -1(2x - 4) \\ -1(2x) - (-1)(4) \end{array}} = -2x + 4$$

- Use the Distributive Property. Do these steps mentally.

Note: When a negative sign immediately precedes the parentheses, the sign of each term inside the parentheses is changed.

Simplify: $-\frac{1}{2}(8x - 12y)$

$$-\frac{1}{2}(8x - 12y) = \boxed{-\frac{1}{2}(8x) - \left(-\frac{1}{2}\right)(12y)} = -4x + 6y$$

- Use the Distributive Property. Do this step mentally.

Simplify: $4(x - y) - 2(-3x + 6y)$

$$4(x - y) - 2(-3x + 6y) = 4x - 4y + 6x - 12y = 10x - 16y$$

- Use the Distributive Property twice.
- Combine like terms.

An extension of the Distributive Property is used when an expression inside parentheses contains more than two terms. See the example below.

Simplify: $3(4x - 2y - z)$

$$3(4x - 2y - z) = \boxed{3(4x) - 3(2y) - 3(z)} = 12x - 6y - 3z$$

- Use the Distributive Property. Do this step mentally.

Example 8 Simplify: $-3(-5a + 7b)$

Solution $-3(-5a + 7b) = 15a - 21b$

You Try It 8 Simplify: $-8(-2a + 7b)$

Your solution

Example 9 Simplify: $(2x - 6)2$

Solution $(2x - 6)2 = 4x - 12$

You Try It 9 Simplify: $(3a - 1)5$

Your solution

Solutions on p. S3

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Example 10 Simplify: $3(x^2 - x - 5)$

Solution $3(x^2 - x - 5) = 3x^2 - 3x - 15$

You Try It 10 Simplify: $2(x^2 - x + 7)$

Your solution

Example 11 Simplify: $2x - 3(2x - 7y)$

Solution
$$2x - 3(2x - 7y) = 2x - 6x + 21y$$
$$= -4x + 21y$$

You Try It 11 Simplify: $3y - 2(y - 7x)$

Your solution

Example 12 Simplify: $7(x - 2y) - (-x - 2y)$

Solution
$$7(x - 2y) - (-x - 2y)$$
$$= 7x - 14y + x + 2y$$
$$= 8x - 12y$$

You Try It 12 Simplify: $-2(x - 2y) - (-x + 3y)$

Your solution

Example 13 Simplify: $2x - 3[2x - 3(x + 7)]$

Solution
$$2x - 3[2x - 3(x + 7)]$$
$$= 2x - 3[2x - 3x - 21]$$
$$= 2x - 3[-x - 21]$$
$$= 2x + 3x + 63$$
$$= 5x + 63$$

You Try It 13 Simplify: $3y - 2[x - 4(2 - 3y)]$

Your solution

Solutions on p. S3

Objective E

To translate a verbal expression into a variable expression



One of the major skills required in applied mathematics is the ability to translate a verbal expression into a variable expression. This requires recognizing the verbal phrases that translate into mathematical operations. A partial list of the verbal phrases used to indicate the different mathematical operations follows.

Addition

added to	6 added to y	$y + 6$
more than	8 more than x	$x + 8$
the sum of	the sum of x and z	$x + z$
increased by	t increased by 9	$t + 9$
the total of	the total of 5 and y	$5 + y$

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Point of Interest

The way in which expressions are symbolized has changed over time. Here is how some of the expressions shown at the right may have appeared in the early 16th century.

R p. 8 for $x + 8$. The symbol R was used for a variable to the first power. The symbol p. was used for plus.

R m. 2 for $x - 2$. The symbol R is still used for the variable. The symbol m. was used for minus.

The square of a variable was designated by Q and the cube was designated by C. The expression $x^3 + x^2$ was written **C p. Q.**

Subtraction	minus	x minus 2	$x - 2$
	less than	7 less than t	$t - 7$
	decreased by	m decreased by 3	$m - 3$
	the difference between	the difference between y and 4	$y - 4$
Multiplication	times	10 times t	$10t$
	of	one-half of x	$\frac{1}{2}x$
	the product of	the product of y and z	yz
	multiplied by	y multiplied by 11	$11y$
	twice	twice d	$2d$
Division	divided by	x divided by 12	$\frac{x}{12}$
	the quotient of	the quotient of y and z	$\frac{y}{z}$
	the ratio of	the ratio of t to 9	$\frac{t}{9}$
Power	the square of	the square of x	x^2
	the cube of	the cube of a	a^3

Translate "14 less than the cube of x" into a variable expression.

14 *less than* the *cube* of x

- Identify the words that indicate the mathematical operations.

$x^3 - 14$

- Use the identified operations to write the variable expression.

In most applications that involve translating phrases into variable expressions, the variable to be used is not given. To translate these phrases, a variable must be assigned to an unknown quantity before the variable expression can be written.

Translate "the sum of two consecutive integers" into a variable expression. Then simplify.

the first integer: n

- Assign a variable to one of the unknown quantities.

the next consecutive integer: $n + 1$

- Use the assigned variable to write an expression for any other unknown quantity.

$n + (n + 1)$

- Use the assigned variable to write the variable expression.

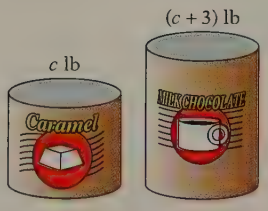
$(n + n) + 1$

$2n + 1$

- Simplify the variable expression.

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Many of the applications of mathematics require that you identify an unknown quantity, assign a variable to that quantity, and then attempt to express another unknown quantity in terms of the variable.



A confectioner makes a mixture of candy that contains 3 lb more of milk chocolate than of caramel. Express the amount of milk chocolate in the mixture in terms of the amount of caramel in the mixture.

Amount of caramel in the mixture: c

- Assign a variable to the amount of caramel in the mixture.

Amount of milk chocolate in the mixture: $c + 3$

- Express the amount of milk chocolate in the mixture in terms of c.

Example 14

Translate "four times the sum of half of a number and fourteen" into a variable expression. Then simplify.

Solution

the unknown number: n

half of the number: $\frac{1}{2}n$

the sum of half of the number and fourteen: $\frac{1}{2}n + 14$

$4\left(\frac{1}{2}n + 14\right)$

$2n + 56$

You Try It 14

Translate "five times the difference between a number and sixty" into a variable expression. Then simplify.

Your solution

Example 15

The length of a swimming pool is 4 ft less than two times the width. Express the length of the pool in terms of the width.

Solution

the width of the pool: w

the length is 4 ft less than two times the width: $2w - 4$

You Try It 15

The speed of a new printer is twice the speed of an older model. Express the speed of the new model in terms of the speed of the older model.

Your solution

Example 16

A banker divided $5000 between two accounts, one paying 10% annual interest and the second paying 8% annual interest. Express the amount invested in the 10% account in terms of the amount invested in the 8% account.

Solution

the amount invested at 8%: x

the amount invested at 10%: $5000 - x$

You Try It 16

A guitar string 6 ft long was cut into two pieces. Express the length of the shorter piece in terms of the length of the longer piece.

Your solution

Solutions on p. S3

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1.4 Exercises

Objective A

Evaluate the variable expression when $a = 2$, $b = 3$, and $c = -4$.

1. $6b \div (-a)$
2. $bc \div (2a)$
3. $b^2 - 4ac$
4. $a^2 - b^2$
5. $b^2 - c^2$
6. $(a + b)^2$
7. $a^2 + b^2$
8. $2a - (c + a)^2$
9. $(b - a)^2 + 4c$
10. $b^2 - \frac{ac}{8}$
11. $\frac{5ab}{6} - 3cb$
12. $(b - 2a)^2 + bc$

Evaluate the variable expression when $a = -2$, $b = 4$, $c = -1$, and $d = 3$.

13. $\frac{b + c}{d}$
14. $\frac{d - b}{c}$
15. $\frac{2d + b}{-a}$
16. $\frac{b + 2d}{b}$
17. $\frac{b - d}{c - a}$
18. $\frac{2c - d}{-ad}$
19. $(b + d)^2 - 4a$
20. $(d - a)^2 - 3c$
21. $(d - a)^2 \div 5$
22. $3(b - a) - bc$
23. $\frac{b - 2a}{bc^2 - d}$
24. $\frac{b^2 - a}{ad + 3c}$
25. $\frac{1}{3}d^2 - \frac{3}{8}b^2$
26. $\frac{5}{8}a^4 - c^2$
27. $\frac{-4bc}{2a - b}$
28. $-\frac{3}{4}b + \frac{1}{2}(ac + bd)$
29. $-\frac{2}{3}d - \frac{1}{5}(bd - ac)$
30. $(b - a)^2 - (d - c)^2$
31. $(b + c)^2 + (a + d)^2$
32. $4ac + (2a)^2$
33. $3dc - (4c)^2$

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Objective B

Simplify.

34. $6x + 8x$

35. $12x + 13x$

36. $9a - 4a$

37. $12a - 3a$

38. $4y + (-10y)$

39. $8y + (-6y)$

40. $-3b - 7$

41. $-12y - 3$

42. $-12a + 17a$

43. $-3a + 12a$

44. $5ab - 7ab$

45. $9ab - 3ab$

46. $-12xy + 17xy$

47. $-15xy + 3xy$

48. $-3ab + 3ab$

49. $-7ab + 7ab$

50. $-\frac{1}{2}x - \frac{1}{3}x$

51. $-\frac{2}{5}y + \frac{3}{10}y$

52. $\frac{3}{8}x^2 - \frac{5}{12}x^2$

53. $\frac{2}{3}y^2 - \frac{4}{9}y^2$

54. $3x + 5x + 3x$

55. $8x + 5x + 7x$

56. $5a - 3a + 5a$

57. $10a - 17a + 3a$

58. $-5x^2 - 12x^2 + 3x^2$

59. $-y^2 - 8y^2 + 7y^2$

60. $7x + (-8x) + 3y$

61. $8y + (-10x) + 8x$

62. $7x - 3y + 10x$

63. $8y + 8x - 8y$

64. $3a + (-7b) - 5a + b$

65. $-5b + 7a - 7b + 12a$

66. $3x + (-8y) - 10x + 4x$

67. $3y + (-12x) - 7y + 2y$

68. $x^2 - 7x + (-5x^2) + 5x$

69. $3x^2 + 5x - 10x^2 - 10x$

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Objective C

Simplify.

70. $4(3x)$ **71.** $12(5x)$ **72.** $-3(7a)$ **73.** $-2(5a)$ **74.** $-2(-3y)$

75. $-5(-6y)$ **76.** $(4x)2$ **77.** $(6x)12$ **78.** $(3a)(-2)$ **79.** $(7a)(-4)$

80. $(-3b)(-4)$ **81.** $(-12b)(-9)$ **82.** $-5(3x^2)$ **83.** $-8(7x^2)$ **84.** $\frac{1}{3}(3x^2)$

85. $\frac{1}{6}(6x^2)$ **86.** $\frac{1}{5}(5a)$ **87.** $\frac{1}{8}(8x)$ **88.** $-\frac{1}{2}(-2x)$ **89.** $-\frac{1}{4}(-4a)$

90. $-\frac{1}{7}(-7n)$ **91.** $-\frac{1}{9}(-9b)$ **92.** $(3x)\left(\frac{1}{3}\right)$ **93.** $(12x)\left(\frac{1}{12}\right)$ **94.** $(-6y)\left(-\frac{1}{6}\right)$

95. $(-10n)\left(-\frac{1}{10}\right)$ **96.** $\frac{1}{3}(9x)$ **97.** $\frac{1}{7}(14x)$ **98.** $-\frac{1}{5}(10x)$ **99.** $-\frac{1}{8}(16x)$

100. $-\frac{2}{3}(12a^2)$ **101.** $-\frac{5}{8}(24a^2)$ **102.** $-\frac{1}{2}(-16y)$ **103.** $-\frac{3}{4}(-8y)$ **104.** $(16y)\left(\frac{1}{4}\right)$

105. $(33y)\left(\frac{1}{11}\right)$ **106.** $(-6x)\left(\frac{1}{3}\right)$ **107.** $(-10x)\left(\frac{1}{5}\right)$ **108.** $(-8a)\left(-\frac{3}{4}\right)$ **109.** $(21y)\left(-\frac{3}{7}\right)$

Objective D

Simplify.

110. $-(x+2)$ **111.** $-(x+7)$ **112.** $2(4x-3)$ **113.** $5(2x-7)$

114. $-2(a+7)$ **115.** $-5(a+16)$ **116.** $-3(2y-8)$ **117.** $-5(3y-7)$

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118. $(5 - 3b)7$ **119.** $(10 - 7b)2$ **120.** $\frac{1}{3}(6 - 15y)$ **121.** $\frac{1}{2}(-8x + 4y)$

122. $3(5x^2 + 2x)$ **123.** $6(3x^2 + 2x)$ **124.** $-2(-y + 9)$ **125.** $-5(-2x + 7)$

126. $(-3x - 6)5$ **127.** $(-2x + 7)7$ **128.** $2(-3x^2 - 14)$ **129.** $5(-6x^2 - 3)$

130. $-3(2y^2 - 7)$ **131.** $-8(3y^2 - 12)$ **132.** $3(x^2 - y^2)$ **133.** $5(x^2 + y^2)$

134. $-\frac{2}{3}(6x - 18y)$ **135.** $-\frac{1}{2}(x - 4y)$ **136.** $-(6a^2 - 7b^2)$

137. $3(x^2 + 2x - 6)$ **138.** $4(x^2 - 3x + 5)$ **139.** $-2(y^2 - 2y + 4)$

140. $\frac{1}{2}(2x - 6y + 8)$ **141.** $-\frac{1}{3}(6x - 9y + 1)$ **142.** $4(-3a^2 - 5a + 7)$

143. $-5(-2x^2 - 3x + 7)$ **144.** $-3(-4x^2 + 3x - 4)$ **145.** $3(2x^2 + xy - 3y^2)$

146. $5(2x^2 - 4xy - y^2)$ **147.** $-(3a^2 + 5a - 4)$ **148.** $-(8b^2 - 6b + 9)$

149. $4x - 2(3x + 8)$ **150.** $6a - (5a + 7)$ **151.** $9 - 3(4y + 6)$

152. $10 - (11x - 3)$ **153.** $5n - (7 - 2n)$ **154.** $8 - (12 + 4y)$

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155. $3(x + 2) - 5(x - 7)$

156. $2(x - 4) - 4(x + 2)$

157. $12(y - 2) + 3(7 - 3y)$

158. $6(2y - 7) - (3 - 2y)$

159. $3(a - b) - (a + b)$

160. $2(a + 2b) - (a - 3b)$

161. $4[x - 2(x - 3)]$

162. $2[x + 2(x + 7)]$

163. $-2[3x + 2(4 - x)]$

164. $-5[2x + 3(5 - x)]$

165. $-3[2x - (x + 7)]$

166. $-2[3x - (5x - 2)]$

167. $2x - 3[x - (4 - x)]$

168. $-7x + 3[x - (3 - 2x)]$

169. $-5x - 2[2x - 4(x + 7)] - 6$

Objective E

Translate into a variable expression. Then simplify.

170. twelve minus a number

171. a number divided by eighteen

172. two-thirds of a number

173. twenty more than a number

174. the quotient of twice a number and nine

175. ten times the difference between a number and fifty

176. eight less than the product of eleven and a number

177. the sum of five-eighths of a number and six

178. nine less than the total of a number and two

179. the difference between a number and three more than the number

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180. the quotient of seven and the total of five and a number

181. four times the sum of a number and nineteen

182. five increased by one-half of the sum of a number and three

183. the quotient of fifteen and the sum of a number and twelve

184. a number added to the difference between twice the number and four

185. the product of two-thirds and the sum of a number and seven

186. the product of five less than a number and seven

187. the difference between forty and the quotient of a number and twenty

188. the quotient of five more than twice a number and the number

189. the sum of the square of a number and twice the number

190. a number decreased by the difference between three times the number and eight

191. the sum of eight more than a number and one-third of the number

192. a number added to the product of three and the number

193. a number increased by the total of the number and nine

194. five more than the sum of a number and six

195. a number decreased by the difference between eight and the number

196. a number minus the sum of the number and ten

197. the difference between one-third of a number and five-eighths of the number

198. the sum of one-sixth of a number and four-ninths of the number

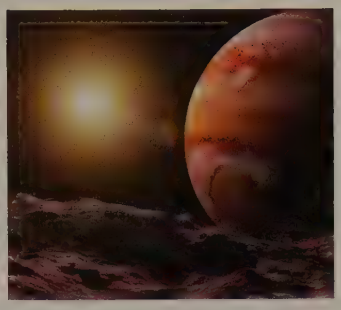
199. two more than the total of a number and five

200. the sum of a number divided by three and the number

201. twice the sum of six times a number and seven

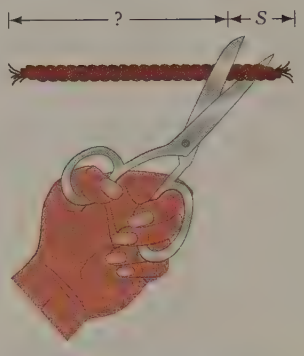
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202. The planet Saturn has 9 more moons than Jupiter (Source: NASA). Express the number of moons Saturn has in terms of the number of moons Jupiter has.



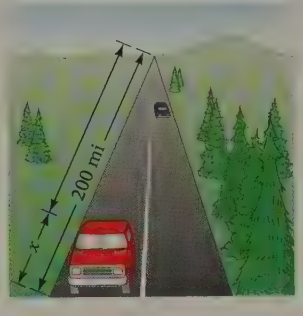
203. According to the National Oceanic and Atmospheric Administration, the number of tornadoes in 1990 was 235 more than the number of tornadoes in 2000. Express the number of tornadoes in 1990 in terms of the number of tornadoes in 2000.

204. A rope 12 ft long was cut into two pieces of different lengths. Use one variable to express the lengths of the two pieces.



205. Twenty gallons of crude oil were poured into two containers of different sizes. Use one variable to express the amount of oil poured into each container.

206. Two cars start at the same place and travel at different rates in opposite directions. Two hours later the cars are 200 mi apart. Express the distance traveled by the faster car in terms of distance traveled by the slower car.



207. In a recent year, Alabama produced one-half the number of pounds of pecans that Texas produced that same year (Source: National Agricultural Statistics Service). Express the amount of pecans produced in Alabama in terms of the amount produced in Texas.

208. According to the Internal Revenue Service, it takes about one-fifth as much time to fill out Schedule B (interest and dividends) as to fill out Schedule A (itemized deductions). Express the amount of time it takes to fill out Schedule B in terms of the time it takes to fill out Schedule A.

209. The diameter of a basketball is approximately 4 times the diameter of a baseball. Express the diameter of a basketball in terms of the diameter of a baseball.

210. According to the U.S. Bureau of the Census, the world population in the year 2050 is expected to be twice the world population in 1980. Express the world population in 2050 in terms of the world population in 1980.

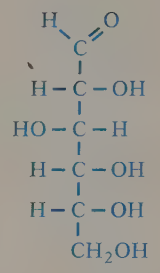
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APPLYING THE CONCEPTS

211. Does every number have an additive inverse? If not, which real numbers do not have an additive inverse?

212. Does every number have a multiplicative inverse? If not, which real numbers do not have a multiplicative inverse?

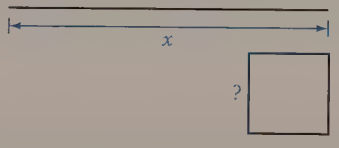
213. The chemical formula for glucose (sugar) is $C_6H_{12}O_6$. This formula means that there are twelve hydrogen atoms, six carbon atoms, and six oxygen atoms in each molecule of glucose. If x represents the number of atoms of oxygen in a pound of sugar, express the number of hydrogen atoms in the pound of sugar.



214. Determine whether the statement is true or false. If the statement is false, give an example that illustrates that it is false.

a. Division is a commutative operation.
b. Division is an associative operation.
c. Subtraction is an associative operation.
d. Subtraction is a commutative operation.
e. Addition is a commutative operation.

215. A wire whose length is given as x inches is bent into a square. Express the length of a side of the square in terms of x.

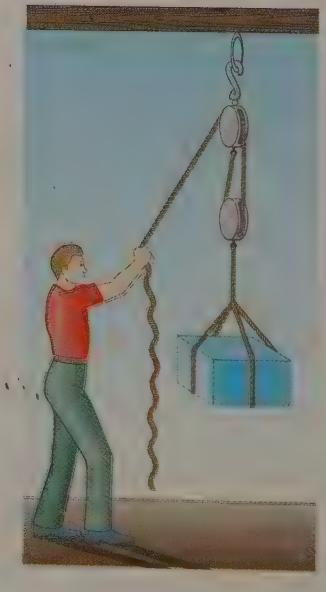


216. For each of the following, determine the first natural number x, greater than 2, for which the second expression is larger than the first.

a. x^3, 3^x **b.** x^4, 4^x **c.** x^5, 5^x **d.** x^6, 6^x

On the basis of your answers, make a conjecture that appears to be true about the expressions x^n and n^x, where $n = 3, 4, 5, 6, 7, \ldots$ and x is a natural number greater than 2.

217. A block-and-tackle system is designed so that pulling five feet on one end of a rope will move a weight on the other end a distance of three feet. If x represents the distance the rope is pulled, express the distance the weight moves in terms of x.



218. Give examples of two operations that occur in everyday experience that are not commutative (for example, putting on socks and then shoes).

219. Choose any number a. Evaluate the expressions $6a^2 + 2a - 10$ and $2a(3a - 4) + 10(a - 1)$. Now choose a different number and evaluate the expressions again. Repeat this two more times with different numbers. What conclusions might you draw from your evaluations?

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1.5 Sets

Objective A

To write a set using the roster method









A **set** is a collection of objects, which are called the **elements** of the set. The **roster method** of writing a set encloses a list of the elements in braces.

The set of the last three letters of the alphabet is written $\{x, y, z\}$.

The set of the positive integers less than 5 is written $\{1, 2, 3, 4\}$.

➧ Use the roster method to write the set of integers between 0 and 10.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

A set can be designated by a capital letter.
Note that 0 and 10 are not elements of set A.

➧ Use the roster method to write the set of natural numbers.

$A = \{1, 2, 3, 4, \ldots\}$ • The three dots mean that the pattern of numbers continues without end.

The **empty set**, or **null set**, is the set that contains no elements. The symbol $\varnothing$ or $\{\ \}$ is used to represent the empty set.

The set of people who have run a two-minute mile is the empty set.

The **union** of two sets, written $A \cup B$, is the set that contains the elements of A and the elements of B.

➧ Find $A \cup B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cup B = \{1, 2, 3, 4, 5, 6\}$ • The union of A and B contains all the elements of A and all the elements of B. Any elements that are in both A and B are listed only once.

The **intersection** of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B.

➧ Find $A \cap B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cap B = \{3, 4\}$ • The intersection of A and B contains the elements common to A and B.

Example 1
Use the roster method to write the set of the odd positive integers less than 12.

Solution
$A = \{1, 3, 5, 7, 9, 11\}$

You Try It 1
Use the roster method to write the set of the odd negative integers greater than -10.

Your solution

Solution on p. S3

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Example 2
Use the roster method to write the set of the even positive integers.

Solution
$A = \{2, 4, 6, \ldots\}$

You Try It 2
Use the roster method to write the set of the odd positive integers.

Your solution

Example 3
Find $D \cup E$, given $D = \{6, 8, 10, 12\}$ and $E = \{-8, -6, 10, 12\}$.

Solution
$D \cup E = \{-8, -6, 6, 8, 10, 12\}$

You Try It 3
Find $A \cup B$, given $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 1, 2, 3, 4\}$.

Your solution

Example 4
Find $A \cap B$, given $A = \{5, 6, 9, 11\}$ and $B = \{5, 9, 13, 15\}$.

Solution
$A \cap B = \{5, 9\}$

You Try It 4
Find $C \cap D$, given $C = \{10, 12, 14, 16\}$ and $D = \{10, 16, 20, 26\}$.

Your solution

Example 5
Find $A \cap B$ given $A = \{1, 2, 3, 4\}$ and $B = \{8, 9, 10, 11\}$.

Solution
$A \cap B = \varnothing$

You Try It 5
Find $A \cap B$, given $A = \{-5, -4, -3, -2\}$ and $B = \{2, 3, 4, 5\}$.

Your solution

Solutions on p. S3

Objective 5 To write a set using set-builder notation



Point of Interest
The symbol $\in$ was first used in the book *Arithmeticae Principia*, published in 1889. It was the first letter of the Greek word εστι, which means "is." The symbols for union and intersection were also introduced at that time.

Another method of representing sets is called **set-builder notation.** Using set-builder notation, the set of all positive integers less than 10 is as follows:

$\{x | x < 10, x \in \text{positive integers}\}$, which is read "the set of all x such that x is less than 10 and x is an element of the positive integers."

Use set-builder notation to write the set of real numbers greater than 4.

$\{x | x > 4, x \in \text{real numbers}\}$

- "$x \in$ real numbers" is read "x is an element of the real numbers."

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Example 6

Use set-builder notation to write the set of negative integers greater than -100.

Solution

$\{x|x > -100, x \in \text{negative integers}\}$

You Try It 6

Use set-builder notation to write the set of positive even integers less than 59.

Your solution

Example 7

Use set-builder notation to write the set of real numbers less than 60.

Solution

$\{x|x < 60, x \in \text{real numbers}\}$

You Try It 7

Use set-builder notation to write the set of real numbers greater than -3.

Your solution

Solutions on p. S3

Objective C

To graph an inequality on the number line

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Point of Interest

The symbols for "is less than" and "is greater than" were introduced by Thomas Harriot around 1630. Before that, ⌐ and ¬ were used for $>$ and $<$, respectively.

An expression that contains the symbol $>$, $<$, $\geq$, or $\leq$ is called an **inequality.** An inequality expresses the relative order of two mathematical expressions. The expressions can be either numerical or variable.

$4 > 2$
$3x \leq 7$
$x^2 - 2x > y + 4$
} Inequalities

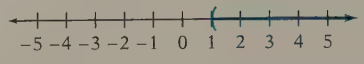
An inequality can be graphed on the number line.

TAKE NOTE

In many cases, we assume that the real numbers are being used and omit "$x \in$ real numbers" from set builder notation. Using this convention, $\{x|x > 1, x \in \text{real numbers}\}$ is written $\{x|x > 1\}$.

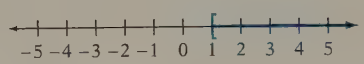
➡ Graph: $\{x|x > 1\}$

The graph is the real numbers greater than 1. The parenthesis at 1 indicates that 1 is not included in the graph.



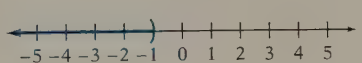
➡ Graph: $\{x|x \geq 1\}$

The bracket at 1 indicates that 1 is included in the graph.



➡ Graph: $\{x|x < -1\}$

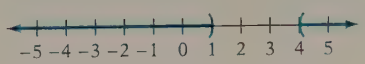
The numbers less than -1 are to the left of -1 on the number line.



The union of two sets is the set that contains all the elements of each set.

➡ Graph: $\{x|x > 4\} \cup \{x|x < 1\}$

The graph is the numbers greater than 4 and the numbers less than 1.

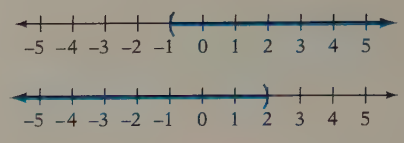


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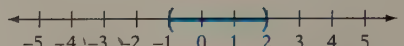
The intersection of two sets is the set that contains the elements common to both sets.

Graph: $\{x|x > -1\} \cap \{x|x < 2\}$

The graphs of $\{x|x > -1\}$ and $\{x|x < 2\}$ are shown at the right.



The graph of $\{x|x > -1\} \cap \{x|x < 2\}$ is the numbers between -1 and 2.

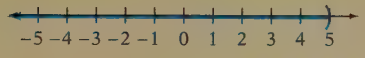


Example 8

Graph: $\{x|x < 5\}$

Solution

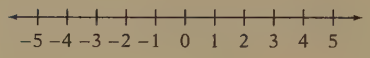
The graph is the numbers less than 5.



You Try It 8

Graph: $\{x|x > -2\}$

Your solution

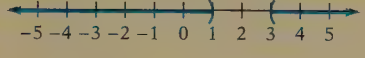


Example 9

Graph: $\{x|x > 3\} \cup \{x|x < 1\}$

Solution

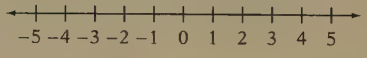
The graph is the numbers greater than 3 and the numbers less than 1.



You Try It 9

Graph: $\{x|x > -1\} \cup \{x|x < -3\}$

Your solution

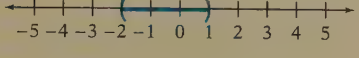


Example 10

Graph: $\{x|x > -2\} \cap \{x|x < 1\}$

Solution

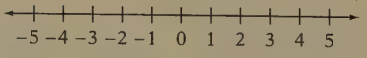
The graph is the numbers between -2 and 1.



You Try It 10

Graph: $\{x|x \leq 4\} \cap \{x|x \geq -4\}$

Your solution

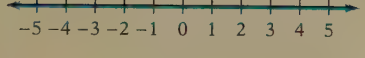


Example 11

Graph: $\{x|x \leq 5\} \cup \{x|x \geq -3\}$

Solution

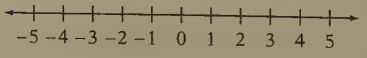
The graph is the real numbers.



You Try It 11

Graph: $\{x|x < 2\} \cup \{x|x \geq -2\}$

Your solution



Solutions on p. S3

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1.5 Exercises

Objective A

Use the roster method to write the set.

1. the integers between 15 and 22
2. the integers between −10 and −4
3. the odd integers between 8 and 18
4. the even integers between −11 and −1
5. the letters of the alphabet between a and d
6. the letters of the alphabet between p and v
7. Explain how to find the union of two sets.
8. Explain how to find the intersection of two sets.

Find $A \cup B$.

9. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$
10. $A = \{-3, -2, -1\}$ $B = \{-2, -1, 0\}$
11. $A = \{-10, -9, -8\}$ $B = \{8, 9, 10\}$
12. $A = \{a, b, c\}$ $B = \{x, y, z\}$
13. $A = \{a, b, d, e\}$ $B = \{c, d, e, f\}$
14. $A = \{m, n, p, q\}$ $B = \{m, n, o\}$
15. $A = \{1, 3, 7, 9\}$ $B = \{7, 9, 11, 13\}$
16. $A = \{-3, -2, -1\}$ $B = \{-1, 1, 2\}$

Find $A \cap B$.

17. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$
18. $A = \{-4, -3, -2\}$ $B = \{-6, -5, -4\}$
19. $A = \{-4, -3, -2\}$ $B = \{2, 3, 4\}$
20. $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4\}$
21. $A = \{a, b, c, d, e\}$ $B = \{c, d, e, f, g\}$
22. $A = \{m, n, o, p\}$ $B = \{k, l, m, n\}$

Objective B

Use set-builder notation to write the set.

23. the negative integers greater than −5
24. the positive integers less than 5

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25. the integers greater than 30

26. the integers less than -70

27. the even integers greater than 5

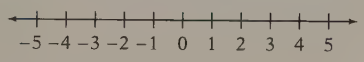
28. the odd integers less than -2

29. the real numbers greater than 8

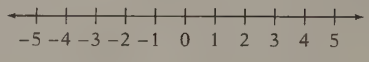
30. the real numbers less than 57

Graph.

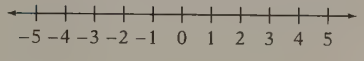
31. $\{x|x > 2\}$



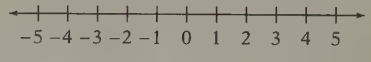
32. $\{x|x \geq -1\}$



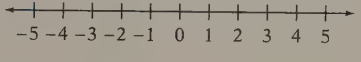
33. $\{x|x \leq 0\}$



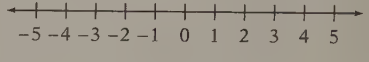
34. $\{x|x < 4\}$



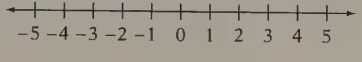
35. $\{x|x > -2\} \cup \{x|x < -4\}$



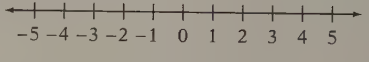
36. $\{x|x > 4\} \cup \{x|x < -2\}$



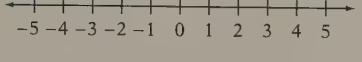
37. $\{x|x > -2\} \cap \{x < 4\}$



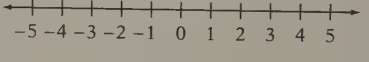
38. $\{x|x > -3\} \cap \{x|x < 3\}$



39. $\{x|x \geq -2\} \cup \{x|x < 4\}$



40. $\{x|x > 0\} \cup \{x|x \leq 4\}$



APPLYING THE CONCEPTS

41. Determine whether the statement is always true, sometimes true, or never true.

a. Given that $a > 0$ and $b < 0$, then $ab > 0$.

b. Given that $a < 0$, then $a^2 > 0$.

c. Given that $a > 0$ and $b < 0$, then $a^2 > b$.

42. By trying various sets, make a conjecture as to whether the union of two sets is

a. a commutative operation

b. an associative operation

43. By trying various sets, make a conjecture as to whether the intersection of two sets is

a. a commutative operation

b. an associative operation

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Focus on Problem Solving

Inductive Reasoning

Suppose you take 9 credit hours each semester. The total number of credit hours you have taken at the end of each semester can be described in a list of numbers.

$$9, 18, 27, 36, 45, 54, 63, \ldots$$

The list of numbers that indicates the total credit hours is an ordered list of numbers, called a **sequence.** Each number in a sequence is called a **term** of the sequence. The list is ordered because the position of a number in the list indicates the semester in which that number of credit hours has been taken. For example, the 7th term of the sequence is 63, and a total of 63 credit hours have been taken after the 7th semester.

Assuming the pattern is continued, find the next three numbers in the pattern

$$-6, -10, -14, -18, \ldots$$

This list of numbers is a sequence. The first step in solving this problem is to observe the pattern in the list of numbers. In this case, each number in the list is 4 less than the previous number. The next three numbers are $-22, -26, -30$.

This process of discovering the pattern in a list of numbers is inductive reasoning. **Inductive reasoning** involves making generalizations from specific examples; in other words, we reach a conclusion by making observations about particular facts or cases.

Try the following exercises. Each exercise requires inductive reasoning.

Name the next two terms in the sequence.

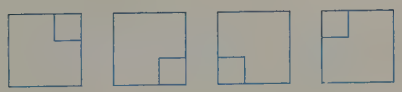
1. 1, 3, 5, 7, 1, 3, 5, 7, 1, . . .

2. 1, 4, 2, 5, 3, 6, 4, . . .

3. 1, 2, 4, 7, 11, 16, . . .

4. A, B, C, G, H, I, M, . . .

Draw the next shape in the sequence.

5. 

6. 

Solve.

7. Convert $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$, and $\frac{5}{11}$ to decimals. Then use the pattern you observe to convert $\frac{6}{11}, \frac{7}{11}$, and $\frac{9}{11}$ to decimals.

8. Convert $\frac{1}{33}, \frac{2}{33}, \frac{4}{33}, \frac{5}{33}$, and $\frac{7}{33}$ to decimals. Then use the pattern you observe to convert $\frac{8}{33}, \frac{13}{33}$, and $\frac{19}{33}$ to decimals.

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Projects and Group Activities

Calculators



Does your calculator use the Order of Operations Agreement? To find out, try this problem:

$$2 + 4 \cdot 7$$

If your answer is 30, then the calculator uses the Order of Operations Agreement. If your answer is 42, it does not use that agreement.

Even if your calculator does not use the Order of Operations Agreement, you can still correctly evaluate numerical expressions. The parentheses keys, [(] and [)], are used for this purpose.

Remember that $2 + 4 \cdot 7$ means $2 + (4 \cdot 7)$ because the multiplication must be completed before the addition. To evaluate this expression, enter the following:

Enter:	2	[+]	[(]	4	[×]	7	[)]	[=]
Display:	2	2	[(]	4	4	7	28	30

When using your calculator to evaluate numerical expressions, insert parentheses around multiplications or divisions. This has the effect of forcing the calculator to do the operations in the order you want rather than in the order the calculator wants.

Evaluate.

1. $3 \cdot (15 - 2 \cdot 3) - 36 \div 3$

2. $4 \cdot 2^2 - (12 + 24 \div 6) - 5$

3. $16 \div 4 \cdot 3 + (3 \cdot 4 - 5) + 2$

4. $15 \cdot 3 \div 9 + (2 \cdot 6 - 3) + 4$

Using your calculator to simplify numerical expressions sometimes requires use of the [+/−] key or, on some calculators, the negative key, which is frequently shown as [(−)]. These keys change the sign of the number currently in the display. To enter −4:

- For those calculators with [+/−], press 4 and then [+/−].
- For those calculators with [(−)], press [(−)] and then 4.

Here are the keystrokes for evaluating the expression $3(-4) - (-5)$.

Calculators with [+/−] key: 3 [×] 4 [+/−] − 5 [+/−] [=]

Calculators with [(−)] key: 3 [×] [(−)] 4 [−] [(−)] 5 [=]

This example illustrates that calculators make a distinction between negative and minus. To perform the operation $3 - (-3)$, you cannot enter 3 [−] [−] 3. This would result in 0, which is not the correct answer. You must enter

3 [−] 3 [+/−] [=] or 3 [−] [(−)] 3 [=]

Use a calculator to evaluate each of the following exercises.

5. $-16 \div 2$

6. $3(-8)$

7. $47 - (-9)$

8. $-50 - (-14)$

9. $4 - (-3)^2$

10. $-8 + (-6)^2 - 7$

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Addition and Multiplication Properties

The chart below is an addition table. Use it to answer Exercises 1 to 7.

+	Δ	‡	◇
Δ	‡	◇	Δ
‡	◇	Δ	‡
◇	Δ	‡	◇

1. Find the sum of Δ and ‡.

2. What is ◇ plus ◇?

3. In our number system, 0 can be added to any number without changing that number; 0 is called the **additive identity.** What is the additive identity for the system in the chart above? Explain your answer.

4. Does the Commutative Property of Addition apply to this system? Explain your answer.

5. What is $-\Delta$ (the opposite of Δ) equal to? Explain your answer.

6. What is $-‡$ (the opposite of ‡) equal to? Explain your answer.

7. Simplify $-\Delta + ‡ - ◇$. Explain how you arrived at your answer.

The chart below is a multiplication table. Use it to answer Exercises 8 to 14.

×	£	¿	&
£	£	¿	&
¿	¿	¿	¿
&	&	¿	*

8. Find the product of £ and &.

9. What is ¿ times £?

10. Find the square of &.

11. Does the Commutative Property of Multiplication apply to this system? Explain your answer.

12. In our number system, the product of a number and 0 is 0. Is there an element in this system that corresponds to 0 in our system? Explain your answer.

13. In our number system, 1 can be multiplied by any number without changing that number; 1 is called the **multiplicative identity.** What is the multiplicative identity for the system in the chart above? Explain your answer.

14. Simplify & ÷ £ × ¿. Explain how you arrived at your answer.

Balance of Payments

An **export** is a good or service produced in one's own country and sold for consumption in another country. An **import** is a good or service that is consumed in one's own country but was bought from another country. A nation's **balance of trade** is the difference between the value of its exports and the value of its imports during a particular period of time.

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A **favorable balance of trade** exists when the value of the exports is greater than the value of the imports. In this case, the balance of trade is a positive number. An **unfavorable balance of trade** exists when the value of the imports is greater than the value of the exports. In this case, the balance of trade is a negative number. An unfavorable balance of trade is referred to as a **trade deficit.** A trade deficit is considered unfavorable because more money is going out of the country to pay for goods imported than is coming into the country to pay for goods exported.

The U.S. government provides data on international trade. On the Internet, go to **www.fedstats.gov** and do a search for "balance of payments." Find tables that provide data on the value of U.S. imports and exports. All figures in the tables are in millions of dollars. The first two columns are annual figures. Subsequent columns are for quarters (3-month periods).

We located the following data for the first quarter of the year 2000.

Exports: Total, all countries	183,659
Imports: Total, all countries	289,699

We then calculated the balance of trade as follows:

$$\begin{aligned}\text{Balance of trade} &= \text{value of exports} - \text{value of imports}\\ &= 183{,}659 - 289{,}699\\ &= 183{,}659 + (-289{,}699)\\ &= -106{,}040\end{aligned}$$

The balance of trade for the first quarter of 2000 was −$106,040 million. This figure is provided in the table under "Balance: Total, all countries."

a. Show the calculation of the balance of trade for each of the four quarters of last year. Use the above calculation as a model.

b. Show the calculation of the annual balance of trade for last year.

c. Show that the sum of the four quarterly figures is equal to the annual figure.

Chapter Summary

Key Words

A *set* is a collection of objects. The objects in the set are called the *elements* of the set. The *roster method* of writing sets encloses a list of the elements in braces. [p. 55]

TAKE NOTE
"To the Student" on page xxi provides suggestions on how best to make use of the "Chapter Summary," the "Chapter Review," and the "Chapter Test."

The set of *natural numbers* is {1, 2, 3, 4, 5, 6, 7, . . .}. The set of *integers* is {. . . , −4, −3, −2, −1, 0, 1, 2, 3, 4, . . .}. [p. 3]

The *empty set* or *null set,* written ∅ or { }, is the set that contains no elements. [p. 55]

A number a *is less than* another number b, written $a < b$, if a is to the left of b on the number line. A number a *is greater than* another number b, written $a > b$, if a is to the right of b on the number line. The symbol ≤ means *is less than or equal to.* The symbol ≥ means *is greater than or equal to.* An expression that contains the symbol >, <, ≥, or ≤ is an *inequality.* [pp. 3, 57]

Two numbers that are the same distance from zero on the number line but on opposite sides of zero are *opposite numbers,* or *opposites.* The *additive inverse* of a number is the opposite of the number. [p. 4]

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The *multiplicative inverse* of a number is the reciprocal of the number. [p. 41]

The *absolute value* of a number is its distance from zero on the number line. [p. 4]

A *rational number* is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. An *irrational number* is a number that has a decimal representation that never terminates or repeats. The rational numbers and the irrational numbers taken together are called the *real numbers*. [pp. 17, 24]

Percent means "parts of 100." [p. 18]

An expression of the form a^n is in *exponential form*, where a is the base and n is the exponent. [p. 22]

A *square root* of a positive number x is a number whose square is x. The *principal square root* of a number is the positive square root. The symbol $\sqrt{\ }$ is called a *radical sign* and is used to indicate the principal square root of a number. The *radicand* is the number under the radical sign. [p. 24]

The square of an integer is a *perfect square*. If a number is not a perfect square, its square root can only be approximated. [p. 24]

A *variable* is a letter that is used to stand for a quantity that is unknown or that varies. A *variable expression* is an expression that contains one or more variables. [p. 37]

The *terms* of a variable expression are the addends of the expression. A *variable term* is composed of a numerical coefficient and a variable part. *Like terms* of a variable expression are terms with the same variable part. [pp. 37, 38]

The *union* of two sets, written $A \cup B$, is the set that contains all the elements of A and all the elements of B. (Any elements that are in both set A and set B are listed only once.) The *intersection* of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B. [p. 55]

Essential Rules

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends. [p. 5]

To add two numbers with different signs, find the absolute value of each number. Subtract the smaller of these from the larger. Attach the sign of the addend with the larger absolute value. [p. 5]

To subtract two numbers, add the opposite of the second number to the first number. [p. 6]

To multiply two numbers with the same sign, multiply the absolute values of the factors. The product is positive. [p. 8]

To multiply two numbers with different signs, multiply the absolute values of the factors. The product is negative. [p. 8]

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive. [p. 9]

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To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative. [p. 9]

To convert a percent to a decimal, remove the percent sign and multiply by 0.01. [p. 18]

To convert a percent to a fraction, remove the percent sign and multiply by $\frac{1}{100}$. [p. 18]

To convert a decimal or a fraction to a percent, multiply by 100%. [p. 18]

Product Property of Square Roots [p. 25] $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Addition Property of Zero [p. 39] $a + 0 = a$ or $0 + a = a$

Commutative Property of Addition [p. 39] $a + b = b + a$

Associative Property of Addition [p. 39] $(a + b) + c = a + (b + c)$

Inverse Property of Addition [p. 40] $a + (-a) = 0$ or $-a + a = 0$

Multiplication Property of One [p. 41] $a \cdot 1 = a$ or $1 \cdot a = a$

Commutative Property of Multiplication [p. 41] $a \cdot b = b \cdot a$

Associative Property of Multiplication [p. 40] $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Inverse Property of Multiplication [p. 41] $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1, a \neq 0$

Division Properties of Zero and One [p. 9]

If $a \neq 0$, $0 \div a = 0$.
If $a \neq 0$, $a \div a = 1$.
$a \div 1 = a$
$a \div 0$ is undefined.

Distributive Property [p. 38] $a(b + c) = ab + ac$

The Order of Operations Agreement [p. 33]

Step 1 Perform operations inside grouping symbols. The grouping symbols include parentheses, brackets, braces, absolute value symbols, and the fraction bar.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

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Chapter Review

1. Let $x \in \{-4, 0, 11\}$. For what values of x is the inequality $x < 1$ a true statement?

2. Find the additive inverse of -4.

3. Evaluate $-|-5|$.

4. Add: $-3 + (-12) + 6 + (-4)$

5. Subtract: $16 - (-3) - 18$

6. Multiply: $(-6)(7)$

7. Divide: $-100 \div 5$

8. Write $\frac{7}{25}$ as a decimal.

9. Write 6.2% as a decimal.

10. Write $\frac{5}{8}$ as a percent.

11. Simplify: $\frac{1}{3} - \frac{1}{6} + \frac{5}{12}$

12. Subtract: $5.17 - 6.238$

13. Divide: $-\frac{18}{35} \div \frac{17}{28}$

14. Multiply: $4.32(-1.07)$

15. Evaluate $\left(-\frac{2}{3}\right)^4$.

16. Simplify: $2\sqrt{36}$

17. Simplify: $-3\sqrt{120}$

18. Evaluate $-3^2 + 4[18 + (12 - 20)]$.

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19. Evaluate $(b - a)^2 + c$ when $a = -2$, $b = 3$, and $c = 4$.

20. Simplify: $6a - 4b + 2a$

21. Simplify: $-3(-12y)$

22. Simplify: $5(2x - 7)$

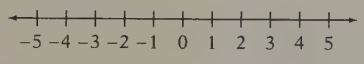
23. Simplify: $-4(2x - 9) + 5(3x + 2)$

24. Simplify: $5[2 - 3(6x - 1)]$

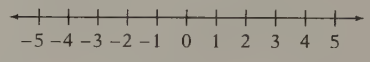
25. Use the roster method to write the set of odd positive integers less than 8.

26. Find $A \cap B$, given $A = \{1, 5, 9, 13\}$ and $B = \{1, 3, 5, 7, 9\}$.

27. Graph: $\{x|x > 3\}$

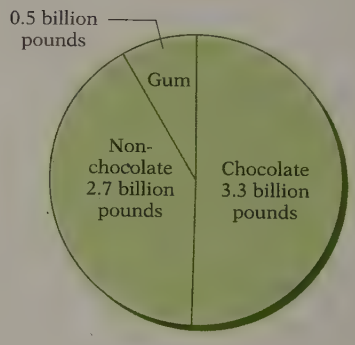


28. Graph $\{x|x \leq 3\} \cup \{x|x < -2\}$.



29. To discourage random guessing on a multiple-choice exam, a professor assigns 6 points for a correct answer, −4 points for an incorrect answer, and −2 points for leaving a question blank. What is the score for a student who had 21 correct answers, had 5 incorrect answers, and left 4 questions blank?

30. The circle graph shows the amount of candy consumed by Americans during a recent year (Source: Candy USA). What percent of the candy consumed was chocolate? Round to the nearest tenth of a percent.



31. Translate "the difference between twice a number and one-half of the number" into a variable expression. Then simplify.

32. A baseball card collection contains five times as many National League players' cards as American League players' cards. Express the number of National League players' cards in terms of the number of American League players' cards.

33. A club treasurer has some five-dollar bills and some ten-dollar bills. The treasurer has a total of 35 bills. Express the number of five-dollar bills in terms of the number of ten-dollar bills.

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Chapter Test

1. Place the correct symbol, $<$ or $>$, between the two numbers.

$-2 \quad -40$

2. Find the opposite of -4.

3. Evaluate $-|-4|$.

4. Subtract: $16 - 30$

5. Add: $-22 + 14 + (-8)$

6. Subtract: $16 - (-30) - 42$

7. Divide: $-561 \div (-33)$

8. Write $\frac{7}{9}$ as a decimal. Place a bar over the repeating digit of the decimal.

9. Write 45% as a fraction and as a decimal.

10. Add: $-\frac{2}{5} + \frac{7}{15}$

11. Multiply: $6.02(-0.89)$

12. Divide: $\frac{5}{12} \div \left(-\frac{5}{6}\right)$

13. Evaluate $\frac{3}{4} \cdot (4)^2$

14. Simplify: $-2\sqrt{45}$

15. Evaluate $16 \div 2[8 - 3(4 - 2)] + 1$.

16. Evaluate $b^2 - 3ab$ when $a = 3$ and $b = -2$.

17. Simplify: $3x - 5x + 7x$

18. Simplify: $\frac{1}{5}(10x)$

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19. Simplify: $-3(2x^2 - 7y^2)$

20. Simplify: $2x - 3(x - 2)$

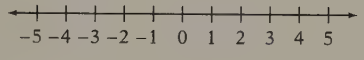
21. Simplify: $2x + 3[4 - (3x - 7)]$

22. Use the roster method to write the set of integers between -3 and 4.

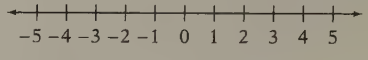
23. Use set-builder notation to write the set of real numbers less than -3.

24. Find $A \cup B$ given $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$.

25. Graph $\{x \mid x < 1\}$.



26. Graph $\{x \mid x \leq -3\} \cup \{x \mid x > 0\}$.



27. Translate "ten times the difference between a number and 3" into a variable expression. Then simplify.

28. The speed of a pitcher's fastball is twice the speed of the catcher's return throw. Express the speed of the fastball in terms of the speed of the return throw.

29. The table at the right shows the U.S. balance of trade, in billions of dollars, for the years 1980 to 2000 (Source: U.S. Dept. of Commerce). See the Projects and Group Activities for a discussion of balance of trade.

a. In which years did the trade balance increase from the previous year?

b. Calculate the difference between the trade balance in 1990 and that in 2000.

c. During which two consecutive years was the difference in the trade balance greatest?

d. How many times greater was the trade balance in 1990 than in 1980? Round to the nearest whole number.

e. Calculate the average trade balance per quarter for the year 2000.

Year	Trade Balance
1980	−19.4
1981	−16.2
1982	−24.2
1983	−57.8
1984	−109.2
1985	−122.1
1986	−140.6
1987	−153.3
1988	−115.9
1989	−92.2
1990	−81.1
1991	−30.7
1992	−35.7
1993	−68.9
1994	−97.0
1995	−95.9
1996	−102.1
1997	−104.7
1998	−166.9
1999	−265.0
2000	−369.7

30. The boiling point of mercury is 356.58°C. The melting point of mercury is -38.87°C. Find the difference between the boiling point and the melting point of mercury.

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Chapter 2

First-Degree Equations and Inequalities



To ensure that heavy equipment, such as a truck transmission, functions properly, each component must be built precisely to the manufacturer's specifications. For example, the thickness and length of the teeth in a gear is specified. A vernier dial caliper, as shown above, is used to measure the components to the nearest thousandth of an inch. A slight variance from the given measurement is allowed. This margin of error is known as tolerance. Given the tolerance of a component, you can calculate the upper and lower limits, or the maximum and minimum allowable measurements. This is illustrated in **Exercises 91–96 on pages 131 and 132.**

Objectives

Section 2.1

A To determine whether a given number is a solution of an equation
B To solve an equation of the form $x + a = b$
C To solve an equation of the form $ax = b$
D To solve application problems using the basic percent equation

Section 2.2

A To solve an equation of the form $ax + b = c$
B To solve an equation of the form $ax + b = cx + d$
C To solve an equation containing parentheses
D To translate a sentence into an equation and solve

Section 2.3

A To solve value mixture problems
B To solve percent mixture problems
C To solve investment problems
D To solve uniform motion problems

Section 2.4

A To solve an inequality in one variable
B To solve a compound inequality
C To solve application problems

Section 2.5

A To solve an absolute value equation
B To solve an absolute value inequality
C To solve application problems

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

For Exercises 1 to 5, add, subtract, multiply, or divide.

1. $8 - 12$

2. $-9 + 3$

3. $\dfrac{-18}{-6}$

4. $-\dfrac{3}{4}\left(-\dfrac{4}{3}\right)$

5. $-\dfrac{5}{8}\left(\dfrac{4}{5}\right)$

For Exercises 6 to 9, simplify.

6. $3x - 5 + 7x$

7. $6(x - 2) + 3$

8. $n + (n + 2) + (n + 4)$

9. $0.08x + 0.05(400 - x)$

10. A 20-ounce package of a snack mixture contains nuts and pretzels. Let n represent the number of ounces of nuts in the mixture. Express the number of ounces of pretzels in the mixture in terms of n.

Go Figure

Two fractions are inserted between $\frac{1}{4}$ and $\frac{1}{2}$ so that the difference between any two successive fractions is the same. Find the sum of the four fractions.

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2.1 Introduction to Equations

Objective A

To determine whether a given number is a solution of an equation

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Point of Interest

One of the most famous equations ever stated is $E = mc^2$. This equation, stated by Albert Einstein, shows that there is a relationship between mass m and energy E. As a side note, the chemical element einsteinium was named in honor of Einstein.

An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

$9 + 3 = 12$
$3x - 2 = 10$
$y^2 + 4 = 2y - 1$
$z = 2$
} Equations

The equation at the right is true if the variable is replaced by 5.

$x + 8 = 13$
$5 + 8 = 13$ A true equation

The equation is false if the variable is replaced by 7.

$7 + 8 = 13$ A false equation

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation. 5 is a solution of the equation $x + 8 = 13$. 7 is not a solution of the equation $x + 8 = 13$.

➪ Is -2 a solution of $2x + 5 = x^2 - 3$?

$2x + 5 = x^2 - 3$	
$2(-2) + 5$	$(-2)^2 - 3$
$-4 + 5$	$4 - 3$
$1 = 1$	

Yes, -2 is a solution of the equation.

- Replace x by -2.
- Evaluate the numerical expressions.
- If the results are equal, -2 is a solution of the equation. If the results are not equal, -2 is not a solution of the equation.

TAKE NOTE

The Order of Operations Agreement applies to evaluating $2(-2) + 5$ and $(-2)^2 - 3$.

Example 1 Is -4 a solution of $5x - 2 = 6x + 2$?

Solution

$5x - 2 = 6x + 2$	
$5(-4) - 2$	$6(-4) + 2$
$-20 - 2$	$-24 + 2$
$-22 = -22$	

Yes, -4 is a solution.

You Try It 1

Is $\frac{1}{4}$ a solution of $5 - 4x = 8x + 2$?

Your solution

Example 2 Is -4 a solution of $4 + 5x = x^2 - 2x$?

Solution

$4 + 5x = x^2 - 2x$	
$4 + 5(-4)$	$(-4)^2 - 2(-4)$
$4 + (-20)$	$16 - (-8)$
$-16 \neq 24$	

($\neq$ means "is not equal to")

No, -4 is not a solution.

You Try It 2 Is 5 a solution of $10x - x^2 = 3x - 10$?

Your solution

Solutions on p. S3

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Objective B — To solve an equation of the form $x + a = b$

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Point of Interest

Finding solutions of equations has been a principal aim of mathematics for thousands of years. However, the equals sign did not occur in any text until 1557.

To **solve an equation** means to find a solution of the equation. The simplest equation to solve is an equation of the form *variable* = *constant*, because the constant is the solution.

The solution of the equation $x = 5$ is 5 because $5 = 5$ is a true equation.

The solution of the equation at the right is 7 because $7 + 2 = 9$ is a true equation.

$$x + 2 = 9 \qquad 7 + 2 = 9$$

Note that if 4 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + 4 &= 9 + 4 \\ x + 6 &= 13 \end{aligned} \qquad 7 + 6 = 13$$

If -5 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + (-5) &= 9 + (-5) \\ x - 3 &= 4 \end{aligned} \qquad 7 - 3 = 4$$

Equations that have the same solution are **equivalent equations**. The equations $x + 2 = 9$, $x + 6 = 13$, and $x - 3 = 4$ are equivalent equations; each equation has 7 as its solution. These examples suggest that adding the same number to each side of an equation produces an equivalent equation. This is called the *Addition Property of Equations*.

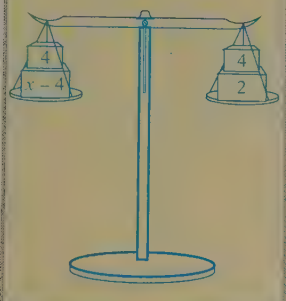
Addition Property of Equations

The same number can be added to each side of an equation without changing its solution. In symbols, the equation $a = b$ has the same solution as the equation $a + c = b + c$.

In solving an equation, the goal is to rewrite the given equation in the form *variable* = *constant*. The Addition Property of Equations is used to remove a *term* from one side of the equation by adding the opposite of that term to each side of the equation.

TAKE NOTE

Think of an equation as a balance scale. If the weights added to each side of an equation are not the same, the "pans" no longer balance.



Solve: $x - 4 = 2$

$$x - 4 = 2$$ • The goal is to rewrite the equation as *variable* = *constant*.

$$x - 4 + 4 = 2 + 4$$ • Add 4 to each side of the equation.

$$x + 0 = 6$$ • Simplify.

$$x = 6$$ • The equation is in the form *variable* = *constant*.

Check: $x - 4 = 2$

$6 - 4 \mid 2$

$2 = 2$ A true equation

The solution is 6.

Because subtraction is defined in terms of addition, the Addition Property of Equations also makes it possible to subtract the same number from each side of an equation without changing the solution of the equation.

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➡ Solve: $y + \frac{3}{4} = \frac{1}{2}$

$$y + \frac{3}{4} = \frac{1}{2}$$

- The goal is to rewrite the equation in the form *variable = constant.*

$$y + \frac{3}{4} - \frac{3}{4} = \frac{1}{2} - \frac{3}{4}$$

- Subtract $\frac{3}{4}$ from each side of the equation.

$$y + 0 = \frac{2}{4} - \frac{3}{4}$$

- Simplify.

$$y = -\frac{1}{4}$$

- The equation is in the form *variable = constant.*

The solution is $-\frac{1}{4}$. You should check this solution.

Example 3

Solve: $x + \frac{3}{4} = \frac{1}{3}$

Solution

$$x + \frac{3}{4} = \frac{1}{3}$$

$$x + \frac{3}{4} - \frac{3}{4} = \frac{1}{3} - \frac{3}{4}$$

$$x + 0 = \frac{4}{12} - \frac{9}{12}$$

$$x = -\frac{5}{12}$$

The solution is $-\frac{5}{12}$.

You Try It 3

Solve: $\frac{5}{6} = y - \frac{3}{8}$

Your solution

Solution on p. S4

Objective C

To solve an equation of the form $ax = b$

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The solution of the equation at the right is 3 because $2 \cdot 3 = 6$ is a true equation.

$$2x = 6 \qquad 2 \cdot 3 = 6$$

Note that if each side of $2x = 6$ is multiplied by 5, the solution is still 3.

$$2x = 6$$
$$5(2x) = 5 \cdot 6$$
$$10x = 30 \qquad 10 \cdot 3 = 30$$

If each side of $2x = 6$ is multiplied by -4, the solution is still 3.

$$2x = 6$$
$$(-4)(2x) = (-4) \cdot 6$$
$$-8x = -24 \qquad -8 \cdot 3 = -24$$

The equations $2x = 6$, $10x = 30$, and $-8x = -24$ are equivalent equations; each equation has 3 as its solution. These examples suggest that multiplying each side of an equation by the same number produces an equivalent equation.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

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The Multiplication Property of Equations is used to remove a coefficient by multiplying each side of the equation by the reciprocal of the coefficient.

Solve: $\frac{3}{4}z = 9$

$$\frac{3}{4}z = 9$$ • The goal is to rewrite the equation in the form *variable = constant.*

$$\frac{4}{3} \cdot \frac{3}{4}z = \frac{4}{3} \cdot 9$$ • Multiply each side of the equation by $\frac{4}{3}$.

$$1 \cdot z = 12$$ • Simplify.

$$z = 12$$ • The equation is in the form *variable = constant.*

The solution is 12. You should check this solution.

Because division is defined in terms of multiplication, each side of an equation can be divided by the same nonzero number without changing the solution of the equation.

Solve: $6x = 14$

$$6x = 14$$ • The goal is to rewrite the equation in the form *variable = constant.*

$$\frac{6x}{6} = \frac{14}{6}$$ • Divide each side of the equation by 6.

$$x = \frac{7}{3}$$ • Simplify. The equation is in the form *variable = constant.*

The solution is $\frac{7}{3}$.

TAKE NOTE
Remember to check the solution.

Check:

$6x = 14$	
$6\left(\frac{7}{3}\right)$	14
$14 = 14$	

When using the Multiplication Property of Equations, multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction. Divide each side of the equation by the coefficient when the coefficient is an integer or decimal.

Example 4

Solve: $\frac{3x}{4} = -9$

Solution

$$\frac{3x}{4} = -9$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9) \quad \left[\frac{3x}{4} = \frac{3}{4}x\right]$$

$$x = -12$$

The solution is -12.

You Try It 4

Solve: $-\frac{2x}{5} = 6$

Your solution

Example 5 Solve: $5x - 9x = 12$

Solution

$$5x - 9x = 12$$

$$-4x = 12$$ • Combine like terms.

$$\frac{-4x}{-4} = \frac{12}{-4}$$

$$x = -3$$

The solution is -3.

You Try It 5 Solve: $4x - 8x = 16$

Your solution

Solutions on p. S4

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Objective D

To solve application problems using the basic percent equation

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An equation that is used frequently in mathematics applications is the basic percent equation.

Basic Percent Equation

$$\text{Percent} \cdot \text{Base} = \text{Amount}$$
$$P \cdot B = A$$

In many application problems involving percent, the base follows the word "of."

➡ 20% of what number is 30?

$$\begin{aligned} P \cdot B &= A \\ 0.20B &= 30 \\ \frac{0.20B}{0.20} &= \frac{30}{0.20} \\ B &= 150 \end{aligned}$$

- Use the basic percent equation.
- $P = 20\% = 0.20$, $A = 30$, and B is unknown.
- Solve for B.

The number is 150.

➡ 70 is what percent of 80?

$$\begin{aligned} P \cdot B &= A \\ P(80) &= 70 \\ \frac{P(80)}{80} &= \frac{70}{80} \\ P &= 0.875 \\ P &= 87.5\% \end{aligned}$$

- Use the basic percent equation.
- $B = 80$, $A = 70$, and P is unknown.
- Solve for P.
- The question asked for a percent. Convert the decimal to a percent.

70 is 87.5% of 80.

TAKE NOTE

We have written $P(80) = 70$ because that is the form of the basic percent equation. We could have written $80P = 70$. The important point is that each side of the equation is divided by 80, the coefficient of P.

Point of Interest

As noted at the right, 172,841 of the dogs or litters registered were Labrador retrievers. Listed below are the next most popular breeds and their registrations.
Golden retrievers: 66,300
German shepherds: 57,660
Dachshunds: 54,773
Beagles: 52,026

➡ During a recent year, nearly 1.2 million dogs or litters were registered with the American Kennel Club. The most popular breed was the Labrador retriever, with 172,841 registered (Source: American Kennel Club). What percent of the registrations were Labrador retrievers? Round to the nearest tenth of a percent.

Strategy To find the percent, use the basic percent equation.
$B = 1.2$ million $= 1{,}200{,}000$; $A = 172{,}841$; P is unknown.

Solution

$$\begin{aligned} P \cdot B &= A \\ P(1{,}200{,}000) &= 172{,}841 \\ \frac{P(1{,}200{,}000)}{1{,}200{,}000} &= \frac{172{,}841}{1{,}200{,}000} \\ P &\approx 0.144 = 14.4\% \end{aligned}$$

Approximately 14.4% of the registrations were Labrador retrievers.

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In most cases, you should write the percent as a decimal before solving the basic percent equation. However, some percents are more easily written as a fraction. For example,

$$33\frac{1}{3}\% = \frac{1}{3} \qquad 66\frac{2}{3}\% = \frac{2}{3} \qquad 16\frac{2}{3}\% = \frac{1}{6} \qquad 83\frac{1}{3}\% = \frac{5}{6}$$

Example 6

12 is $33\frac{1}{3}\%$ of what number?

Solution

$$P \cdot B = A$$

$$\frac{1}{3}B = 12 \qquad \bullet\ 33\tfrac{1}{3}\% = \tfrac{1}{3}$$

$$3 \cdot \frac{1}{3}B = 3 \cdot 12$$

$$B = 36$$

12 is $33\frac{1}{3}\%$ of 36.

You Try It 6

18 is $16\frac{2}{3}\%$ of what number?

Your solution

Example 7

In a recent year, 238 U.S. airports collected \$1,100 million in passenger taxes. Of this amount, \$88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$

$$P(1100) = 88$$

$$\frac{P(1100)}{1100} = \frac{88}{1100}$$

$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was \$158.3 million. (Source: *San Diego Union-Tribune*, January 18, 2001). Of that amount, the Big Ten conference received \$22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent.

Your strategy

Your solution

Solutions on p. S4

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2.1 Exercises

Objective A

1. What is the difference between an equation and an expression?

2. How can you determine whether a number is a solution of an equation?

3. Is 4 a solution of $2x = 8$?

4. Is 3 a solution of $y + 4 = 7$?

5. Is -1 a solution of $2b - 1 = 3$?

6. Is 5 a solution of $2x + 5 = 3x$?

7. Is 4 a solution of $3y - 4 = 2y$?

8. Is 0 a solution of $4a + 5 = 3a + 5$?

9. Is 3 a solution of $z^2 + 1 = 4 + 3z$?

10. Is 2 a solution of $2x^2 - 1 = 4x - 1$?

11. Is -1 a solution of $y^2 - 1 = 4y + 3$?

12. Is -2 a solution of $m^2 - 4 = m + 3$?

13. Is 5 a solution of $x^2 + 2x + 1 = (x + 1)^2$?

14. Is -6 a solution of $(n - 2)^2 = n^2 - 4n + 4$?

15. Is $\frac{1}{2}$ a solution of $4y + 1 = 3$?

16. Is $\frac{2}{5}$ a solution of $5m + 1 = 10m - 3$?

17. Is $\frac{3}{4}$ a solution of $8x - 1 = 12x + 3$?

Objective B

18. How is the Addition Property of Equations used to solve an equation?

19. Explain why the goal in solving an equation is to write the equation in the form *variable* = *constant*.

Solve and check.

20. $x + 5 = 7$

21. $y + 3 = 9$

22. $b - 4 = 11$

23. $z - 6 = 10$

24. $2 + a = 8$

25. $5 + x = 12$

26. $b + 7 = 7$

27. $y - 5 = -5$

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28. $a - 3 = -5$ **29.** $x - 6 = -3$ **30.** $z + 9 = 2$ **31.** $n + 11 = 1$

32. $10 + m = 3$ **33.** $8 + x = 5$ **34.** $9 + x = -3$ **35.** $10 + y = -4$

36. $b - 5 = -3$ **37.** $t - 6 = -4$ **38.** $4 + x = 10$ **39.** $9 + a = 20$

40. $2 = x + 7$ **41.** $-8 = n + 1$ **42.** $4 = m - 11$ **43.** $-6 = y - 5$

44. $12 = 3 + w$ **45.** $-9 = 5 + x$ **46.** $4 = -10 + b$ **47.** $-7 = -2 + x$

48. $x - \frac{2}{5} = \frac{3}{5}$ **49.** $\frac{5}{8} + y = \frac{1}{8}$ **50.** $\frac{4}{9} + a = -\frac{2}{9}$

51. $m + \frac{1}{2} = -\frac{1}{4}$ **52.** $b + \frac{1}{6} = -\frac{1}{3}$ **53.** $x + \frac{2}{3} = \frac{3}{4}$

54. $n + \frac{2}{5} = \frac{2}{3}$ **55.** $-\frac{5}{6} = x - \frac{1}{4}$ **56.** $-\frac{1}{4} = c - \frac{2}{3}$

57. $d + 1.3619 = 2.0148$ **58.** $w + 2.932 = 4.801$ **59.** $-0.813 + x = -1.096$

60. $-1.926 + t = -1.042$ **61.** $6.149 = -3.108 + z$ **62.** $5.237 = -2.014 + x$

Objective C

63. How is the Multiplication Property of Equations used to solve an equation?

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64. Why, when the Multiplication Property of Equations is used, must the number that multiplies each side of the equation be nonzero?

Solve and check.

65. $5x = -15$

66. $4y = -28$

67. $3b = 0$

68. $2a = 0$

69. $-3x = 6$

70. $-5m = 20$

71. $-3x = -27$

72. $-\frac{1}{6}n = -30$

73. $20 = \frac{1}{4}c$

74. $18 = 2t$

75. $-32 = 8w$

76. $-56 = 7x$

77. $0 = -5x$

78. $0 = -8a$

79. $-32 = -4y$

80. $-54 = 6c$

81. $49 = -7t$

82. $\frac{x}{3} = 2$

83. $\frac{x}{4} = 3$

84. $-\frac{y}{2} = 5$

85. $-\frac{b}{3} = 6$

86. $\frac{3}{4}y = 9$

87. $\frac{2}{5}x = 6$

88. $-\frac{2}{3}d = 8$

89. $-\frac{3}{5}m = 12$

90. $\frac{2n}{3} = 0$

91. $\frac{5x}{6} = 0$

92. $\frac{-3z}{8} = 9$

93. $\frac{-4x}{5} = -12$

94. $-6 = -\frac{2}{3}y$

95. $-15 = -\frac{1}{5}x$

96. $\frac{2}{5}a = 3$

97. $\frac{3x}{4} = 2$

98. $\frac{3}{4}c = \frac{3}{5}$

99. $\frac{2}{9} = \frac{2}{3}y$

100. $-\frac{6}{7} = -\frac{3}{4}b$

101. $\frac{1}{5}x = -\frac{1}{10}$

102. $-\frac{2}{3}y = -\frac{8}{9}$

103. $-1 = \frac{2n}{3}$

104. $-\frac{3}{4} = \frac{a}{8}$

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105. $-\frac{2}{5}m = -\frac{6}{7}$ **106.** $5x + 2x = 14$ **107.** $3n + 2n = 20$ **108.** $7d - 4d = 9$

109. $10y - 3y = 21$ **110.** $2x - 5x = 9$ **111.** $\frac{x}{1.46} = 3.25$ **112.** $\frac{z}{2.95} = -7.88$

113. $3.47a = 7.1482$ **114.** $2.31m = 2.4255$ **115.** $-3.7x = 7.881$ **116.** $\frac{n}{2.65} = 9.08$

Objective B

117. Employee A had an annual salary of \$22,000, Employee B had an annual salary of \$28,000, and Employee C had an annual salary of \$26,000 before each employee was given a 5% raise. Which of the three employees' annual salaries is now the highest? Explain how you arrived at your answer.

118. Each of three employees earned an annual salary of \$25,000 before Employee A was given a 3% raise, Employee B was given a 6% raise, and the Employee C was given a 4.5% raise. Which of the three employees' annual salaries is now the highest? Explain how you arrived at your answer.

119. What is 35% of 80? **120.** What percent of 8 is 0.5? **121.** Find 1.2% of 60.

122. 8 is what percent of 5? **123.** 125% of what is 80? **124.** What percent of 20 is 30?

125. 12 is what percent of 50? **126.** What percent of 125 is 50? **127.** Find 18% of 40.

128. What is 25% of 60? **129.** 12% of what is 48? **130.** 45% of what is 9?

131. 12 is what percent of 6? **132.** 20 is what percent of 16? **133.** $5\frac{1}{4}$% of what is 21?

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134. $37\frac{1}{2}\%$ of what is 15?

135. Find 15.4% of 50.

136. What is 18.5% of 46?

137. 1 is 0.5% of what?

138. 3 is 1.5% of what?

139. $\frac{3}{4}\%$ of what is 3?

140. $\frac{1}{2}\%$ of what is 3?

141. Find 125% of 16.

142. What is 250% of 12?

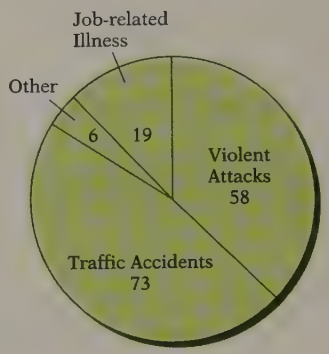
143. 16.43 is what percent of 20.45? Round to the nearest hundredth of a percent.

144. Find 18.37% of 625.43. Round to the nearest hundredth.

145. A university consists of three colleges: business, engineering, and fine arts. There are 2900 students in the business college, 1500 students in the engineering college, and 1000 students in the fine arts college. What percent of the total number of students in the university are in the fine arts college? Round to the nearest percent.

146. Approximately 21% of air is oxygen. Using this estimate, determine how many liters of oxygen there are in a room containing 21,600 L of air.

147. The graph at the right shows the causes of death for all police officers killed in the line of duty during a recent year (Source: International Union of Police Associations). What percent of the deaths were due to traffic accidents? Round to the nearest tenth of a percent.



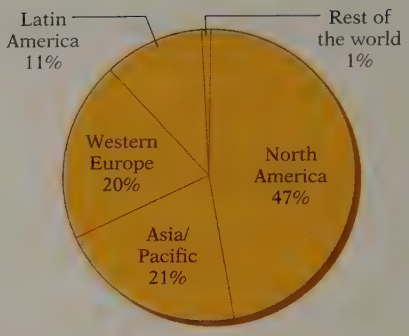
148. According to the Centers for Disease Control and Prevention, 30.8% of the adult population of Kentucky smokes. How many adults in Kentucky smoke? Use a figure of 3,000,000 for the number of adults in Kentucky.

149. During a recent year, 276 billion product coupons were issued by manufacturers. Shoppers redeemed 4.8 billion of these coupons (Source: NCH NuWorld Consumer Behavior Study, America Coupon Council). What percent of the coupons issued were redeemed by customers? Round to the nearest tenth of a percent.

150. At least $66\frac{2}{3}\%$ of the Senate must vote to override a presidential veto. There are 100 senators in the Senate. What is the minimum number of votes needed to override a veto?

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151. The graph at the right, based on data from Forrester Research, shows the percent share of the global e-commerce market for various regions of the world. The total projected revenues for e-commerce business in 2004 is $7500 billion. How many billion dollars more is Western Europe's share than Latin America's share?



152. Suppose that 125 million people saw a particular 30-second commercial for a soft drink during Super Bowl XXXV. The cost for that commercial was approximately $2.2 million.

a. If the soft drink manufacturer makes a profit of $.08 for each can sold, what percent of the people watching that commercial would have to buy one can of that soft drink for the company to recover the cost of the one commercial?

b. What percent of the people watching the commercial would have to purchase one six-pack of the soft drink for the company to recover the cost of the commercial? Round to the nearest tenth of a percent.

153. In a recent year, 53.5 million children were enrolled in kindergarten through 12th grade in the United States. Of those enrolled, 47.5 million were enrolled in public school (Source: Department of Education). What percent of the children enrolled in school were enrolled in private school? Round to the nearest tenth of a percent.

154. On average, a person uses 13.2 gal of water per day for showering. This is 17.8% of the total amount of water used per person per day in the average single-family home. Find the total amount of water used per person per day in the average single-family home. Round to the nearest whole number.

APPLYING THE CONCEPTS

155. Solve the equation $ax = b$ for x. Is the solution you have written valid for all real numbers a and b?

156. **a.** Make up an equation of the form $x + a = b$ that has 2 as a solution.

b. Make up an equation of the form $ax = b$ that has -1 as a solution.

157. Solve. **a.** $\dfrac{3}{\frac{1}{x}} = 5$ **b.** $\dfrac{2}{\frac{1}{y}} = -2$ **c.** $\dfrac{3x + 2x}{3} = 2$

158. Write out the steps for solving the equation $\frac{1}{2}x = -3$. Identify each Property of Real Numbers or Property of Equations as you use it.

159. If a quantity increases by 100%, how many times its original value is the new value?

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2.2 General Equations

Objective A

To solve an equation of the form $ax + b = c$

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Point of Interest

Evariste Galois, despite being killed in a duel at the age of 21, made significant contributions to solving equations. In fact, there is a branch of mathematics called Galois Theory showing what kinds of equations can and what kinds cannot be solved.

In solving an equation of the form $ax + b = c$, the goal is to rewrite the equation in the form *variable* = *constant*. This requires the application of both the Addition and the Multiplication Properties of Equations.

Solve: $\frac{3}{4}x - 2 = -11$

The goal is to write the equation in the form *variable* = *constant*.

$$\frac{3}{4}x - 2 = -11$$

$$\frac{3}{4}x - 2 + 2 = -11 + 2$$ • Add 2 to each side of the equation.

$$\frac{3}{4}x = -9$$ • Simplify.

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$ • Multiply each side of the equation by $\frac{4}{3}$.

$$x = -12$$ • The equation is of the form *variable* = *constant*.

Check: • Check the solution.

$$\frac{3}{4}x - 2 = -11$$

$\frac{3}{4}(-12) - 2$	-11
$-9 - 2$	-11
$-11 = -11$	

• A true equation

The solution is -12.

Example 1 Solve: $3x - 7 = -5$

Solution

$$3x - 7 = -5$$
$$3x - 7 + 7 = -5 + 7$$
$$3x = 2$$
$$\frac{3x}{3} = \frac{2}{3}$$
$$x = \frac{2}{3}$$

The solution is $\frac{2}{3}$.

You Try It 1 Solve: $5x + 7 = 10$

Your solution

Solution on p. S4

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Example 2

Solve: $5 = 9 - 2x$

Solution

$$5 = 9 - 2x$$
$$5 - 9 = 9 - 9 - 2x$$
$$-4 = -2x$$
$$\frac{-4}{-2} = \frac{-2x}{-2}$$
$$2 = x$$

The solution is 2.

You Try It 2

Solve: $2 = 11 + 3x$

Your solution

Example 3

Solve: $2x + 4 - 5x = 10$

Solution

$$2x + 4 - 5x = 10$$
$$-3x + 4 = 10 \quad \text{• Combine like terms.}$$
$$-3x + 4 - 4 = 10 - 4$$
$$-3x = 6$$
$$\frac{-3x}{-3} = \frac{6}{-3}$$
$$x = -2$$

The solution is -2.

You Try It 3

Solve: $x - 5 + 4x = 25$

Your solution

Solutions on p. S4

Objective B

To solve an equation of the form $ax + b = cx + d$



In solving an equation of the form $ax + b = cx + d$, the goal is to rewrite the equation in the form *variable* = *constant*. Begin by rewriting the equation so that there is only one variable term in the equation. Then rewrite the equation so that there is only one constant term.

Solve: $2x + 3 = 5x - 9$

$$2x + 3 = 5x - 9$$
$$2x - 5x + 3 = 5x - 5x - 9 \quad \text{• Subtract } 5x \text{ from each side of the equation.}$$
$$-3x + 3 = -9 \quad \text{• Simplify.}$$
$$-3x + 3 - 3 = -9 - 3 \quad \text{• Subtract 3 from each side of the equation.}$$
$$-3x = -12 \quad \text{• Simplify.}$$
$$\frac{-3x}{-3} = \frac{-12}{-3} \quad \text{• Divide each side of the equation by } -3.$$
$$x = 4 \quad \text{• The equation is in the form } \textit{variable} = \textit{constant}.$$

The solution is 4. You should verify this by checking this solution.

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Example 4

Solve: $4x - 5 = 8x - 7$

Solution

$$4x - 5 = 8x - 7$$
$$4x - 8x - 5 = 8x - 8x - 7$$
$$-4x - 5 = -7$$
$$-4x - 5 + 5 = -7 + 5$$
$$-4x = -2$$
$$\frac{-4x}{-4} = \frac{-2}{-4}$$
$$x = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 4

Solve: $5x + 4 = 6 + 10x$

Your solution

Example 5

Solve: $3x + 4 - 5x = 2 - 4x$

Solution

$$3x + 4 - 5x = 2 - 4x$$
$$-2x + 4 = 2 - 4x$$
$$-2x + 4x + 4 = 2 - 4x + 4x$$
$$2x + 4 = 2$$
$$2x + 4 - 4 = 2 - 4$$
$$2x = -2$$
$$\frac{2x}{2} = \frac{-2}{2}$$
$$x = -1$$

The solution is -1.

You Try It 5

Solve: $5x - 10 - 3x = 6 - 4x$

Your solution

Solutions on p. S4

Objective C

To solve an equation containing parentheses



When an equation contains parentheses, one of the steps in solving the equation requires the use of the Distributive Property. The Distributive Property is used to remove parentheses from a variable expression.

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Solve: $4 + 5(2x - 3) = 3(4x - 1)$

$$4 + 5(2x - 3) = 3(4x - 1)$$

$$4 + 10x - 15 = 12x - 3$$ • Use the Distributive Property. Then simplify.

$$10x - 11 = 12x - 3$$

$$10x - 12x - 11 = 12x - 12x - 3$$ • Subtract $12x$ from each side of the equation.

$$-2x - 11 = -3$$ • Simplify.

$$-2x - 11 + 11 = -3 + 11$$ • Add 11 to each side of the equation.

$$-2x = 8$$ • Simplify.

$$\frac{-2x}{-2} = \frac{8}{-2}$$ • Divide each side of the equation by -2.

$$x = -4$$ • The equation is in the form *variable* = *constant*.

The solution is -4. You should verify this by checking this solution.

Example 6

Solve: $3x - 4(2 - x) = 3(x - 2) - 4$

Solution

$$3x - 4(2 - x) = 3(x - 2) - 4$$
$$3x - 8 + 4x = 3x - 6 - 4$$
$$7x - 8 = 3x - 10$$
$$7x - 3x - 8 = 3x - 3x - 10$$
$$4x - 8 = -10$$
$$4x - 8 + 8 = -10 + 8$$
$$4x = -2$$
$$\frac{4x}{4} = \frac{-2}{4}$$
$$x = -\frac{1}{2}$$

The solution is $-\frac{1}{2}$.

You Try It 6

Solve: $5x - 4(3 - 2x) = 2(3x - 2) + 6$

Your solution

Solution on p. S5

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Example 7

Solve: $3[2 - 4(2x - 1)] = 4x - 10$

Solution

$$\begin{aligned}
3[2 - 4(2x - 1)] &= 4x - 10 \\
3[2 - 8x + 4] &= 4x - 10 \\
3[6 - 8x] &= 4x - 10 \\
18 - 24x &= 4x - 10 \\
18 - 24x - 4x &= 4x - 4x - 10 \\
18 - 28x &= -10 \\
18 - 18 - 28x &= -10 - 18 \\
-28x &= -28 \\
\frac{-28x}{-28} &= \frac{-28}{-28} \\
x &= 1
\end{aligned}$$

The solution is 1.

You Try It 7

Solve: $-2[3x - 5(2x - 3)] = 3x - 8$

Your solution

Solution on p. S5

Objective D **To translate a sentence into an equation and solve**

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An equation states that two mathematical expressions are equal. Therefore, to translate a sentence into an equation requires recognition of the words or phrases that mean "equals." Some of these words and phrases are listed below.

equals *is* *totals* *is the same as*

Point of Interest

Number puzzle problems similar to the one on this page have appeared in textbooks for hundreds of years. Here is one from a 1st-century Chinese textbook: "When a number is divided by 3, the remainder is 2; when it is divided by 5, the remainder is 3; when it is divided by 7, the remainder is 2. Find the number." There are actually an infinite number of solutions to this problem. See if you can find one of them.

➡ Translate "five less than four times a number is four more than the number" into an equation and solve.

the unknown number: n

- Assign a variable to the unknown number.

five less than four times a number	is	four more than the number

- Find two verbal expressions for the same value.

$$\begin{aligned}
4n - 5 &= n + 4 \\
4n - n - 5 &= n - n + 4 \\
3n - 5 &= 4 \\
3n - 5 + 5 &= 4 + 5 \\
3n &= 9 \\
\frac{3n}{3} &= \frac{9}{3} \\
n &= 3
\end{aligned}$$

- Write an equation.
- Solve the equation.

The number is 3.

- *Check:*

5 less than 4 times 3	4 more than 3
$4 \cdot 3 - 5$	$3 + 4$
$12 - 5$	7
$7 = 7$	

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Many of the applications of mathematics require that you identify an unknown quantity, assign a variable to that quantity, and then attempt to express another unknown quantity in terms of that variable.

TAKE NOTE

The sum of two numbers is 10.

One number	*Other number*
4	$10 - 4 = 6$
7	$10 - 7 = 3$
2	$10 - 2 = 8$
x	$10 - x$

Suppose we know that the sum of two numbers is 10 and that one of the two numbers is 4. We can find the other number by subtracting 4 from 10.

one number: 4
other number: $10 - 4 = 6$
The two numbers are 4 and 6.

Now suppose we know that the sum of two numbers is 10, we don't know either number, and we want to express *both* numbers in terms of the *same* variable. Let one number be x. Again, we can find the other number by subtracting x from 10.

one number: x
other number: $10 - x$
The two numbers are x and $10 - x$.

Note that the sum of x and $10 - x$ is 10.

$x + (10 - x) = x + 10 - x = 10$

Example 8

The sum of two numbers is 9. Eight times the smaller number is five less than three times the larger number. Find the numbers.

Solution

the smaller number: p
the larger number: $9 - p$

eight times the smaller number	is	five less than three times the larger number

$$\begin{aligned} 8p &= 3(9 - p) - 5 \\ 8p &= 27 - 3p - 5 \\ 8p &= 22 - 3p \\ 8p + 3p &= 22 - 3p + 3p \\ 11p &= 22 \\ \frac{11p}{11} &= \frac{22}{11} \\ p &= 2 \end{aligned}$$

$9 - p = 9 - 2 = 7$

These numbers check as solutions.

The smaller number is 2.
The larger number is 7.

You Try It 8

The sum of two numbers is 14. One more than three times the smaller number equals the sum of the larger number and three. Find the two numbers.

Your solution

Solution on p. S5

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2.2 Exercises

Objective A

Solve and check.

1. $3x + 1 = 10$

2. $4y + 3 = 11$

3. $2a - 5 = 7$

4. $5m - 6 = 9$

5. $5 = 4x + 9$

6. $2 = 5b + 12$

7. $2x - 5 = -11$

8. $3n - 7 = -19$

9. $4 - 3w = -2$

10. $5 - 6x = -13$

11. $8 - 3t = 2$

12. $12 - 5x = 7$

13. $4a - 20 = 0$

14. $3y - 9 = 0$

15. $6 + 2b = 0$

16. $10 + 5m = 0$

17. $-2x + 5 = -7$

18. $-5d + 3 = -12$

19. $-12x + 30 = -6$

20. $-13 = -11y + 9$

21. $2 = 7 - 5a$

22. $3 = 11 - 4n$

23. $-35 = -6b + 1$

24. $-8x + 3 = -29$

25. $-3m - 21 = 0$

26. $-5x - 30 = 0$

27. $-4y + 15 = 15$

28. $-3x + 19 = 19$

29. $9 - 4x = 6$

30. $3t - 2 = 0$

31. $9x - 4 = 0$

32. $7 - 8z = 0$

33. $1 - 3x = 0$

34. $9d + 10 = 7$

35. $12w + 11 = 5$

36. $6y - 5 = -7$

37. $8b - 3 = -9$

38. $5 - 6m = 2$

39. $7 - 9a = 4$

40. $9 = -12c + 5$

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41. $10 = -18x + 7$

42. $2y + \frac{1}{3} = \frac{7}{3}$

43. $4a + \frac{3}{4} = \frac{19}{4}$

44. $2n - \frac{3}{4} = \frac{13}{4}$

45. $3x - \frac{5}{6} = \frac{13}{6}$

46. $5y + \frac{3}{7} = \frac{3}{7}$

47. $9x + \frac{4}{5} = \frac{4}{5}$

48. $8 = 7d - 1$

49. $8 = 10x - 5$

50. $4 = 7 - 2w$

51. $7 = 9 - 5a$

52. $8t + 13 = 3$

53. $12x + 19 = 3$

54. $-6y + 5 = 13$

55. $-4x + 3 = 9$

56. $\frac{1}{2}a - 3 = 1$

57. $\frac{1}{3}m - 1 = 5$

58. $\frac{2}{5}y + 4 = 6$

59. $\frac{3}{4}n + 7 = 13$

60. $-\frac{2}{3}x + 1 = 7$

61. $-\frac{3}{8}b + 4 = 10$

62. $\frac{x}{4} - 6 = 1$

63. $\frac{y}{5} - 2 = 3$

64. $\frac{2x}{3} - 1 = 5$

65. $\frac{3c}{7} - 1 = 8$

66. $4 - \frac{3}{4}z = -2$

67. $3 - \frac{4}{5}w = -9$

68. $5 + \frac{2}{3}y = 3$

69. $17 + \frac{5}{8}x = 7$

70. $17 = 7 - \frac{5}{6}t$

71. $9 = 3 - \frac{2x}{7}$

72. $3 = \frac{3a}{4} + 1$

73. $7 = \frac{2x}{5} + 4$

74. $5 - \frac{4c}{7} = 8$

75. $7 - \frac{5}{9}y = 9$

76. $6a + 3 + 2a = 11$

77. $5y + 9 + 2y = 23$

78. $7x - 4 - 2x = 6$

79. $11z - 3 - 7z = 9$

80. $2x - 6x + 1 = 9$

81. $b - 8b + 1 = -6$

82. $3 = 7x + 9 - 4x$

83. $-1 = 5m + 7 - m$

84. $8 = 4n - 6 + 3n$

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Objective B

Solve and check.

85. $8x + 5 = 4x + 13$

86. $6y + 2 = y + 17$

87. $5x - 4 = 2x + 5$

88. $13b - 1 = 4b - 19$

89. $15x - 2 = 4x - 13$

90. $7a - 5 = 2a - 20$

91. $3x + 1 = 11 - 2x$

92. $n - 2 = 6 - 3n$

93. $2x - 3 = -11 - 2x$

94. $4y - 2 = -16 - 3y$

95. $2b + 3 = 5b + 12$

96. $m + 4 = 3m + 8$

97. $4y - 8 = y - 8$

98. $5a + 7 = 2a + 7$

99. $6 - 5x = 8 - 3x$

100. $10 - 4n = 16 - n$

101. $5 + 7x = 11 + 9x$

102. $3 - 2y = 15 + 4y$

103. $2x - 4 = 6x$

104. $2b - 10 = 7b$

105. $8m = 3m + 20$

106. $9y = 5y + 16$

107. $8b + 5 = 5b + 7$

108. $6y - 1 = 2y + 2$

109. $7x - 8 = x - 3$

110. $2y - 7 = -1 - 2y$

111. $2m - 1 = -6m + 5$

Objective C

Solve and check.

112. $5x + 2(x + 1) = 23$

113. $6y + 2(2y + 3) = 16$

114. $9n - 3(2n - 1) = 15$

115. $12x - 2(4x - 6) = 28$

116. $7a - (3a - 4) = 12$

117. $9m - 4(2m - 3) = 11$

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118. $5(3 - 2y) + 4y = 3$

119. $4(1 - 3x) + 7x = 9$

120. $5y - 3 = 7 + 4(y - 2)$

121. $5 + 2(3b + 1) = 3b + 5$

122. $6 - 4(3a - 2) = 2(a + 5)$

123. $7 - 3(2a - 5) = 3a + 10$

124. $2a - 5 = 4(3a + 1) - 2$

125. $5 - (9 - 6x) = 2x - 2$

126. $7 - (5 - 8x) = 4x + 3$

127. $3[2 - 4(y - 1)] = 3(2y + 8)$

128. $5[2 - (2x - 4)] = 2(5 - 3x)$

129. $3a + 2[2 + 3(a - 1)] = 2(3a + 4)$

130. $5 + 3[1 + 2(2x - 3)] = 6(x + 5)$

131. $-2[4 - (3b + 2)] = 5 - 2(3b + 6)$

132. $-4[x - 2(2x - 3)] + 1 = 2x - 3$

Objective D

Translate into an equation and solve.

133. The sum of a number and twelve is twenty. Find the number.

134. The difference between nine and a number is seven. Find the number.

135. Three-fifths of a number is negative thirty. Find the number.

136. The quotient of a number and six is twelve. Find the number.

137. Four more than three times a number is thirteen. Find the number.

138. The sum of twice a number and five is fifteen. Find the number.

139. The difference between nine times a number and six is twelve. Find the number.

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140. Six less than four times a number is twenty-two. Find the number.

141. The sum of a number and twice the number is nine. Find the number.

142. Eleven more than negative four times a number is three. Find the number.

143. Seventeen less than the product of five and a number is two. Find the number.

144. Eight less than the product of eleven and a number is negative nineteen. Find the number.

145. Seven more than the product of six and a number is eight less than the product of three and the number. Find the number.

146. Fifteen less than the product of four and a number is the difference between six times the number and eleven. Find the number.

147. Thirty equals nine less than the product of seven and a number. Find the number.

148. Twenty-three equals the difference between eight and the product of five and a number. Find the number.

149. The sum of two numbers is twenty-one. Twice the smaller number is three more than the larger number. Find the two numbers.

150. The sum of two numbers is thirty. Three times the smaller number is twice the larger number. Find the two numbers.

151. The sum of two numbers is twenty-three. The larger number is five more than twice the smaller number. Find the two numbers.

152. The sum of two numbers is twenty-five. The larger number is ten less than four times the smaller number. Find the two numbers.

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153. Anthropologists can approximate the height of a primate from the size of its humerus (the bone extending from the shoulder to the elbow) by using the equation $H = 1.2L + 27.8$, where L is the length of the humerus in inches and H is the height of the primate in inches. If the height of a primate is estimated to be 62 in., what is the approximate length of the humerus of this primate?



154. The fare, F, to be charged a customer by a taxi company is calculated using the formula $F = 1.50 + 0.95(m - 1)$, where m is the number of miles traveled. If a passenger is charged \$9.10, how many miles was the passenger driven?

155. Black ice is an ice covering on roads that is especially difficult to see and therefore extremely dangerous for motorists. The distance that a car traveling 30 mph will slide after its brakes are applied is related to the outside temperature by the formula $C = \frac{1}{4}D - 45$, where C is the Celsius temperature and D is the distance in feet that the car will slide. Determine the distance a car will slide on black ice when the outside temperature is $-11°C$.



APPLYING THE CONCEPTS

156. If $2x - 3 = 7$, evaluate $3x + 4$.

157. If $5x = 3x - 8$, evaluate $4x + 2$.

158. Solve: $x \div 28 = 1481$ remainder 25

159. If $3 + 2(4a - 3) = 5$ and $4 - 3(2 - 3b) = 11$, which is larger, a or b?

Solve. If the equation has no solution, write "no solution."

160. $3(2x - 1) - (6x - 4) = -9$

161. $7(3x + 6) - 4(3 + 5x) = 13 + x$

162. $\frac{1}{5}(25 - 10a) + 4 = \frac{1}{3}(12a - 15) + 14$

163. $5[m + 2(3 - m)] = 3[2(4 - m) - 5]$

164. Solve the equation $x + a = b$ for x. Is the solution you have written valid for all real numbers a and b?

165. Does the sentence "Solve $2x - 3(4x + 1)$" make sense? Why or why not?

166. The equation $x = x + 1$ has no solution, whereas the solution of the equation $2x + 3 = 3$ is zero. Is there a difference between no solution and a solution of zero? Explain your answer.

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2.3 Mixture, Investment, and Motion Problems

Objective A To solve value mixture problems

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TAKE NOTE
The equation $AC = V$ is used to find the value of an ingredient. For example, the value of 4 lb of cashews costing \$6 per pound is

$$AC = V$$
$$4 \cdot 6 = V$$
$$\$24 = V$$

A value mixture problem involves combining two ingredients that have different prices into a single blend. For example, a coffee merchant may blend two types of coffee into a single blend, or a candy manufacturer may combine two types of candy to sell as a "variety pack."

The solution of a value mixture problem is based on the equation $V = AC$, where V is the value of an ingredient, A is the amount of the ingredient, and C is the cost per unit of the ingredient.

➡ A coffee merchant wants to make 6 lb of a blend of coffee costing \$5 per pound. The blend is made using a \$6-per-pound grade of coffee and a \$3-per-pound grade. How many pounds of each of these grades should be used?



Strategy for Solving a Value Mixture Problem

1. For each ingredient in the mixture, write a numerical or variable expression for the amount of the ingredient used, the unit cost of the ingredient, and the value of the amount used. For the blend, write a numerical or variable expression for the amount, the unit cost of the blend, and the value of the amount. The results can be recorded in a table.

Amount of \$6 coffee: x
Amount of \$3 coffee: $6 - x$

• The sum of the amounts is 6 lb.

TAKE NOTE
Use the information given in the problem to fill in the "Amount" and "Unit Cost" columns of the table. Fill in the "Value" column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

	Amount, A	·	*Unit Cost, C*	=	*Value, V*
\$6 grade	x	·	6	=	$6x$
\$3 grade	$6 - x$	·	3	=	$3(6 - x)$
\$5 blend	6	·	5	=	$5(6)$

2. Determine how the values of the individual ingredients are related. Use the fact that the sum of the values of these ingredients is equal to the value of the blend.

$$6x + 3(6 - x) = 5(6)$$
$$6x + 18 - 3x = 30$$
$$3x + 18 = 30$$
$$3x = 12$$
$$x = 4$$

• The sum of the values of the \$6 grade and the \$3 grade is equal to the value of the \$5 blend.

$$6 - x = 6 - 4 = 2$$

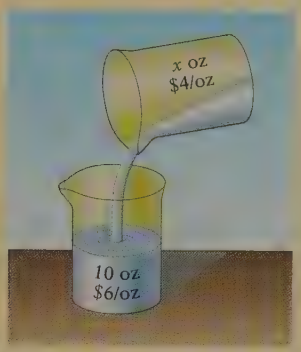
• Find the amount of \$3 coffee.

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Example 1

How many ounces of a silver alloy that costs \$4 an ounce must be mixed with 10 oz of an alloy that costs \$6 an ounce to make a mixture that costs \$4.32 an ounce?

Strategy



- Ounces of \$4 alloy: x

	Amount	*Cost*	*Value*
\$4 alloy	x	4	$4x$
\$6 alloy	10	6	$6(10)$
\$4.32 mixture	$10 + x$	4.32	$4.32(10 + x)$

- The sum of the values before mixing equals the value after mixing.

Solution

$$4x + 6(10) = 4.32(10 + x)$$
$$4x + 60 = 43.2 + 4.32x$$
$$-0.32x + 60 = 43.2$$
$$-0.32x = -16.8$$
$$x = 52.5$$

52.5 oz of the \$4 silver alloy must be used.

You Try It 1

A gardener has 20 lb of a lawn fertilizer that costs \$.80 per pound. How many pounds of a fertilizer that costs \$.55 per pound should be mixed with this 20 lb of lawn fertilizer to produce a mixture that costs \$.75 per pound?

Your strategy

Your solution

Solution on p. S5

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Objective B To solve percent mixture problems

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The amount of a substance in a solution can be given as a percent of the total solution. For example, a 5% salt-water solution means that 5% of the total solution is salt. The remaining 95% is water.

TAKE NOTE
The equation $Ar = Q$ is used to find the amount of a substance in a mixture. For example, the number of grams of silver in 50 g of a 40% alloy is

$$Ar = Q$$
$$(50\text{ g})(0.40) = Q$$
$$20\text{ g} = Q$$

The solution of a percent mixture problem is based on the equation $Q = Ar$, where Q is the quantity of a substance in the solution, r is the percent of concentration, and A is the amount of solution.

➡ A 500-milliliter bottle contains a 4% solution of hydrogen peroxide. Find the amount of hydrogen peroxide in the solution.

$Q = Ar$ • Given: $A = 500$; $r = 4\% = 0.04$
$Q = 500(0.04)$
$Q = 20$

The bottle contains 20 ml of hydrogen peroxide.



➡ How many gallons of a 20% salt solution must be mixed with 6 gal of a 30% salt solution to make a 22% salt solution?

Strategy for Solving a Percent Mixture Problem

1. For each solution, use the equation $Ar = Q$. Write a numerical or variable expression for the amount of solution, the percent of concentration, and the quantity of the substance in the solution. The results can be recorded in a table.

TAKE NOTE
Use the information given in the problem to fill in the "Amount" and "Percent" columns of the table. Fill in the "Quantity" column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

The unknown quantity of 20% solution: x

	Amount of Solution, A	·	*Percent of Concentration, r*	=	*Quantity of Substance, Q*
20% solution	x	·	0.20	=	$0.20x$
30% solution	6	·	0.30	=	$0.30(6)$
22% solution	$x + 6$	·	0.22	=	$0.22(x + 6)$

2. Determine how the quantities of the substance in the individual solutions are related. Use the fact that the sum of the quantities of the substances being mixed is equal to the quantity of the substance after mixing.

$$0.20x + 0.30(6) = 0.22(x + 6)$$
$$0.20x + 1.80 = 0.22x + 1.32$$
$$-0.02x + 1.80 = 1.32$$
$$-0.02x = -0.48$$
$$x = 24$$

• The sum of the quantities of the substance in the 20% solution and the 30% solution is equal to the quantity of the substance in the 22% solution.

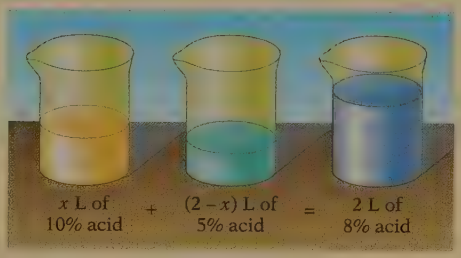
24 gal of the 20% solution must be used.

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Example 2

A chemist wishes to make 2 L of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

Strategy



- Liters of 10% solution: x
  Liters of 5% solution: $2 - x$

	Amount	*Percent*	*Quantity*
10% solution	x	0.10	$0.10x$
5% solution	$2 - x$	0.05	$0.05(2 - x)$
8% solution	2	0.08	0.08(2)

- The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$0.10x + 0.05(2 - x) = 0.08(2)$$
$$0.10x + 0.10 - 0.05x = 0.16$$
$$0.05x + 0.10 = 0.16$$
$$0.05x = 0.06$$
$$x = 1.2$$

$2 - x = 2 - 1.2 = 0.8$

The chemist needs 1.2 L of the 10% solution and 0.8 L of the 5% solution.

You Try It 2

A pharmacist dilutes 5 L of a 12% solution with a 6% solution. How many liters of the 6% solution are added to make an 8% solution?

Your strategy

Your solution

Solution on p. S5

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Objective C To solve investment problems

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Point of Interest

You may be familiar with the simple interest formula $I = Prt$. If so, you know that t represents time. In the problems in this section, time is always 1 (one year), so the formula $I = Prt$ simplifies to

$I = Pr(1)$
$I = Pr$

The annual simple interest that an investment earns is given by the equation $I = Pr$, where I is the simple interest, P is the principal, or the amount invested, and r is the simple interest rate.

➡ The annual interest rate on a \$2500 investment is 8%. Find the annual simple interest earned on the investment.

$I = Pr$ • Given: $P = \$2500$; $r = 8\% = 0.08$
$I = 2500(0.08)$
$I = 200$

The annual simple interest is \$200.



➡ An investor has a total of \$10,000 deposited in two simple interest accounts. On one account, the annual simple interest rate is 6%. On the second account, the annual simple interest rate is 10%. How much is invested in the 6% account if the total annual interest earned is \$900?

Strategy for Solving a Problem Involving Money Deposited in Two Simple Interest Accounts

1. For each amount invested, use the equation $Pr = I$. Write a numerical or variable expression for the principal, the interest rate, and the interest earned. The results can be recorded in a table.

Amount invested at 6%: x
Amount invested at 10%: $\$10{,}000 - x$ • The sum of the amounts invested is \$10,000.

TAKE NOTE

Use the information given in the problem to fill in the "Principal" and "Interest Rate" columns of the table. Fill in the "Interest Earned" column by multiplying the two expressions you wrote in each row.

	Principal, P	·	*Interest Rate, r*	=	*Interest Earned, I*
Amount at 6%	x	·	0.06	=	$0.06x$
Amount at 10%	$10{,}000 - x$	·	0.10	=	$0.10(10{,}000 - x)$

2. Determine how the amounts of interest earned on the individual amounts are related. For example, the total interest earned by both accounts may be known, or it may be known that the interest earned on one account is equal to the interest earned on the other account.

Point of Interest

One way to invest money is to buy U.S. Treasury Bills, often called T-Bills. T-Bills are issued by the U.S. Treasury, and the purchaser of a T-Bill is actually lending money to the federal government. The simple interest formula is used to compute the interest on a T-Bill.

$$0.06x + 0.10(10{,}000 - x) = 900$$
$$0.06x + 1000 - 0.10x = 900$$
$$-0.04x + 1000 = 900$$
$$-0.04x = -100$$
$$x = 2500$$

• The sum of the interest earned on the two accounts is \$900.

The amount invested at 6% is \$2500.

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Example 3

An investment counselor invested 75% of a client's money in a 9% annual simple interest money market fund. The remainder was invested in 7% annual simple interest government securities. Find the amount invested in each if the total annual interest earned is $3825.

Strategy

- Amount invested: x
  Amount invested at 7%: $0.25x$
  Amount invested at 9%: $0.75x$

	Principal	*Rate*	*Interest*
Amount at 7%	$0.25x$	0.07	$0.0175x$
Amount at 9%	$0.75x$	0.09	$0.0675x$

- The sum of the interest earned by the two investments equals the total annual interest earned ($3825).

Solution

$$0.0175x + 0.0675x = 3825$$
$$0.085x = 3825$$
$$x = 45{,}000$$

$0.25x = 0.25(45{,}000) = 11{,}250$

$0.75x = 0.75(45{,}000) = 33{,}750$

The amount invested at 7% is $11,250.
The amount invested at 9% is $33,750.

You Try It 3

An investment of $5000 is made at an annual simple interest rate of 8%. How much additional money must be invested at 11% so that the total interest earned will be 9% of the total investment?

Your strategy

Your solution

Solution on pp. S5–S6

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Objective D To solve uniform motion problems

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A train that travels constantly in a straight line at 50 mph is in *uniform motion.* **Uniform motion** means that the speed or direction of an object does not change.

The solution of a uniform motion problem is based on the equation $d = rt$, where d is the distance traveled, r is the rate of travel, and t is the time traveled.

A train traveled at a speed of 55 mph for 3 h. The distance traveled by the train can be found by using the equation $d = rt$, as shown at the right.

$$d = rt$$
$$d = 55(3)$$
$$d = 165$$

The distance traveled by the train was 165 mi.

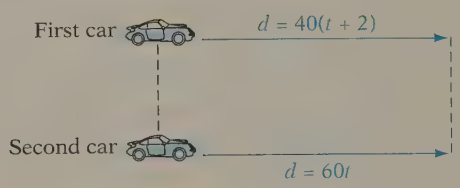
Example A car leaves a town traveling at 40 mph. Two hours later, a second car leaves the same town, on the same road, traveling at 60 mph. In how many hours will the second car pass the first car?

Strategy for Solving a Uniform Motion Problem

1. For each object, use the equation $d = rt$. Write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

The first car traveled 2 h longer than the second car.

Unknown time for the second car: t
Time for the first car: $t + 2$



TAKE NOTE
Use the information given in the problem to fill in the "Rate" and "Time" columns of the table. Fill in the "Distance" column by multiplying the two expressions you wrote in each row.

	Rate, r	·	*Time, t*	=	*Distance, d*
First car	40	·	$t + 2$	=	$40(t + 2)$
Second car	60	·	t	=	$60t$

2. Determine how the distances traveled by the individual objects are related. For example, the total distance traveled by both objects may be known, or it may be known that the two objects traveled the same distance.

The two cars travel the same distance.

$$40(t + 2) = 60t$$
$$40t + 80 = 60t$$
$$80 = 20t$$
$$4 = t$$

The second car will pass the first car in 4 h.

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Example 4

Two cars, one traveling 10 mph faster than the other, start at the same time from the same point and travel in opposite directions. In 3 h they are 300 mi apart. Find the rate of each car.

Strategy

- Rate of 1st car: r
- Rate of 2nd car: $r + 10$

	Rate	*Time*	*Distance*
1st car	r	3	$3r$
2nd car	$r + 10$	3	$3(r + 10)$

- The total distance traveled by the two cars is 300 mi.

Solution

$$3r + 3(r + 10) = 300$$
$$3r + 3r + 30 = 300$$
$$6r + 30 = 300$$
$$6r = 270$$
$$r = 45$$

$r + 10 = 45 + 10 = 55$

The first car is traveling 45 mph.
The second car is traveling 55 mph.

You Try It 4

Two trains, one traveling at twice the speed of the other, start at the same time on parallel tracks from stations that are 288 mi apart and travel toward each other. In 3 h, the trains pass each other. Find the rate of each train.

Your strategy

Your solution

Example 5

How far can the members of a bicycling club ride out into the country at a speed of 12 mph and return over the same road at 8 mph if they travel a total of 10 h?

Strategy

- Time spent riding out: t
  Time spent riding back: $10 - t$

	Rate	*Time*	*Distance*
Out	12	t	$12t$
Back	8	$10 - t$	$8(10 - t)$

- The distance out equals the distance back.

Solution

$$12t = 8(10 - t)$$
$$12t = 80 - 8t$$
$$20t = 80$$
$$t = 4 \quad \text{(The time is 4 h.)}$$

The distance out $= 12t = 12(4) = 48$ mi.
The club can ride 48 mi into the country.

You Try It 5

A pilot flew out to a parcel of land and back in 5 h. The rate out was 150 mph, and the rate returning was 100 mph. How far away was the parcel of land?

Your strategy

Your solution

Solutions on p. S6

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2.3 Exercises

Objective A *Application Problems*

1. An herbalist has 30 oz of herbs costing $2 per ounce. How many ounces of herbs costing $1 per ounce should be mixed with the 30 oz to produce a mixture costing $1.60 per ounce?

2. The manager of a farmer's market has 500 lb of grain that costs $1.20 per pound. How many pounds of meal costing $.80 per pound should be mixed with the 500 lb of grain to produce a mixture that costs $1.05 per pound?

3. Find the cost per pound of a meatloaf mixture made from 3 lb of ground beef costing $3.99 per pound and 1 lb of ground turkey costing $1.79 per pound.

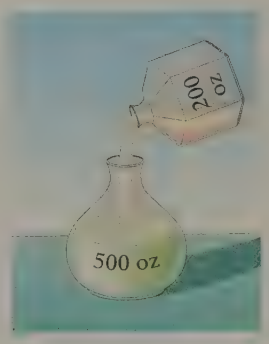
4. Find the cost per ounce of a sunscreen made from 100 oz of a lotion that costs $2.50 per ounce and 50 oz of a lotion that costs $4.00 per ounce.

5. A snack food is made by mixing 5 lb of popcorn that costs $.80 per pound with caramel that costs $2.40 per pound. How much caramel is needed to make a mixture that costs $1.40 per pound?

6. A wild birdseed mix is made by combining 100 lb of millet seed costing $.60 per pound with sunflower seeds costing $1.10 per pound. How many pounds of sunflower seeds are needed to make a mixture that costs $.70 per pound?

7. Ten cups of a restaurant's house Italian dressing is made by blending olive oil costing $1.50 per cup with vinegar that costs $.25 per cup. How many cups of each are used if the cost of the blend is $.50 per cup?

8. A high-protein diet supplement that costs $6.75 per pound is mixed with a vitamin supplement that costs $3.25 per pound. How many pounds of each should be used to make 5 lb of a mixture that costs $4.65 per pound?



9. Find the cost per ounce of a mixture of 200 oz of a cologne that costs $5.50 per ounce and 500 oz of a cologne that costs $2.00 per ounce.

10. Find the cost per pound of a trail mix made from 40 lb of raisins that cost $4.40 per pound and 100 lb of granola that costs $2.30 per pound.

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11. Twenty ounces of a platinum alloy that costs $220 per ounce are mixed with an alloy that costs $400 per ounce. How many ounces of the $400 alloy should be used to make an alloy that costs $300 per ounce?

12. How many liters of a blue dye that costs $1.60 per liter must be mixed with 18 L of anil that costs $2.50 per liter to make a mixture that costs $1.90 per liter?

13. The manager of a specialty food store combined almonds that cost $4.50 per pound with walnuts that cost $2.50 per pound. How many pounds of each were used to make a 100-pound mixture that costs $3.24 per pound?

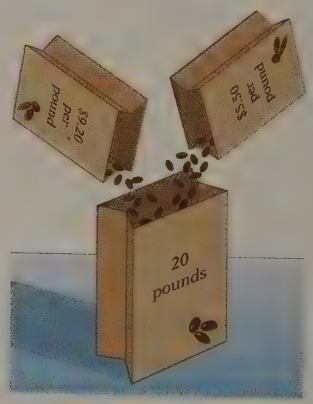
14. A goldsmith combined an alloy that cost $4.30 per ounce with an alloy that cost $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing $2.50 per ounce?

15. Adult tickets for a play cost $6.00 and children's tickets cost $2.50. For one performance, 370 tickets were sold. Receipts for the performance were $1723. Find the number of adult tickets sold.

16. Tickets for a piano concert sold for $4.50 for each adult. Student tickets sold for $2.00 each. The total receipts for 1720 tickets were $5980. Find the number of adult tickets sold.

17. Find the cost per pound of sugar-coated breakfast cereal made from 40 lb of sugar that costs $1.00 per pound and 120 lb of corn flakes that cost $.60 per pound.

18. Find the cost per pound of a coffee mixture made from 8 lb of coffee that costs $9.20 per pound and 12 lb of coffee that costs $5.50 per pound.



Objective B Application Problems

19. Forty ounces of a 30% gold alloy are mixed with 60 oz of a 20% gold alloy. Find the percent concentration of the resulting gold alloy.

20. One hundred ounces of juice that is 50% tomato juice are added to 200 oz of a vegetable juice that is 25% tomato juice. What is the percent concentration of tomato juice in the resulting mixture?

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21. How many gallons of a 15% acid solution must be mixed with 5 gal of a 20% acid solution to make a 16% acid solution?

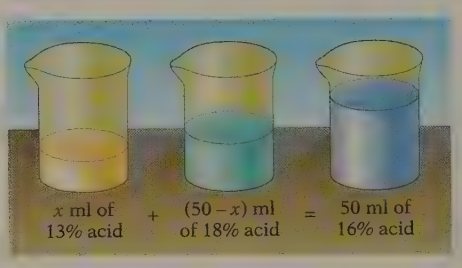
22. How many pounds of a chicken feed that is 50% corn must be mixed with 400 lb of a feed that is 80% corn to make a chicken feed that is 75% corn?

23. A rug is made by weaving 20 lb of yarn that is 50% wool with a yarn that is 25% wool. How many pounds of the yarn that is 25% wool are used if the finished rug is 35% wool?

24. Five gallons of a dark green latex paint that is 20% yellow paint are combined with a lighter green latex paint that is 40% yellow paint. How many gallons of the lighter green paint must be used to create a green paint that is 25% yellow paint?

25. How many gallons of a plant food that is 9% nitrogen must be combined with another plant food that is 25% nitrogen to make 10 gal of a solution that is 15% nitrogen?

26. A chemist wants to make 50 ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?



27. Five grams of sugar are added to a 45-gram serving of a breakfast cereal that is 10% sugar. What is the percent concentration of sugar in the resulting mixture?

28. A goldsmith mixes 8 oz of a 30% gold alloy with 12 oz of a 25% gold alloy. What is the percent concentration of the resulting alloy?

29. How many pounds of coffee that is 40% java beans must be mixed with 80 lb of coffee that is 30% java beans to make a coffee blend that is 32% java beans?

30. The manager of a garden shop mixes grass seed that is 60% rye grass with 70 lb of grass seed that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% rye grass is used?

31. A hair dye is made by blending a 7% hydrogen peroxide solution and a 4% hydrogen peroxide solution. How many milliliters of each are used to make a 300-milliliter solution that is 5% hydrogen peroxide?

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32. A tea that is 20% jasmine is blended with a tea that is 15% jasmine. How many pounds of each tea are used to make 5 lb of tea that is 18% jasmine?

33. How many ounces of pure chocolate must be added to 150 oz of chocolate topping that is 50% chocolate to make a topping that is 75% chocolate?

34. How many ounces of pure bran flakes must be added to 50 oz of cereal that is 40% bran flakes to produce a mixture that is 50% bran flakes?

35. Thirty ounces of pure silver are added to 50 oz of a silver alloy that is 20% silver. What is the percent concentration of the resulting alloy?

36. A clothing manufacturer has some pure silk thread and some thread that is 85% silk. How many kilograms of each must be woven together to make 75 kg of cloth that is 96% silk?

Objective C *Application Problems*

37. An investment of $3000 is made at an annual simple interest rate of 5%. How much additional money must be invested at an annual simple interest rate of 9% so that the total annual interest earned is 7.5% of the total investment?

38. A total of $6000 is invested into two simple interest accounts. The annual simple interest rate on one account is 9%; on the second account, the annual simple interest rate is 6%. How much should be invested in each account so that both accounts earn the same amount of annual interest?

39. An engineer invested a portion of $15,000 in a 7% annual simple interest account and the remainder in a 6.5% annual simple interest government bond. The amount of interest earned for one year was $1020. How much was invested in each account?

40. An investment club invested part of $20,000 in preferred stock that pays 8% annual simple interest and the remainder in a municipal bond that pays 7% annual simple interest. The amount of interest earned each year is $1520. How much was invested in each account?

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41. A grocery checker deposited an amount of money into a high-yield mutual fund that returns a 9% annual simple interest rate. A second deposit, $2500 more than the first, was placed in a certificate of deposit that returns a 5% annual simple interest rate. The total interest earned on both investments for one year was $475. How much money was deposited in the mutual fund?

42. A deposit was made into a 7% annual simple interest account. Another deposit, $1500 less than the first deposit, was placed in a 9% annual simple interest certificate of deposit. The total interest earned on both accounts for one year was $505. How much money was deposited in the certificate of deposit?

43. A corporation gave a university $300,000 to support product safety research. The university deposited some of the money in a 10% simple interest account and the remainder in an 8.5% simple interest account. How much was deposited in each account if the annual interest earned is $28,500?

44. A financial consultant advises a client to invest part of $30,000 in municipal bonds that earn 6.5% annual simple interest and the remainder of the money in 8.5% corporate bonds. How much should be invested in each type of bond so that the total annual interest earned each year is $2190?

45. To provide for retirement income, an auto mechanic purchases a $5000 bond that earns 7.5% annual simple interest. How much money must be invested in additional bonds that have an interest rate of 8% so that the total annual interest earned from the two investments is $615?



46. The portfolio manager for an investment group invested $40,000 in a certificate of deposit that earns 7.25% annual simple interest. How much money must be invested in additional certificates that have an interest rate of 8.5% so that the total annual interest earned from the two investments is $5025?

47. A charity deposited a total of $54,000 into two simple interest accounts. The annual simple interest rate on one account is 8%. The annual simple interest rate on the second account is 12%. How much was invested in each account if the total interest earned is 9% of the total investment?

48. A college sports foundation deposited a total of $24,000 into two simple interest accounts. The annual simple interest rate on one account is 7%. The annual simple interest rate on the second account is 11%. How much was invested in each account if the total annual interest earned is 10% of the total investment?

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49. An investment banker invested 55% of the bank's available cash in an account that earns 8.25% annual simple interest. The remainder of the cash was placed in an account that earns 10% annual simple interest. The interest earned in one year was $58,743.75. Find the total amount invested.

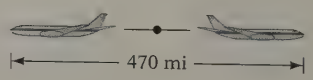
50. A financial planner invested 40% of a client's cash account in preferred stock that earns 9% annual simple interest. The remainder of the client's cash was placed in treasury bonds that earn 7% annual interest. The total annual interest earned from the two investments was $2496. What was the total amount invested?

51. The manager of a mutual fund placed 30% of the fund's available cash in a 6% simple interest account, 25% in 8% corporate bonds, and the remainder in a money market fund that earns 7.5% annual simple interest. The total annual interest from the investments was $35,875. What was the total amount invested?

52. The manager of a trust decided to invest 30% of a client's cash in government bonds that earn 6.5% annual simple interest. Another 30% was placed in utility stocks that earn 7% annual simple interest. The remainder of the cash was placed in an account earning 8% annual simple interest. The total annual interest earned from the investments was $5437.50. What was the total amount invested?

Objective D *Application Problems*

53. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.



54. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In 3 h, they are 81 mi apart. Find the rate of each cyclist.

55. Two planes leave an airport at 8 A.M., one flying north at 480 km/h and the other flying south at 520 km/h. At what time will they be 3000 km apart?

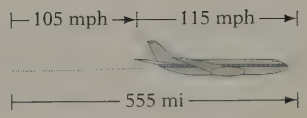
56. A long-distance runner started on a course running at an average speed of 6 mph. One-half hour later, a second runner began the same course at an average speed of 7 mph. How long after the second runner started will the second runner overtake the first runner?



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57. A motorboat leaves a harbor and travels at an average speed of 9 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

58. A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. For the remainder of the trip, the average speed was 115 mph. For how long did the plane fly at each speed?

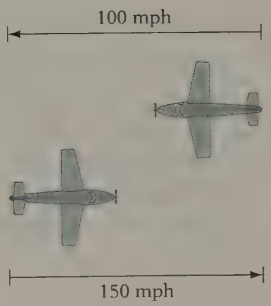


59. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

60. After a sailboat had been on the water for 3 h, a change in the wind direction reduced the average speed of the boat by 5 mph. The entire distance sailed was 57 mi. The total time spent sailing was 6 h. How far did the sailboat travel in the first 3 h?

61. A car and a bus set out at 3 P.M. from the same point headed in the same direction. The average speed of the car is twice the average speed of the bus. In 2 h the car is 68 mi ahead of the bus. Find the rate of the car.

62. A passenger train leaves a train depot 2 h after a freight train leaves the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.



63. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flying time was 5 h.

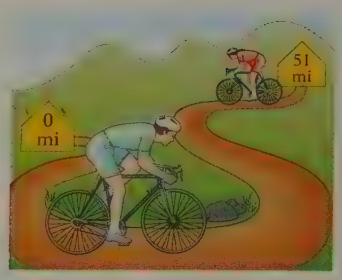
64. A ship traveling east at 25 mph is 10 mi from a harbor when another ship leaves the harbor traveling east at 35 mph. How long does it take the second ship to catch up to the first ship?

65. At 10 A.M. a plane leaves Boston, Massachusetts, for Seattle, Washington, a distance of 3000 mi. One hour later a plane leaves Seattle for Boston. Both planes are traveling at a speed of 500 mph. How many hours after the plane leaves Seattle will the planes pass each other?

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66. At noon a train leaves Washington, D.C., headed for Charleston, South Carolina, a distance of 500 mi. The train travels at a speed of 60 mph. At 1 P.M. a second train leaves Charleston headed for Washington, D.C., traveling at 50 mph. How long after the train leaves Charleston will the two trains pass each other?

67. Two cyclists start at the same time from opposite ends of a course that is 51 mi long. One cyclist is riding at a rate of 16 mph, and the second cyclist is riding at a rate of 18 mph. How long after they begin will they meet?



68. A bus traveled on a straight road for 2 h at an average speed that was 20 mph faster than its average speed on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 210 mi.

69. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1-hour head start, how far from the starting point does the bus overtake the car?

70. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, had a 3-hour head start. How far from the starting point does the car overtake the cyclist?

APPLYING THE CONCEPTS

71. A radiator contains 15 gal of a 20% antifreeze solution. How many gallons must be drained from the radiator and replaced by pure antifreeze so that the radiator will contain 15 gal of a 40% antifreeze solution?

72. A plant manager invested $3000 more in preferred stocks than in bonds. The stocks paid 8% annual simple interest, and the bonds paid 9.5% annual simple interest. Both investments yielded the same income. Find the total annual interest received on both investments.

73. A bicyclist rides for 2 h at a speed of 10 mph and then returns at a speed of 20 mph. Find the cyclist's average speed for the trip.

74. A car travels a 1-mile track at an average speed of 30 mph. At what average speed must the car travel the next mile so that the average speed for the 2 mi is 60 mph?

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2.4 Inequalities in One Variable

Objective A

To solve an inequality in one variable

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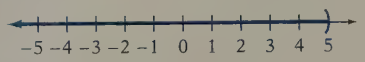
The **solution set of an inequality** is a set of numbers, each element of which, when substituted for the variable, results in a true inequality.

The inequality at the right is true if the variable is replaced by (for instance) 3, −1.98, or $\frac{2}{3}$.

$$x - 1 < 4$$
$$3 - 1 < 4$$
$$-1.98 - 1 < 4$$
$$\frac{2}{3} - 1 < 4$$

There are many values of the variable x that will make the inequality $x - 1 < 4$ true. The solution set of the inequality is any number less than 5. The solution set can be written in set-builder notation as $\{x | x < 5\}$.

The graph of the solution set of $x - 1 < 4$ is shown at the right.



In solving an inequality, we use the Addition and Multiplication Properties of Inequalities to rewrite the inequality in the form *variable* < *constant* or in the form *variable* > *constant*.

The Addition Property of Inequalities

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

The Addition Property of Inequalities states that the same number can be added to each side of an inequality without changing the solution set of the inequality. This property is also true for an inequality that contains the symbol ≤ or ≥.

The Addition Property of Inequalities is used to remove a term from one side of an inequality by adding the additive inverse of that term to each side of the inequality. Because subtraction is defined in terms of addition, the same number can be subtracted from each side of an inequality without changing the solution set of the inequality.

Solve: $x + 2 \geq 4$

$$x + 2 \geq 4$$
$$x + 2 - 2 \geq 4 - 2 \quad \text{• Subtract 2 from each side of the inequality.}$$
$$x \geq 2 \quad \text{• Simplify.}$$

The solution set is $\{x | x \geq 2\}$.

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➡ Solve: $3x - 4 < 2x - 1$

$$3x - 4 < 2x - 1$$
$$3x - 2x - 4 < 2x - 2x - 1$$ • Subtract $2x$ from each side of the inequality.
$$x - 4 < -1$$
$$x - 4 + 4 < -1 + 4$$ • Add 4 to each side of the inequality.
$$x < 3$$

The solution set is $\{x|x < 3\}$.

The Multiplication Property of Inequalities is used to remove a coefficient from one side of an inequality by multiplying each side of the inequality by the reciprocal of the coefficient.

TAKE NOTE

$c > 0$ means c is a positive number.

$c < 0$ means c is a negative number.

The Multiplication Property of Inequalities

Rule 1 If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.

Rule 2 If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Here are some examples of this property.

Rule 1		Rule 2	
$3 > 2$	$2 < 5$	$3 > 2$	$2 < 5$
$3(4) > 2(4)$	$2(4) < 5(4)$	$3(-4) < 2(-4)$	$2(-4) > 5(-4)$
$12 > 8$	$8 < 20$	$-12 < -8$	$-8 > -20$

Rule 1 states that **when each side of an inequality is multiplied by a positive number, the inequality symbol remains the same.** However, Rule 2 states that **when each side of an inequality is multiplied by a negative number, the inequality symbol must be reversed.** Because division is defined in terms of multiplication, **when each side of an inequality is divided by a positive number, the inequality symbol remains the same. But when each side of an inequality is divided by a negative number, the inequality symbol must be reversed.**

The Multiplication Property of Inequalities is also true for the symbols $\leq$ and $\geq$.

➡ Solve: $-3x > 9$

TAKE NOTE

Each side of the inequality is *divided by* a negative number; the inequality symbol must be reversed.

$$-3x > 9$$
$$\frac{-3x}{-3} < \frac{9}{-3}$$ • Divide each side of the inequality by the coefficient -3. Because -3 is a negative number, the inequality symbol must be reversed.
$$x < -3$$

The solution set is $\{x|x < -3\}$.

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TAKE NOTE

Any time an inequality is multiplied or divided by a negative number, the inequality symbol must be reversed. Compare the next two examples.

$2x < -4$ Divide each side by *positive* 2.

$\frac{2x}{2} < \frac{-4}{2}$

$x < -2$ Inequality *is not* reversed.

$-2x < 4$ Divide each side by *negative* 2.

$\frac{-2x}{-2} > \frac{4}{-2}$

$x > -2$ Inequality *is* reversed.

➡ Solve: $3x + 2 < -4$

$3x + 2 < -4$

$3x < -6$ • Subtract 2 from each side of the inequality.

$\frac{3x}{3} < \frac{-6}{3}$ • Divide each side of the inequality by the coefficient 3.

$x < -2$

The solution set is $\{x|x < -2\}$.

➡ Solve: $2x - 9 > 4x + 5$

$2x - 9 > 4x + 5$

$-2x - 9 > 5$ • Subtract $4x$ from each side of the inequality.

$-2x > 14$ • Add 9 to each side of the inequality.

$\frac{-2x}{-2} < \frac{14}{-2}$ • Divide each side of the inequality by the coefficient -2. Reverse the inequality symbol.

$x < -7$

The solution set is $\{x|x < -7\}$.

TAKE NOTE

Solving these inequalities is similar to solving the equations solved in Section 2 *except* that when you multiply or divide the inequality by a negative number, you must reverse the inequality symbol.

➡ Solve: $5(x - 2) \geq 9x - 3(2x - 4)$

$5(x - 2) \geq 9x - 3(2x - 4)$

$5x - 10 \geq 9x - 6x + 12$ • Use the Distributive Property to remove parentheses.

$5x - 10 \geq 3x + 12$ • Combine like terms.

$2x - 10 \geq 12$ • Subtract $3x$ from each side of the inequality.

$2x \geq 22$ • Add 10 to each side of the inequality.

$\frac{2x}{2} \geq \frac{22}{2}$ • Divide each side of the inequality by the coefficient 2.

$x \geq 11$

The solution set is $\{x|x \geq 11\}$.

Example 1

Solve: $x + 3 > 4x + 6$

Solution

$x + 3 > 4x + 6$

$-3x + 3 > 6$ • Subtract $4x$ from each side.

$-3x > 3$ • Subtract 3 from each side.

$\frac{-3x}{-3} < \frac{3}{-3}$ • Divide each side by -3.

$x < -1$

The solution set is $\{x|x < -1\}$.

You Try It 1

Solve: $2x - 1 < 6x + 7$

Your solution

Solution on p. S6

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Example 2 Solve:
$3x - 5 \le 3 - 2(3x + 1)$

Solution

$$\begin{aligned} 3x - 5 &\le 3 - 2(3x + 1) \\ 3x - 5 &\le 3 - 6x - 2 \\ 3x - 5 &\le 1 - 6x \\ 9x - 5 &\le 1 \\ 9x &\le 6 \\ \frac{9x}{9} &\le \frac{6}{9} \\ x &\le \frac{2}{3} \end{aligned}$$

$\left\{x \mid x \le \frac{2}{3}\right\}$

You Try It 2 Solve:
$5x - 2 \le 4 - 3(x - 2)$

Your solution

Solution on p. S6

Objective B To solve a compound inequality

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A **compound inequality** is formed by joining two inequalities with a connective word such as *and* or *or*. The inequalities at the right are compound inequalities.

$2x < 4$ and $3x - 2 > -8$

$2x + 3 > 5$ or $x + 2 < 5$

The solution set of a compound inequality with the connective word *and* is the set of all elements that appear in the solution sets of both inequalities. Therefore, it is the intersection of the solution sets of the two inequalities.

TAKE NOTE
The intersection $\{x \mid x < 3\} \cap \{x \mid x > -2\}$ can be written $\{x \mid x > -2 \text{ and } x < 3\}$. However, it is more commonly written $\{x \mid -2 < x < 3\}$. This is read "the set of all x such that x is greater than -2 *and* less than 3."

➡ Solve: $2x < 6$ and $3x + 2 > -4$

$2x < 6$	and	$3x + 2 > -4$	• Solve each inequality.
$x < 3$		$3x > -6$	
$\{x \mid x < 3\}$		$x > -2$	
		$\{x \mid x > -2\}$	

The solution of a compound inequality with *and* is the intersection of the solution sets of the two inequalities.

$\{x \mid x < 3\} \cap \{x \mid x > -2\} = \{x \mid -2 < x < 3\}$

➡ Solve: $-3 < 2x + 1 < 5$

This inequality is equivalent to the compound inequality shown at the right.

$-3 < 2x + 1$ and $2x + 1 < 5$

$-3 < 2x + 1$	and	$2x + 1 < 5$	• Solve each inequality.
$-4 < 2x$		$2x < 4$	
$-2 < x$		$x < 2$	
$\{x \mid x > -2\}$		$\{x \mid x < 2\}$	

$\{x \mid x > -2\} \cap \{x \mid x < 2\} = \{x \mid -2 < x < 2\}$

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There is an alternative method for solving the inequality in the last example.

Solve: $-3 < 2x + 1 < 5$

$$-3 < 2x + 1 < 5$$

$$-3 - 1 < 2x + 1 - 1 < 5 - 1$$ • Subtract 1 from each of the three parts of the inequality.

$$-4 < 2x < 4$$

$$\frac{-4}{2} < \frac{2x}{2} < \frac{4}{2}$$ • Divide each of the three parts of the inequality by the coefficient 2.

$$-2 < x < 2$$

The solution set is $\{x \mid -2 < x < 2\}$.

The solution set of a compound inequality with the connective word *or* is the union of the solution sets of the two inequalities.

Solve: $2x + 3 > 7$ or $4x - 1 < 3$

$2x + 3 > 7$	or	$4x - 1 < 3$	• Solve each inequality.
$2x > 4$		$4x < 4$	
$x > 2$		$x < 1$	
$\{x \mid x > 2\}$		$\{x \mid x < 1\}$	

$\{x \mid x > 2\} \cup \{x \mid x < 1\} = \{x \mid x > 2 \text{ or } x < 1\}$ • Find the union of the solution sets.

Example 3

Solve: $1 < 3x - 5 < 4$

Solution

$$1 < 3x - 5 < 4$$

$$1 + 5 < 3x - 5 + 5 < 4 + 5$$

$$6 < 3x < 9$$

$$\frac{6}{3} < \frac{3x}{3} < \frac{9}{3}$$

$$2 < x < 3$$

$\{x \mid 2 < x < 3\}$

You Try It 3

Solve: $-2 \le 5x + 3 \le 13$

Your solution

Example 4

Solve: $11 - 2x > -3$ and $7 - 3x < 4$

Solution

$11 - 2x > -3$	and	$7 - 3x < 4$
$-2x > -14$		$-3x < -3$
$x < 7$		$x > 1$
$\{x \mid x < 7\}$		$\{x \mid x > 1\}$

$\{x \mid x < 7\} \cap \{x \mid x > 1\} = \{x \mid 1 < x < 7\}$

You Try It 4

Solve: $2 - 3x > 11$ or $5 + 2x > 7$

Your solution

Solutions on p. S6

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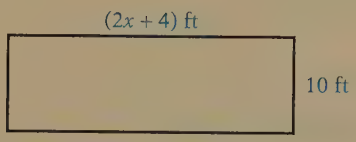
Objective C To solve application problems

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Example 5

A rectangle is 10 ft wide and $(2x + 4)$ ft long. Express as an integer the maximum length of the rectangle when the area is less than 200 ft^2. (The area of a rectangle is equal to its length times its width.)

Strategy



To find the maximum length:

- Replace the variables in the area formula by the given values and solve for x.
- Replace the variable in the expression $2x + 4$ with the value found for x.

Solution

Length times width	is	less than	200 ft^2

$$\begin{aligned} (2x + 4)10 &< 200 \\ 20x + 40 &< 200 \\ 20x + 40 - 40 &< 200 - 40 \\ 20x &< 160 \\ \frac{20x}{20} &< \frac{160}{20} \\ x &< 8 \end{aligned}$$

The length is $(2x + 4)$ ft. Because $x < 8$, $2x + 4 < 2(8) + 4 = 20$. Therefore, the length is less than 20 ft.

The maximum length is 19 ft.

You Try It 5

Company A rents cars for \$24 a day and 10¢ for every mile driven. Company B rents cars for \$30 a day and 8¢ per mile driven. You want to rent a car for one week. What is the maximum number of miles you can drive a Company A car if it is to cost you less than a Company B car?

Your strategy

Your solution

Solution on pp. S6–S7

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2.4 Exercises

Objective A

1. State the Addition Property of Inequalities and give numerical examples of its use.

2. State the Multiplication Property of Inequalities and give numerical examples of its use.

3. Which numbers are solutions of the inequality $x + 7 \leq -3$?
   a. -17 **b.** 8 **c.** -10 **d.** 0

4. Which numbers are solutions of the inequality $2x - 1 > 5$?
   a. 6 **b.** -4 **c.** 3 **d.** 5

Solve.

5. $x - 3 < 2$ **6.** $x + 4 \geq 2$ **7.** $4x \leq 8$

8. $6x > 12$ **9.** $-2x > 8$ **10.** $-3x \leq -9$

11. $3x - 1 > 2x + 2$ **12.** $5x + 2 \geq 4x - 1$ **13.** $2x - 1 > 7$

14. $3x + 2 < 8$ **15.** $5x - 2 \leq 8$ **16.** $4x + 3 \leq -1$

17. $6x + 3 > 4x - 1$ **18.** $7x + 4 < 2x - 6$ **19.** $8x + 1 \geq 2x + 13$

20. $5x - 4 < 2x + 5$ **21.** $4 - 3x < 10$ **22.** $2 - 5x > 7$

23. $7 - 2x \geq 1$ **24.** $3 - 5x \leq 18$ **25.** $-3 - 4x > -11$

26. $-2 - x < 7$ **27.** $4x - 2 < x - 11$ **28.** $6x + 5 \leq x - 10$

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29. $x + 7 \geq 4x - 8$

30. $3x + 1 \leq 7x - 15$

31. $3x + 2 \leq 7x + 4$

32. $3x - 5 \geq -2x + 5$

33. $\frac{3}{5}x - 2 < \frac{3}{10} - x$

34. $\frac{5}{6}x - \frac{1}{6} < x - 4$

35. $\frac{2}{3}x - \frac{3}{2} < \frac{7}{6} - \frac{1}{3}x$

36. $\frac{7}{12}x - \frac{3}{2} < \frac{2}{3}x + \frac{5}{6}$

37. $\frac{1}{2}x - \frac{3}{4} < \frac{7}{4}x - 2$

38. $6 - 2(x - 4) \leq 2x + 10$

39. $4(2x - 1) > 3x - 2(3x - 5)$

40. $2(1 - 3x) - 4 > 10 + 3(1 - x)$

41. $2 - 5(x + 1) \geq 3(x - 1) - 8$

42. $2 - 2(7 - 2x) < 3(3 - x)$

43. $3 + 2(x + 5) \geq x + 5(x + 1) + 1$

44. $3x - 2(3x - 5) \leq 2 - 5(x - 4)$

45. $12 - 2(3x - 2) \geq 5x - 2(5 - x)$

Objective B

46. **a.** Which set operation is used when a compound inequality is combined with *or*?

b. Which set operation is used when a compound inequality is combined with *and*?

47. Explain why writing $-3 > x > 4$ does not make sense.

Solve.

48. $3x < 6$ and $x + 2 > 1$

49. $x - 3 \leq 1$ and $2x \geq -4$

50. $x + 2 \geq 5$ or $3x \leq 3$

51. $2x < 6$ or $x - 4 > 1$

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52. $-2x > -8$ and $-3x < 6$

53. $\frac{1}{2}x > -2$ and $5x < 10$

54. $\frac{1}{3}x < -1$ or $2x > 0$

55. $\frac{2}{3}x > 4$ or $2x < -8$

56. $x + 4 \geq 5$ and $2x \geq 6$

57. $3x < -9$ and $x - 2 < 2$

58. $-5x > 10$ and $x + 1 > 6$

59. $7x < 14$ and $1 - x < 4$

60. $2x - 3 > 1$ and $3x - 1 < 2$

61. $4x + 1 < 5$ and $4x + 7 > -1$

62. $3x + 7 < 10$ or $2x - 1 > 5$

63. $6x - 2 < -14$ or $5x + 1 > 11$

64. $-5 < 3x + 4 < 16$

65. $5 < 4x - 3 < 21$

66. $0 < 2x - 6 < 4$

67. $-2 < 3x + 7 < 1$

68. $4x - 1 > 11$ or $4x - 1 \leq -11$

69. $3x - 5 > 10$ or $3x - 5 < -10$

70. $2x - 3 \geq 5$ and $3x - 1 > 11$

71. $6x - 2 < 5$ or $7x - 5 < 16$

72. $9x - 2 < 7$ and $3x - 5 > 10$

73. $8x + 2 \leq -14$ and $4x - 2 > 10$

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74. $3x - 11 < 4$ or $4x + 9 \geq 1$

75. $5x + 12 \geq 2$ or $7x - 1 \leq 13$

76. $-6 \leq 5x + 14 \leq 24$

77. $3 \leq 7x - 14 \leq 31$

78. $3 - 2x > 7$ and $5x + 2 > -18$

79. $1 - 3x < 16$ and $1 - 3x > -16$

80. $9 - x \geq 7$ and $9 - 2x < 3$

81. $6x + 5 < -1$ or $1 - 2x < 7$

Objective C *Application Problems*

82. Five times the difference between a number and two is greater than the quotient of two times the number and three. Find the smallest integer that will satisfy the inequality.

83. Two times the difference between a number and eight is less than or equal to five times the sum of the number and four. Find the smallest number that will satisfy the inequality.

84. The length of a rectangle is 2 ft more than four times the width. Express as an integer the maximum width of the rectangle when the perimeter is less than 34 ft.

85. The length of a rectangle is 5 cm less than twice the width. Express as an integer the maximum width of the rectangle when the perimeter is less than 60 cm.

86. In 2002, the computer service America Online offered its customers the option of paying \$23.90 per month for unlimited use. Another option was a rate of \$4.95 per month with 3 free hours plus \$2.50 per hour thereafter. How many hours per month can you use this second option if it is to cost you less than the first option? Round to the nearest whole number.

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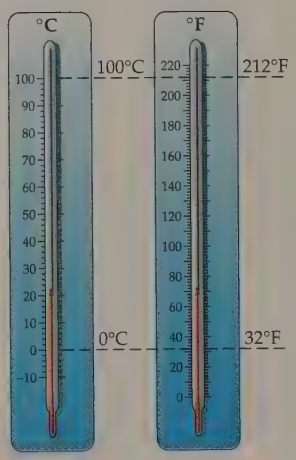
87. TopPage advertises local paging service for \$6.95 per month for up to 400 pages and \$.10 per page thereafter. A competitor advertises service for \$3.95 per month for up to 400 pages and \$.15 per page thereafter. For what number of pages per month is the TopPage plan less expensive?

88. Suppose PayRite Rental Cars rents compact cars for \$32 per day with unlimited mileage and Otto Rentals offers compact cars for \$19.99 per day but charges \$.19 for each mile beyond 100 mi driven per day. You want to rent a car for one week. How many miles can you drive during the week if Otto Rentals is to be less expensive than PayRite?

89. During a weekday, to call a city 40 mi away from a certain pay phone costs \$.70 for the first 3 min and \$.15 for each additional minute. If you use a calling card, there is a \$.35 fee and then the rates are \$.196 for the first minute and \$.126 for each additional minute. How long must a call be for it to be cheaper to pay with coins rather than a calling card?

90. The temperature range for a week was between 14°F and 77°F. Find the temperature range in Celsius degrees. $F = \frac{9}{5}C + 32$

91. The temperature range for a week in a mountain town was between 0°C and 30°C. Find the temperature range in Fahrenheit degrees.
$C = \frac{5(F - 32)}{9}$



92. You are a sales account executive earning \$1200 per month plus 6% commission on the amount of sales. Your goal is to earn a minimum of \$6000 per month. What amount of sales will enable you to earn \$6000 or more per month?

93. George Stoia earns \$1000 per month plus 5% commission on the amount of sales. George's goal is to earn a minimum of \$3200 per month. What amount of sales will enable George to earn \$3200 or more per month?

94. Heritage National Bank offers two different checking accounts. The first charges \$3 per month and \$.50 per check after the first 10 checks. The second account charges \$8 per month with unlimited check writing. How many checks can be written per month if the first account is to be less expensive than the second account?

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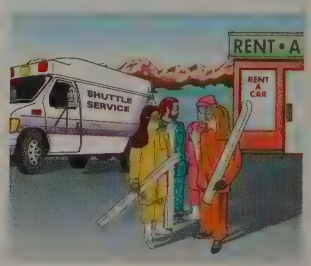
95. Glendale Federal Bank offers a checking account to small businesses. The charge is \$8 per month plus \$.12 per check after the first 100 checks. A competitor is offering an account for \$5 per month plus \$.15 per check after the first 100 checks. If a business chooses the first account, how many checks does the business write monthly if it is assumed that the first account will cost less than the competitor's account?

96. An average score of 90 or above in a history class receives an A grade. You have grades of 95, 89, and 81 on three exams. Find the range of scores on the fourth exam that will give you an A grade for the course.

97. An average of 70 to 79 in a mathematics class receives a C grade. A student has grades of 56, 91, 83, and 62 on four tests. Find the range of scores on the fifth test that will give the student a C for the course.

98. Grade A hamburger cannot contain more than 20% fat. How much fat can a butcher mix with 300 lb of lean meat to meet the 20% requirement?

99. A shuttle service taking skiers to a ski area charges \$8 per person each way. Four skiers are debating whether to take the shuttle bus or rent a car for \$45 plus \$.25 per mile. The skiers will share the cost of the car, and they want the least expensive method of transportation. How far away is the ski area if they choose the shuttle service?



APPLYING THE CONCEPTS

100. Determine whether the statement is always true, sometimes true, or never true, given that a, b, and c are real numbers.

a. If $a > b$, then $-a > -b$.

b. If $a < b$, then $ac < bc$.

c. If $a > b$, then $a + c > b + c$.

d. If $a \neq 0$, $b \neq 0$, and $a > b$, then $\frac{1}{a} > \frac{1}{b}$.

101. Use the roster method to list the set of positive integers that are solutions of the inequality $7 - 2b \leq 15 - 5b$.

102. Determine the solution set of $2 - 3(x + 4) < 5 - 3x$.

103. Determine the solution set of $3x + 2(x - 1) > 5(x + 1)$.

104. The average of two negative integers is less than or equal to -15. The smaller integer is 7 less than the larger integer. Find the greatest possible value for the smaller integer.

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2.5 Absolute Value Equations and Inequalities

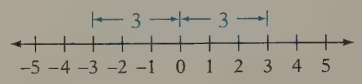
Objective A

To solve an absolute value equation

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The **absolute value** of a number is its distance from zero on the number line. Distance is always a positive number or zero. Therefore, the absolute value of a number is always a positive number or zero.

The distance from 0 to 3 or from 0 to −3 is 3 units.



$|3| = 3 \qquad |-3| = 3$

An equation that contains an absolute value symbol is called an **absolute value equation**. The solution of an absolute value equation is based on the following property:

If $a \geq 0$ and $|x| = a$, then $x = a$ or $x = -a$.

For instance, given $|x| = 3$, then $x = 3$ or $x = -3$, because $|3| = 3$ and $|-3| = 3$.

➡ Solve: $|x + 2| = 8$

$$|x + 2| = 8$$

$$x + 2 = 8 \qquad x + 2 = -8$$

- Remove the absolute value sign and rewrite as two equations.
- Solve each equation.

$$x = 6 \qquad x = -10$$

TAKE NOTE
Because the absolute value of $x + 2$ is 8, the number $x + 2$ is 8 units from 0 on the number line. Therefore, $x + 2$ is equal to 8 or −8.

Check:

$\lvert x + 2\rvert = 8$	
$\lvert 6 + 2\rvert$	8
$\lvert 8\rvert$	8
$8 = 8$	

$\lvert x + 2\rvert = 8$	
$\lvert -10 + 2\rvert$	8
$\lvert -8\rvert$	8
$8 = 8$	

The solutions are 6 and −10.

➡ Solve: $|5 - 3x| - 8 = -4$

$$|5 - 3x| - 8 = -4$$

$$|5 - 3x| = 4$$

- Solve for the absolute value.
- Remove the absolute value sign and rewrite as two equations.
- Solve each equation.

$$5 - 3x = 4 \qquad 5 - 3x = -4$$

$$-3x = -1 \qquad -3x = -9$$

$$x = \frac{1}{3} \qquad x = 3$$

Check:

$\lvert 5 - 3x\rvert - 8 = -4$	
$\left\lvert 5 - 3\left(\frac{1}{3}\right)\right\rvert - 8$	−4
$\lvert 5 - 1\rvert - 8$	−4
$4 - 8$	−4
$-4 = -4$	

$\lvert 5 - 3x\rvert - 8 = -4$	
$\lvert 5 - 3(3)\rvert - 8$	−4
$\lvert 5 - 9\rvert - 8$	−4
$4 - 8$	−4
$-4 = -4$	

The solutions are $\frac{1}{3}$ and 3.

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Example 1

Solve: $|2 - x| = 12$

Solution

$$|2 - x| = 12$$

$2 - x = 12$ | $2 - x = -12$

$-x = 10$ | $-x = -14$

$x = -10$ | $x = 14$

The solutions are -10 and 14.

You Try It 1

Solve: $|2x - 3| = 5$

Your solution

Example 2

Solve: $3 - |2x - 4| = -5$

Solution

$$3 - |2x - 4| = -5$$
$$-|2x - 4| = -8$$
$$|2x - 4| = 8$$

$2x - 4 = 8$ | $2x - 4 = -8$

$2x = 12$ | $2x = -4$

$x = 6$ | $x = -2$

The solutions are 6 and -2.

You Try It 2

Solve: $5 - |3x + 5| = 3$

Your solution

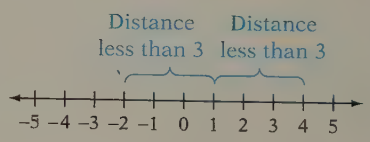
Solutions on p. S7

Objective B To solve an absolute value inequality

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Recall that absolute value represents the distance between two points. For example, the solutions of the absolute value equation $|x - 1| = 3$ are the numbers whose distance from 1 is 3. Therefore, the solutions are -2 and 4.

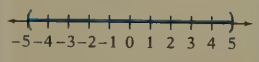
The solutions of the absolute value inequality $|x - 1| < 3$ are the numbers whose distance from 1 is less than 3. Therefore, the solutions are the numbers greater than -2 and less than 4. The solution set is $\{x \mid -2 < x < 4\}$.



To solve an absolute value inequality of the form $|ax + b| < c$, solve the equivalent compound inequality $-c < ax + b < c$.

TAKE NOTE

Because the absolute value of $3x - 1$ is less than 5, the number $3x - 1$ is less than 5 units from 0 on the number line.



Therefore, $3x - 1$ is between -5 and 5.

➡ Solve: $|3x - 1| < 5$

$$|3x - 1| < 5$$
$$-5 < 3x - 1 < 5$$

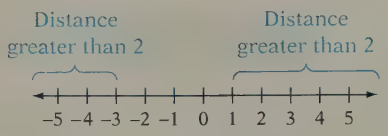
• Solve the equivalent compound inequality.

$$-5 + 1 < 3x - 1 + 1 < 5 + 1$$
$$-4 < 3x < 6$$
$$\frac{-4}{3} < \frac{3x}{3} < \frac{6}{3}$$
$$-\frac{4}{3} < x < 2$$

$$\left\{x \mid -\frac{4}{3} < x < 2\right\}$$

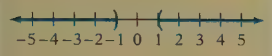
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The solutions of the absolute value inequality $|x + 1| > 2$ are the numbers whose distance from -1 is greater than 2. Therefore, the solutions are the numbers that are less than -3 or greater than 1. The solution set of $|x + 1| > 2$ is $\{x|x < -3 \text{ or } x > 1\}$.



TAKE NOTE

Because the absolute value of $3 - 2x$ is greater than 1, the number $3 - 2x$ is more than 1 unit from 0 on the number line.



Therefore, $3 - 2x$ is less than -1 or greater than 1.

To solve an absolute value inequality of the form $|ax + b| > c$, solve the equivalent compound inequality $ax + b < -c$ or $ax + b > c$.

Solve: $|3 - 2x| > 1$

$3 - 2x < -1$ or $3 - 2x > 1$

$-2x < -4$ $\qquad -2x > -2$ • Solve each inequality.

$x > 2$ $\qquad x < 1$

$\{x|x > 2\}$ $\qquad \{x|x < 1\}$

$\{x|x > 2\} \cup \{x|x < 1\}$ • Find the union of the solution sets of the two inequalities.

$= \{x|x > 2 \text{ or } x < 1\}$

Example 3 Solve: $|4x - 3| < 5$

Solution Solve the equivalent compound inequality.

$$-5 < 4x - 3 < 5$$
$$-5 + 3 < 4x - 3 + 3 < 5 + 3$$
$$-2 < 4x < 8$$
$$\frac{-2}{4} < \frac{4x}{4} < \frac{8}{4}$$
$$-\frac{1}{2} < x < 2$$
$$\left\{x \middle| -\frac{1}{2} < x < 2\right\}$$

You Try It 3 Solve: $|3x + 2| < 8$

Your solution

Example 4 Solve: $|2x - 1| > 7$

Solution Solve the equivalent compound inequality.

$2x - 1 < -7$ or $2x - 1 > 7$

$2x < -6$ $\qquad 2x > 8$

$x < -3$ $\qquad x > 4$

$\{x|x < -3\}$ $\qquad \{x|x > 4\}$

$\{x|x < -3\} \cup \{x|x > 4\}$

$= \{x|x < -3 \text{ or } x > 4\}$

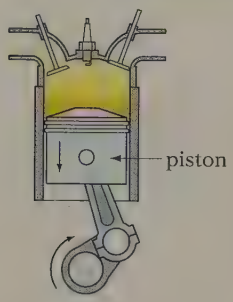
You Try It 4 Solve: $|5x + 3| > 8$

Your solution

Solutions on p. S7

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Objective C To solve application problems



The **tolerance** of a component, or part, is the acceptable amount by which the component may vary from a given measurement. For example, the diameter of a piston may vary from the given measurement of 9 cm by 0.001 cm. This is written 9 cm ± 0.001 cm and is read "9 centimeters plus or minus 0.001 centimeter." The maximum diameter, or **upper limit**, of the piston is 9 cm + 0.001 cm = 9.001 cm. The minimum diameter, or **lower limit**, is 9 cm − 0.001 cm = 8.999 cm.

The lower and upper limits of the diameter of the piston could also be found by solving the absolute value inequality $|d - 9| \le 0.001$, where d is the diameter of the piston.

$$|d - 9| \le 0.001$$
$$-0.001 \le d - 9 \le 0.001$$
$$-0.001 + 9 \le d - 9 + 9 \le 0.001 + 9$$
$$8.999 \le d \le 9.001$$

The lower and upper limits of the diameter of the piston are 8.999 cm and 9.001 cm.

Example 5

A doctor has prescribed 2 cc of medication for a patient. The tolerance is 0.03 cc. (In the medical field, cubic centimeter is usually abbreviated cc.) Find the lower and upper limits of the amount of medication to be given.

Strategy

Let p represent the prescribed amount of medication, T the tolerance, and m the given amount of medication. Solve the absolute value inequality $|m - p| \le T$ for m.

Solution

$|m - p| \le T$
$|m - 2| \le 0.03$

$$-0.03 \le m - 2 \le 0.03$$
$$-0.03 + 2 \le m - 2 + 2 \le 0.03 + 2$$
$$1.97 \le m \le 2.03$$

The lower and upper limits of the amount of medication to be given are 1.97 cc and 2.03 cc.

You Try It 5

A machinist must make a bushing that has a diameter of 2.55 in. The tolerance of the bushing is 0.003 in. Find the lower and upper limits of the diameter of the bushing.

Your strategy

Your solution

Solution on p. S7

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2.5 Exercises

Objective A

1. Is 2 a solution of $|x - 8| = 6$?

2. Is -2 a solution of $|2x - 5| = 9$?

3. Is -1 a solution of $|3x - 4| = 7$?

4. Is 1 a solution of $|6x - 1| = -5$?

Solve.

5. $|x| = 7$

6. $|a| = 2$

7. $|b| = 4$

8. $|c| = 12$

9. $|-y| = 6$

10. $|-t| = 3$

11. $|-a| = 7$

12. $|-x| = 3$

13. $|x| = -4$

14. $|y| = -3$

15. $|x + 2| = 3$

16. $|x + 5| = 2$

17. $|y - 5| = 3$

18. $|y - 8| = 4$

19. $|x - 2| = -4$

20. $|x + 8| = -2$

21. $|3 - 4x| = 9$

22. $|2 - 5x| = 3$

23. $|2x - 3| = 0$

24. $|5x + 5| = 0$

25. $|3x - 2| = -4$

26. $|2x + 5| = -2$

27. $|x - 2| - 2 = 3$

28. $|x - 9| - 3 = 2$

29. $|3a + 2| - 4 = 4$

30. $|2a + 9| + 4 = 5$

31. $|2 - y| + 3 = 4$

32. $|8 - y| - 3 = 1$

33. $|2x - 3| + 3 = 3$

34. $|4x - 7| - 5 = -5$

35. $|2x - 3| + 4 = -4$

36. $|3x - 2| + 1 = -1$

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37. $|6x - 5| - 2 = 4$

38. $|4b + 3| - 2 = 7$

39. $|3t + 2| + 3 = 4$

40. $|5x - 2| + 5 = 7$

41. $3 - |x - 4| = 5$

42. $2 - |x - 5| = 4$

43. $8 - |2x - 3| = 5$

44. $8 - |3x + 2| = 3$

45. $|2 - 3x| + 7 = 2$

46. $|1 - 5a| + 2 = 3$

47. $|8 - 3x| - 3 = 2$

48. $|6 - 5b| - 4 = 3$

49. $|2x - 8| + 12 = 2$

50. $|3x - 4| + 8 = 3$

51. $2 + |3x - 4| = 5$

52. $5 + |2x + 1| = 8$

53. $5 - |2x + 1| = 5$

54. $3 - |5x + 3| = 3$

55. $6 - |2x + 4| = 3$

56. $8 - |3x - 2| = 5$

57. $8 - |1 - 3x| = -1$

58. $3 - |3 - 5x| = -2$

59. $5 + |2 - x| = 3$

60. $6 + |3 - 2x| = 2$

Objective B

Solve.

61. $|x| > 3$

62. $|x| < 5$

63. $|x + 1| > 2$

64. $|x - 2| > 1$

65. $|x - 5| \leq 1$

66. $|x - 4| \leq 3$

67. $|2 - x| \geq 3$

68. $|3 - x| \geq 2$

69. $|2x + 1| < 5$

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70. $|3x - 2| < 4$

71. $|5x + 2| > 12$

72. $|7x - 1| > 13$

73. $|4x - 3| \leq -2$

74. $|5x + 1| \leq -4$

75. $|2x + 7| > -5$

76. $|3x - 1| > -4$

77. $|4 - 3x| \geq 5$

78. $|7 - 2x| > 9$

79. $|5 - 4x| \leq 13$

80. $|3 - 7x| < 17$

81. $|6 - 3x| \leq 0$

82. $|10 - 5x| \geq 0$

83. $|2 - 9x| > 20$

84. $|5x - 1| < 16$

85. $|2x - 3| + 2 < 8$

86. $|3x - 5| + 1 < 7$

87. $|2 - 5x| - 4 > -2$

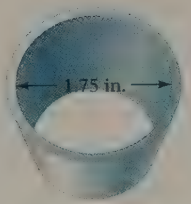
88. $|4 - 2x| - 9 > -3$

89. $8 - |2x - 5| < 3$

90. $12 - |3x - 4| > 7$

Objective C Application Problems

91. The diameter of a bushing is 1.75 in. The bushing has a tolerance of 0.008 in. Find the lower and upper limits of the diameter of the bushing.



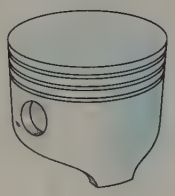
92. A machinist must make a bushing that has a tolerance of 0.004 in. The diameter of the bushing is 3.48 in. Find the lower and upper limits of the diameter of the bushing.

93. An electric motor is designed to run on 220 volts plus or minus 25 volts. Find the lower and upper limits of voltage on which the motor will run.

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94. A power strip is utilized on a computer to prevent the loss of programming by electrical surges. The power strip is designed to allow 110 volts plus or minus 16.5 volts. Find the lower and upper limits of voltage to the computer.

95. A piston rod for an automobile is $9\frac{5}{8}$ in. long with a tolerance of $\frac{1}{32}$ in. Find the lower and upper limits of the length of the piston rod.



96. The diameter of a piston for an automobile is $3\frac{5}{16}$ in. with a tolerance of $\frac{1}{64}$ in. Find the lower and upper limits of the diameter of the piston.

The tolerance of the resistors used in electronics is given as a percent. Use your calculator for the following exercises.

97. Find the lower and upper limits of a 29,000-ohm resistor with a 2% tolerance.

98. Find the lower and upper limits of a 15,000-ohm resistor with a 10% tolerance.

99. Find the lower and upper limits of a 25,000-ohm resistor with a 5% tolerance.

100. Find the lower and upper limits of a 56-ohm resistor with a 5% tolerance.

APPLYING THE CONCEPTS

101. For what values of the variable is the equation true? Write the solution set in set-builder notation.

a. $|x + 3| = x + 3$ **b.** $|a - 4| = 4 - a$

102. Write an absolute value inequality to represent all real numbers within 5 units of 2.

103. Replace the question mark with $\leq$, $\geq$, or $=$.

a. $|x + y| \;?\; |x| + |y|$ **b.** $|x - y| \;?\; |x| - |y|$

c. $||x| - |y|| \;?\; |x| - |y|$ **d.** $\left|\frac{x}{y}\right| ? \frac{|x|}{|y|}, y \neq 0$

e. $|xy| \;?\; |x||y|$

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Focus on Problem Solving

From Concrete to Abstract

In your study of algebra, you will find that the problems are less concrete than those you studied in arithmetic. Problems that are concrete provide information pertaining to a specific instance. Algebra is more abstract. Abstract problems are theoretical; they are stated without reference to a specific instance. Let's look at an example of an abstract problem.

How many minutes are in h hours?

A strategy that can be used to solve this problem is to solve the same problem after substituting a number for the variable.

How many minutes are in 5 hours?

You know that there are 60 minutes in 1 hour. To find the number of minutes in 5 hours, multiply 5 by 60.

$60 \cdot 5 = 300$ There are 300 minutes in 5 hours.

Use the same procedure to find the number of minutes in h hours: multiply h by 60.

$60 \cdot h = 60h$ There are $60h$ minutes in h hours.

This problem might be taken a step further:

If you walk 1 mile in x minutes, how far can you walk in h hours?

Consider the same problem using numbers in place of the variables.

If you walk 1 mile in 20 minutes, how far can you walk in 3 hours?

To solve this problem, you need to calculate the number of minutes in 3 hours (multiply 3 by 60), and divide the result by the number of minutes it takes to walk one mile (20 minutes).

$\frac{60 \cdot 3}{20} = \frac{180}{20} = 9$ If you walk 1 mile in 20 minutes, you can walk 9 miles in 3 hours.

Use the same procedure to solve the related abstract problem. Calculate the number of minutes in h hours (multiply h by 60), and divide the result by the number of minutes it takes to walk 1 mile (x minutes).

$\frac{60 \cdot h}{x} = \frac{60h}{x}$ If you walk 1 mile in x minutes, you can walk $\frac{60h}{x}$ miles in h hours.

At the heart of the study of algebra is the use of variables. It is the variables in the problems above that make them abstract. But it is variables that allow us to generalize situations and state rules about mathematics.

Try each of the following problems.

1. How many hours are in d days?
2. You earn d dollars an hour. What are your wages for working h hours?

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3. If p is the price of one share of stock, how many shares can you purchase with d dollars?
4. A company pays a television station d dollars to air a commercial lasting s seconds. What is the cost per second?
5. After every v videotape rentals, you are entitled to one free rental. You have rented t tapes, where $t < v$. How many more do you need to rent before you are entitled to a free rental?
6. Your car gets g miles per gallon. How many gallons of gasoline does your car consume traveling t miles?
7. If you drink j ounces of juice each day, how many days will q quarts of the juice last?
8. A TV station has m minutes of commercials each hour. How many ads lasting s seconds each can be sold for each hour of programming?
9. A factory worker can assemble p products in m minutes. How many products can the factory worker assemble in h hours?
10. If one candy bar costs n nickels, how many candy bars can be purchased with q quarters?

Projects and Group Activities

Prime and Composite Numbers

A **prime number** is a natural number greater than 1 whose only natural-number factors are itself and 1. The number 11 is a prime number because the only natural-number factors of 11 are 11 and 1.

Eratosthenes, a Greek philosopher and astronomer who lived from 270 to 190 B.C., devised a method of identifying prime numbers. It is called the **Sieve of Eratosthenes**. The procedure is illustrated below.

~~1~~	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	㊸	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	㊾	~~60~~
㊶	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	㊼	~~68~~	~~69~~	~~70~~
㊶	~~72~~	㊸	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	㊾	~~80~~
~~81~~	~~82~~	㊸	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	㊾	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	㊼	~~98~~	~~99~~	~~100~~

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List all the natural numbers from 1 to 100. Cross out the number 1, because it is not a prime number. The number 2 is prime; circle it. Cross out all the other multiples of 2 (4, 6, 8, . . .), because they are not prime. The number 3 is prime; circle it. Cross out all the other multiples of 3 (6, 9, 12, . . .) that are not already crossed out. The number 4, the next consecutive number in the list, has already been crossed out. The number 5 is prime; circle it. Cross out all the other multiples of 5 that are not already crossed out. Continue in this manner until all the prime numbers less than 100 are circled.

A **composite number** is a natural number greater than 1 that has a natural-number factor other than itself and 1. The number 21 is a composite number because it has factors of 3 and 7. All the numbers crossed out in the preceding table, except the number 1, are composite numbers.

1. Use the Sieve of Eratosthenes to find the prime numbers between 100 and 200.
2. How many prime numbers are even numbers?
3. Find the "twin primes" between 1 and 200. Twin primes are two prime numbers whose difference is 2. For instance, 3 and 5 are twin primes; 5 and 7 are also twin primes.
4. **a.** List two prime numbers that are consecutive natural numbers.
   b. Can there be any other pairs of prime numbers that are consecutive natural numbers?
5. Some primes are the sum of a square and 1. For example, $5 = 2^2 + 1$. Find another prime p such that $p = n^2 + 1$, where n is a natural number.
6. Find a prime number p such that $p = n^2 - 1$, where n is a natural number.
7. **a.** 4! (which is read "4 factorial") is equal to $4 \cdot 3 \cdot 2 \cdot 1$. Show that $4! + 2$, $4! + 3$, and $4! + 4$ are all composite numbers.
   b. 5! (which is read "5 factorial") is equal to $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Will $5! + 2$, $5! + 3$, $5! + 4$, and $5! + 5$ generate four consecutive composite numbers?
   c. Use the notation 6! to represent a list of five consecutive composite numbers.

Investigation into Operations with Even and Odd Integers

Complete each statement with the word *even* or *odd*.

1. If k is an odd integer, then $k + 1$ is an _____ integer.
2. If k is an odd integer, then $k - 2$ is an _____ integer.
3. If n is an integer, then $2n$ is an _____ integer.
4. If m and n are even integers, then $m - n$ is an _____ integer.
5. If m and n are even integers, then mn is an _____ integer.
6. If m and n are odd integers, then $m + n$ is an _____ integer.
7. If m and n are odd integers, then $m - n$ is an _____ integer.
8. If m and n are odd integers, then mn is an _____ integer.
9. If m is an even integer and n is an odd integer, then $m - n$ is an _____ integer.
10. If m is an even integer and n is an odd integer, then $m + n$ is an _____ integer.

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Chapter Summary

Key Words

An *equation* expresses the equality of two mathematical expressions. A *solution* of an equation is a number that, when substituted for the variable, results in a true equation. To *solve* an equation means to find a solution of the equation. [pp. 73, 74]

An *inequality* is an expression that contains the symbol $>$, $<$, $\geq$, or $\leq$. The *solution set of an inequality* is a set of numbers, each element of which, when substituted for the variable, results in a true inequality. [p. 113]

A *compound inequality* is formed by joining two inequalities with a connective word such as *and* or *or*. [p. 116]

The *absolute value* of a number is its distance from zero on the number line. An equation that contains an absolute value symbol is an *absolute value equation.* [p. 125]

Essential Rules

Addition Property of Equations [p. 74]

If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations [p. 75]

If $a = b$ and $c \neq 0$, then $ac = bc$.

Addition Property of Inequalities [p. 113]

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities [p. 114]

Rule 1
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
Rule 2
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Basic Percent Equation [p. 77]

$$\text{Percent} \cdot \text{base} = \text{amount}$$
$$P \cdot B = A$$

Value Mixture Equation [p. 97]

$$\text{Amount} \cdot \text{unit cost} = \text{value}$$
$$A \cdot C = V$$

Percent Mixture Equation [p. 99]

$$\text{Amount of solution} \cdot \text{percent of concentration} = \text{quantity of substance}$$
$$A \cdot r = Q$$

Annual Simple Interest Equation [p. 101]

$$\text{Principal} \cdot \text{interest rate} = \text{interest earned}$$
$$P \cdot r = I$$

Uniform Motion Equation [p. 103]

$$\text{Rate} \cdot \text{time} = \text{distance}$$
$$r \cdot t = d$$

To solve an absolute value inequality of the form $|ax + b| < c$, solve the equivalent compound inequality $-c < ax + b < c$. [p. 126]

To solve an absolute value inequality of the form $|ax + b| > c$, solve the equivalent compound inequality $ax + b < -c$ or $ax + b > c$. [p. 127]

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Chapter Review

1. Is 3 a solution of $5x - 2 = 4x + 5$?

2. Solve: $x + 3 = 24$

3. Solve: $\frac{3}{5}a = 12$

4. Solve: $-4x - 2 = 10$

5. Solve: $14x + 7x + 8 = -10$

6. Solve: $12y - 1 = 3y + 2$

7. Solve: $-6x + 16 = -2x$

8. Solve: $6x + 3(2x - 1) = -27$

9. Solve: $x + 5(3x - 20) = 10(x - 4)$

10. Solve: $3x - 7 > -2$

11. Solve: $4 - 3(x + 2) < 2(2x + 3) - 1$

12. Solve: $3x < 4$ and $x + 2 > -1$

13. Solve: $3x - 2 > x - 4$ or $7x - 5 < 3x + 3$

14. Solve: $|3 - 5x| = 12$

15. Solve: $|x - 4| - 8 = -3$

16. Solve: $|2x - 5| < 3$

17. Solve: $|4x - 5| \geq 3$

18. 30 is what percent of 12?

19. $\frac{1}{2}\%$ of what is 8?

20. Translate "four less than the product of five and a number is sixteen" into an equation and solve.

21. The sum of two numbers is twenty-one. Three times the smaller number is two less than twice the larger number. Translate into an equation. Then find the two numbers.

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22. An airline knowingly overbooks certain flights by selling 18% more tickets than there are available seats. How many tickets would this airline sell for an airplane that has 150 seats?

23. An auto manufacturer offers a rebate of $1000 on each car sold by a dealership. A customer bought a car from the dealership for $16,500. What percent of the cost is the $1000 rebate? Round to the nearest tenth of a percent.

24. A health food store combined cranberry juice that costs $2.79 per quart with apple juice that costs $2.19 per quart. How many quarts of each were used to make 10 qt of a cranapple juice mixture that costs $2.31 per quart?

25. Find the cost per ounce of a sun screen made from 100 oz of a lotion that costs $2.50 per ounce and 50 oz of a lotion that costs $4.00 per ounce.

26. A dairy mixed 5 gal of cream that is 30% butterfat with 8 gal of milk that is 4% butterfat. What is the percent concentration of butterfat in the resulting mixture?

27. An alloy containing 30% tin is mixed with an alloy containing 70% tin. How many pounds of each were used to make 500 lb of an alloy containing 40% tin?

28. An investment banker invested 45% of the bank's available cash in an account earning 8.5% annual simple interest. The remainder of the cash was placed in an account earning 10% annual simple interest. The interest earned in one year was $41,962.50. What was the total amount invested?

29. A club treasurer deposited $2400 into two simple interest accounts. On one account the annual simple interest rate was 6.75%. The annual simple interest rate on the other account was 9.45%. How much was deposited in each account if both accounts earned the same amount of interest?

30. A jet plane traveling at 600 mph overtakes a propeller-driven plane that had a 2-hour headstart. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven plane?

31. A bus traveled on a straight road for 2 h at an average speed that was 20 mph faster than it traveled on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 200 mi.

32. An average score of 80 to 90 in a psychology class receives a B grade. A student has grades of 92, 66, 72, and 88 on four tests. Find the range of scores on the fifth test that will earn the student a B for the course.

33. A doctor has prescribed 2 cc of medication for a patient. The tolerance is 0.25 cc. Find the lower and upper limits of the amount of medication to be given.

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Chapter Test

1. Solve: $3x - 2 = 5x + 8$

2. Solve: $x - 3 = -8$

3. Solve: $3x - 5 = -14$

4. Solve: $4 - 2(3 - 2x) = 2(5 - x)$

5. Is -2 a solution of $x^2 - 3x = 2x - 6$?

6. Solve: $7 - 4x = -13$

7. What is 0.5% of 8?

8. Solve: $5x - 2(4x - 3) = 6x + 9$

9. Solve: $5x + 3 - 7x = 2x - 5$

10. Solve: $\frac{3}{4}x = -9$

11. Solve: $3x - 2 \geq 6x + 7$

12. Solve: $4 - 3(x + 2) < 2(2x + 3) - 1$

13. Solve: $4x - 1 > 5$ or $2 - 3x < 8$

14. Solve: $4 - 3x \geq 7$ and $2x + 3 \geq 7$

15. Solve: $|3 - 5x| = 12$

16. Solve: $2 - |2x - 5| = -7$

17. Solve: $|3x - 5| \leq 4$

18. Solve: $|4x - 3| > 5$

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19. The sum of two integers is fifteen. Eight times the smaller integer is one less than three times the larger integer. Find the integers.

20. A butcher combines 100 lb of hamburger that costs \$2.40 per pound with 60 lb of hamburger that costs \$3.20 per pound. Find the cost of the hamburger mixture.

21. How many gallons of water must be mixed with 5 gal of a 20% salt solution to make a 16% salt solution?

22. A trust administrator divided \$20,000 between two accounts. One account earns an annual simple interest rate of 3%, and the second account earns an annual simple interest rate of 7%. The total annual income from the two accounts is \$1200. How much is invested in each account?

23. A cross-country skier leaves a camp to explore a wilderness area. Two hours later a friend leaves the camp in a snowmobile, traveling 4 mph faster than the skier, and meets the skier 1 h later. Find the rate of the snowmobile.

24. Gambelli Agency rents cars for \$24 a day and 10¢ for every mile driven. McDougal Rental rents cars for \$48 a day with unlimited mileage. How many miles a day can you drive a Gambelli Agency car if it is to cost you less than a McDougal Rental car?

25. A machinist must make a shoulder bushing that has a tolerance of 0.002 in. The outer diameter of the bushing is 2.65 in. Find the lower and upper limits of the outer diameter of the bushing.



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Cumulative Review

1. Subtract: $-6 - (-20) - 8$

2. Multiply: $(-2)(-6)(-4)$

3. Subtract: $-\frac{5}{6} - \left(-\frac{7}{16}\right)$

4. Simplify: $-4^2 \cdot \left(-\frac{3}{2}\right)^3$

5. Simplify: $\frac{5}{8} - \left(\frac{1}{2}\right)^2 \div \left(\frac{1}{3} - \frac{3}{4}\right)$

6. Evaluate $3(a - c) - 2ab$ when $a = 2$, $b = 3$, and $c = -4$.

7. Simplify: $3x - 8x + (-12x)$

8. Simplify: $2a - (-b) - 7a - 5b$

9. Simplify: $(16x)\left(\frac{1}{8}\right)$

10. Simplify: $-4(-9y)$

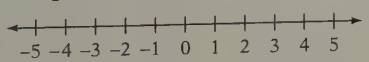
11. Simplify: $-2(-x^2 - 3x + 2)$

12. Simplify: $-2(x - 3) + 2(4 - x)$

13. Simplify: $-3[2x - 4(x - 3)] + 2$

14. Find $A \cap B$ given $A = \{-4, -2, 0, 2\}$ and $B = \{-4, 0, 4, 8\}$.

15. Graph: $\{x|x < 3\} \cap \{x|x > -2\}$



16. Is -3 a solution of $x^2 + 6x + 9 = x + 3$?

17. Solve: $\frac{3}{5}x = -15$

18. Solve: $7x - 8 = -29$

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19. Solve: $13 - 9x = -14$

20. Solve: $5x - 8 = 12x + 13$

21. Solve: $11 - 4x = 2x + 8$

22. Solve: $8x - 3(4x - 5) = -2x - 11$

23. Solve: $3 - 2(2x - 1) \geq 3(2x - 2) + 1$

24. Solve: $3x + 2 \leq 5$ and $x + 5 \geq 1$

25. Solve: $|3 - 2x| = 5$

26. Solve: $|3x - 1| > 5$

27. Write 55% as a fraction.

28. Write 1.03 as a percent.

29. 25% of what number is 30?

30. Translate "the sum of six times a number and thirteen is five less than the product of three and the number" into an equation and solve.

31. How many pounds of an oat flour that costs \$.80 per pound must be mixed with 40 lb of a wheat flour that costs \$.50 per pound to make a blend that costs \$.60 per pound?

32. How many grams of pure gold must be added to 100 g of a 20% gold alloy to make an alloy that is 36% gold?

33. A sprinter ran to the end of a track at an average rate of 8 m/s and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

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Chapter 3 Geometry

Objectives

Section 3.1

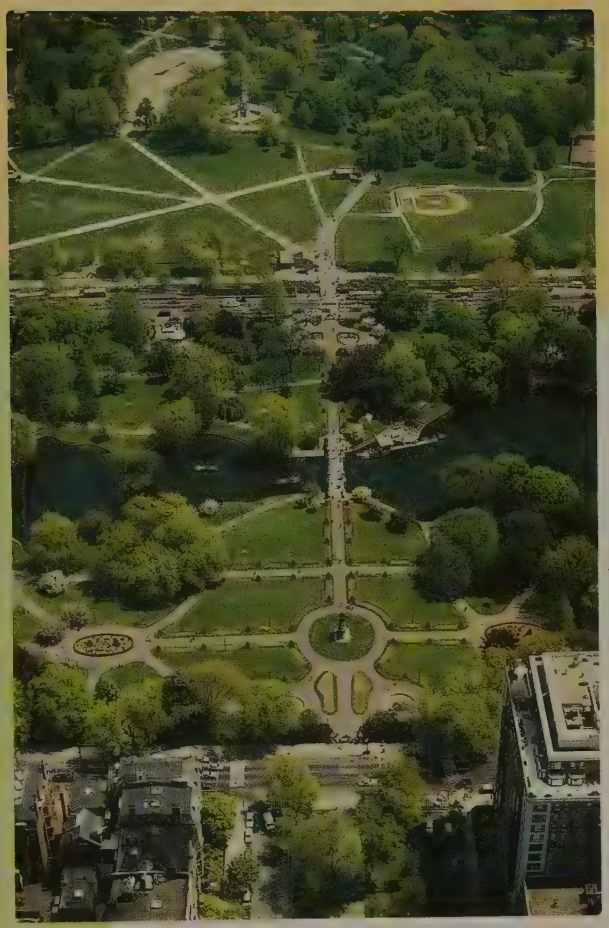
- **A** To solve problems involving lines and angles
- **B** To solve problems involving angles formed by intersecting lines
- **C** To solve problems involving the angles of a triangle

Section 3.2

- **A** To solve problems involving the perimeter of geometric figures
- **B** To solve problems involving the area of geometric figures

Section 3.3

- **A** To solve problems involving the volume of a solid
- **B** To solve problems involving the surface area of a solid



The best way to appreciate the different shapes and sizes of grassy areas in Boston's Public Garden is to view the garden from overhead. Each geometric shape, having its own set of dimensions, combines to form the entire park. **Exercise 98 on page 177** illustrates how to use a geometric formula to determine how large an area is and how much grass seed is needed for an area that size.



Need help? For online student resources, such as section quizzes, visit this textbook's website at **college.hmco.com/students.**

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Prep Test

1. Solve: $x + 47 = 90$

2. Solve: $32 + (97 + x) = 180$

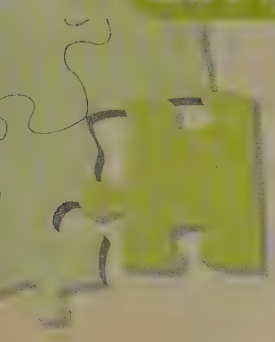
3. Simplify: $2(18) + 2(10)$

4. Evaluate abc when $a = 2$, $b = 3.14$, and $c = 9$.

5. Evaluate xyz^3 when $x = \frac{4}{3}$, $y = 3.14$, and $z = 3$.

6. Evaluate $\frac{1}{2}a(b + c)$ when $a = 6$, $b = 25$, and $c = 15$.

Go Figure



How can you cut a donut into 8 equal pieces with three cuts of the knife?

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3.1 Introduction to Geometry

Objective A **To solve problems involving lines and angles**

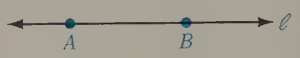
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Point of Interest

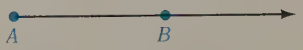
Geometry is one of the oldest branches of mathematics. Around 350 B.C., the Greek mathematician Euclid wrote the *Elements*, which contained all of the known concepts of geometry. Euclid's contribution was to unify various concepts into a single deductive system that was based on a set of axioms.

The word *geometry* comes from the Greek words for "earth" and "measure." The original purpose of geometry was to measure land. Today geometry is used in many fields, such as physics, medicine, and geology, and is applied in such areas as mechanical drawing and astronomy. Geometric forms are also used in art and design.

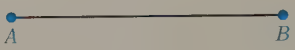
Three basic concepts of geometry are the point, line, and plane. A **point** is symbolized by drawing a dot. A **line** is determined by two distinct points and extends indefinitely in both directions, as the arrows on the line shown at the right indicate. This line contains points A and B and is represented by $\overleftrightarrow{AB}$. A line can also be represented by a single letter, such as ℓ.



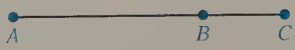
A **ray** starts at a point and extends indefinitely in *one* direction. The point at which a ray starts is called the **endpoint** of the ray. The ray shown at the right is denoted by $\overrightarrow{AB}$. Point A is the endpoint of the ray.



A **line segment** is part of a line and has two endpoints. The line segment shown at the right is denoted by $\overline{AB}$.



The distance between the endpoints of $\overline{AC}$ is denoted by AC. If B is a point on $\overline{AC}$, then AC (the distance from A to C) is the sum of AB (the distance from A to B) and BC (the distance from B to C).



$AC = AB + BC$

➡ Given $AB = 22$ cm and $AC = 31$ cm, find BC.

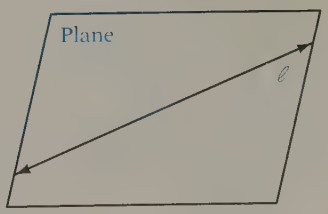
$AC = AB + BC$ • Write an equation for the distances between points on the line segment.

$31 = 22 + BC$ • Substitute the given distances for AB and AC into the equation.

$9 = BC$ • Solve for BC.

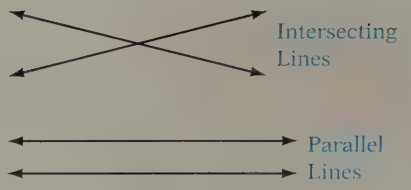
$BC = 9$ cm

In this section we will be discussing figures that lie in a plane. A **plane** is a flat surface and can be pictured as a table top or blackboard that extends in all directions. Figures that lie in a plane are called **plane figures**.

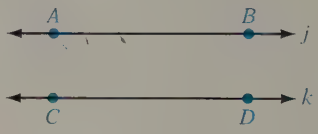


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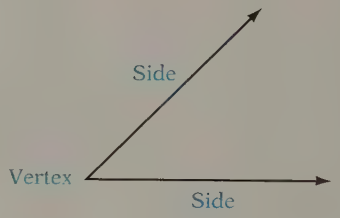
Lines in a plane can be intersecting or parallel. **Intersecting lines** cross at a point in the plane. **Parallel lines** never meet. The distance between them is always the same.



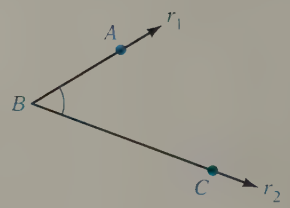
The symbol $\parallel$ means "is parallel to." In the figure at the right, $j \parallel k$ and $\overline{AB} \parallel \overline{CD}$. Note that j contains $\overline{AB}$ and k contains $\overline{CD}$. Parallel lines contain parallel line segments.



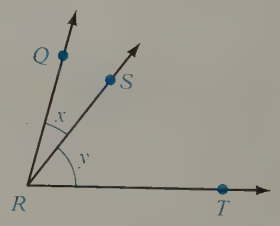
An **angle** is formed by two rays with the same endpoint. The **vertex** of the angle is the point at which the two rays meet. The rays are called the **sides** of the angle.



If A and C are points on rays r_1 and r_2, and B is the vertex, then the angle is called $\angle B$ or $\angle ABC$, where $\angle$ is the symbol for angle. Note that either the angle is named by the vertex, or the vertex is the second point listed when the angle is named by giving three points. $\angle ABC$ could also be called $\angle CBA$.



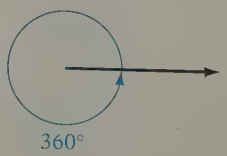
An angle can also be named by a variable written between the rays close to the vertex. In the figure at the right, $\angle x = \angle QRS$ and $\angle y = \angle SRT$. Note that in this figure, more than two rays meet at R. In this case, the vertex cannot be used to name an angle.



Point of Interest

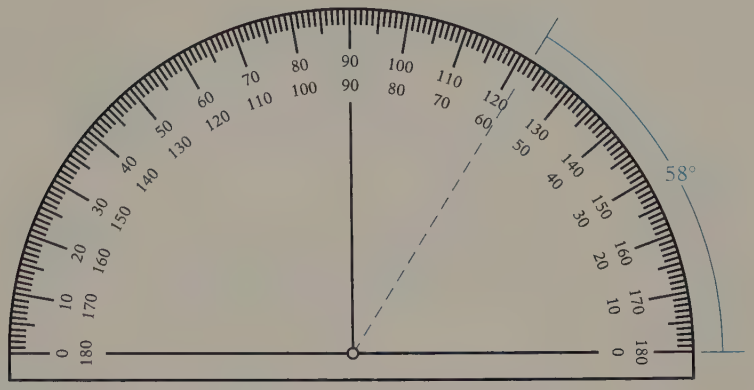
The first woman mathematician for whom documented evidence exists is Hypatia (370–415). She lived in Alexandria, Egypt, and lectured at the Museum, the forerunner of our modern university. She made important contributions in mathematics, astronomy, and philosophy.

An angle is measured in **degrees**. The symbol for degrees is a small raised circle, °. Probably because early Babylonians believed that Earth revolves around the sun in approximately 360 days, the angle formed by a circle has a measure of 360° (360 degrees).



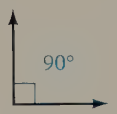
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A **protractor** is used to measure an angle. Place the center of the protractor at the vertex of the angle with the edge of the protractor along a side of the angle. The angle shown in the figure below measures 58°.

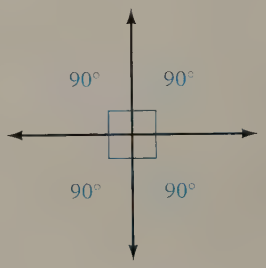


TAKE NOTE
The corner of a page of this book is a good example of a 90° angle.

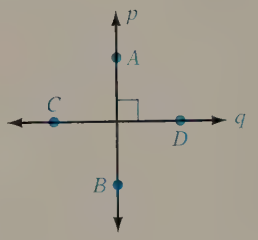
A 90° angle is called a **right angle**. The symbol ⌞ represents a right angle.



Perpendicular lines are intersecting lines that form right angles.



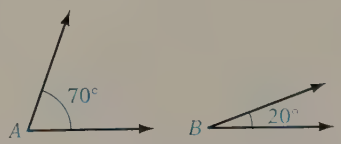
The symbol ⊥ means "is perpendicular to." In the figure at the right, $p \perp q$ and $\overline{AB} \perp \overline{CD}$. Note that line p contains $\overline{AB}$ and line q contains $\overline{CD}$. Perpendicular lines contain perpendicular line segments.



Complementary angles are two angles whose measures have the sum 90°.

$$\angle A + \angle B = 70° + 20° = 90°$$

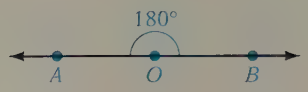
$\angle A$ and $\angle B$ are complementary angles.



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A 180° angle is called a **straight angle**.

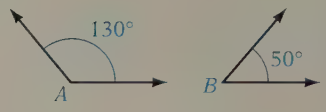
$\angle AOB$ is a straight angle.



Supplementary angles are two angles whose measures have the sum 180°.

$$\angle A + \angle B = 130° + 50° = 180°$$

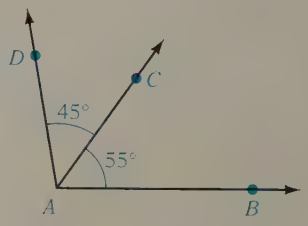
$\angle A$ and $\angle B$ are supplementary angles.



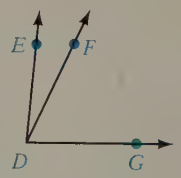
An **acute angle** is an angle whose measure is between 0° and 90°. $\angle B$ above is an acute angle. An **obtuse angle** is an angle whose measure is between 90° and 180°. $\angle A$ above is an obtuse angle.

Two angles that share a common side are **adjacent angles**. In the figure at the right, $\angle DAC$ and $\angle CAB$ are adjacent angles. $\angle DAC = 45°$ and $\angle CAB = 55°$.

$$\angle DAB = \angle DAC + \angle CAB$$
$$= 45° + 55° = 100°$$



In the figure at the right, $\angle EDG = 80°$. $\angle FDG$ is three times the measure of $\angle EDF$. Find the measure of $\angle EDF$.



Let x = the measure of $\angle EDF$. Then $3x$ = the measure of $\angle FDG$. Write an equation and solve for x, the measure of $\angle EDF$.

$$\angle EDF + \angle FDG = \angle EDG$$
$$x + 3x = 80$$
$$4x = 80$$
$$x = 20$$

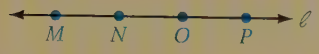
$$\angle EDF = 20°$$

TAKE NOTE

Answers to application problems must have units, such as degrees, feet, dollars, or hours.

Example 1

Given $MN = 15$ mm, $NO = 18$ mm, and $MP = 48$ mm, find OP.



Solution

$$MN + NO + OP = MP$$
$$15 + 18 + OP = 48$$
$$33 + OP = 48$$
$$OP = 15$$

$OP = 15$ mm

You Try It 1

Given $QR = 24$ cm, $ST = 17$ cm, and $QT = 62$ cm, find RS.



Your solution

Solution on p. S7

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Example 2

Given $XY = 9$ m and YZ is twice XY, find XZ.



Solution

$XZ = XY + YZ$
$XZ = XY + 2(XY)$
$XZ = 9 + 2(9)$
$XZ = 9 + 18$
$XZ = 27$

$XZ = 27$ m

You Try It 2

Given $BC = 16$ ft and $AB = \frac{1}{4}(BC)$, find AC.



Your solution

Example 3

Find the complement of a 38° angle.

Strategy

Complementary angles are two angles whose sum is 90°. To find the complement, let x represent the complement of a 38° angle. Write an equation and solve for x.

Solution

$x + 38° = 90°$
$x = 52°$

The complement of a 38° angle is a 52° angle.

You Try It 3

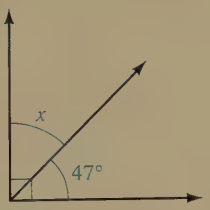
Find the supplement of a 129° angle.

Your strategy

Your solution

Example 4

Find the measure of $\angle x$.



Strategy

To find the measure of $\angle x$, write an equation using the fact that the sum of the measure of $\angle x$ and 47° is 90°. Solve for $\angle x$.

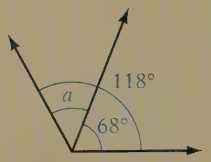
Solution

$\angle x + 47° = 90°$
$\angle x = 43°$

The measure of $\angle x$ is 43°.

You Try It 4

Find the measure of $\angle a$.



Your strategy

Your solution

Solutions on pp. S7–S8

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Objective B

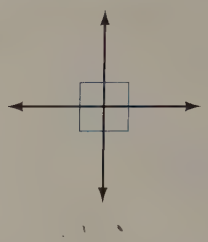
To solve problems involving angles formed by intersecting lines

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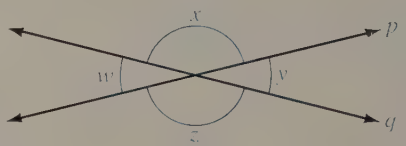
Point of Interest

Many cities in the New World, unlike those in Europe, were designed using rectangular street grids. Washington, D.C. was planned that way except that diagonal avenues were added, primarily for the purpose of enabling quick troop movement in the event the city required defense. As an added precaution, monuments of statuary were constructed at major intersections so that attackers would not have a straight shot down a boulevard.

Four angles are formed by the intersection of two lines. If the two lines are perpendicular, then each of the four angles is a right angle.



If the two lines are not perpendicular, then two of the angles formed are acute angles and two of the angles are obtuse angles. The two acute angles are always opposite each other, and the two obtuse angles are always opposite each other. In the figure at the right, $\angle w$ and $\angle y$ are acute angles. $\angle x$ and $\angle z$ are obtuse angles.



Two angles that are on opposite sides of the intersection of two lines are called **vertical angles**. Vertical angles have the same measure. $\angle w$ and $\angle y$ are vertical angles. $\angle x$ and $\angle z$ are vertical angles.

Vertical angles have the same measure.

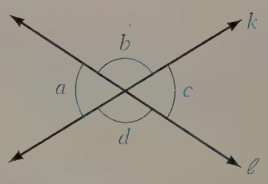
$$\angle w = \angle y$$
$$\angle x = \angle z$$

Two angles that share a common side are called **adjacent angles**. For the figure shown above, $\angle x$ and $\angle y$ are adjacent angles, as are $\angle y$ and $\angle z$, $\angle z$ and $\angle w$, and $\angle w$ and $\angle x$. Adjacent angles of intersecting lines are supplementary angles.

Adjacent angles of intersecting lines are supplementary angles.

$$\angle x + \angle y = 180°$$
$$\angle y + \angle z = 180°$$
$$\angle z + \angle w = 180°$$
$$\angle w + \angle x = 180°$$

➨ Given that $\angle c = 65°$, find the measures of angles a, b, and d.



$\angle a = 65°$

- $\angle a = \angle c$ because $\angle a$ and $\angle c$ are vertical angles.

$\angle b + \angle c = 180°$
$\angle b + 65° = 180°$
$\angle b = 115°$

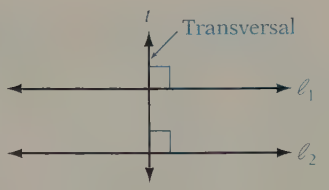
- $\angle b$ is supplementary to $\angle c$ because $\angle b$ and $\angle c$ are adjacent angles of intersecting lines.

$\angle d = 115°$

- $\angle d = \angle b$ because $\angle d$ and $\angle b$ are vertical angles.

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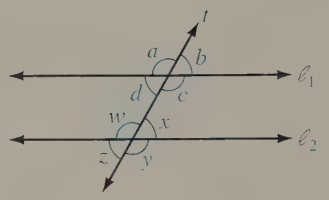
A line that intersects two other lines at different points is called a **transversal**.



If the lines cut by a transversal t are parallel lines and the transversal is perpendicular to the parallel lines, all eight angles formed are right angles.

If the lines cut by a transversal t are parallel lines and the transversal is not perpendicular to the parallel lines, all four acute angles have the same measure and all four obtuse angles have the same measure. For the figure at the right,

$$\angle b = \angle d = \angle x = \angle z$$
$$\angle a = \angle c = \angle w = \angle y$$



Alternate interior angles are two nonadjacent angles that are on opposite sides of the transversal and lie between the parallel lines. In the figure above, $\angle c$ and $\angle w$ are alternate interior angles; $\angle d$ and $\angle x$ are alternate interior angles. Alternate interior angles have the same measure.

Alternate interior angles have the same measure.

$$\angle c = \angle w$$
$$\angle d = \angle x$$

Alternate exterior angles are two nonadjacent angles that are on opposite sides of the transversal and lie outside the parallel lines. In the figure above, $\angle a$ and $\angle y$ are alternate exterior angles; $\angle b$ and $\angle z$ are alternate exterior angles. Alternate exterior angles have the same measure.

Alternate exterior angles have the same measure.

$$\angle a = \angle y$$
$$\angle b = \angle z$$

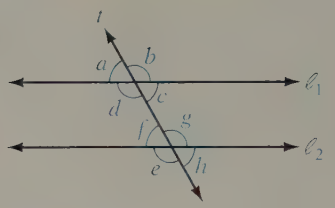
Corresponding angles are two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. For the figure above, the following pairs of angles are corresponding angles: $\angle a$ and $\angle w$, $\angle d$ and $\angle z$, $\angle b$ and $\angle x$, and $\angle c$ and $\angle y$. Corresponding angles have the same measure.

Corresponding angles have the same measure.

$$\angle a = \angle w$$
$$\angle d = \angle z$$
$$\angle b = \angle x$$
$$\angle c = \angle y$$

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Given that $\ell_1 \parallel \ell_2$ and $\angle c = 58°$, find the measures of $\angle f$, $\angle h$, and $\angle g$.



$\angle f = \angle c = 58°$ • $\angle c$ and $\angle f$ are alternate interior angles.

$\angle h = \angle c = 58°$ • $\angle c$ and $\angle h$ are corresponding angles.

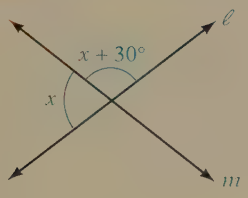
$\angle g + \angle h = 180°$ • $\angle g$ is supplementary to $\angle h$.

$\angle g + 58° = 180°$

$\angle g = 122°$

Example 5

Find x.



Strategy

The angles labeled are adjacent angles of intersecting lines and are, therefore, supplementary angles. To find x, write an equation and solve for x.

Solution

$x + (x + 30°) = 180°$

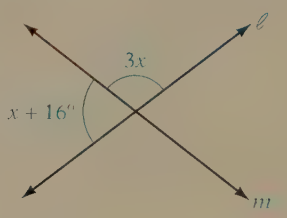
$2x + 30° = 180°$

$2x = 150°$

$x = 75°$

You Try It 5

Find x.

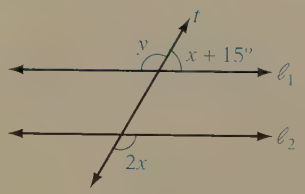


Your strategy

Your solution

Example 6

Given $\ell_1 \parallel \ell_2$, find x.



Strategy

$2x = y$ because alternate exterior angles have the same measure. $(x + 15°) + y = 180°$ because adjacent angles of intersecting lines are supplementary angles. Substitute $2x$ for y and solve for x.

Solution

$(x + 15°) + 2x = 180°$

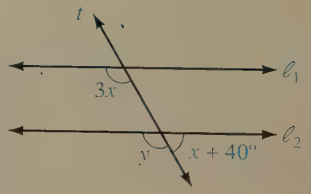
$3x + 15° = 180°$

$3x = 165°$

$x = 55°$

You Try It 6

Given $\ell_1 \parallel \ell_2$, find x.



Your strategy

Your solution

Solutions on p. S8

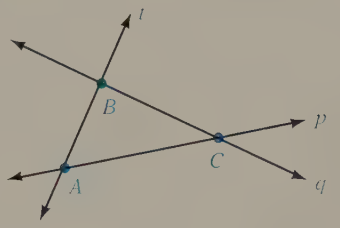
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Objective C

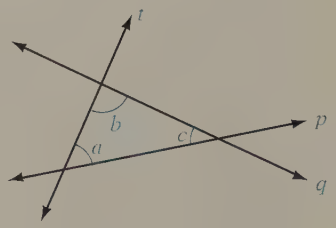
To solve problems involving the angles of a triangle

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If the lines cut by a transversal are not parallel lines, the three lines will intersect at three points. In the figure at the right, the transversal t intersects lines p and q. The three lines intersect at points A, B, and C. These three points define three line segments: $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. The plane figure formed by these three line segments is called a **triangle**.



Each of the three points of intersection is the vertex of four angles. The angles within the region enclosed by the triangle are called **interior angles**. In the figure at the right, angles a, b, and c are interior angles. The sum of the measures of the interior angles of a triangle is 180°.

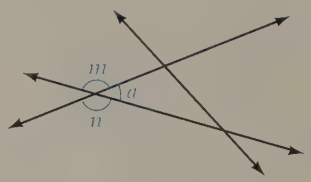


$\angle a + \angle b + \angle c = 180°$

The Sum of the Measures of the Interior Angles of a Triangle

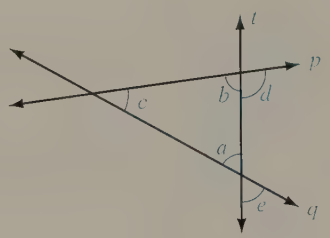
The sum of the measures of the interior angles of a triangle is 180°.

An angle adjacent to an interior angle is an **exterior angle**. In the figure at the right, angles m and n are exterior angles for angle a. The sum of the measures of an interior and an exterior angle is 180°.



$\angle a + \angle m = 180°$
$\angle a + \angle n = 180°$

Given that $\angle c = 40°$ and $\angle d = 100°$, find the measure of $\angle e$.



$\angle d$ and $\angle b$ are supplementary angles.

$$\begin{aligned} \angle d + \angle b &= 180° \\ 100° + \angle b &= 180° \\ \angle b &= 80° \end{aligned}$$

The sum of the interior angles is 180°.

$$\begin{aligned} \angle c + \angle b + \angle a &= 180° \\ 40° + 80° + \angle a &= 180° \\ 120° + \angle a &= 180° \\ \angle a &= 60° \end{aligned}$$

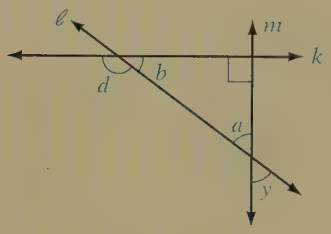
$\angle a$ and $\angle e$ are vertical angles.

$$\angle e = \angle a = 60°$$

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Example 7

Given that $\angle y = 55°$, find the measures of angles a, b, and d.



Strategy

- To find the measure of angle a, use the fact that $\angle a$ and $\angle y$ are vertical angles.
- To find the measure of angle b, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
- To find the measure of angle d, use the fact that the sum of an interior and an exterior angle is 180°.

Solution

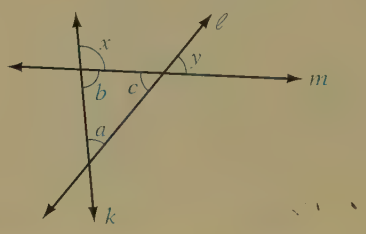
$\angle a = \angle y = 55°$

$\angle a + \angle b + 90° = 180°$
$55° + \angle b + 90° = 180°$
$\angle b + 145° = 180°$
$\angle b = 35°$

$\angle d + \angle b = 180°$
$\angle d + 35° = 180°$
$\angle d = 145°$

You Try It 7

Given that $\angle a = 45°$ and $\angle x = 100°$, find the measures of angles b, c, and y.



Your strategy

Your solution

Example 8

Two angles of a triangle measure 53° and 78°. Find the measure of the third angle.

Strategy

To find the measure of the third angle, use the fact that the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution

$x + 53° + 78° = 180°$
$x + 131° = 180°$
$x = 49°$

The measure of the third angle is 49°.

You Try It 8

One angle in a triangle is a right angle, and one angle measures 34°. Find the measure of the third angle.

Your strategy

Your solution

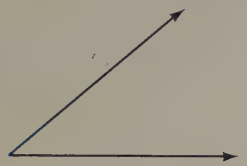
Solutions on p. S8

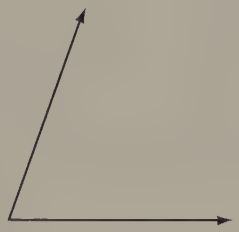
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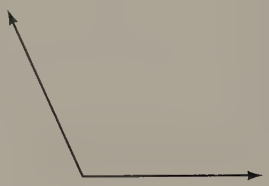
3.1 Exercises

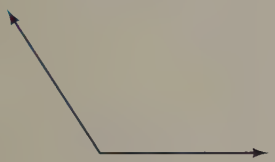
Objective A

Use a protractor to measure the angle. State whether the angle is acute, obtuse, or right.

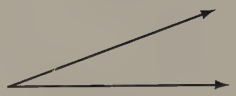
1. 

2. 

3. 

4. 

5. 

6. 

Solve.

7. Find the complement of a 62° angle.

8. Find the complement of a 31° angle.

9. Find the supplement of a 162° angle.

10. Find the supplement of a 72° angle.

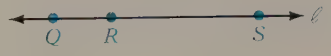
11. Given $AB = 12$ cm, $CD = 9$ cm, and $AD = 35$ cm, find the length of BC.



12. Given $AB = 21$ mm, $BC = 14$ mm, and $AD = 54$ mm, find the length of CD.



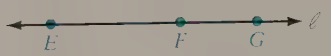
13. Given $QR = 7$ ft and RS is three times the length of QR, find the length of QS.



14. Given $QR = 15$ in. and RS is twice the length of QR, find the length of QS.



15. Given $EF = 20$ m and FG is $\frac{1}{2}$ the length of EF, find the length of EG.

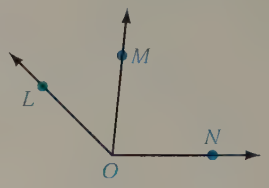


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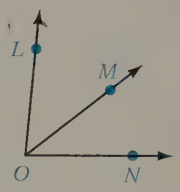
16. Given $EF = 18$ cm and FG is $\frac{1}{3}$ the length of EF, find the length of EG.



17. Given $\angle LOM = 53°$ and $\angle LON = 139°$, find the measure of $\angle MON$.

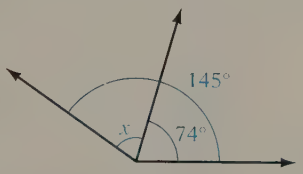


18. Given $\angle MON = 38°$ and $\angle LON = 85°$, find the measure of $\angle LOM$.

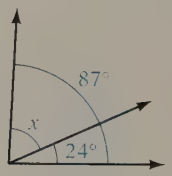


Find the measure of $\angle x$.

19.

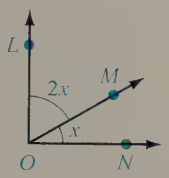


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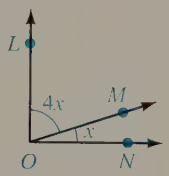


Given that $\angle LON$ is a right angle, find the measure of $\angle x$.

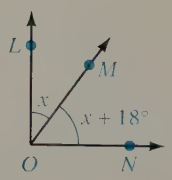
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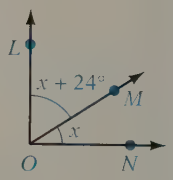
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23.

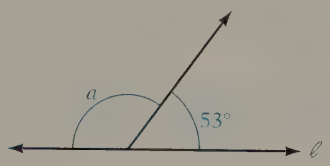


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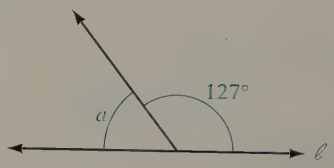


Find the measure of $\angle a$.

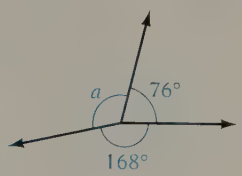
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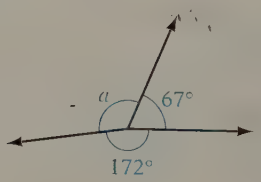
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27.



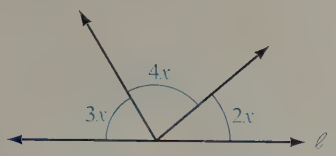
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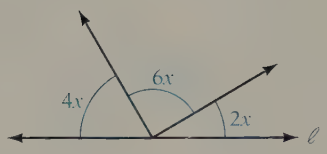
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Find x.

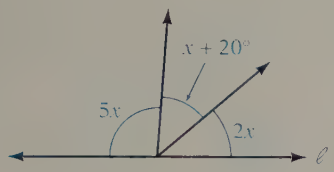
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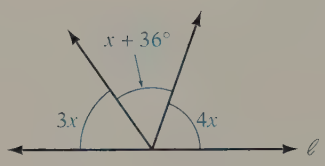
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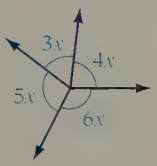
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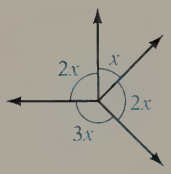
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33.

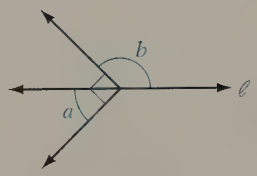


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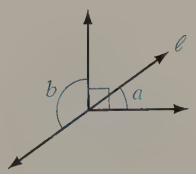


Solve.

35. Given $\angle a = 51°$, find the measure of $\angle b$.



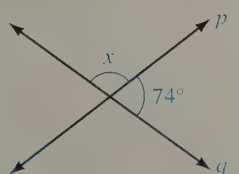
36. Given $\angle a = 38°$, find the measure of $\angle b$.



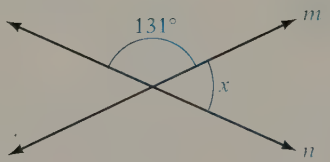
Objective B

Find the measure of $\angle x$.

37.



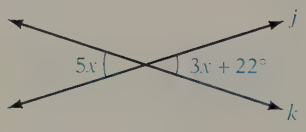
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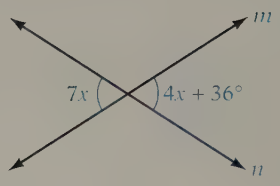
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Find x.

39.

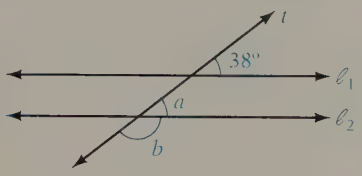


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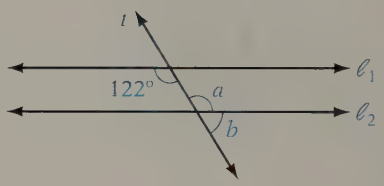


Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.

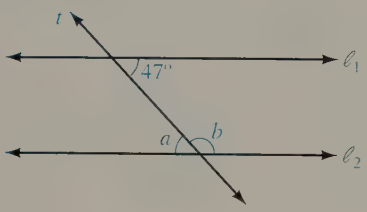
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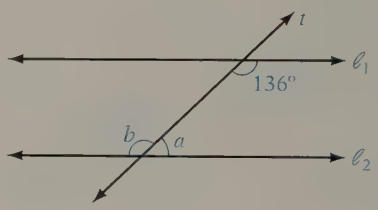
42.



43.

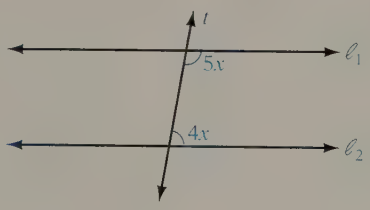


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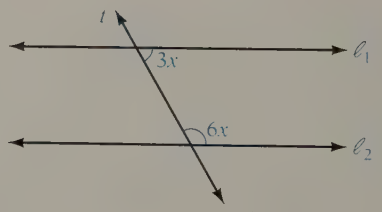


Given that $\ell_1 \parallel \ell_2$, find x.

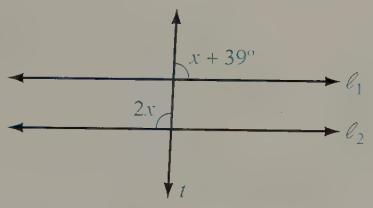
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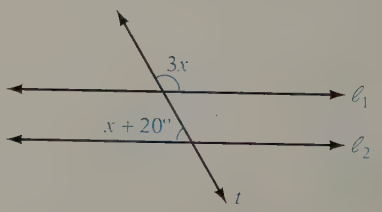
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47.



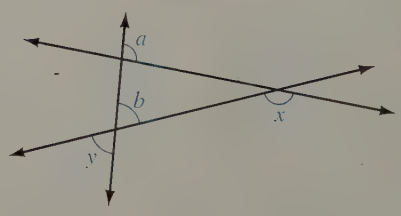
48.



Objective C

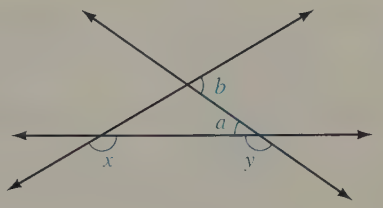
Solve.

49. Given that $\angle a = 95°$ and $\angle b = 70°$, find the measures of angles x and y.

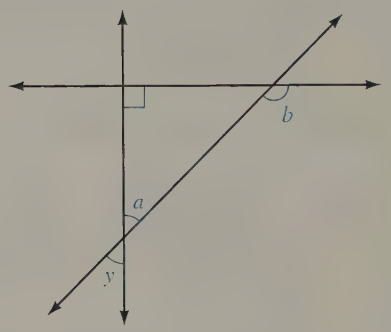


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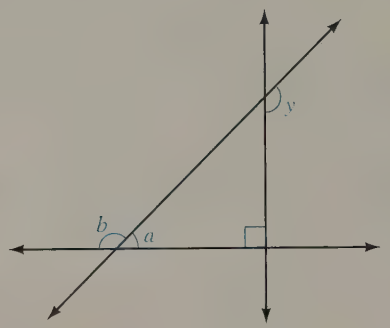
50. Given that $\angle a = 35°$ and $\angle b = 55°$, find the measures of angles x and y.



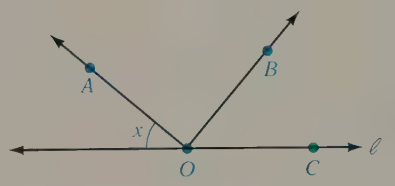
51. Given that $\angle y = 45°$, find the measures of angles a and b.



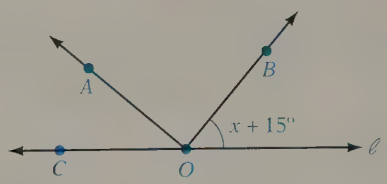
52. Given that $\angle y = 130°$, find the measures of angles a and b.



53. Given that $\overline{AO} \perp \overline{OB}$, express in terms of x the number of degrees in $\angle BOC$.



54. Given that $\overline{AO} \perp \overline{OB}$, express in terms of x the number of degrees in $\angle AOC$.



55. One angle in a triangle is a right angle, and one angle is equal to 30°. What is the measure of the third angle?

56. A triangle has a 45° angle and a right angle. Find the measure of the third angle.

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57. Two angles of a triangle measure 42° and 103°. Find the measure of the third angle.

58. Two angles of a triangle measure 62° and 45°. Find the measure of the third angle.

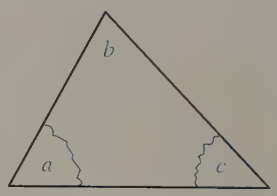
59. A triangle has a 13° angle and a 65° angle. What is the measure of the third angle?

60. A triangle has a 105° angle and a 32° angle. What is the measure of the third angle?

APPLYING THE CONCEPTS

61. **a.** What is the smallest possible whole number of degrees in an angle of a triangle?
b. What is the largest possible whole number of degrees in an angle of a triangle?

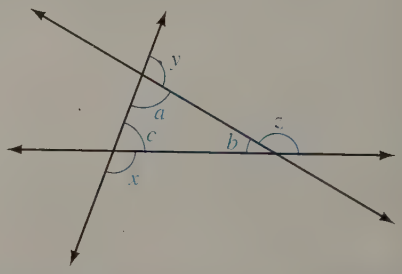
62. Cut out a triangle and then tear off two of the angles, as shown at the right. Position the pieces you tore off so that angle a is adjacent to angle b and angle c is adjacent to angle b (on the other side). Describe what you observe. What does this demonstrate?



63. Construct a triangle with the given angle measures.
a. 45°, 45°, and 90°
b. 30°, 60°, and 90°
c. 40°, 40°, and 100°

64. Determine whether the statement is always true, sometimes true, or never true.
a. Two lines that are parallel to a third line are parallel to each other.
b. A triangle contains two acute angles.
c. Vertical angles are complementary angles.

65. For the figure at the right, find the sum of the measures of angles x, y, and z.



66. For the figure at the right, explain why $\angle a + \angle b = \angle x$. Write a rule that describes the relationship between an exterior angle of a triangle and the opposite interior angles. Use the rule to write an equation involving angles a, c, and z.

67. If $\overline{AB}$ and $\overline{CD}$ intersect at point O, and $\angle AOC = \angle BOC$, explain why $\overline{AB} \perp \overline{CD}$.

68. Do some research on the principle of reflection. Explain how this principle applies to the operation of a periscope and to the game of billiards.



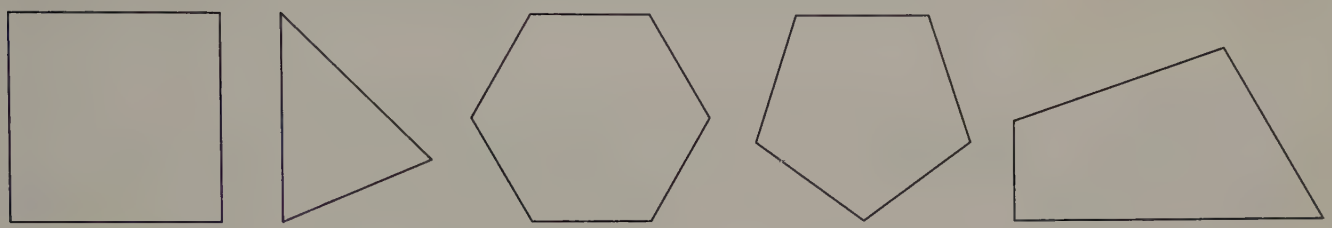
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3.2 Plane Geometric Figures

Objective A **To solve problems involving the perimeter of geometric figures**

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A **polygon** is a closed figure determined by three or more line segments that lie in a plane. The line segments that form the polygon are called its **sides**. The figures below are examples of polygons.



A *B* *C* *D* *E*

Point of Interest

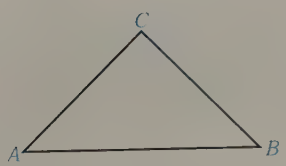
Although a polygon is defined in terms of its *sides* (see the definition above), the word actually comes from the Latin word *polygonum*, which means "having many *angles*." This is certainly the case for a polygon.

A **regular polygon** is one in which each side has the same length and each angle has the same measure. The polygons in Figures *A*, *C*, and *D* above are regular polygons.

The name of a polygon is based on the number of its sides. The table below lists the names of polygons that have from 3 to 10 sides.

Number of Sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

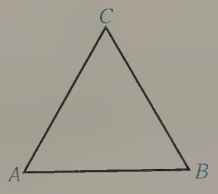
Triangles and quadrilaterals are two of the most common types of polygons. Triangles are distinguished by the number of equal sides and also by the measures of their angles.



An **isosceles triangle** has two sides of equal length. The angles opposite the equal sides are of equal measure.

$AC = BC$

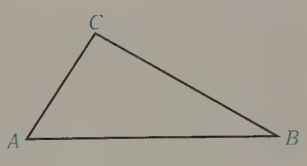
$\angle A = \angle B$



The three sides of an **equilateral triangle** are of equal length. The three angles are of equal measure.

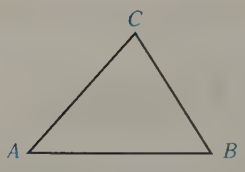
$AB = BC = AC$

$\angle A = \angle B = \angle C$

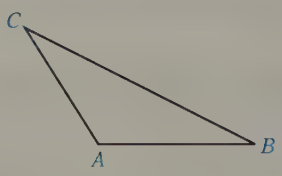


A **scalene triangle** has no two sides of equal length. No two angles are of equal measure.

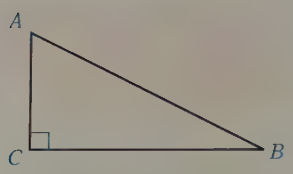
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An **acute triangle** has three acute angles.



An **obtuse triangle** has one obtuse angle.

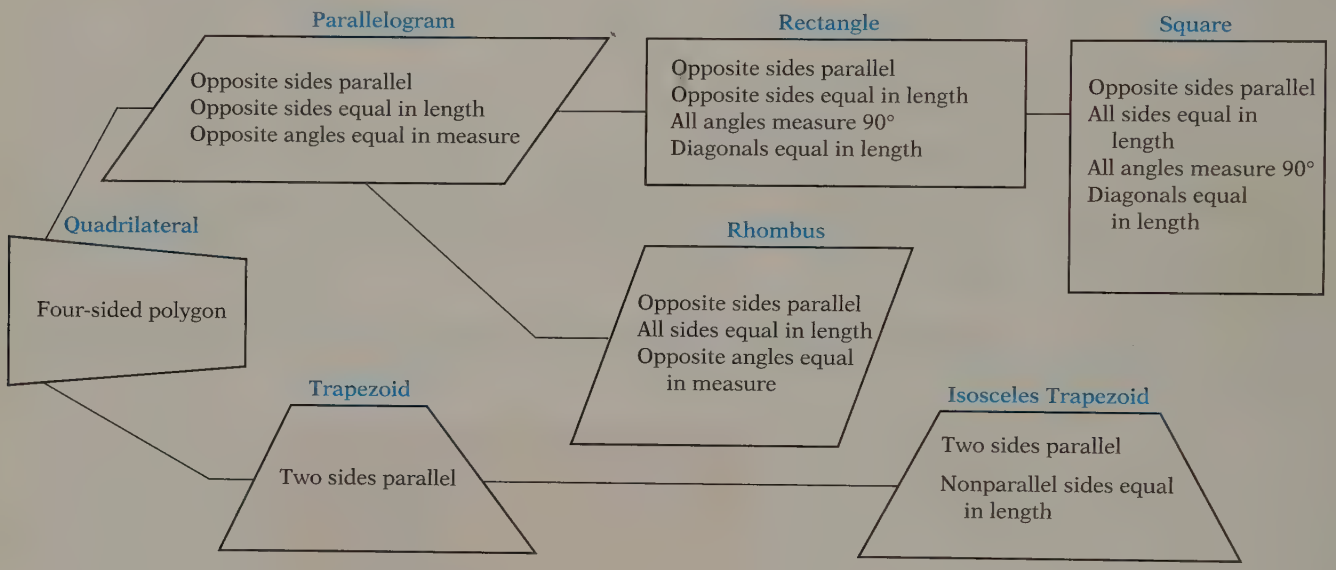


A **right triangle** has a right angle.

TAKE NOTE

The diagram below shows the relationships among all quadrilaterals. The description of each quadrilateral is within an example of that quadrilateral.

Quadrilaterals are also distinguished by their sides and angles, as shown below. Note that a rectangle, a square, and a rhombus are different forms of a parallelogram.

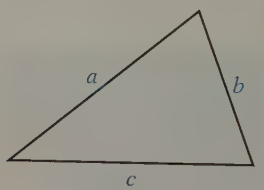


The **perimeter** of a plane geometric figure is a measure of the distance around the figure. Perimeter is used in buying fencing for a lawn or determining how much baseboard is needed for a room.

The perimeter of a triangle is the sum of the length of the three sides.

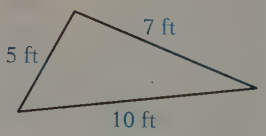
Perimeter of a Triangle

Let a, b, and c be the lengths of the sides of a triangle. The perimeter, P, of the triangle is given by $P = a + b + c$.



$P = a + b + c$

➡ Find the perimeter of the triangle shown at the right.



$P = 5 + 7 + 10 = 22$

The perimeter is 22 ft.

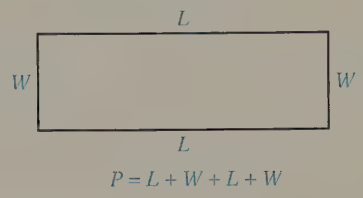
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The perimeter of a quadrilateral is the sum of the lengths of its four sides.

Point of Interest

Leonardo DaVinci painted the *Mona Lisa* on a rectangular canvas whose height was approximately 1.6 times its width. Rectangles with these proportions, called golden rectangles, were used extensively in Renaissance art.

A rectangle is a quadrilateral with opposite sides of equal length. Usually the length, L, of a rectangle refers to the length of one of the longer sides of the rectangle, and the width, W, refers to the length of one of the shorter sides. The perimeter can then be represented $P = L + W + L + W$.



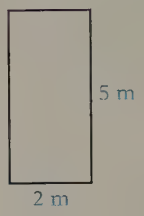
The formula for the perimeter of a rectangle is derived by combining like terms.

$$P = 2L + 2W$$

Perimeter of a Rectangle

Let L represent the length and W the width of a rectangle. The perimeter, P, of the rectangle is given by $P = 2L + 2W$.

➨ Find the perimeter of the rectangle shown at the right.



$P = 2L + 2W$

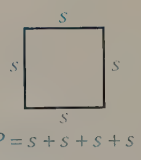
$P = 2(5) + 2(2)$ • The length is 5 m. Substitute 5 for L. The width is 2 m. Substitute 2 for W.

$P = 10 + 4$ • Solve for P.

$P = 14$

The perimeter is 14 m.

A square is a rectangle in which each side has the same length. Let s represent the length of each side of a square. Then the perimeter of a square can be represented $P = s + s + s + s$.



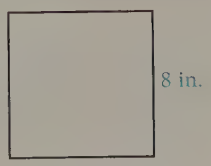
The formula for the perimeter of a square is derived by combining like terms.

$$P = 4s$$

Perimeter of a Square

Let s represent the length of a side of a square. The perimeter, P, of the square is given by $P = 4s$.

➨ Find the perimeter of the square shown at the right.



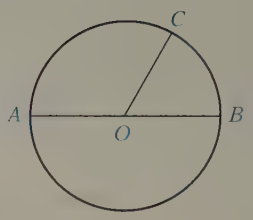
$P = 4s = 4(8) = 32$

The perimeter is 32 in.

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A **circle** is a plane figure in which all points are the same distance from point O, which is called the **center** of the circle.

The **diameter** of a circle is a line segment across the circle through point O. AB is a diameter of the circle at the right. The variable d is used to designate the diameter of a circle.



The **radius** of a circle is a line segment from the center of the circle to a point on the circle. OC is a radius of the circle at the right. The variable r is used to designate a radius of a circle.

The length of the diameter is twice the length of the radius.

$$d = 2r \text{ or } r = \frac{1}{2}d$$

The distance around a circle is called the **circumference**. The circumference, C, of a circle is equal to the product of π (pi) and the diameter.

$$C = \pi d$$

Because $d = 2r$, the formula for the circumference can be written in terms of r.

$$C = 2\pi r$$

Point of Interest

Archimedes (c. 287–212 B.C.) was the mathematician who gave us the approximate value of π as $\frac{22}{7} = 3\frac{1}{7}$. He actually showed that π was between $3\frac{10}{71}$ and $3\frac{1}{7}$. The approximation $3\frac{10}{71}$ is closer to the exact value of π, but it is more difficult to use.

The Circumference of a Circle

The circumference, C, of a circle with diameter d and radius r is given by $C = \pi d$ or $C = 2\pi r$.

The formula for circumference uses the number π, which is an irrational number. The value of π can be approximated by a fraction or by a decimal.

$$\pi \approx \frac{22}{7} \text{ or } \pi \approx 3.14$$

The π key on a scientific calculator gives a closer approximation of π than 3.14. Use a scientific calculator to find approximate values in calculations involving π.

➡ Find the circumference of a circle with a diameter of 6 in.

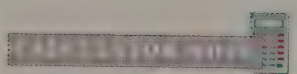
$C = \pi d$ • The diameter of the circle is given. Use the circumference formula that involves the diameter. $d = 6$.

$C = \pi(6)$

$C = 6\pi$ • The exact circumference of the circle is 6π in.

$C \approx 18.85$ • An approximate measure is found by using the π key on a calculator.

The circumference is approximately 18.85 in.



The π key on your calculator can be used to find decimal approximations for expressions that contain π. To perform the calculation at the right, enter $6 \boxed{\times} \boxed{\pi} \boxed{=}$.

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Example 1

A carpenter is designing a square patio with a perimeter of 44 ft. What is the length of each side?

Strategy

To find the length of each side, use the formula for the perimeter of a square. Substitute 44 for P and solve for s.

Solution

$P = 4s$
$44 = 4s$
$11 = s$

The length of each side of the patio is 11 ft.

You Try It 1

The infield for a softball field is a square with each side of length 60 ft. Find the perimeter of the infield.

Your strategy

Your solution

Example 2

The dimensions of a triangular sail are 18 ft, 11 ft, and 15 ft. What is the perimeter of the sail?

Strategy

To find the perimeter, use the formula for the perimeter of a triangle. Substitute 18 for a, 11 for b, and 15 for c. Solve for P.

Solution

$P = a + b + c$
$P = 18 + 11 + 15$
$P = 44$

The perimeter of the sail is 44 ft.

You Try It 2

What is the perimeter of a standard piece of typing paper that measures $8\frac{1}{2}$ in. by 11 in.?

Your strategy

Your solution

Example 3

Find the circumference of a circle with a radius of 15 cm. Round to the nearest hundredth.

Strategy

To find the circumference, use the circumference formula that involves the radius. An approximation is asked for; use the π key on a calculator. $r = 15$.

Solution

$C = 2\pi r = 2\pi(15) = 30\pi \approx 94.25$

The circumference is 94.25 cm.

You Try It 3

Find the circumference of a circle with a diameter of 9 in. Give the exact measure.

Your strategy

Your solution

Solutions on pp. S8–S9

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Objective B

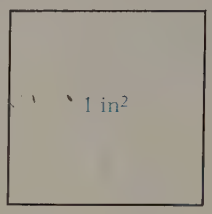
To solve problems involving the area of geometric figures

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Area is the amount of surface in a region. Area can be used to describe the size of a rug, a parking lot, a farm, or a national park. Area is measured in square units.

A square that measures 1 in. on each side has an area of 1 square inch, written 1 in^2.

A square that measures 1 cm on each side has an area of 1 square centimeter, written 1 cm^2.





Point of Interest

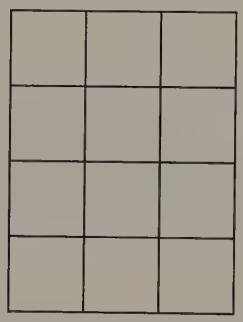
Polygonal numbers are whole numbers that can be represented as regular geometric figures. For example, a square number is one that can be represented as a square array.



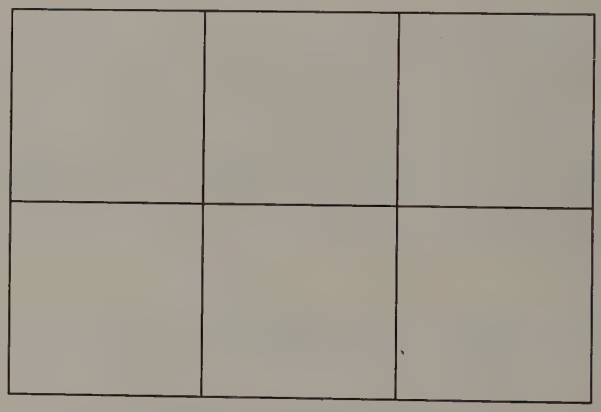
The square numbers are 1, 4, 9, 16, 25, . . . They can be represented as 1^2, 2^2, 3^2, 4^2, 5^2, . . .

Larger areas can be measured in square feet (ft^2), square meters (m^2), square miles (mi^2), acres (43,560 ft^2), or any other square unit.

The area of a geometric figure is the number of squares that are necessary to cover the figure. In the figures below, two rectangles have been drawn and covered with squares. In the figure on the left, 12 squares, each of area 1 cm^2, were used to cover the rectangle. The area of the rectangle is 12 cm^2. In the figure on the right, 6 squares, each of area 1 in^2, were used to cover the rectangle. The area of the rectangle is 6 in^2.



The area of the rectangle is 12 cm^2.



The area of the rectangle is 6 in^2.

Note from the above figures that the area of a rectangle can be found by multiplying the length of the rectangle by its width.

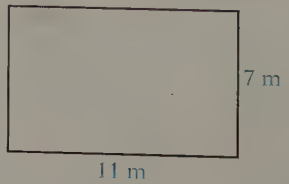
Area of a Rectangle

Let L represent the length and W the width of a rectangle. The area, A, of the rectangle is given by $A = LW$.

➡ Find the area of the rectangle shown at the right.

$A = LW = 11(7) = 77$

The area is 77 m^2.



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A square is a rectangle in which all sides are the same length. Therefore, both the length and the width of a square can be represented by s, and $A = LW = s \cdot s = s^2$.

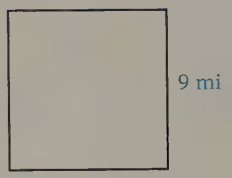
Area of a Square

Let s represent the length of a side of a square. The area, A, of the square is given by $A = s^2$.



$A = s \cdot s = s^2$

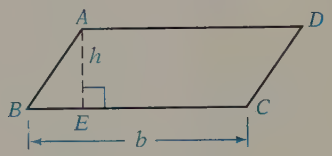
➡ Find the area of the square shown at the right.



$A = s^2 = 9^2 = 81$

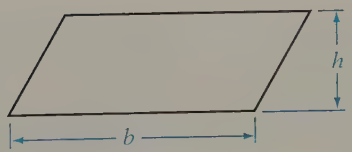
The area is 81 mi^2.

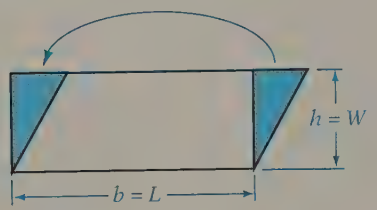
Figure $ABCD$ is a parallelogram. BC is the **base**, b, of the parallelogram. AE, perpendicular to the base, is the **height**, h, of the parallelogram.



Any side of a parallelogram can be designated as the base. The corresponding height is found by drawing a line segment perpendicular to the base from the opposite side.

A rectangle can be formed from a parallelogram by cutting a right triangle from one end of the parallelogram and attaching it to the other end. The area of the resulting rectangle will equal the area of the original parallelogram.

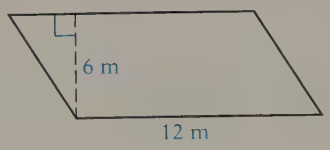




Area of a Parallelogram

Let b represent the length of the base and h the height of a parallelogram. The area, A, of the parallelogram is given by $A = bh$.

➡ Find the area of the parallelogram shown at the right.

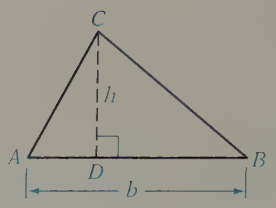


$A = bh = 12 \cdot 6 = 72$

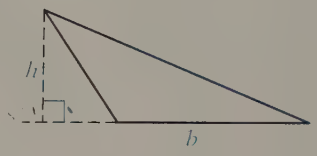
The area is 72 m^2.

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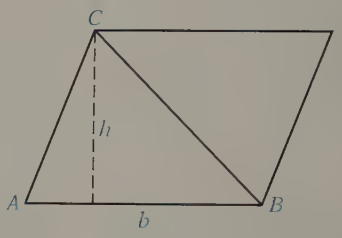
Figure ABC is a triangle. AB is the **base**, b, of the triangle. CD, perpendicular to the base, is the **height**, h, of the triangle.



Any side of a triangle can be designated as the base. The corresponding height is found by drawing a line segment perpendicular to the base from the vertex opposite the base.



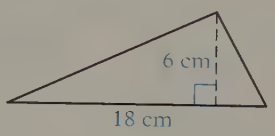
Consider the triangle with base b and height h shown at the right. By extending a line from C parallel to the base AB and equal in length to the base, and extending a line from B parallel to AC and equal in length to AC, a parallelogram is formed. The area of the parallelogram is bh and is twice the area of the triangle. Therefore, the area of the triangle is one-half the area of the parallelogram, or $\frac{1}{2}bh$.



Area of a Triangle

Let b represent the length of the base and h the height of a triangle. The area, A, of the triangle is given by $A = \frac{1}{2}bh$.

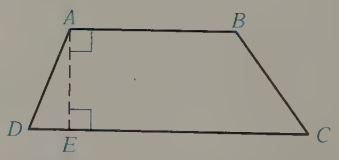
➡ Find the area of a triangle with a base of 18 cm and a height of 6 cm.



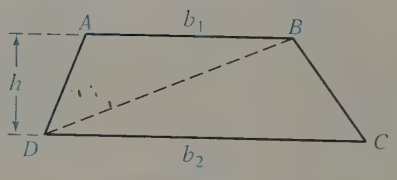
$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 18 \cdot 6 = 54$$

The area is 54 cm^2.

Figure $ABCD$ is a trapezoid. AB is one **base**, b_1, of the trapezoid, and CD is the other base, b_2. AE, perpendicular to the two bases, is the **height**, h.



In the trapezoid at the right, the line segment BD divides the trapezoid into two triangles, ABD and BCD. In triangle ABD, b_1 is the base and h is the height. In triangle BCD, b_2 is the base and h is the height. The area of the trapezoid is the sum of the areas of the two triangles.



Area of trapezoid $ABCD$ = area of triangle ABD + area of triangle BCD

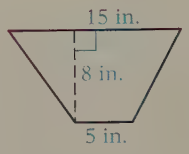
$$= \frac{1}{2}b_1h + \frac{1}{2}b_2h = \frac{1}{2}h(b_1 + b_2)$$

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Area of a Trapezoid

Let b_1 and b_2 represent the lengths of the bases and h the height of a trapezoid. The area, A, of the trapezoid is given by $A = \frac{1}{2}h(b_1 + b_2)$.

Find the area of a trapezoid that has bases measuring 15 in. and 5 in. and a height of 8 in.

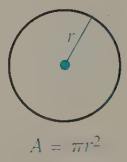


$$A = \frac{1}{2}h(b_1 + b_2)$$

$$= \frac{1}{2} \cdot 8(15 + 5) = 4(20) = 80$$

The area is 80 in^2.

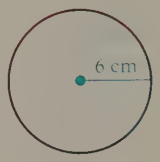
The area of a circle is equal to the product of π and the square of the radius.



Area of a Circle

The area, A, of a circle with radius r is given by $A = \pi r^2$.

Find the area of a circle that has a radius of 6 cm.



$A = \pi r^2$
$A = \pi(6)^2$
$A = \pi(36)$

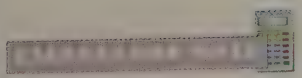
- Use the formula for the area of a circle. $r = 6$

$A = 36\pi$

- The exact area of the circle is 36π cm^2.

$A \approx 113.10$

- An approximate measure is found by using the π key on a calculator.



To approximate 36π on your calculator, enter 36 [×] [π] [=].

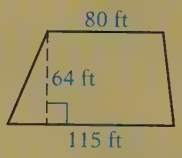
The approximate area of the circle is 113.10 cm^2.

For your reference, all of the formulas for the perimeters and areas of the geometric figures presented in this section are listed in the Chapter Summary located at the end of this chapter.

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Example 4

The Parks and Recreation Department of a city plans to plant grass seed in a playground that has the shape of a trapezoid, as shown below. Each bag of grass seed will seed 1500 ft². How many bags of grass seed should the department purchase?



Strategy

To find the number of bags to be purchased:

- Use the formula for the area of a trapezoid to find the area of the playground.
- Divide the area of the playground by the area one bag will seed (1500).

Solution

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2} \cdot 64(80 + 115)$$

$A = 6240$ • The area of the playground is 6240 ft².

$6240 \div 1500 = 4.16$

Because a portion of a fifth bag is needed, 5 bags of grass seed should be purchased.

You Try It 4

An interior designer decides to wallpaper two walls of a room. Each roll of wallpaper will cover 30 ft². Each wall measures 8 ft by 12 ft. How many rolls of wallpaper should be purchased?

Your strategy

Your solution

Example 5

Find the area of a circle with a diameter of 5 ft. Give the exact measure.

Strategy

To find the area:

- Find the radius of the circle.
- Use the formula for the area of a circle. Leave the answer in terms of π.

Solution

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5$$

$$A = \pi r^2 = \pi(2.5)^2 = \pi(6.25) = 6.25\pi$$

The area of the circle is 6.25π ft².

You Try It 5

Find the area of a circle with a radius of 11 cm. Round to the nearest hundredth.

Your strategy

Your solution

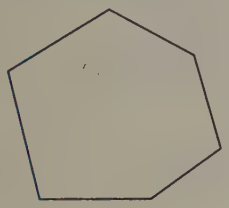
Solutions on p. S9

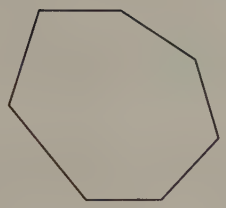
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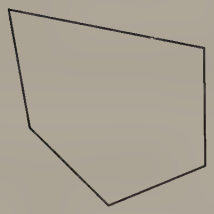
3.2 Exercises

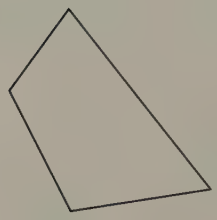
Objective A

Name each polygon.

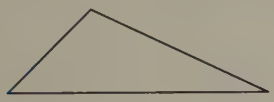
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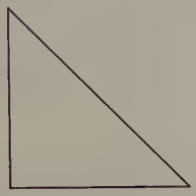
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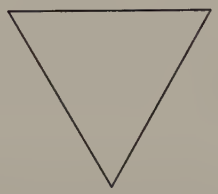
3. 

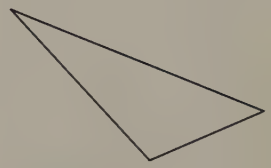
4. 

Classify the triangle as isosceles, equilateral, or scalene.

5. 

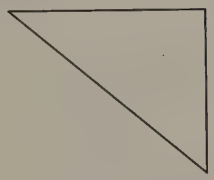
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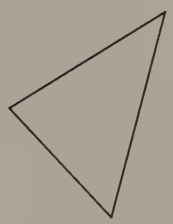
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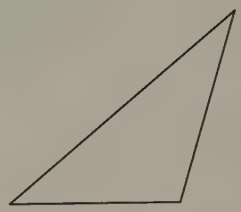
8. 

Classify the triangle as acute, obtuse, or right.

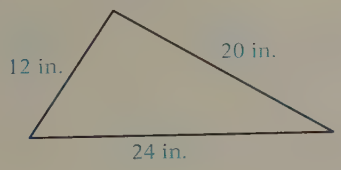
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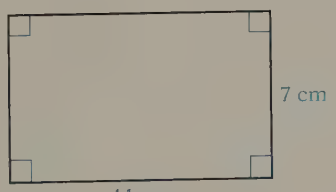
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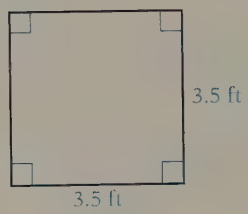
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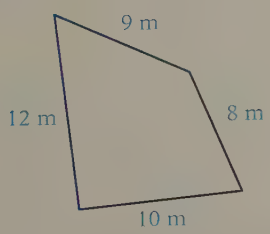
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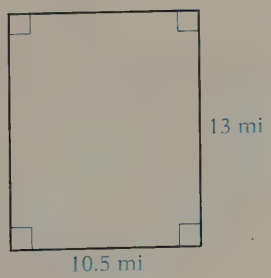
Find the perimeter of the figure.

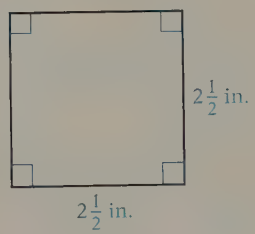
13. 

14. 

15. 

16. 

17. 

18. 

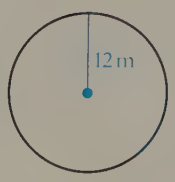
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Find the circumference of the figure. Give both the exact value and an approximation to the nearest hundredth.

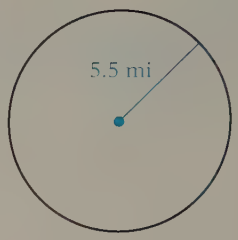
19.



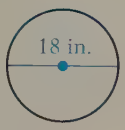
20.



21.



22.



23.



24.



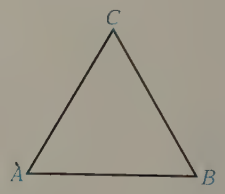
Solve.

25. The lengths of the three sides of a triangle are 3.8 cm, 5.2 cm, and 8.4 cm. Find the perimeter of the triangle.

26. The lengths of the three sides of a triangle are 7.5 m, 6.1 m, and 4.9 m. Find the perimeter of the triangle.

27. The length of each of two sides of an isosceles triangle is $2\frac{1}{2}$ cm. The third side measures 3 cm. Find the perimeter of the triangle.

28. The length of each side of an equilateral triangle is $4\frac{1}{2}$ in. Find the perimeter of the triangle.



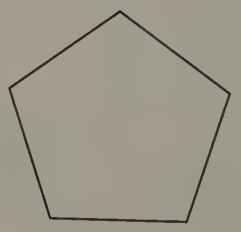
29. A rectangle has a length of 8.5 m and a width of 3.5 m. Find the perimeter of the rectangle.

30. Find the perimeter of a rectangle that has a length of $5\frac{1}{2}$ ft and a width of 4 ft.

31. The length of each side of a square is 12.2 cm. Find the perimeter of the square.

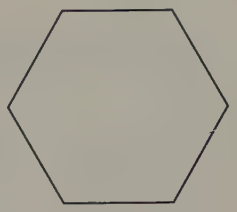
32. Find the perimeter of a square that measures 0.5 m on each side.

33. Find the perimeter of a regular pentagon that measures 3.5 in. on each side.



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34. What is the perimeter of a regular hexagon that measures 8.5 cm on each side?



35. The radius of a circle is 4.2 cm. Find the length of a diameter of the circle.

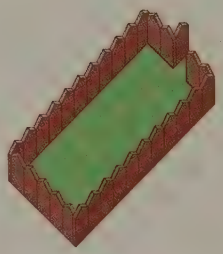
36. The diameter of a circle is 0.56 m. Find the length of a radius of the circle.

37. Find the circumference of a circle that has a diameter of 1.5 in. Give the exact value.

38. The diameter of a circle is 4.2 ft. Find the circumference of the circle. Round to the nearest hundredth.

39. The radius of a circle is 36 cm. Find the circumference of the circle. Round to the nearest hundredth.

40. Find the circumference of a circle that has a radius of 2.5 m. Give the exact value.

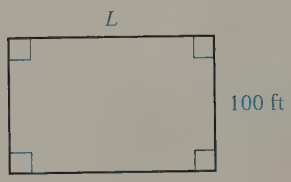


41. How many feet of fencing should be purchased for a rectangular garden that is 18 ft long and 12 ft wide?

42. How many meters of binding are required to bind the edge of a rectangular quilt that measures 3.5 m by 8.5 m?

43. Wall-to-wall carpeting is installed in a room that is 12 ft long and 10 ft wide. The edges of the carpet are nailed to the floor. Along how many feet must the carpet be nailed down?

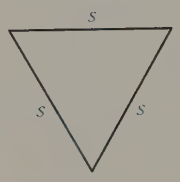
44. The length of a rectangular park is 55 yd. The width is 47 yd. How many yards of fencing are needed to surround the park?



45. The perimeter of a rectangular playground is 440 ft. If the width is 100 ft, what is the length of the playground?

46. A rectangular vegetable garden has a perimeter of 64 ft. The length of the garden is 20 ft. What is the width of the garden?

47. Each of two sides of a triangular banner measures 18 in. If the perimeter of the banner is 46 in., what is the length of the third side of the banner?



48. The perimeter of an equilateral triangle is 13.2 cm. What is the length of each side of the triangle?

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49. The perimeter of a square picture frame is 48 in. Find the length of each side of the frame.

50. A square rug has a perimeter of 32 ft. Find the length of each edge of the rug.

51. The circumference of a circle is 8 cm. Find the length of a diameter of the circle. Round to the nearest hundredth.

52. The circumference of a circle is 15 in. Find the length of a radius of the circle. Round to the nearest hundredth.

53. Find the length of molding needed to put around a circular table that is 4.2 ft in diameter. Round to the nearest hundredth.

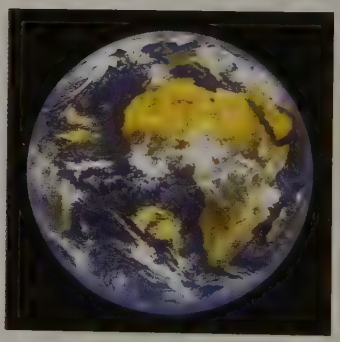
54. How much binding is needed to bind the edge of a circular rug that is 3 m in diameter? Round to the nearest hundredth.

55. A bicycle tire has a diameter of 24 in. How many feet does the bicycle travel when the wheel makes eight revolutions? Round to the nearest hundredth.



56. A tricycle tire has a diameter of 12 in. How many feet does the tricycle travel when the wheel makes twelve revolutions? Round to the nearest hundredth.

57. The distance from the surface of Earth to its center is 6356 km. What is the circumference of Earth? Round to the nearest hundredth.

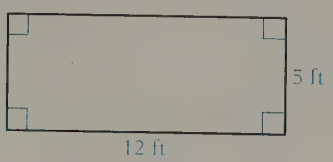


58. Bias binding is to be sewed around the edge of a rectangular tablecloth measuring 72 in. by 45 in. If the bias binding comes in packages containing 15 ft of binding, how many packages of bias binding are needed for the tablecloth?

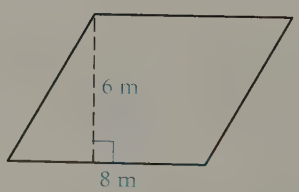
Objective B

Find the area of the figure.

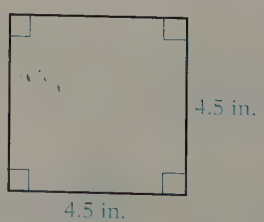
59.



60.

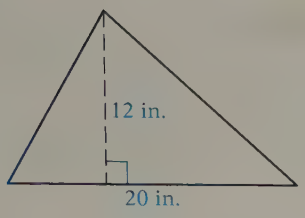


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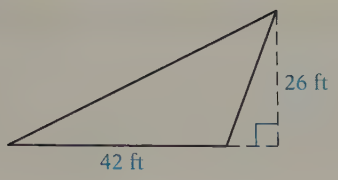


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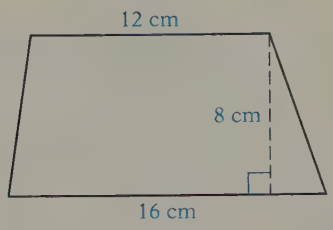
62.



63.



64.

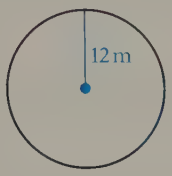


Find the area of the figure. Give both the exact value and an approximation to the nearest hundredth.

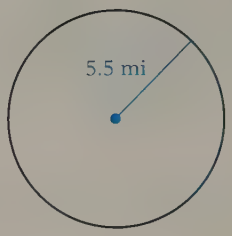
65.



66.



67.



68.



69.



70.



Solve.

71. The length of a side of a square is 12.5 cm. Find the area of the square.

72. Each side of a square measures $3\frac{1}{2}$ in. Find the area of the square.

73. The length of a rectangle is 38 in., and the width is 15 in. Find the area of the rectangle.

74. Find the area of a rectangle that has a length of 6.5 m and a width of 3.8 m.

75. The length of the base of a parallelogram is 16 in., and the height is 12 in. Find the area of the parallelogram.

76. The height of a parallelogram is 3.4 m, and the length of the base is 5.2 m. Find the area of the parallelogram.

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77. The length of the base of a triangle is 6 ft. The height is 4.5 ft. Find the area of the triangle.

78. The height of a triangle is 4.2 cm. The length of the base is 5 cm. Find the area of the triangle.

79. The length of one base of a trapezoid is 35 cm, and the length of the other base is 20 cm. If the height is 12 cm, what is the area of the trapezoid?

80. The height of a trapezoid is 5 in. The bases measure 16 in. and 18 in. Find the area of the trapezoid.

81. The radius of a circle is 5 in. Find the area of the circle. Give the exact value.

82. Find the area of a circle with a radius of 14 m. Round to the nearest hundredth.

83. Find the area of a circle that has a diameter of 3.4 ft. Round to the nearest hundredth.

84. The diameter of a circle is 6.5 m. Find the area of the circle. Give the exact value.



85. The dome of the Hale telescope at Mount Palomar, California, has a diameter of 200 in. Find the area across the dome. Give the exact value.

86. An irrigation system waters a circular field that has a 50-foot radius. Find the area watered by the irrigation system. Give the exact value.

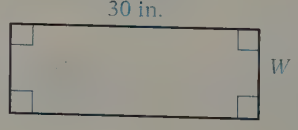
87. Find the area of a rectangular flower garden that measures 14 ft by 9 ft.

88. What is the area of a square patio that measures 8.5 m on each side?

89. Artificial turf is being used to cover a playing field. If the field is rectangular with a length of 100 yd and a width of 75 yd, how much artificial turf must be purchased to cover the field?

90. A fabric wall hanging is to fill a space that measures 5 m by 3.5 m. Allowing for 0.1 m of the fabric to be folded back along each edge, how much fabric must be purchased for the wall hanging?

91. The area of a rectangle is 300 in^2. If the length of the rectangle is 30 in., what is the width?

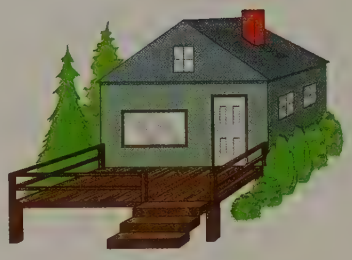


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92. The width of a rectangle is 12 ft. If the area is 312 ft^2, what is the length of the rectangle?

93. The height of a triangle is 5 m. The area of the triangle is 50 m^2. Find the length of the base of the triangle.

94. The area of a parallelogram is 42 m^2. If the height of the parallelogram is 7 m, what is the length of the base?



95. You plan to stain the wooden deck attached to your house. The deck measures 10 ft by 8 ft. If a quart of stain will cover 50 ft^2, how many quarts of stain should you buy?

96. You want to tile your kitchen floor. The floor measures 12 ft by 9 ft. How many tiles, each a square with side $1\frac{1}{2}$ ft, should you purchase for the job?

97. You are wallpapering two walls of a child's room, one measuring 9 ft by 8 ft and the other measuring 11 ft by 8 ft. The wallpaper costs \$24.50 per roll, and each roll of the wallpaper will cover 40 ft^2. What will it cost to wallpaper the two walls?

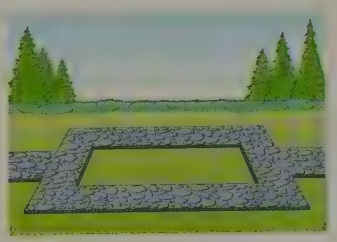
98. An urban renewal project involves reseeding a park that is in the shape of a square 60 ft on each side. Each bag of grass seed costs \$9.75 and will seed 1200 ft^2. How much money should be budgeted for buying grass seed for the park?

99. A circle has a radius of 8 in. Find the increase in area when the radius is increased by 2 in. Round to the nearest hundredth.

100. A circle has a radius of 6 cm. Find the increase in area when the radius is doubled. Round to the nearest hundredth.

101. You want to install wall-to-wall carpeting in your living room, which measures 15 ft by 24 ft. If the cost of the carpet you would like to purchase is \$21.95 per square yard, what will be the cost of the carpeting for your living room? (*Hint:* 9 ft^2 = 1 yd^2)

102. You want to paint the walls of your bedroom. Two walls measure 15 ft by 9 ft, and the other two walls measure 12 ft by 9 ft. The paint you wish to purchase costs \$12.98 per gallon, and each gallon will cover 400 ft^2 of wall. Find the total amount you will spend on paint.



103. A walkway 2 m wide surrounds a rectangular plot of grass. The plot is 30 m long and 20 m wide. What is the area of the walkway?

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104. Pleated draperies for a window must be twice as wide as the width of the window. Draperies are being made for four windows, each 2 ft wide and 4 ft high. Because the drapes will fall slightly below the window sill, and because extra fabric will be needed for hemming the drapes, 1 ft must be added to the height of the window. How much material must be purchased to make the drapes?

105. Find the cost of plastering the walls of a room 22 ft long, 25 ft 6 in. wide, and 8 ft high. Subtract 120 ft^2 for windows and doors. The cost is \$2.50 per square foot.

APPLYING THE CONCEPTS

106. If both the length and the width of a rectangle are doubled, how many times larger is the area of the resulting rectangle?

107. A **hexagram** is a six-pointed star, formed by extending each of the sides of a regular hexagon into an equilateral triangle. A hexagram is shown at the right. Use a pencil, paper, protractor, and ruler to create a hexagram.



108. If the formula $C = \pi d$ is solved for π, the resulting equation is $\pi = \frac{C}{d}$. Therefore, π is the ratio of the circumference of a circle to the length of its diameter. Use several circular objects, such as coins, plates, tin cans, and wheels, to show that the ratio of the circumference of each object to its diameter is approximately 3.14.

109. Derive a formula for the area of a circle in terms of the diameter of the circle.

110. Determine whether the statement is always true, sometimes true, or never true.

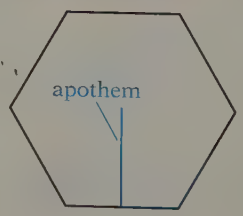
a. If two triangles have the same perimeter, then they have the same area.
b. If two rectangles have the same area, then they have the same perimeter.
c. If two squares have the same area, then the sides of the squares have the same length.
d. An equilateral triangle is also an isosceles triangle.
e. All the radii (plural of radius) of a circle are equal.
f. All the diameters of a circle are equal.





111. Suppose a circle is cut into 16 equal pieces, which are then arranged as shown at the right. The figure formed resembles a parallelogram. What variable expression could describe the base of the parallelogram? What variable could describe its height? Explain how the formula for the area of a circle is derived from this approach.

112. The **apothem** of a regular polygon is the distance from the center of the polygon to a side. Explain how to derive a formula for the area of a regular polygon using the apothem. (*Hint:* Use the formula for the area of a triangle.)



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3.3 Solids

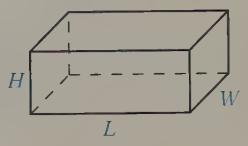
Objective A

To solve problems involving the volume of a solid

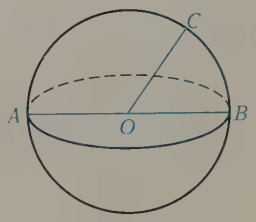
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Geometric solids are figures in space. Five common geometric solids are the rectangular solid, the sphere, the cylinder, the cone, and the pyramid.

A **rectangular solid** is one in which all six sides, called **faces**, are rectangles. The variable L is used to represent the length of a rectangular solid, W its width, and H its height.

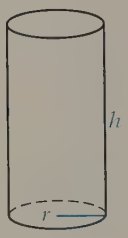


A **sphere** is a solid in which all points are the same distance from point O, which is called the **center** of the sphere. The **diameter**, d, of a sphere is a line across the sphere going through point O. The **radius**, r, is a line from the center to a point on the sphere. AB is a diameter and OC is a radius of the sphere shown at the right.

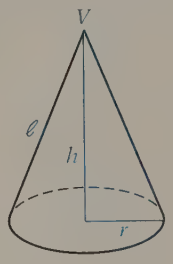


$$d = 2r \text{ or } r = \frac{1}{2}d$$

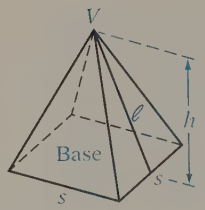
The most common cylinder, called a **right circular cylinder**, is one in which the bases are circles and are perpendicular to the height of the cylinder. The variable r is used to represent the radius of a base of a cylinder, and h represents the height. In this text, only right circular cylinders are discussed.



A **right circular cone** is obtained when one base of a right circular cylinder is shrunk to a point, called a **vertex**, V. The variable r is used to represent the radius of the base of the cone, and h represents the height. The variable ℓ is used to represent the **slant height**, which is the distance from a point on the circumference of the base to the vertex. In this text, only right circular cones are discussed.

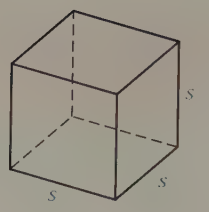


The base of a **regular pyramid** is a regular polygon, and the sides are isosceles triangles. The height, h, is the distance from the vertex, V, to the base and is perpendicular to the base. The variable ℓ is used to represent the **slant height**, which is the height of one of the isosceles triangles on the face of the pyramid. The regular square pyramid at the right has a square base. This is the only type of pyramid discussed in this text.



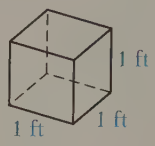
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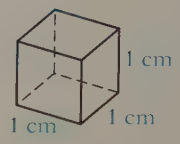
A **cube** is a special type of rectangular solid. Each of the six faces of a cube is a square. The variable s is used to represent the length of one side of a cube.



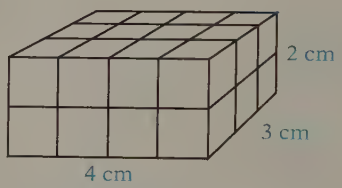
Volume is a measure of the amount of space inside a figure in space. Volume can be used to describe the amount of heating gas used for cooking, the amount of concrete delivered for the foundation of a house, or the amount of water in storage for a city's water supply.

A cube that is 1 ft on each side has a volume of 1 cubic foot, which is written 1 ft^3. A cube that measures 1 cm on each side has a volume of 1 cubic centimeter, which is written 1 cm^3.





The volume of a solid is the number of cubes that are necessary to exactly fill the solid. The volume of the rectangular solid at the right is 24 cm^3 because it will hold exactly 24 cubes, each 1 cm on a side. Note that the volume can be found by multiplying the length times the width times the height.



$4 \cdot 3 \cdot 2 = 24$

The formulas for the volumes of the geometric solids described above are given below.

Volumes of Geometric Solids

The volume, V, of a **rectangular solid** with length L, width W, and height H is given by $V = LWH$.

The volume, V, of a **cube** with side s is given by $V = s^3$.

The volume, V, of a **sphere** with radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume, V, of a **right circular cylinder** is given by $V = \pi r^2 h$, where r is the radius of the base and h is the height.

The volume, V, of a **right circular cone** is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base and h is the height.

The volume, V, of a **regular square pyramid** is given by $V = \frac{1}{3}s^2 h$, where s is the length of a side of the base and h is the height.

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➡ Find the volume of a sphere with a diameter of 6 in.

$r = \frac{1}{2}d = \frac{1}{2}(6) = 3$ • First find the radius of the sphere.

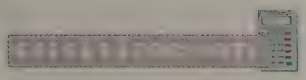
$V = \frac{4}{3}\pi r^3$ • Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi(3)^3$

$V = \frac{4}{3}\pi(27)$

$V = 36\pi$ • The exact volume of the sphere is 36π in^3.

$V \approx 113.10$ • An approximate measure can be found by using the π key on a calculator.



To approximate 36π on your calculator, enter 36 [×] [π] [=].

The approximate volume is 113.10 in^3.

Example 1

The length of a rectangular solid is 5 m, the width is 3.2 m, and the height is 4 m. Find the volume of the solid.

Strategy

To find the volume, use the formula for the volume of a rectangular solid. $L = 5$, $W = 3.2$, $H = 4$.

Solution

$V = LWH = 5(3.2)(4) = 64$

The volume of the rectangular solid is 64 m^3.

You Try It 1

Find the volume of a cube that measures 2.5 m on a side.

Your strategy

Your solution

Example 2

The radius of the base of a cone is 8 cm. The height is 12 cm. Find the volume of the cone. Round to the nearest hundredth.

Strategy

To find the volume, use the formula for the volume of a cone. An approximation is asked for; use the π key on a calculator. $r = 8$, $h = 12$.

Solution

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(8)^2(12) = \frac{1}{3}\pi(64)(12) = 256\pi$

≈ 804.25

The volume is approximately 804.25 cm^3.

Your Try It 2

The diameter of the base of a cylinder is 8 ft. The height of the cylinder is 22 ft. Find the exact volume of the cylinder.

Your strategy

Your solution

Solutions on p. S9

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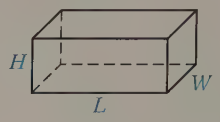
Objective B

To solve problems involving the surface area of a solid

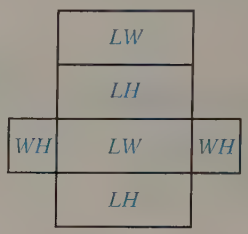
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The **surface area** of a solid is the total area on the surface of the solid.

When a rectangular solid is cut open and flattened out, each face is a rectangle. The surface area, SA, of the rectangular solid is the sum of the areas of the six rectangles:



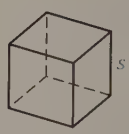
$$SA = LW + LH + WH + LW + WH + LH$$

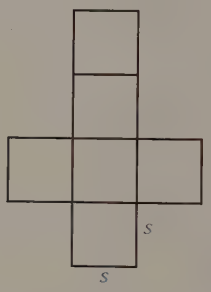


which simplifies to

$$\boldsymbol{SA = 2LW + 2LH + 2WH}$$

The surface area of a cube is the sum of the areas of the six faces of the cube. The area of each face is s^2. Therefore, the surface area, SA, of a cube is given by the formula $\boldsymbol{SA = 6s^2}$.



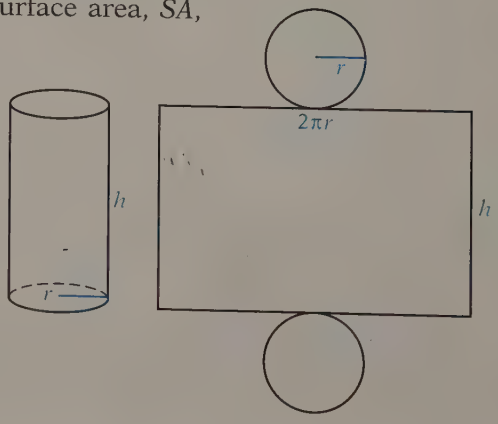


When a cylinder is cut open and flattened out, the top and bottom of the cylinder are circles. The side of the cylinder flattens out to a rectangle. The length of the rectangle is the circumference of the base, which is $2\pi r$; the width is h, the height of the cylinder. Therefore, the area of the rectangle is $2\pi rh$. The surface area, SA, of the cylinder is

$$SA = \pi r^2 + 2\pi rh + \pi r^2$$

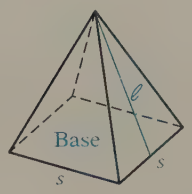
which simplifies to

$$\boldsymbol{SA = 2\pi r^2 + 2\pi rh}$$



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The surface area of a pyramid is the area of the base plus the area of the four isosceles triangles. A side of the square base is s; therefore, the area of the base is s^2. The slant height, ℓ, is the height of each triangle, and s is the base of each triangle. The surface area, SA, of a pyramid is



$$SA = s^2 + 4\left(\frac{1}{2}s\ell\right)$$

which simplifies to

$$\boldsymbol{SA = s^2 + 2s\ell}$$

Formulas for the surface areas of geometric solids are given below.

Surface Areas of Geometric Solids

The surface area, SA, of a **rectangular solid** with length L, width W, and height H is given by $SA = 2LW + 2LH + 2WH$.

The surface area, SA, of a **cube** with side s is given by $SA = 6s^2$.

The surface area, SA, of a **sphere** with radius r is given by $SA = 4\pi r^2$.

The surface area, SA, of a **right circular cylinder** is given by $SA = 2\pi r^2 + 2\pi rh$, where r is the radius of the base and h is the height.

The surface area, SA, of a **right circular cone** is given by $SA = \pi r^2 + \pi r\ell$, where r is the radius of the circular base and ℓ is the slant height.

The surface area, SA, of a **regular pyramid** is given by $SA = s^2 + 2s\ell$, where s is the length of a side of the base and ℓ is the slant height.

Find the surface area of a sphere with a diameter of 18 cm.

$r = \frac{1}{2}d = \frac{1}{2}(18) = 9$ • First find the radius of the sphere.

$SA = 4\pi r^2$

$SA = 4\pi(9)^2$ • Use the formula for the surface area of a sphere.

$SA = 4\pi(81)$

$SA \approx 324\pi$ • The exact surface area of the sphere is 324π cm^2.

$SA \approx 1017.88$ • An approximate measure can be found by using the π key on a calculator.

The approximate surface area is 1017.88 cm^2.

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Example 3

The diameter of the base of a cone is 5 m, and the slant height is 4 m. Find the surface area of the cone. Give the exact measure.

Strategy

To find the surface area of the cone:

- Find the radius of the base of the cone.
- Use the formula for the surface area of a cone. Leave the answer in terms of π.

Solution

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5$$

$$SA = \pi r^2 + \pi r\ell$$
$$SA = \pi(2.5)^2 + \pi(2.5)(4)$$
$$SA = \pi(6.25) + \pi(2.5)(4)$$
$$SA = 6.25\pi + 10\pi$$
$$SA = 16.25\pi$$

The surface area of the cone is 16.25π m^2.

You Try It 3

The diameter of the base of a cylinder is 6 ft, and the height is 8 ft. Find the surface area of the cylinder. Round to the nearest hundredth.

Your strategy

Your solution

Example 4

Find the area of a label used to cover a soup can that has a radius of 4 cm and a height of 12 cm. Round to the nearest hundredth.

Strategy

To find the area of the label, use the fact that the surface area of the sides of a cylinder is given by $2\pi rh$. An approximation is asked for; use the π key on a calculator. $r = 4$, $h = 12$.

Solution

Area of the label $= 2\pi rh$

Area of the label $= 2\pi(4)(12) = 96\pi$

≈ 301.59

The area is approximately 301.59 cm^2.

You Try It 4

Which has a larger surface area, a cube with a side measuring 10 cm or a sphere with a diameter measuring 8 cm?

Your strategy

Your solution

Solutions on pp. S9–S10

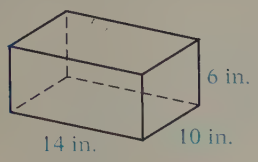
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3.3 Exercises

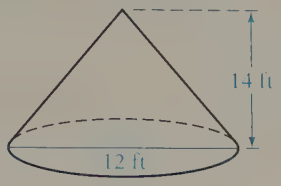
Objective A

Find the volume of the figure. For calculations involving π, give both the exact value and an approximation to the nearest hundredth.

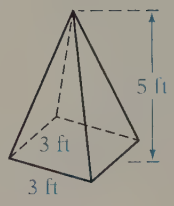
1.



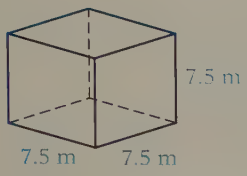
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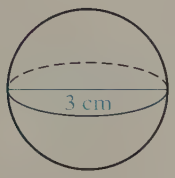
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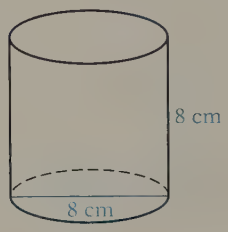
4.



5.



6.



Solve.

7. A rectangular solid has a length of 6.8 m, a width of 2.5 m, and a height of 2 m. Find the volume of the solid.

8. Find the volume of a rectangular solid that has a length of 4.5 ft, a width of 3 ft, and a height of 1.5 ft.

9. Find the volume of a cube whose side measures 3.5 in.

10. The length of a side of a cube is 7 cm. Find the volume of the cube.

11. The diameter of a sphere is 6 ft. Find the volume of the sphere. Give the exact measure.

12. Find the volume of a sphere that has a radius of 1.2 m. Round to the nearest tenth.

13. The diameter of the base of a cylinder is 24 cm. The height of the cylinder is 18 cm. Find the volume of the cylinder. Round to the nearest hundredth.

14. The height of a cylinder is 7.2 m. The radius of the base is 4 m. Find the volume of the cylinder. Give the exact measure.

15. The radius of the base of a cone is 5 in. The height of the cone is 9 in. Find the volume of the cone. Give the exact measure.

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16. The height of a cone is 15 cm. The diameter of the cone is 10 cm. Find the volume of the cone. Round to the nearest hundredth.

17. The length of a side of the base of a pyramid is 6 in., and the height is 10 in. Find the volume of the pyramid.

18. The height of a pyramid is 8 m, and the length of a side of the base is 9 m. What is the volume of the pyramid?

19. The volume of a freezer with a length of 7 ft and a height of 3 ft is 52.5 ft^3. Find the width of the freezer.

20. The length of an aquarium is 18 in., and the width is 12 in. If the volume of the aquarium is 1836 in^3, what is the height of the aquarium?

21. The volume of a cylinder with a height of 10 in. is 502.4 in^3. Find the radius of the base of the cylinder. Round to the nearest hundredth.

22. The diameter of the base of a cylinder is 14 cm. If the volume of the cylinder is 2310 cm^3, find the height of the cylinder. Round to the nearest hundredth.

23. A rectangular solid has a square base and a height of 5 in. If the volume of the solid is 125 in^3, find the length and the width.

24. The volume of a rectangular solid is 864 m^3. The rectangular solid has a square base and a height of 6 m. Find the dimensions of the solid.

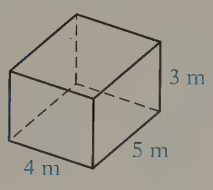
25. An oil storage tank, which is in the shape of a cylinder, is 4 m high and has a diameter of 6 m. The oil tank is two-thirds full. Find the number of cubic meters of oil in the tank. Round to the nearest hundredth.

26. A silo, which is in the shape of a cylinder, is 16 ft in diameter and has a height of 30 ft. The silo is three-fourths full. Find the volume of the portion of the silo that is not being used for storage. Round to the nearest hundredth.

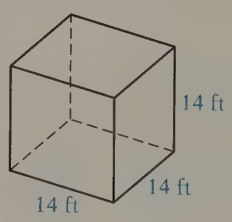
Objective B

Find the surface area of the figure.

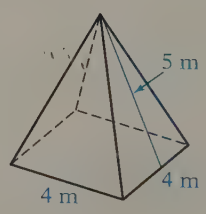
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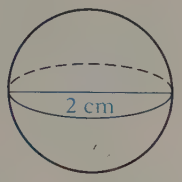
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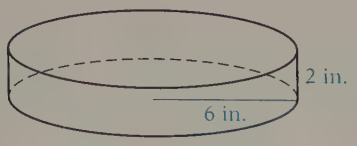
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Find the surface area of the figure. Give both the exact value and an approximation to the nearest hundredth.

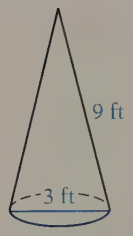
30.



31.



32.

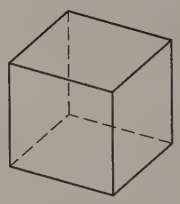


Solve.

33. The height of a rectangular solid is 5 ft. The length is 8 ft, and the width is 4 ft. Find the surface area of the solid.

34. The width of a rectangular solid is 32 cm. The length is 60 cm, and the height is 14 cm. What is the surface area of the solid?

35. The side of a cube measures 3.4 m. Find the surface area of the cube.



36. Find the surface area of a cube that has a side measuring 1.5 in.

37. Find the surface area of a sphere with a diameter of 15 cm. Give the exact value.

38. The radius of a sphere is 2 in. Find the surface area of the sphere. Round to the nearest hundredth.

39. The radius of the base of a cylinder is 4 in. The height of the cylinder is 12 in. Find the surface area of the cylinder. Round to the nearest hundredth.

40. The diameter of the base of a cylinder is 1.8 m. The height of the cylinder is 0.7 m. Find the surface area of the cylinder. Give the exact value.

41. The slant height of a cone is 2.5 ft. The radius of the base is 1.5 ft. Find the surface area of the cone. Give the exact value.

42. The diameter of the base of a cone is 21 in. The slant height is 16 in. What is the surface area of the cone? Round to the nearest hundredth.

43. The length of a side of the base of a pyramid is 9 in., and the slant height is 12 in. Find the surface area of the pyramid.

44. The slant height of a pyramid is 18 m, and the length of a side of the base is 16 m. What is the surface area of the pyramid?

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45. The surface area of a rectangular solid is 108 cm^2. The height of the solid is 4 cm, and the length is 6 cm. Find the width of the rectangular solid.

46. The length of a rectangular solid is 12 ft. The width is 3 ft. If the surface area is 162 ft^2, find the height of the rectangular solid.

47. A can of paint will cover 300 ft^2. How many cans of paint should be purchased in order to paint a cylinder that has a height of 30 ft and a radius of 12 ft?



48. A hot air balloon is in the shape of a sphere. Approximately how much fabric was used to construct the balloon if its diameter is 32 ft? Round to the nearest whole number.

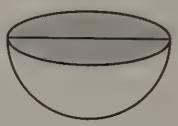
49. How much glass is needed to make a fish tank that is 12 in. long, 8 in. wide, and 9 in. high? The fish tank is open at the top.

50. Find the area of a label used to cover a can of juice that has a diameter of 16.5 cm and a height of 17 cm. Round to the nearest hundredth.

51. The length of a side of the base of a pyramid is 5 cm, and the slant height is 8 cm. How much larger is the surface area of this pyramid than the surface area of a cone with a diameter of 5 cm and a slant height of 8 cm? Round to the nearest hundredth.

APPLYING THE CONCEPTS

52. Half of a sphere is called a **hemisphere**. Derive formulas for the volume and surface area of a hemisphere.



53. Determine whether the statement is always true, sometimes true, or never true.

a. The slant height of a regular pyramid is longer than the height.
b. The slant height of a cone is shorter than the height.
c. The four triangular faces of a regular pyramid are equilateral triangles.

54. **a.** What is the effect on the surface area of a rectangular solid when the width and height are doubled?
b. What is the effect on the volume of a rectangular solid when both the length and the width are doubled?
c. What is the effect on the volume of a cube when the length of each side of the cube is doubled?
d. What is the effect on the surface area of a cylinder when the radius and height are doubled?

55. Explain how you could cut through a cube so that the face of the resulting solid is

a. a square
b. an equilateral triangle
c. a trapezoid
d. a hexagon

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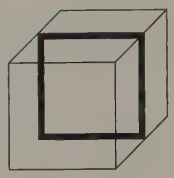
Focus on Problem Solving

Trial and Error

Some problems in mathematics are solved by using **trial and error**. The trial-and-error method of arriving at a solution to a problem involves repeated tests or experiments until a satisfactory conclusion is reached.

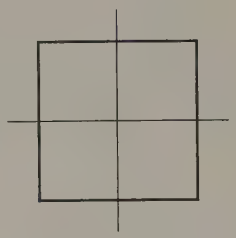
Many of the Applying the Concepts exercises in this text require a trial and error method of solution. For example, an exercise in Section 3 of this chapter reads:

Explain how you could cut through a cube so that the face of the resulting solid is **(a)** a square, **(b)** an equilateral triangle, **(c)** a trapezoid, **(d)** a hexagon.



There is no formula to apply to this problem; there is no computation to perform. This problem requires picturing a cube and the results after cutting through it at different places on its surface and at different angles. For part (a), cutting perpendicular to the top and bottom of the cube and parallel to two of its sides will result in a square. The other shapes may prove more difficult.

When solving problems of this type, keep an open mind. Sometimes when using the trial-and-error method, we are hampered by narrowness of vision; we cannot expand our thinking to include other possibilities. Then when we see someone else's solution, it appears so obvious to us! For example, for the Applying the Concepts question above, it is necessary to conceive of cutting through the cube at places other than the top surface; we need to be open to the idea of beginning the cut at one of the corner points of the cube.



A topic of the Projects and Group Activities in this chapter is symmetry. Here again, trial and error is used to determine the lines of symmetry inherent in an object. For example, in determining lines of symmetry for a square, begin by drawing a square. The horizontal line of symmetry and the vertical line of symmetry may be immediately obvious to you.

But there are two others. Do you see that a line drawn through opposite corners of the square is also a line of symmetry?

Many of the questions in this text that require an answer of "always true, sometimes true, or never true" are best solved by the trial-and-error method. For example, consider the statement presented in Section 2 of this chapter.

If two rectangles have the same area, then they have the same perimeter.

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Try some numbers. Each of two rectangles, one measuring 6 units by 2 units and another measuring 4 units by 3 units, has an area of 12 square units, but the perimeter of the first is 16 units and the perimeter of the second is 14 units. So the answer "always true" has been eliminated. We still need to determine whether there is a case when it is true. After experimenting with a lot of numbers, you may come to realize that we are trying to determine if it is possible for two different pairs of factors of a number to have the same sum. Is it?

Don't be afraid to make many experiments, and remember that *errors*, or tests that "don't work," are a part of the trial-and-*error* process.

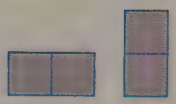
Projects and Group Activities

Investigating Perimeter

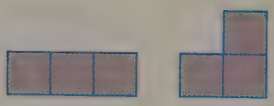
The perimeter of the square at the right is 4 units.



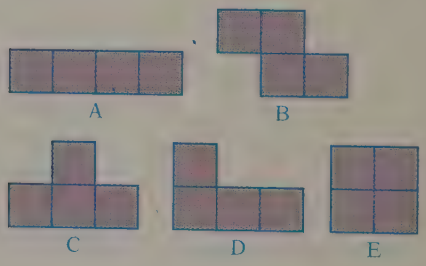
If two squares are joined along one of the sides, the perimeter is 6 units. Note that it does not matter which sides are joined; the perimeter is still 6 units.



If three squares are joined, the perimeter of the resulting figure is 8 units for each possible placement of the squares.



Four squares can be joined in five different ways as shown. There are two possible perimeters, 10 units for A, B, C, and D, and 8 units for E.



1. If five squares are joined, what is the maximum perimeter possible?

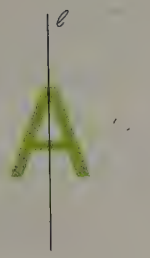
2. If five squares are joined, what is the minimum perimeter possible?

3. If six squares are joined, what is the maximum perimeter possible?

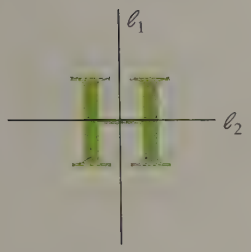
4. If six squares are joined, what is the minimum perimeter possible?

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Symmetry Look at the letter A printed at the left. If the letter were folded along line ℓ, the two sides of the letter would match exactly. This letter has **symmetry** with respect to line ℓ. Line ℓ is called the **axis of symmetry**.



Now consider the letter H printed below at the left. Both lines ℓ_1 and ℓ_2 are axes of symmetry for this letter; the letter could be folded along either line and the two sides would match exactly.



1. Does the letter A have more than one axis of symmetry?
2. Find axes of symmetry for other capital letters of the alphabet.
3. Which lowercase letters have one axis of symmetry?
4. Do any of the lowercase letters have more than one axis of symmetry?
5. Find the number of axes of symmetry for each of the plane geometric figures presented in this chapter.
6. There are other types of symmetry. Look up the meaning of point symmetry and rotational symmetry. Which plane geometric figures provide examples of these types of symmetry?
7. Find examples of symmetry in nature, art, and architecture.

Chapter Summary

Key Words

A *line* is determined by two distinct points and extends indefinitely in both directions. A *line segment* is part of a line that has two endpoints. *Parallel lines* never meet; the distance between them is always the same. *Perpendicular lines* are intersecting lines that form right angles. [pp. 145–147]

A *ray* starts at a point and extends indefinitely in one direction. The point at which a ray starts is the *endpoint* of the ray. An *angle* is formed by two rays with the same endpoint. The *vertex* of an angle is the point at which the two rays meet. An angle is measured in *degrees*. A 90° angle is a *right angle*. A 180° angle is a *straight angle*. An *acute angle* is an angle whose measure is between 0° and 90°. An *obtuse angle* is an angle whose measure is between 90° and 180°. *Complementary angles* are two angles whose measures have the sum 90°. *Supplementary angles* are two angles whose measures have the sum 180°. [pp. 145–148]

Two angles that are on opposite sides of the intersection of two lines are *vertical angles;* vertical angles have the same measure. Two angles that share a common side are *adjacent angles;* adjacent angles of intersecting lines are supplementary angles. [p. 150]

A line that intersects two other lines at two different points is a *transversal.* If the lines cut by a transversal are parallel lines, equal angles are formed: *alternate interior angles, alternate exterior angles,* and *corresponding angles*. [p. 151]

A *polygon* is a closed figure determined by three or more line segments. The line segments that form the polygon are its *sides*. A *regular polygon* is one in which each side has the same length and each angle has the same measure. Polygons are classified by the number of sides. [p. 161]

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A *triangle* is a plane figure formed by three line segments. An *isosceles triangle* has two sides of equal length. The three sides of an *equilateral triangle* are of equal length. A *scalene triangle* has no two sides of equal length. An *acute triangle* has three acute angles. An *obtuse triangle* has one obtuse angle. A *right triangle* has a right angle. [pp. 161–162]

A *quadrilateral* is a four-sided polygon. A parallelogram, a rectangle, a square, a rhombus, and a trapezoid are all quadrilaterals. [p. 162]

A *circle* is a plane figure in which all points are the same distance from the center of the circle. A *diameter* of a circle is a line segment across the circle through the center. A *radius* of a circle is a line segment from the center of the circle to a point on the circle. [p. 164]

The *perimeter* of a plane geometric figure is a measure of the distance around the figure. The distance around a circle is called the *circumference*. *Area* is the amount of surface in a region. *Volume* is a measure of the amount of space inside a figure in space. The *surface area* of a solid is the total area on the surface of the solid. [pp. 162, 164, 166, 179, 182]

Essential Rules

Triangles [p. 153]

Sum of the measures of the interior angles = 180°

Sum of an interior and corresponding exterior angle = 180°

Perimeter [pp. 162–164]

Triangle:	$P = a + b + c$
Rectangle:	$P = 2L + 2W$
Square:	$P = 4s$
Circle:	$C = \pi d$ or $C = 2\pi r$

Area [pp. 166–169]

Triangle:	$A = \frac{1}{2}bh$
Rectangle:	$A = LW$
Square:	$A = s^2$
Circle:	$A = \pi r^2$
Parallelogram:	$A = bh$
Trapezoid:	$A = \frac{1}{2}h(b_1 + b_2)$

Volume [p. 180]

Rectangular solid:	$V = LWH$
Cube:	$V = s^3$
Sphere:	$V = \frac{4}{3}\pi r^3$
Right circular cylinder:	$V = \pi r^2 h$
Right circular cone:	$V = \frac{1}{3}\pi r^2 h$
Regular pyramid:	$V = \frac{1}{3}s^2 h$

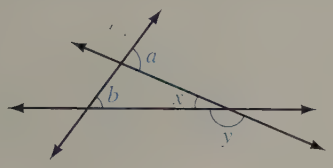
Surface Area [p. 183]

Rectangular solid:	$SA = 2LW + 2LH + 2WH$
Cube:	$SA = 6s^2$
Sphere:	$SA = 4\pi r^2$
Right circular cylinder:	$SA = 2\pi r^2 + 2\pi rh$
Right circular cone:	$SA = \pi r^2 + \pi r\ell$
Regular pyramid:	$SA = s^2 + 2s\ell$

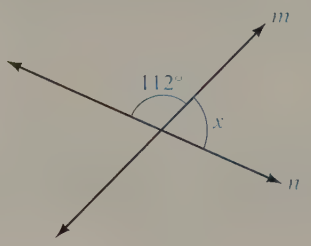
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Chapter Review

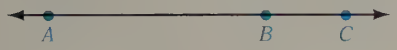
1. Given that $\angle a = 74°$ and $\angle b = 52°$, find the measures of angles x and y.



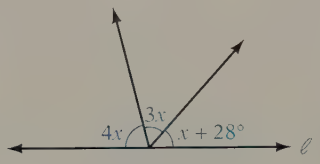
2. Find the measure of $\angle x$.



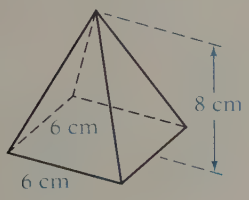
3. Given that $BC = 11$ cm and AB is three times the length of BC, find the length of AC.



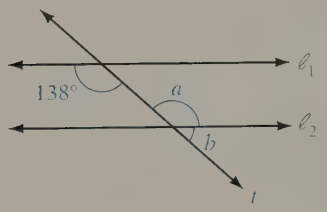
4. Find x.



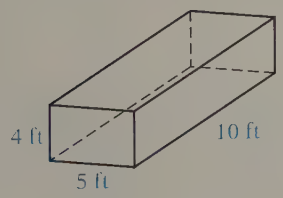
5. Find the volume of the figure.



6. Given that $\ell \parallel_1 \ell_2$, find the measures of angles a and b.



7. Find the surface area of the figure.



8. Find the supplement of a 32° angle.

9. Determine the area of a rectangle with a length of 12 cm and a width of 6.5 cm.

10. Determine the area of a triangle whose base is 9 m and whose height is 14 m.

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11. Find the volume of a rectangular solid with a length of 6.5 ft, a width of 2 ft, and a height of 3 ft.

12. Two angles of a triangle measure 37° and 48°. Find the measure of the third angle.

13. The height of a triangle is 7 cm. The area of the triangle is 28 cm^2. Find the length of the base of the triangle.

14. Find the volume of a sphere that has a diameter of 12 mm. Find the exact value.

15. Determine the exact volume of a right circular cone whose radius is 7 cm and whose height is 16 cm.

16. The perimeter of a square picture frame is 86 cm. Find the length of each side of the frame.

17. A can of paint will cover 200 ft^2. How many cans of paint should be purchased in order to paint a cylinder that has a height of 15 ft and a radius of 6 ft?

18. The length of a rectangular park is 56 yd. The width is 48 yd. How many yards of fencing are needed to surround the park?

19. What is the area of a square patio that measures 9.5 m on each side?

20. A walkway 2 m wide surrounds a rectangular plot of grass. The plot is 40 m long and 25 m wide. What is the area of the walkway?

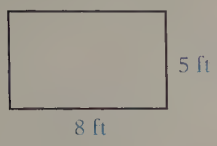
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Chapter Test

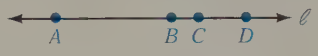
1. The diameter of a sphere is 1.5 m. Find the radius of the sphere.

2. Find the circumference of a circle with a radius of 5 cm. Round to the nearest hundredth.

3. Find the perimeter of the rectangle in the figure below.

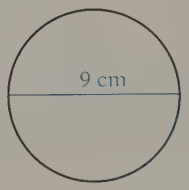


4. Given $AB = 15$, $CD = 6$, and $AD = 24$, find the length of BC.

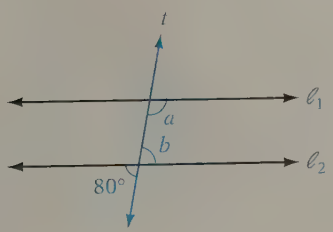


5. Find the volume of a sphere with a diameter of 8 ft. Round to the nearest hundredth.

6. Find the area of the circle shown below. Round to the nearest hundredth.

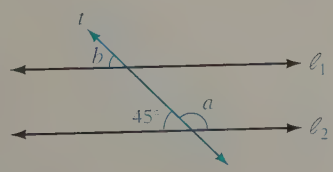


7. Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.

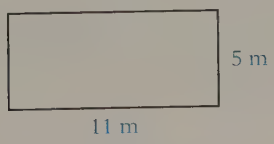


8. Find the supplement of a 105° angle.

9. Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.



10. Find the area of the rectangle shown below.



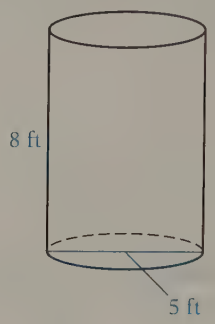
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11. Find the volume of a cylinder with a height of 6 m and a radius of 3 m. Round to the nearest hundredth.

12. Find the perimeter of a rectangle that has a length of 2 m and a width of 1.4 m.

13. Find the complement of a 32° angle.

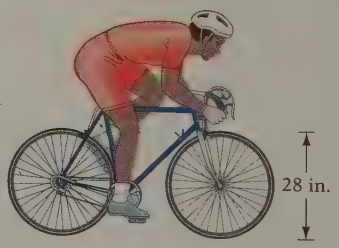
14. Find the surface area of the figure. Round to the nearest hundredth.



15. How much more pizza is contained in a pizza with radius 10 in. than in one with radius 8 in.? Round to the nearest hundredth.

16. A right triangle has a 32° angle. Find the measures of the other two angles.

17. A bicycle tire has a diameter of 28 in. How many feet does the bicycle travel if the wheel makes 10 revolutions? Round to the nearest tenth.



18. New carpet is installed in a room measuring 18 ft by 14 ft. Find the area of the room in square yards. ($9\ ft^2 = 1\ yd^2$)

19. A silo, which is in the shape of a cylinder, is 9 ft in diameter and has a height of 18 ft. Find the volume of the silo. Round to the nearest hundredth.

20. Find the area of a right triangle with a base of 8 m and a height of 2.75 m.

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Cumulative Review

1. Let $x \in \{-3, 0, 1\}$. For what values of x is the inequality $x \leq 1$ a true statement?

2. Write 8.9% as a decimal.

3. Write $\frac{7}{20}$ as a percent.

4. Divide: $-\frac{4}{9} \div \frac{2}{3}$

5. Multiply: $5.7\ (-4.3)$

6. Simplify: $-\sqrt{125}$

7. Evaluate $5 - 3[10 + (5 - 6)^2]$.

8. Evaluate $a(b - c)^3$ when $a = -1$, $b = -2$, and $c = -4$.

9. Simplify: $5m + 3n - 8m$

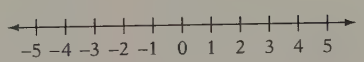
10. Simplify: $-7(-3y)$

11. Simplify: $4(3x + 2) - (5x - 1)$

12. Use the roster method to write the set of negative integers greater than or equal to -2.

13. Find $C \cup D$, given $C = \{0, 10, 20, 30\}$ and $D = \{-10, 0, 10\}$.

14. Graph: $x \leq 1$



15. Solve: $4x + 2 = 6x - 8$

16. Solve: $3(2x + 5) = 18$

17. Solve: $4y - 3 \geq 6y + 5$

18. Solve: $8 - 4(3x + 5) \leq 6(x - 8)$

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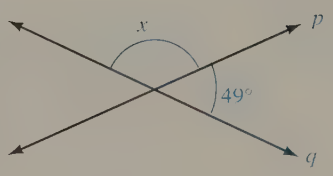
19. Solve: $2x - 3 > 5$ or $x + 4 < 1$

20. Solve: $-3 \leq 2x - 7 \leq 5$

21. Solve: $|3x - 1| = 2$

22. Solve: $|x - 8| \leq 2$

23. Find the measure of $\angle x$.



24. Translate "the difference between four times a number and ten is two" into an equation and solve.

25. Two angles of a triangle measure 37° and 21°. Find the measure of the third angle of the triangle.

26. An engineering consultant invested $14,000 in a 5.5% annual simple interest account. How much should the consultant deposit in an account that pays 9.5% annual simple interest if the total interest earned on both accounts is to be $1245?

27. Two sides of an isosceles triangle measure 7.5 m. The perimeter of the triangle is 19.5 m. Find the measure of the third side of the triangle.

28. According to the Census Bureau, the median annual earnings of a man with a bachelor's degree is $49,982, and the median earnings of a woman with a bachelor's degree is $35,408. What percent of the men's median annual earnings is the women's median annual earnings? Round to the nearest tenth of a percent. (Median is a type of average.)

29. Find the exact area of a circle that has a diameter of 9 cm.

30. The volume of a box is 144 ft^3. The length of the box is 12 ft, and the width is 4 ft. Find the height of the box.

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Chapter 4

Linear Equations and Inequalities in Two Variables



During portions of a flight, a passenger jet, like the one shown here, is in uniform motion, meaning that its speed and direction do not change. Uniform motion can be depicted in a graph to illustrate how fast or slow an object is moving and how much distance it has covered. By plotting the time and distance of two objects in uniform motion, it is clear which one is moving faster and has traveled the greater distance. The **Project on page 265** discusses objects in uniform motion and how to analyze speed and distance.

Objectives

Section 4.1

A To graph points in a rectangular coordinate system

B To determine ordered-pair solutions of an equation in two variables

C To graph a scatter diagram

Section 4.2

A To evaluate a function

Section 4.3

A To graph an equation of the form $y = mx + b$

B To graph an equation of the form $Ax + By = C$

C To solve application problems

Section 4.4

A To find the slope of a straight line

B To graph a line using the slope and the y-intercept

Section 4.5

A To find the equation of a line given a point and the slope

B To find the equation of a line given two points

C To solve application problems

Section 4.6

A To find equations of parallel and perpendicular lines

Section 4.7

A To graph an inequality in two variables

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Prep Test

For Exercises 1 to 3, simplify.

1. $-4(x - 3)$

2. $\sqrt{(-6)^2 + (-8)^2}$

3. $\dfrac{3 - (-5)}{2 - 6}$

4. Evaluate $-2x + 5$ for $x = -3$.

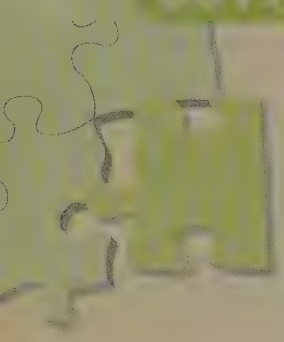
5. Evaluate $\dfrac{2r}{r - 1}$ for $r = 5$.

6. Evaluate $2p^3 - 3p + 4$ for $p = -1$.

7. Evaluate $\dfrac{x_1 + x_2}{2}$ for $x_1 = 7$ and $x_2 = -5$.

8. Given $3x - 4y = 12$, find the value of x when $y = 0$.

Go Figure



If $\boxed{5} = 4$ and $(5) = 6$ and $y = x - 1$, which of the following has the largest value?

$\boxed{x}$ (x) $\boxed{y}$ (y)

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4.1 The Rectangular Coordinate System

Objective A

To graph points in a rectangular coordinate system

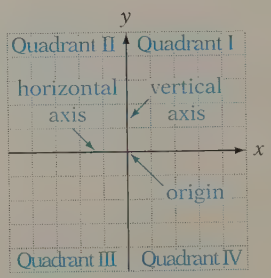
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Point of Interest

A rectangular coordinate system is also called a **Cartesian** coordinate system in honor of Descartes.

Before the 15th century, geometry and algebra were considered separate branches of mathematics. That all changed when René Descartes, a French mathematician who lived from 1596 to 1650, developed **analytic geometry**. In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin**. The two lines are called **coordinate axes**, or simply **axes**. Generally, the horizontal axis is labeled the x-axis and the vertical axis is labeled the y-axis.

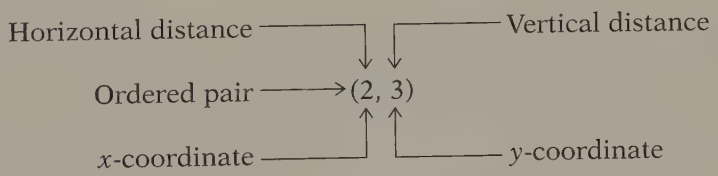


The axes determine a **plane**, which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants**, which are numbered counterclockwise from I to IV starting from the upper right.

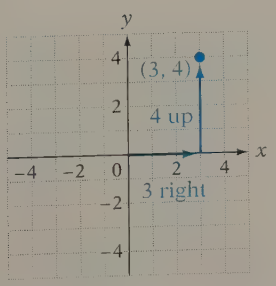
Point of Interest

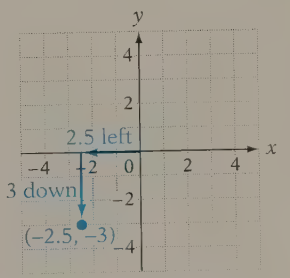
Gottfried Leibnitz introduced the words *abscissa* and *ordinate*. *Abscissa* is from a Latin word meaning "to cut off." Originally, Leibnitz used *abscissa linea*, "cut off a line" (axis). The root of *ordinate* is also a Latin word used to suggest the sense of order.

Each point in the plane can be identified by a pair of numbers called an **ordered pair**. The first number of the ordered pair measures a horizontal distance and is called the **abscissa**, or ***x*-coordinate**. The second number of the pair measures a vertical distance and is called the **ordinate**, or ***y*-coordinate**. The ordered pair (x, y) associated with a point is also called the **coordinates** of the point.



To **graph** or **plot** a point in the plane, place a dot at the location given by the ordered pair. The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the plane. The points whose coordinates are $(3, 4)$ and $(-2.5, -3)$ are graphed in the figures below.

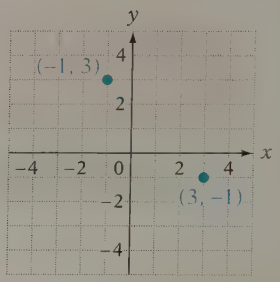




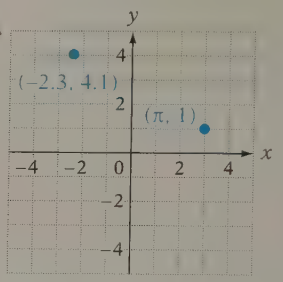
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TAKE NOTE
This is very important. An **ordered pair** is a *pair* of coordinates, and the *order* in which the coordinates appear is important.

The points whose coordinates are $(3, -1)$ and $(-1, 3)$ are graphed at the right. Note that the graphed points are in different locations. *The order of the coordinates of an ordered pair is important.*

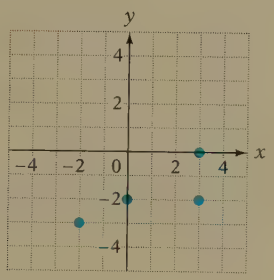


Each point in the plane is associated with an ordered pair, and each ordered pair is associated with a point in the plane. Although only the labels for integers are given on a coordinate grid, the graph of any ordered pair can be approximated. For example, the points whose coordinates are $(-2.3, 4.1)$ and $(\pi, 1)$ are shown on the graph at the right.



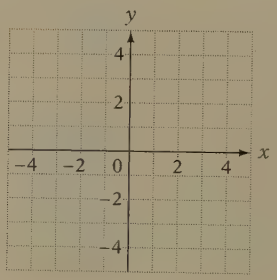
Example 1 Graph the ordered pairs $(-2, -3)$, $(3, -2)$, $(0, -2)$, and $(3, 0)$.

Solution

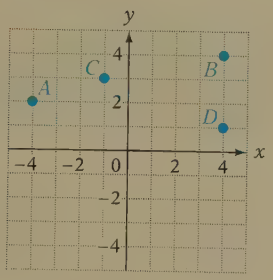


You Try It 1 Graph the ordered pairs $(-4, 1)$, $(3, -3)$, $(0, 4)$, and $(-3, 0)$.

Your solution

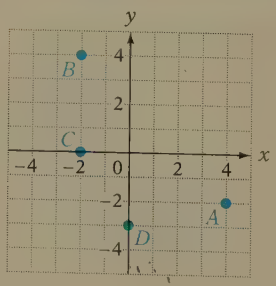


Example 2 Give the coordinates of the points labeled A and B. Give the abscissa of point C and the ordinate of point D.



Solution
The coordinates of A are $(-4, 2)$.
The coordinates of B are $(4, 4)$.
The abscissa of C is -1.
The ordinate of D is 1.

You Try It 2 Give the coordinates of the points labeled A and B. Give the abscissa of point D and the ordinate of point C.



Your solution

Solutions on p. S10

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Objective B

To determine ordered-pair solutions of an equation in two variables

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When drawing a rectangular coordinate system, we often label the horizontal axis x and the vertical axis y. In this case, the coordinate system is called the ***xy*-coordinate system.** The coordinates of the points are given by ordered pairs (x, y), where the abscissa is called the x-coordinate and the ordinate is called the y-coordinate.

A coordinate system is used to study the relationship between two variables. Frequently this relationship is given by an equation. Examples of equations in two variables include

$$y = 2x - 3 \qquad 3x + 2y = 6 \qquad x^2 - y = 0$$

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

TAKE NOTE
An ordered pair is of the form (x, y). For the ordered pair $(-3, 7)$, -3 is the x value and 7 is the y value. Substitute -3 for x and 7 for y.

➡ Is the ordered pair $(-3, 7)$ a solution of the equation $y = -2x + 1$?

$$\begin{array}{r|l} y = -2x + 1 & \\ \hline 7 & -2(-3) + 1 \\ 7 & 6 + 1 \\ 7 = 7 & \end{array}$$

- Replace x by -3 and y by 7.
- Simplify.
- Compare the results. If the resulting equation is a true statement, the ordered pair is a solution of the equation. If it is not a true statement, the ordered pair is not a solution of the equation.

Yes, the ordered pair $(-3, 7)$ is a solution of the equation.

Besides $(-3, 7)$, there are many other ordered-pair solutions of $y = -2x + 1$. For example, $(0, 1)$, $\left(-\frac{3}{2}, 4\right)$, and $(4, -7)$ are also solutions.

In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

➡ Find the ordered-pair solution of $y = \frac{2}{3}x - 3$ that corresponds to $x = 6$.

$$\begin{aligned} y &= \frac{2}{3}x - 3 \\ &= \frac{2}{3}(6) - 3 \qquad \text{• Replace } x \text{ by 6.} \\ &= 4 - 3 \qquad \text{• Solve for } y. \\ &= 1 \end{aligned}$$

The ordered-pair solution is $(6, 1)$.

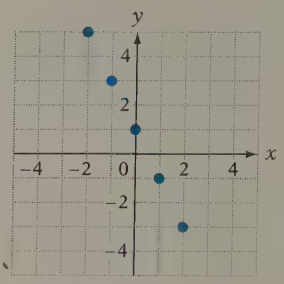
The solution of an equation in two variables can be graphed in an xy-coordinate system.

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Graph the ordered-pair solutions of $y = -2x + 1$ when $x = -2, -1, 0, 1,$ and 2.

Use the values of x to determine ordered-pair solutions of the equation. It is convenient to record these in a table.

x	$y = -2x + 1$	y	(x, y)
-2	$-2(-2) + 1$	5	$(-2, 5)$
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$



Example 3

Is $(3, -2)$ a solution of $3x - 4y = 15$?

Solution

$$\begin{array}{r|l} 3x - 4y = 15 & \\ \hline 3(3) - 4(-2) & 15 \\ 9 + 8 & 15 \\ 17 \neq 15 & \end{array}$$

• Replace x by 3 and y by -2.

No, $(3, -2)$ is not a solution of $3x - 4y = 15$.

You Try It 3

Is $(-2, 4)$ a solution of $x - 3y = -14$?

Your solution

Example 4

Find the ordered-pair solution of $y = \frac{x}{x - 2}$ corresponding to $x = 4$.

Solution

Replace x by 4 and solve for y.

$$y = \frac{x}{x - 2} = \frac{4}{4 - 2} = \frac{4}{2} = 2$$

The ordered-pair solution is $(4, 2)$.

You Try It 4

Find the ordered-pair solution of $y = \frac{3x}{x + 1}$ corresponding to $x = -2$.

Your solution

Solutions on p. S10

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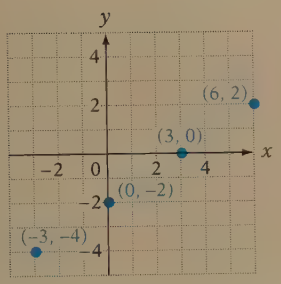
Example 5

Graph the ordered-pair solutions of $y = \frac{2}{3}x - 2$ when $x = -3, 0, 3, 6$.

Solution

Replace x in $y = \frac{2}{3}x - 2$ by -3, 0, 3, and 6. For each value of x, determine the value of y.

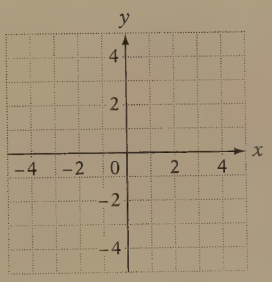
x	$y = \frac{2}{3}x - 2$	y	(x, y)
-3	$\frac{2}{3}(-3) - 2$	-4	$(-3, -4)$
0	$\frac{2}{3}(0) - 2$	-2	$(0, -2)$
3	$\frac{2}{3}(3) - 2$	0	$(3, 0)$
6	$\frac{2}{3}(6) - 2$	2	$(6, 2)$



You Try It 5

Graph the ordered-pair solutions of $y = -\frac{1}{2}x + 2$ when $x = -4, -2, 0, 2$.

Your solution



Solution on p. S10

Objective C To graph a scatter diagram

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Discovering a relationship between two variables is an important task in the study of mathematics. These relationships occur in many forms and in a wide variety of applications. Here are some examples.

- A botanist wants to know the relationship between the number of bushels of wheat yielded per acre and the amount of watering per acre.
- An environmental scientist wants to know the relationship between the incidence of skin cancer and the amount of ozone in the atmosphere.
- A business analyst wants to know the relationship between the price of a product and the number of products that are sold at that price.

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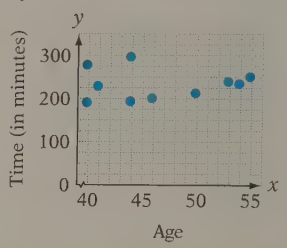
A researcher may investigate the relationship between two variables by means of *regression analysis,* which is a branch of statistics. The study of the relationship between the two variables may begin with a **scatter diagram,** which is a graph of the ordered pairs of the known data.

The following table shows randomly selected data from the participants 40 years old and older and their times (in minutes) for a recent Boston Marathon.

Age (x)	55	46	53	40	40	44	54	44	41	50
Time (y)	254	204	243	194	281	197	238	300	232	216

TAKE NOTE
The jagged portion of the horizontal axis in the figure at the right indicates that the numbers between 0 and 40 are missing.

The scatter diagram for these data is shown at the right. Each ordered pair represents the age and time for a participant. For instance, the ordered pair (53, 243) indicates that a 53-year-old participant ran the marathon in 243 min.



Example 6

The grams of sugar and the grams of fiber in a 1-ounce serving of six breakfast cereals are shown in the table below.

	Sugar (x)	*Fiber (y)*
Wheaties	4	3
Rice Krispies	3	0
Total	5	3
Life	6	2
Kix	3	1
Grape-Nuts	7	5

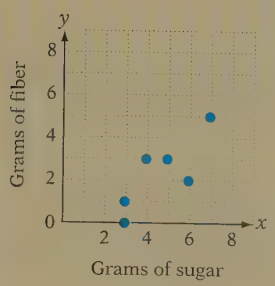
Draw a scatter diagram of these data.

Strategy

To draw a scatter diagram:

- Draw a coordinate grid with the horizontal axis representing the grams of sugar and the vertical axis the grams of fiber.
- Graph the ordered pairs (4, 3), (3, 0), (5, 3), (6, 2), (3, 1), and (7, 5).

Solution



You Try It 6

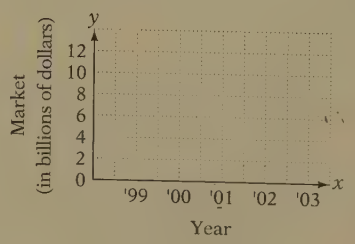
According to Forester Research, the growth of the software rental market, in billions of dollars, is projected to be as shown in the table below.

Year	*Market (in billions of dollars)*
1999	1
2000	2
2001	4
2002	7
2003	11

Draw a scatter diagram of these data.

Your strategy

Your solution



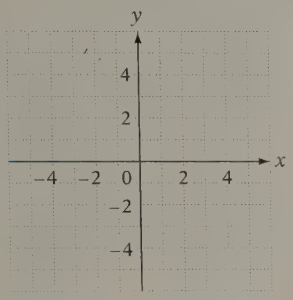
Solution on p. S10

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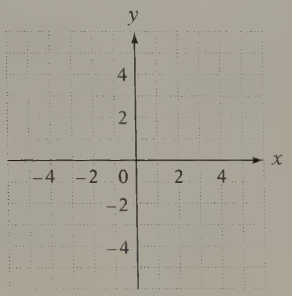
4.1 Exercises

Objective A

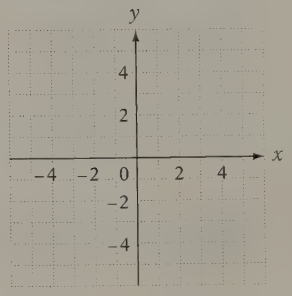
1. Graph $(-2, 1)$, $(3, -5)$, $(-2, 4)$, and $(0, 3)$.



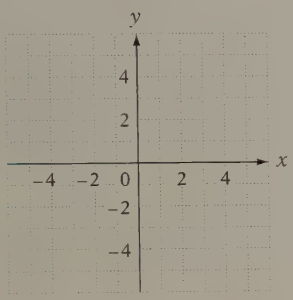
2. Graph $(5, -1)$, $(-3, -3)$, $(-1, 0)$, and $(1, -1)$.



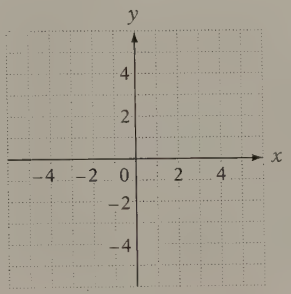
3. Graph $(0, 0)$, $(0, -5)$, $(-3, 0)$, and $(0, 2)$.



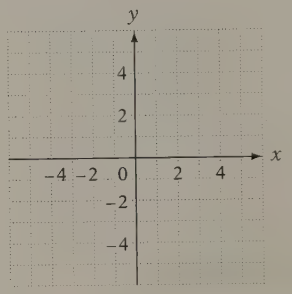
4. Graph $(-4, 5)$, $(-3, 1)$, $(3, -4)$, and $(5, 0)$.



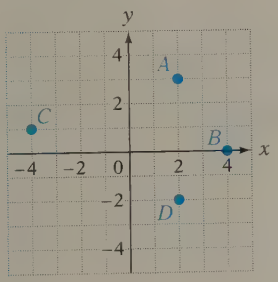
5. Graph $(-1, 4)$, $(-2, -3)$, $(0, 2)$, and $(4, 0)$.



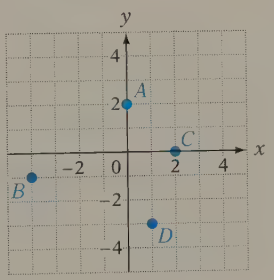
6. Graph $(5, 2)$, $(-4, -1)$, $(0, 0)$, and $(0, 3)$.



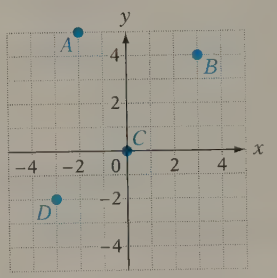
7. Find the coordinates of each of the points.



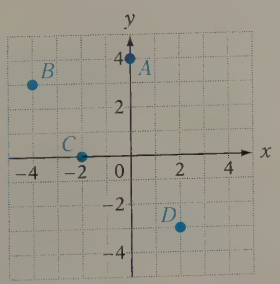
8. Find the coordinates of each of the points.



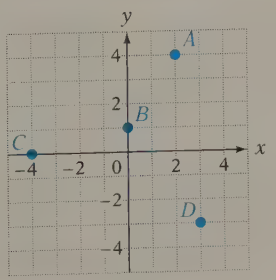
9. Find the coordinates of each of the points.



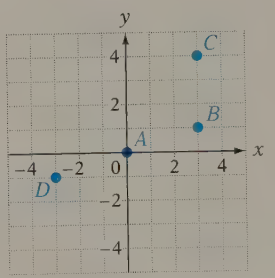
10. Find the coordinates of each of the points.



11. Find the coordinates of each of the points.

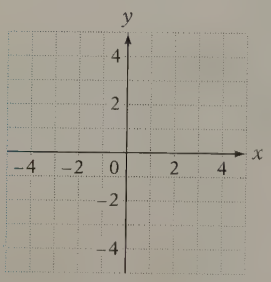


12. Find the coordinates of each of the points.

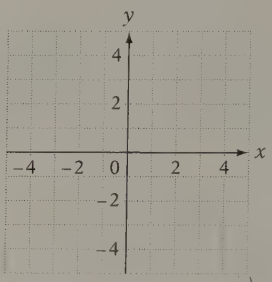


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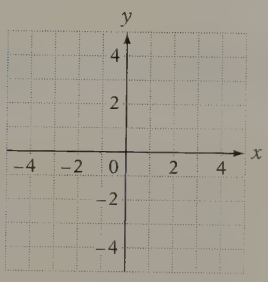
13. Draw a line through all points with an abscissa of 2.



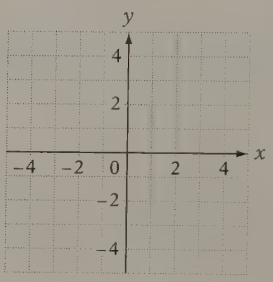
14. Draw a line through all points with an abscissa of −3.



15. Draw a line through all points with an ordinate of −3.



16. Draw a line through all points with an ordinate of 4.



Objective B

17. Is (3, 4) a solution of $y = -x + 7$?

18. Is (2, −3) a solution of $y = x + 5$?

19. Is (−1, 2) a solution of $y = \frac{1}{2}x - 1$?

20. Is (1, −3) a solution of $y = -2x - 1$?

21. Is (4, 1) a solution of $2x - 5y = 4$?

22. Is (−5, 3) a solution of $3x - 2y = 9$?

23. Is (0, 4) a solution of $3x - 4y = -4$?

24. Is (−2, 0) a solution of $x + 2y = -1$?

25. Find the ordered-pair solution of $y = 3x - 2$ corresponding to $x = 3$.

26. Find the ordered-pair solution of $y = 4x + 1$ corresponding to $x = -1$.

27. Find the ordered-pair solution of $y = \frac{2}{3}x - 1$ corresponding to $x = 6$.

28. Find the ordered-pair solution of $y = \frac{3}{4}x - 2$ corresponding to $x = 4$.

29. Find the ordered-pair solution of $y = -3x + 1$ corresponding to $x = 0$.

30. Find the ordered-pair solution of $y = \frac{2}{5}x - 5$ corresponding to $x = 0$.

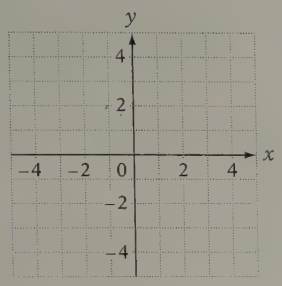
31. Find the ordered-pair solution of $y = \frac{2}{5}x + 2$ corresponding to $x = -5$.

32. Find the ordered-pair solution of $y = -\frac{1}{6}x - 2$ corresponding to $x = 12$.

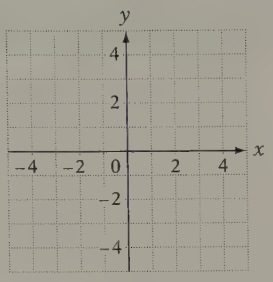
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Graph the ordered-pair solutions for the given values of x.

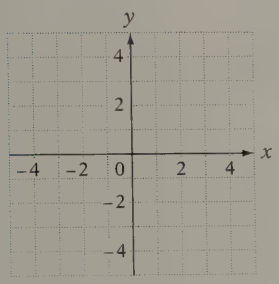
33. $y = 2x; x = -2, -1, 0, 2$



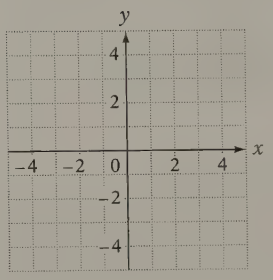
34. $y = -2x; x = -2, -1, 0, 2$



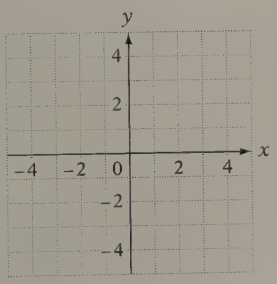
35. $y = x + 2; x = -4, -2, 0, 3$



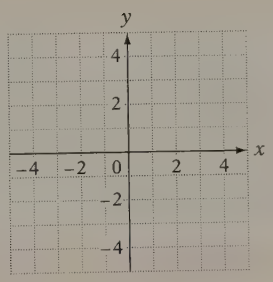
36. $y = \frac{1}{2}x - 1; x = -2, 0, 2, 4$



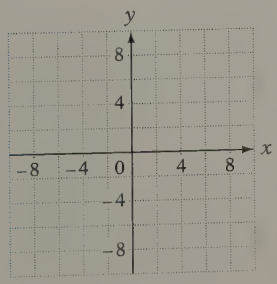
37. $y = \frac{2}{3}x + 1; x = -3, 0, 3$



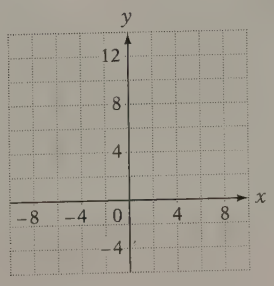
38. $y = -\frac{1}{3}x - 2; x = -3, 0, 3$



39. $y = x^2; x = -2, -1, 0, 1, 2$



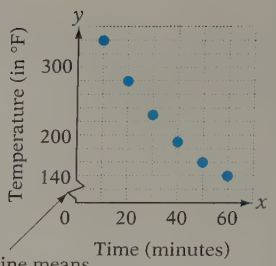
40. $y = |x + 1|; x = -5, -3, 0, 3, 5$



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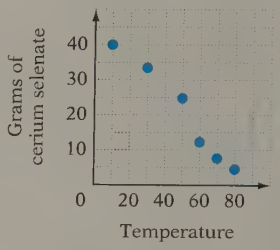
Objective C

41. The temperature of a chemical reaction is measured at intervals of 10 min and recorded in the scatter diagram at the right.
a. Find the temperature of the reaction after 20 min.
b. After how many minutes is the temperature 160°F?



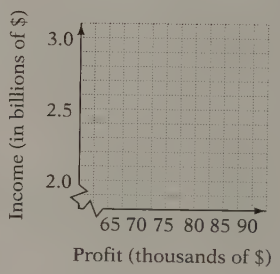
The jagged line means that the numbers between 0 and 140 are missing.

42. The amount of a substance that can be dissolved in a fixed amount of water usually increases as the temperature of the water increases. Cerium selenate, however, does not behave in this manner. The graph at the right shows the number of grams of cerium selenate that will dissolve in 100 mg of water for various temperatures, in degrees Celsius.
a. Determine the temperature at which 25 g of cerium selenate will dissolve.
b. Determine the number of grams of cerium selenate that will dissolve when the temperature is 80°C.



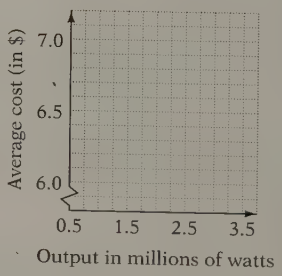
43. Past experience recorded by executives of a car company shows that the profit of a dealership will depend on the total income of all the residents of the town in which the dealership is located. The table below shows the profit of several dealerships and the total income of the towns. Draw the scatter diagram for these data.

Profit (thousands of $)	65	85	81	77	89	69
Total Income (billions of $)	2.2	2.6	2.5	2.4	2.7	2.3



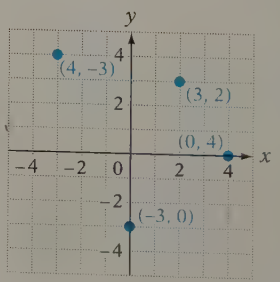
44. A power company suggests that a larger power plant can produce energy more efficiently and therefore at lower cost to consumers. The table below shows the output and average cost for power plants of various sizes. Draw the scatter diagram for these data.

Output (in millions of watts)	0.7	2.2	2.6	3.2	2.8	3.5
Average Cost (in dollars)	6.9	6.5	6.3	6.4	6.5	6.1



APPLYING THE CONCEPTS

45. Suppose you are helping a student who is having trouble graphing ordered pairs. The work of the student is at the right. What can you say to this student to correct the error that is being made?



46. Describe the graph of all the ordered pairs (x, y) that are 5 units from the origin.

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4.2 Introduction to Functions

Objective A

To evaluate a function

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In mathematics and its applications, there are many times when it is necessary to investigate a relationship between two quantities. Here is a financial application: Consider a person who is planning to finance the purchase of a car. If the current interest rate for a five-year loan is 9%, the equation that describes the relationship between the amount that is borrowed B and the monthly payment P is $P = 0.020758B$.

For each amount the purchaser may borrow (B), there is a certain monthly payment (P). The relationship between the amount borrowed and the payment can be recorded as ordered pairs, where the first coordinate is the amount borrowed and the second coordinate is the monthly payment. Some of these ordered pairs are shown at the right.

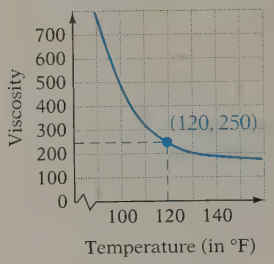
$0.020758B = P$

(5000, 103.79)
(6000, 124.55)
(7000, 145.31)
(8000, 166.06)

A relationship between two quantities is not always given by an equation. The table at the right describes a grading scale that defines a relationship between a score on a test and a letter grade. For each score, the table assigns only one letter grade. The ordered pair (84, B) indicates that a score of 84 receives a letter grade of B.

Score	*Grade*
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

The graph at the right also shows a relationship between two quantities. It is a graph of the viscosity V of SAE 40 motor oil at various temperatures T. Ordered pairs can be approximated from the graph. The ordered pair (120, 250) indicates that the viscosity of the oil at 120°F is 250 units.



In each of these examples, there is a rule (an equation, a table, or a graph) that determines a certain set of ordered pairs.

Definition of Relation

A **relation** is a set of ordered pairs.

Here are some of the ordered pairs for the relations given above.

Relation	*Some of the Ordered Pairs of the Relation*
Car Payment	(2500, 51.90), (3750, 77.84), (4396, 91.25)
Grading Scale	(78, C), (98, A), (70, C), (81, B), (94, A)
Oil Viscosity	(100, 500), (120, 250), (130, 200), (150, 180)

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These three relations are actually a special type of relation called a function. Functions play an important role in mathematics and its applications.

Definition of Function

A **function** is a relation in which no two ordered pairs have the same first coordinate and different second coordinates.

The **domain** of a function is the set of the first coordinates of all the ordered pairs of the function. The **range** is the set of the second coordinates of all the ordered pairs of the function.

For the function defined by the ordered pairs

$$\{(2, 3), (4, 5), (6, 7), (8, 9)\}$$

the domain is $\{2, 4, 6, 8\}$ and the range is $\{3, 5, 7, 9\}$.

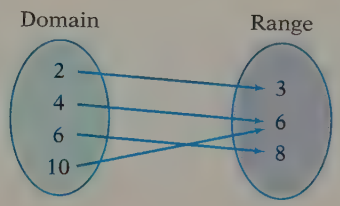
Find the domain and range of the function $\{(2, 3), (4, 6), (6, 8), (10, 6)\}$.

The domain is $\{2, 4, 6, 10\}$.

The range is $\{3, 6, 8\}$.

- The domain of the function is the set of the first components in the ordered pairs.
- The range of the function is the set of the second components in the ordered pairs.

For each element of the domain of a function there is a corresponding element in the range of the function. A possible diagram for the function in the example above is



$\{(2, 3), (4, 6), (6, 8), (10, 6)\}$

Functions defined by tables or graphs, such as those described at the beginning of this section, have important applications. However, a major focus of this text is functions defined by equations in two variables.

The "square" function, which pairs each real number with its square, can be defined by the equation

$$y = x^2$$

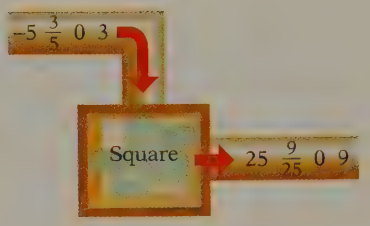
This equation states that for a given value of x in the domain, the value of y in the range is the square of x. For instance, if $x = 6$, then $y = 36$, and if $x = -7$, then $y = 49$. Because the value of y *depends* on the value of x, y is called the **dependent variable** and x is called the **independent variable.**

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TAKE NOTE

A pictorial representation of the square function is shown at the right. The function acts as a number from the domain into the square of the number.

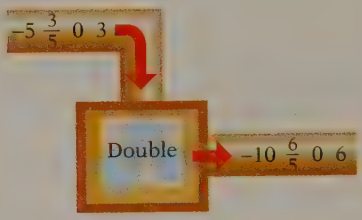
A function can be thought of as a rule that pairs one number with another number. For instance, the **square function** pairs a number with its square. The ordered pairs for the values shown at the right are $(-5, 25)$, $\left(\frac{3}{5}, \frac{9}{25}\right)$, $(0, 0)$, and $(3, 9)$. For this function, the second coordinate is the square of the first coordinate. If we let x represent the first coordinate, then the second coordinate is x^2 and we have the ordered pair (x, x^2).



$f(x) = x^2$

A function cannot have two ordered pairs with *different* second coordinates and the same first coordinate. However, a function can contain ordered pairs with the *same* second coordinate. For instance, the square function has the ordered pairs $(-3, 9)$ and $(3, 9)$: the same second coordinate but different first coordinates.

The **double function** pairs a number with twice that number. The ordered pairs for the values shown at the right are $(-5, -10)$, $\left(\frac{3}{5}, \frac{6}{5}\right)$, $(0, 0)$, and $(3, 6)$. For this function, the second coordinate is twice the first coordinate. If we let x represent the first coordinate, then the second coordinate is $2x$ and we have the ordered pair $(x, 2x)$.



$g(x) = 2x$

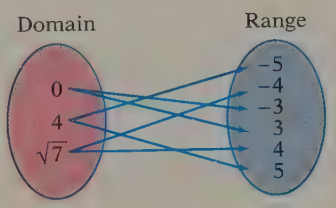
Not every equation in two variables defines a function. For instance, consider

$$y^2 = x^2 + 9$$

This equation does not define a function. Because

$$5^2 = 4^2 + 9 \quad \text{and} \quad (-5)^2 = 4^2 + 9$$

the ordered pairs $(4, 5)$ and $(4, -5)$ are both solutions of the equation. Consequently, there are two ordered pairs that have the same first coordinate, 4, but *different* second coordinates, 5 and −5. Therefore, the equation does not define a function. Other ordered pairs for this equation are $(0, -3)$, $(0, 3)$, $(\sqrt{7}, -4)$, and $(\sqrt{7}, 4)$. A graphical representation of these ordered pairs is shown below.

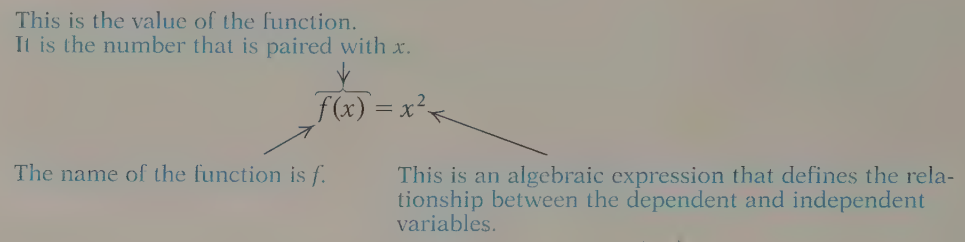


Note from this graphical representation that an element from the domain has two arrows to different elements in the range. Any time this occurs, the situation does not represent a function. However, this diagram does represent a relation. The relation for the values shown is $\{(0, -3), (0, 3), (4, -5), (4, 5), (\sqrt{7}, -4), (\sqrt{7}, 4)\}$.

The phrase "y is a function of x," or a similar phrase with different variables, is used to describe those equations in two variables that define functions. To emphasize that the equation represents a function, **functional notation** is used.

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Just as x is commonly used as a variable to represent a number, the letter f is commonly used to name a function. The square function is written in functional notation as follows:



The symbol $f(x)$ is read "the *value* of f at x" or "f of x."

TAKE NOTE
The dependent variable y and $f(x)$ can be used interchangeably.

It is important to note that $f(x)$ does not mean f times x. The symbol $f(x)$ is the **value of the function** and represents the value of the dependent variable for a given value of the independent variable. We often write $\boldsymbol{y = f(x)}$ to emphasize the relationship between the independent variable, x, and the dependent variable, y. Remember that y and $f(x)$ are different symbols for the same number.

The letters used to represent a function are somewhat arbitrary. All of the following equations represent the same function.

$$\left.\begin{aligned} f(x) &= x^2 \\ s(t) &= t^2 \\ P(v) &= v^2 \end{aligned}\right\} \text{ These equations represent the square function.}$$

The process of determining $f(x)$ for a given value of x is called **evaluating the function.** For instance, to evaluate $f(x) = x^2$ when $x = 4$, replace x by 4 and simplify.

$$\begin{aligned} f(x) &= x^2 \\ f(4) &= 4^2 = 16 \end{aligned}$$

The *value* of the function is 16 when $x = 4$. An ordered pair of the function is $(4, 16)$.

➨ Evaluate $g(t) = 3t^2 - 5t + 1$ when $t = -2$.

$$\begin{aligned} g(t) &= 3t^2 - 5t + 1 \\ g(-2) &= 3(-2)^2 - 5(-2) + 1 \\ &= 3(4) - 5(-2) + 1 \\ &= 12 + 10 + 1 = 23 \end{aligned}$$

• Replace t by -2 and then simplify.

When t is -2, the value of the function is 23.
Therefore, an ordered pair of the function is $(-2, 23)$.

It is possible to evaluate a function for a variable expression.

➨ Evaluate $P(z) = 3z - 7$ when $z = 3 + h$.

$$\begin{aligned} P(z) &= 3z - 7 \\ P(3 + h) &= 3(3 + h) - 7 \\ &= 9 + 3h - 7 \\ &= 3h + 2 \end{aligned}$$

• Replace z by $3 + h$ and then simplify.

When z is $3 + h$, the value of the function is $3h + 2$.
Therefore, an ordered pair of the function is $(3 + h, 3h + 2)$.

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The range of a function contains all the elements that result from applying the function to each element of the domain. If the domain contains an infinite number of elements, then it may be difficult to find the range. However, if the domain has only a finite number of elements, then the range can be found by evaluating the function for each element in the domain.

Find the range of $f(x) = x^3 + x$ if the domain is $\{-2, -1, 0, 1, 2\}$.

$$f(x) = x^3 + x$$
$$f(-2) = (-2)^3 + (-2) = -10$$
$$f(-1) = (-1)^3 + (-1) = -2$$
$$f(0) = 0^3 + 0 = 0$$
$$f(1) = 1^3 + 1 = 2$$
$$f(2) = 2^3 + 2 = 10$$

- Replace x by each member of the domain. The range includes the values of $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$.

The range is $\{-10, -2, 0, 2, 10\}$.

When a function is represented by an equation, the domain of the function is all real numbers for which the value of the function is a real number. For instance,

- The domain of $f(x) = x^2$ is all real numbers, because the square of every real number is a real number.
- The domain of $g(x) = \frac{1}{x - 2}$ is all real numbers except 2, because when $x = 2$, $g(2) = \frac{1}{2 - 2} = \frac{1}{0}$, which is not a real number.

The domain of the grading-scale function is the set of whole numbers from 0 to 100. In set-builder notation, this is written $\{x \mid 0 \le x \le 100, x \in \text{whole numbers}\}$. The range is {A, B, C, D, F}.

Score	*Grade*
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

What values, if any, are excluded from the domain of $f(x) = 2x^2 - 7x + 1$?

Because the value of $2x^2 - 7x + 1$ is a real number for any value of x, the domain of the function is all real numbers. No values are excluded from the domain of $f(x) = 2x^2 - 7x + 1$.

Example 1

Find the domain and range of the function $\{(2, 3), (4, 6), (6, 8), (10, 6)\}$.

Solution

Domain: $\{2, 4, 6, 10\}$;
Range: $\{3, 6, 8\}$

You Try It 1

Find the domain and range of the function $\{(-1, 5), (3, 5), (4, 5), (6, 5)\}$.

Your Solution

Solution on p. S11

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Example 2

Given $p(r) = 5r^3 - 6r - 2$, find $p(-3)$.

Solution

$$\begin{aligned} p(r) &= 5r^3 - 6r - 2 \\ p(-3) &= 5(-3)^3 - 6(-3) - 2 \\ &= 5(-27) + 18 - 2 \\ &= -135 + 18 - 2 = -119 \end{aligned}$$

You Try It 2

Evaluate $G(x) = \frac{3x}{x + 2}$ when $x = -4$.

Your Solution

Example 3

Evaluate $Q(r) = 2r + 5$ when $r = h + 3$.

Solution

$$\begin{aligned} Q(r) &= 2r + 5 \\ Q(h + 3) &= 2(h + 3) + 5 \\ &= 2h + 6 + 5 \\ &= 2h + 11 \end{aligned}$$

You Try It 3

Evaluate $f(x) = x^2 - 11$ when $x = 3h$.

Your solution

Example 4

Find the range of $f(x) = x^2 - 1$ if the domain is $\{-2, -1, 0, 1, 2\}$.

Solution

To find the range, evaluate the function at each element of the domain.

$$\begin{aligned} f(x) &= x^2 - 1 \\ f(-2) &= (-2)^2 - 1 = 4 - 1 = 3 \\ f(-1) &= (-1)^2 - 1 = 1 - 1 = 0 \\ f(0) &= 0^2 - 1 = 0 - 1 = -1 \\ f(1) &= 1^2 - 1 = 1 - 1 = 0 \\ f(2) &= 2^2 - 1 = 4 - 1 = 3 \end{aligned}$$

The range is $\{-1, 0, 3\}$. Note that 0 and 3 are listed only once.

You Try It 4

Find the range of $h(z) = 3z + 1$ if the domain is $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$.

Your solution

Example 5

What is the domain of $f(x) = 2x^2 - 7x + 1$?

Solution

Because $2x^2 - 7x + 1$ evaluates to a real number for any value of x, the domain of the function is all real numbers.

You Try It 5

What value is excluded from the domain of $f(x) = \frac{2}{x - 5}$?

Your solution

Solutions on p. S11

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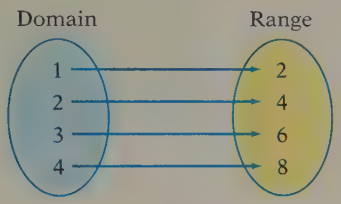
4.2 Exercises

Objective A

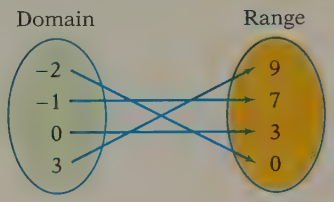
1. In your own words, explain what a function is.

2. What is the domain of a function? What is the range of a function?

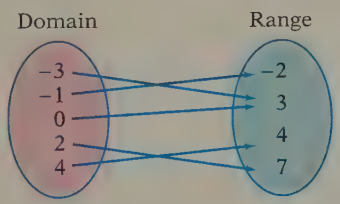
3. Does the diagram below represent a function? Explain your answer.



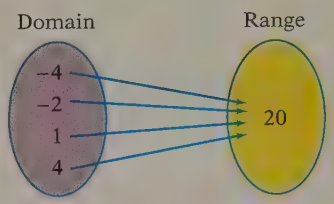
4. Does the diagram below represent a function? Explain your answer.



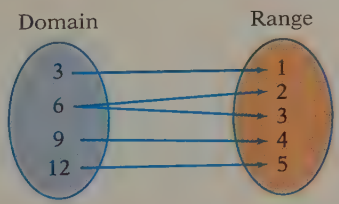
5. Does the diagram below represent a function? Explain your answer.



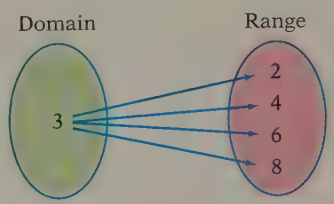
6. Does the diagram below represent a function? Explain your answer.



7. Does the diagram below represent a function? Explain your answer.



8. Does the diagram below represent a function? Explain your answer.



State whether each of the following relations is a function.

9. {(0, 0), (2, 4), (3, 6), (4, 8), (5, 10)}

10. {(1, 3), (3, 5), (5, 7), (7, 9)}

11. {(−2, −1), (−4, −5), (0, −1), (3, 5)}

12. {(−3, −1), (−1, −1), (0, 1), (2, 6)}

13. {(−2, 3), (−1, 3), (0, −3), (1, 3), (2, 3)}

14. {(0, 0), (1, 0), (2, 0), (3, 0), (4, 0)}

15. {(1, 1), (4, 2), (9, 3), (1, −1), (4, −2)}

16. {(3, 1), (3, 2), (3, 3), (3, 4)}

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17. The table at the right shows the cost to send an overnight package using United Parcel Service.
a. Does this table define a function?
b. Given $x = 2.75$ lb, find y.

Weight in pounds (x)	Cost (y)
$0 < x \le 1$	\$24.05
$1 < x \le 2$	\$26.83
$2 < x \le 3$	\$29.62
$3 < x \le 4$	\$32.40
$4 < x \le 5$	\$35.18

18. The table at the right shows the cost to send an "Express Mail" package using the U.S. Postal Service.
a. Does this table define a function?
b. Given $x = 0.5$ lb, find y.

Weight in pounds (x)	Cost (y)
$0 < x \le 0.5$	\$10.70
$0.5 < x \le 2$	\$14.90
$2 < x \le 3$	\$14.90
$3 < x \le 4$	\$18.10
$4 < x \le 5$	\$21.25

Given $f(x) = 5x - 4$, evaluate:

19. $f(3)$ **20.** $f(-2)$ **21.** $f(0)$ **22.** $f(-1)$

Given $G(t) = 4 - 3t$, evaluate:

23. $G(0)$ **24.** $G(-3)$ **25.** $G(-2)$ **26.** $G(4)$

Given $q(r) = r^2 - 4$, evaluate:

27. $q(3)$ **28.** $q(4)$ **29.** $q(-2)$ **30.** $q(-5)$

Given $F(x) = x^2 + 3x - 4$, evaluate:

31. $F(4)$ **32.** $F(-4)$ **33.** $F(-3)$ **34.** $F(-6)$

Given $H(p) = \frac{3p}{p+2}$, evaluate:

35. $H(1)$ **36.** $H(-3)$ **37.** $H(t)$ **38.** $H(v)$

Given $s(t) = t^3 - 3t + 4$, evaluate:

39. $s(-1)$ **40.** $s(2)$ **41.** $s(a)$ **42.** $s(w)$

43. Given $P(x) = 4x + 7$, write $P(-2 + h) - P(-2)$ in simplest form.

44. Given $G(t) = 9 - 2t$, write $G(-3 + h) - G(-3)$ in simplest form.

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45. Game Engineering has just completed the programming and testing for a new computer game. The cost to manufacture and package the game depends on the number of units Game Engineering plans to sell. The table at the right shows the cost per game for packaging various quantities. Evaluate this function when

a. $x = 7000$ **b.** $x = 20{,}000$

Number of Games Manufactured	Cost to Manufacture One Game
$0 < x \le 2500$	\$6.00
$2500 < x \le 5000$	\$5.50
$5000 < x \le 10{,}000$	\$4.75
$10{,}000 < x \le 20{,}000$	\$4.00
$20{,}000 < x \le 40{,}000$	\$3.00

46. Airport administrators have a tendency to price airport parking at a rate that discourages people from using the parking lot for long periods of time. The rate structure for an airport is given in the table at the right. Evaluate this function when

a. $t = 2.5$ h **b.** $t = 7$ h

Hours Parked	Cost
$0 < t \le 1$	\$1.00
$1 < t \le 2$	\$3.00
$2 < t \le 4$	\$6.50
$4 < t \le 7$	\$10.00
$7 < t \le 12$	\$14.00

47. A real estate appraiser charges a fee that depends on the estimated value, V, of the property. A table giving the fees charged for various estimated values of the real estate appears at the right. Evaluate this function when

a. $V = \$5{,}000{,}000$ **b.** $V = \$767{,}000$

Value of Property	Appraisal Fee
$V < 100{,}000$	\$350
$100{,}000 \le V < 500{,}000$	\$525
$500{,}000 \le V < 1{,}000{,}000$	\$950
$1{,}000{,}000 \le V < 5{,}000{,}000$	\$2500
$5{,}000{,}000 \le V < 10{,}000{,}000$	\$3000

48. The cost to mail a priority overnight package by Federal Express depends on the weight, w, of the package. A table of the costs for selected weights is given at the right. Evaluate this function when

a. $w = 2$ lb 3 oz **b.** $w = 1.9$ lb

Weight (lb)	Cost
$0 < w < 1$	\$24.00
$1 \le w < 2$	\$26.75
$2 \le w < 3$	\$29.25
$3 \le w < 4$	\$32.00
$4 \le w < 5$	\$34.00

Find the domain and range of the function.

49. $\{(1, 1), (2, 4), (3, 7), (4, 10), (5, 13)\}$

50. $\{(2, 6), (4, 18), (6, 38), (8, 66), (10, 102)\}$

51. $\{(0, 1), (2, 2), (4, 3), (6, 4)\}$

52. $\{(0, 1), (1, 2), (4, 3), (9, 4)\}$

53. $\{(1, 0), (3, 0), (5, 0), (7, 0), (9, 0)\}$

54. $\{(-2, -4), (2, 4), (-1, 1), (1, 1), (-3, 9), (3, 9)\}$

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55. $\{(0, 0), (1, 1), (-1, 1), (2, 2), (-2, 2)\}$

56. $\{(0, -5), (5, 0), (10, 5), (15, 10)\}$

57. $\{(-2, -3), (-1, 6), (0, 7), (2, 3), (1, 9)\}$

58. $\{(-8, 0), (-4, 2), (-2, 4), (0, -4), (4, 4)\}$

What values are excluded from the domain of the function?

59. $f(x) = \dfrac{1}{x - 1}$

60. $g(x) = \dfrac{1}{x + 4}$

61. $h(x) = \dfrac{x + 3}{x + 8}$

62. $F(x) = \dfrac{2x - 5}{x - 4}$

63. $f(x) = 3x + 2$

64. $g(x) = 4 - 2x$

65. $G(x) = x^2 + 1$

66. $H(x) = \dfrac{1}{2}x^2$

67. $f(x) = \dfrac{x - 1}{x}$

68. $g(x) = \dfrac{2x + 5}{7}$

69. $H(x) = x^2 - x + 1$

70. $f(x) = 3x^2 + x + 4$

71. $f(x) = \dfrac{2x - 5}{3}$

72. $g(x) = \dfrac{3 - 5x}{5}$

73. $H(x) = \dfrac{x - 2}{x + 2}$

74. $h(x) = \dfrac{3 - x}{6 - x}$

75. $f(x) = \dfrac{x - 2}{2}$

76. $G(x) = \dfrac{2}{x - 2}$

Find the range of the function defined by each equation and the given domain.

77. $f(x) = 4x - 3$; domain $= \{0, 1, 2, 3\}$

78. $G(x) = 3 - 5x$; domain $= \{-2, -1, 0, 1, 2\}$

79. $g(x) = 5x - 8$; domain $= \{-3, -1, 0, 1, 3\}$

80. $h(x) = 3x - 7$; domain $= \{-4, -2, 0, 2, 4\}$

81. $h(x) = x^2$; domain $= \{-2, -1, 0, 1, 2\}$

82. $H(x) = 1 - x^2$; domain $= \{-2, -1, 0, 1, 2\}$

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83. $f(x) = 2x^2 - 2x + 2$; domain $= \{-4, -2, 0, 4\}$

84. $G(x) = -2x^2 + 5x - 2$; domain $= \{-3, -1, 0, 1, 3\}$

85. $H(x) = \dfrac{5}{1 - x}$; domain $= \{-2, 0, 2\}$

86. $g(x) = \dfrac{4}{4 - x}$; domain $= \{-5, 0, 3\}$

87. $f(x) = \dfrac{2}{x - 4}$; domain $= \{-2, 0, 2, 6\}$

88. $g(x) = \dfrac{x}{3 - x}$; domain $= \{-2, -1, 0, 1, 2\}$

89. $H(x) = 2 - 3x - x^2$; domain $= \{-5, 0, 5\}$

90. $G(x) = 4 - 3x - x^3$; domain $= \{-3, 0, 3\}$

APPLYING THE CONCEPTS

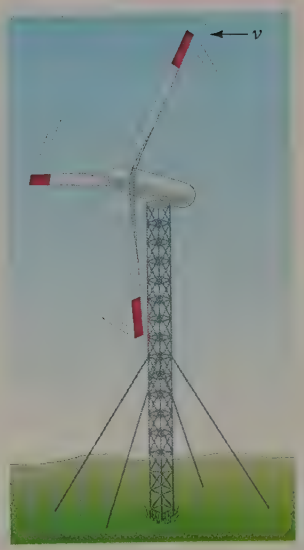
91. Explain the words *relation* and *function*. Include in your explanation how the meanings of the two words differ.

92. Give a real-world example of a relation that is not a function. Is it possible to give an example of a function that is not a relation? If so, give one. If not, explain why it is not possible.

93. Find the set of ordered pairs (x, y) determined by the equation $y = x^3$, where $x \in \{-2, -1, 0, 1, 2\}$. Does the set of ordered pairs define a function? Why or why not?

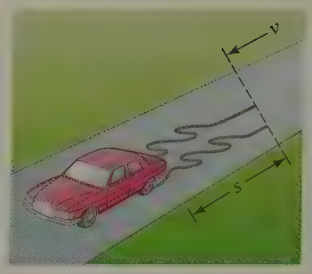
94. Find the set of ordered pairs (x, y) determined by the equation $|y| = x$, where $x \in \{0, 1, 2, 3\}$. Does the set of ordered pairs define a function? Why or why not?

95. The power a windmill can generate is a function of the velocity of the wind. The function can be approximated by $P = f(v) = 0.015v^3$, where P is the power in watts and v is the velocity of the wind in meters per second. How much power will be produced by a windmill when the velocity of the wind is 15 m/s?



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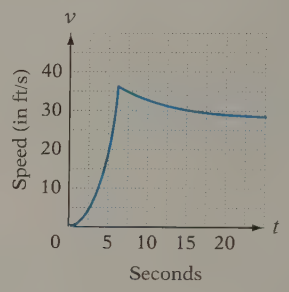
96. The distance s, in feet, that a car will skid on a certain road surface after the brakes are applied is a function of the car's velocity, v (in miles per hour). The function can be approximated by $s = f(v) = 0.017v^2$. How far will a car skid after its brakes are applied if it is traveling 60 mph?



Each of the following graphs defines a function. Evaluate the function by estimating the ordinate (which is the value of the function) for the given value of t.

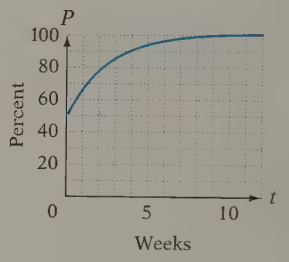
97. The graph at the right shows the speed v, in feet per second, that a parachutist is falling during the first 20 s after jumping out of a plane. Estimate the speed at which the parachutist is falling when

a. $t = 5$ s **b.** $t = 15$ s



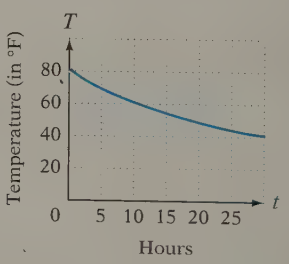
98. The graph at the right shows what an industrial psychologist has determined to be the average percent score, P, for an employee taking a performance test t weeks after training begins. Estimate the score an employee would receive on this test when

a. $t = 4$ weeks **b.** $t = 10$ weeks



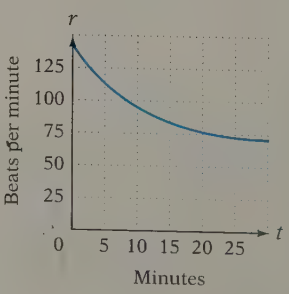
99. The graph at the right shows the temperature T, in degrees Fahrenheit, of a can of cola t hours after it is placed in a refrigerator. Use the graph to estimate the temperature of the cola when

a. $t = 5$ h **b.** $t = 15$ h



100. The graph at the right shows the decrease in the heart rate r, in beats per minute, of a runner t minutes after the completion of a race. Use the graph to estimate the heart rate of a runner when

a. $t = 5$ min **b.** $t = 20$ min



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4.3 Linear Equations in Two Variables

Objective A

To graph an equation of the form $y = mx + b$

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The Projects and Group Activities at the end of this chapter contain information on using calculators to graph an equation.

The **graph of an equation in two variables** is a graph of the ordered-pair solutions of the equation.

Consider $y = 2x + 1$. Choosing $x = -2$, -1, 0, 1, and 2 and determining the corresponding values of y produces some of the ordered pairs of the equation. These are recorded in the table at the right. See the graph of the ordered pairs in Figure 1.

x	$y = 2x + 1$	y	(x, y)
-2	$2(-2) + 1$	-3	$(-2, -3)$
-1	$2(-1) + 1$	-1	$(-1, -1)$
0	$2(0) + 1$	1	$(0, 1)$
1	$2(1) + 1$	3	$(1, 3)$
2	$2(2) + 1$	5	$(2, 5)$

Choosing values of x that are not integers produces more ordered pairs to graph, such as $\left(-\frac{5}{2}, -4\right)$ and $\left(\frac{3}{2}, 4\right)$, as shown in Figure 2. Choosing still other values of x would result in more and more ordered pairs being graphed. The result would be so many dots that the graph would appear as the straight line shown in Figure 3, which is the graph of $y = 2x + 1$.

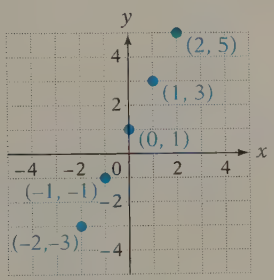


Figure 1

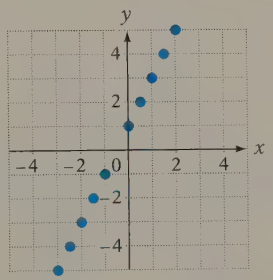


Figure 2

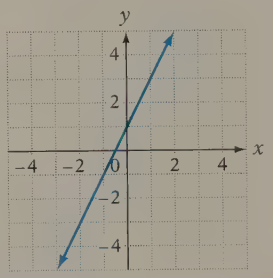


Figure 3

Equations in two variables have characteristic graphs. The equation $y = 2x + 1$ is an example of a *linear equation*, or *linear function*, because its graph is a straight line. It is also called a *first-degree equation* in two variables because the exponent on each variable is the first power.

Linear Equation in Two Variables

Any equation of the form $y = mx + b$, where m is the coefficient of x and b is a constant, is a **linear equation in two variables or a first-degree equation in two variables.** The graph of a linear equation in two variables is a straight line.

Examples of linear equations are shown at the right. These equations represent linear functions because there is only one possible y for each x. Note that for $y = 3 - 2x$, m is the coefficient of x and b is the constant.

$y = 2x + 1$	$(m = 2, b = 1)$
$y = x - 4$	$(m = 1, b = -4)$
$y = -\frac{3}{4}x$	$\left(m = -\frac{3}{4}, b = 0\right)$
$y = 3 - 2x$	$(m = -2, b = 3)$

The equation $y = x^2 + 4x + 3$ is not a linear equation in two variables because there is a term with a variable squared. The equation $y = \frac{3}{x - 4}$ is not a linear equation because a variable occurs in the denominator of a fraction.

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To graph a linear equation, find ordered-pair solutions of the equation. Do this by choosing any value of x and finding the corresponding value of y. Repeat this procedure, choosing different values for x, until you have found the number of solutions desired.

Because the graph of a linear equation in two variables is a straight line, and a straight line is determined by two points, it is necessary to find only two solutions. However, it is recommended that at least three points be used to ensure accuracy.

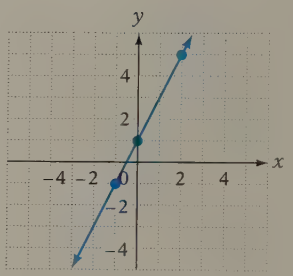
TAKE NOTE
If the three points you graph do not lie on a straight line, then either you have made an arithmetic error in calculating a point or you have plotted a point incorrectly.

➧ Graph $y = 2x + 1$.

Choose any values of x, and then find corresponding values of y. The numbers 0, 2, and -1 were chosen arbitrarily for x. It is convenient to record these solutions in a table.

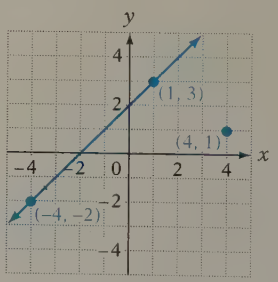
x	$y = 2x + 1$	y
0	$2(0) + 1$	1
2	$2(2) + 1$	5
-1	$2(-1) + 1$	-1

Graph the ordered-pair solutions $(0, 1)$, $(2, 5)$, and $(-1, -1)$. Draw a line through the ordered-pair solutions.



Remember that a graph is a drawing of the ordered-pair solutions of the equation. Therefore, every point on the graph is a solution of the equation, and every solution of the equation is a point on the graph.

The graph at the right is the graph of $y = x + 2$. Note that $(-4, -2)$ and $(1, 3)$ are points on the graph and that these points are solutions of $y = x + 2$. The point whose coordinates are $(4, 1)$ is not a point on the graph and is not a solution of the equation.



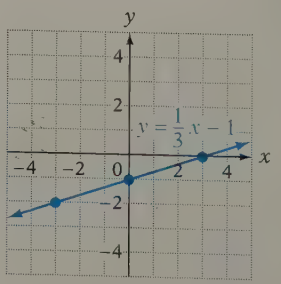
When m is a fraction in the equation $y = mx + b$, choose values of x that will simplify the evaluation.

➧ Graph $y = \frac{1}{3}x - 1$.

m is a fraction. $\left(m = \frac{1}{3}\right)$

Choose values of x that are multiples of the denominator. The numbers 0, 3, and -3 are used here.

x	y
0	-1
3	0
-3	-2

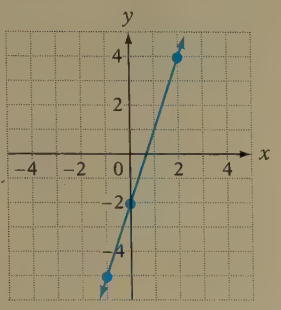


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Example 1 Graph $y = 3x - 2$.

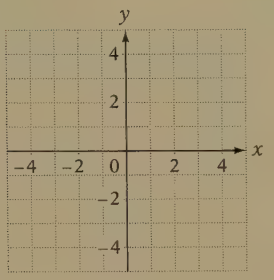
Solution

x	y
0	−2
−1	−5
2	4



You Try It 1 Graph $y = 3x + 1$.

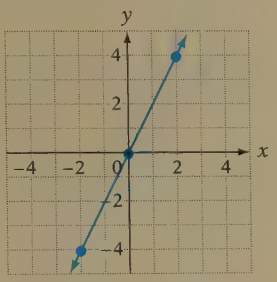
Your solution



Example 2 Graph $y = 2x$.

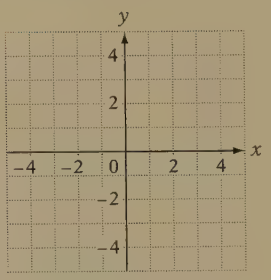
Solution

x	y
0	0
2	4
−2	−4



You Try It 2 Graph $y = -2x$.

Your solution

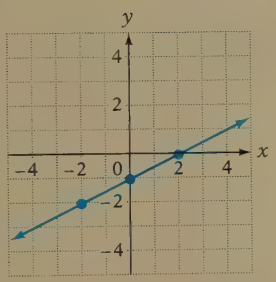


Example 3

Graph $y = \frac{1}{2}x - 1$.

Solution

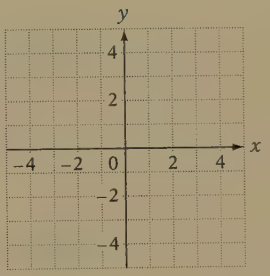
x	y
0	−1
2	0
−2	−2



You Try It 3

Graph $y = \frac{1}{3}x - 3$.

Your solution



Solutions on p. S11

Objective B To graph an equation of the form $Ax + By = C$



An equation in the form $Ax + By = C$ is also a linear equation. Examples of these equations are shown below.

$2x + 3y = 6$ $(A = 2, B = 3, C = 6)$
$x - 2y = -4$ $(A = 1, B = -2, C = -4)$
$2x + y = 0$ $(A = 2, B = 1, C = 0)$
$4x - 5y = 2$ $(A = 4, B = -5, C = 2)$

One method of graphing an equation of the form $Ax + By = C$ involves first solving the equation for y and then following the same procedure used for graphing an equation of the form $y = mx + b$. To solve the equation for y means to rewrite the equation so that y is alone on one side of the equation and the term containing x and the constant are on the other side of the equation. The Addition and Multiplication Properties of Equations are used to rewrite an equation of the form $Ax + By = C$ in the form $y = mx + b$.

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➡ Solve the equation $3x + 2y = 4$ for y.

$$3x + 2y = 4$$

- The equation is in the form $Ax + By = C$.

$$3x - 3x + 2y = -3x + 4$$

- Use the Addition Property of Equations to subtract the term $3x$ from each side of the equation.

$$2y = -3x + 4$$

- Simplify. Note that on the right side of the equation, the term containing x is first, followed by the constant.

$$\frac{1}{2} \cdot 2y = \frac{1}{2}(-3x + 4)$$

- Use the Multiplication Property of Equations to multiply each side of the equation by the reciprocal of the coefficient of y. (The coefficient of y is 2; the reciprocal of 2 is $\frac{1}{2}$.)

$$y = \frac{1}{2}(-3x) + \frac{1}{2}(4)$$

- Simplify. Use the Distributive Property on the right side of the equation.

$$y = -\frac{3}{2}x + 2$$

- The equation is now in the form $y = mx + b$, with $m = -\frac{3}{2}$ and $b = 2$.

In solving the equation $3x + 2y = 4$ for y, where we multiplied both sides of the equation by $\frac{1}{2}$, we could have divided both sides of the equation by 2, as shown at the right. In simplifying the right side after dividing both sides by 2, be sure to divide *each term* by 2.

$$2y = -3x + 4$$
$$\frac{2y}{2} = \frac{-3x + 4}{2}$$
$$y = \frac{-3x}{2} + \frac{4}{2}$$
$$y = -\frac{3}{2}x + 2$$

➡ Graph $3x + 4y = 12$.

$$3x + 4y = 12$$

- Solve the equation for y.

$$4y = -3x + 12$$

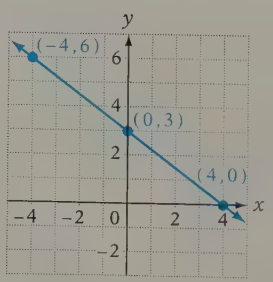
- Subtract $3x$ from each side of the equation.

$$y = -\frac{3}{4}x + 3$$

- Divide each side of the equation by 4.

x	y
0	3
4	0
−4	6

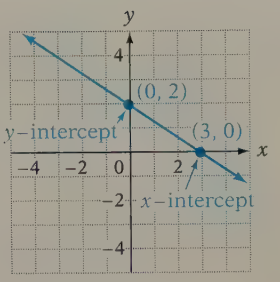
- Find three ordered-pair solutions of the equation.



- Graph the ordered pairs and then draw a line through the points.

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The graph of the equation $2x + 3y = 6$ is shown at the right. The graph crosses the x-axis at the point $(3, 0)$. This point is called the ***x*-intercept**. The graph also crosses the y-axis at the point $(0, 2)$. This point is called the ***y*-intercept**.



We can find the x-intercept and the y-intercept of the graph of the equation $2x + 3y = 6$ algebraically.

TAKE NOTE
Any point on the x-axis has y-coordinate 0. Any point on the y-axis has x-coordinate 0.

To find the x-intercept, let $y = 0$.
(Any point on the x-axis has y-coordinate 0.)

$$\begin{aligned} 2x + 3y &= 6 \\ 2x + 3(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

The x-intercept is $(3, 0)$.

To find the y-intercept, let $x = 0$.
(Any point on the y-axis has x-coordinate 0.)

$$\begin{aligned} 2x + 3y &= 6 \\ 2(0) + 3y &= 6 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

The y-intercept is $(0, 2)$.

Another method of graphing an equation of the form $Ax + By = C$ is to find the x- and y-intercepts, plot both intercepts, and then draw a line through the two points. This method of graphing the equation $3x + 4y = 12$ is shown below. Note that this is the same equation graphed at the bottom of the previous page.

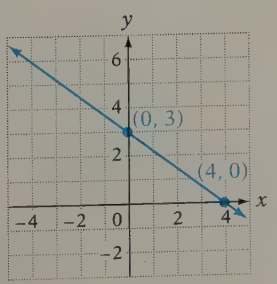
Graph $3x + 4y = 12$ by using the x- and y-intercepts.

x-intercept: $3x + 4y = 12$
$3x + 4(0) = 12$
$3x = 12$
$x = 4$

- To find the x-intercept, let $y = 0$.
- The x-intercept is $(4, 0)$.

y-intercept: $3x + 4y = 12$
$3(0) + 4y = 12$
$4y = 12$
$y = 3$

- To find the y-intercept, let $x = 0$.
- The y-intercept is $(0, 3)$.



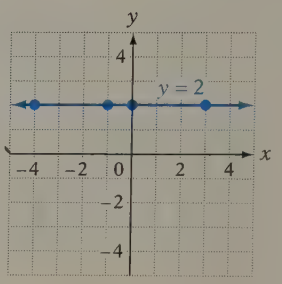
- Graph the ordered pairs $(4, 0)$, and $(0, 3)$. Draw a straight line through the points.

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The graph of a linear equation with one of the variables missing is either a horizontal or a vertical line.

The equation $y = 2$ could be written $0x + y = 2$. Because $0x = 0$ for any value of x, the value of y is always 2, no matter what value of x is chosen. For instance, replace x by -4, -1, 0, or 3. In each case, $y = 2$.

$$\begin{aligned} 0x + y &= 2 \\ 0(-4) + y &= 2 \quad (-4, 2) \text{ is a solution.} \\ 0(-1) + y &= 2 \quad (-1, 2) \text{ is a solution.} \\ 0(0) + y &= 2 \quad (0, 2) \text{ is a solution.} \\ 0(3) + y &= 2 \quad (3, 2) \text{ is a solution.} \end{aligned}$$



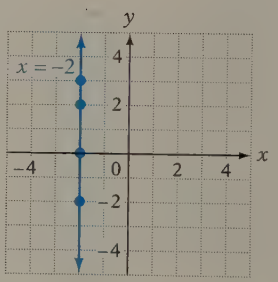
The solutions are plotted in the graph to the right, and a line is drawn through the plotted points. Note that the line is horizontal.

Graph of a Horizontal Line

The graph of $y = b$ is a horizontal line passing through $(0, b)$.

The equation $x = -2$ could be written $x + 0y = -2$. Because $0y = 0$ for any value of y, the value of x is always -2, no matter what value of y is chosen. For instance, replace y by -2, 0, 2, or 3. In each case, $x = -2$.

$$\begin{aligned} x + 0y &= -2 \\ x + 0(-2) &= -2 \quad (-2, -2) \text{ is a solution.} \\ x + 0(0) &= -2 \quad (-2, 0) \text{ is a solution.} \\ x + 0(2) &= -2 \quad (-2, 2) \text{ is a solution.} \\ x + 0(3) &= -2 \quad (-2, 3) \text{ is a solution.} \end{aligned}$$

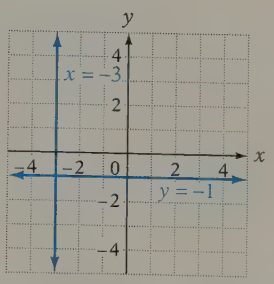


The solutions are plotted in the graph at the right, and a line is drawn through the plotted points. Note that the line is vertical.

Graph of a Vertical Line

The graph of $x = a$ is a vertical line passing through $(a, 0)$.

➡ Graph $x = -3$ and $y = -1$ on the same coordinate grid.



- The graph of $x = -3$ is a vertical line passing through $(-3, 0)$.
- The graph of $y = -1$ is a horizontal line passing through $(0, -1)$.

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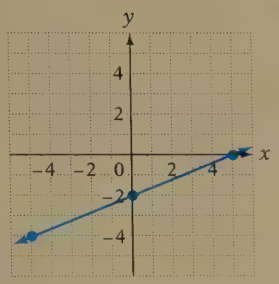
Example 4

Solve the equation $2x - 5y = 10$ for y. Then graph the equation.

Solution

$$2x - 5y = 10$$
$$-5y = -2x + 10$$
$$y = \frac{2}{5}x - 2$$

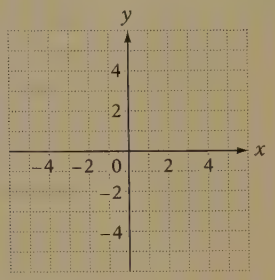
x	y
0	-2
5	0
-5	-4



You Try It 4

Solve the equation $5x - 2y = 10$ for y. Then graph the equation.

Your solution



Example 5

Find the x- and y-intercepts of $x - 2y = 4$. Graph the line.

Solution

x-intercept:

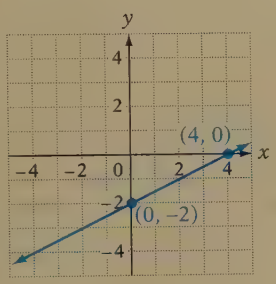
$$x - 2y = 4$$
$$x - 2(0) = 4$$
$$x = 4$$

$(4, 0)$

y-intercept:

$$x - 2y = 4$$
$$0 - 2y = 4$$
$$-2y = 4$$
$$y = -2$$

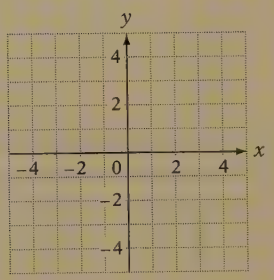
$(0, -2)$



You Try It 5

Find the x- and y-intercepts of $x - 4y = -4$. Graph the line.

Your solution

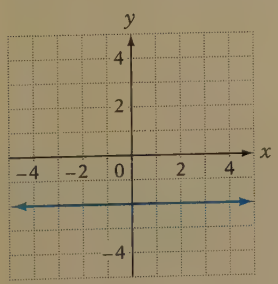


Example 6

Graph $y = -2$.

Solution

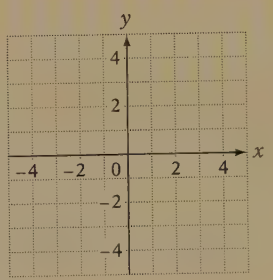
The graph of an equation of the form $y = b$ is a horizontal line passing through the point $(0, b)$.



You Try It 6

Graph $x = -4$.

Your solution



Solutions on p. S11

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Objective C To solve application problems

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TAKE NOTE
In many applications, the domain of the variable is given such that the equation makes sense. For this application, it would not be sensible to have values of t that are less than 0. This would indicate negative time! The number 12 is somewhat arbitrary, but after 12 min, most people's heart rates would level off and a linear function would no longer apply.

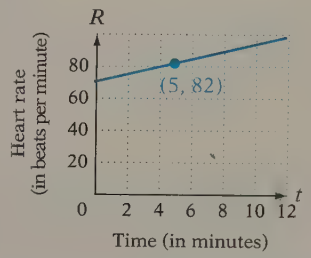
There are a variety of applications of linear functions.

The heart rate, R, after t minutes for a person taking a brisk walk can be approximated by the equation $R = 2t + 72$.

a. Graph this equation for $0 \le t \le 12$.
b. The point whose coordinates are (5, 82) is on the graph. Write a sentence that describes the meaning of this ordered pair.

Solution

a.

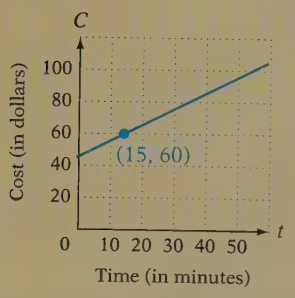


b. The point whose coordinates are (5, 82) means that after 5 min the person's heart rate is 82 beats per minute.

Example 7

A telephone repair technician charges $45 plus $1 per minute to repair defective wiring in a home or apartment. The equation that describes the total cost, C, to have defective wiring repaired is given by $C = t + 45$, where t is the number of minutes the technician works. Graph this equation for $0 \le t \le 60$. The point whose coordinates are (15, 60) is on the graph. Write a sentence that describes the meaning of this ordered pair.

Solution



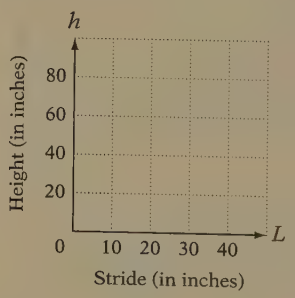
The ordered pair (15, 60) means that it costs $60 for the technician to work 15 min.

You Try It 7

The height h (in inches) of a person and the length L (in inches) of that person's stride while walking are related. The equation $h = \frac{3}{4}L + 50$ approximates that relationship.

Graph this equation for $15 \le L \le 40$. The point whose coordinates are (32, 74) is on this graph. Write a sentence that describes the meaning of this ordered pair.

Your solution



Solution on p. S11

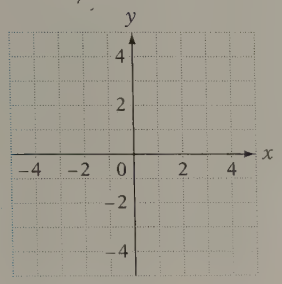
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4.3 Exercises

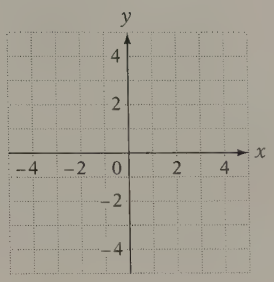
Objective A

Graph.

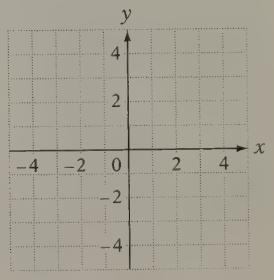
1. $y = 2x - 3$



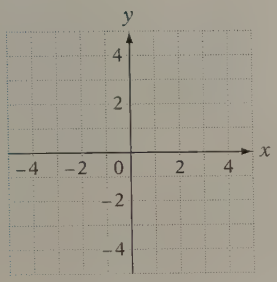
2. $y = -2x + 2$



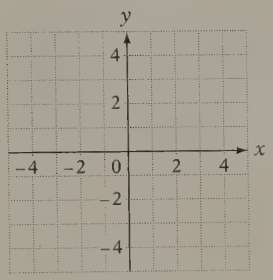
3. $y = \frac{1}{3}x$



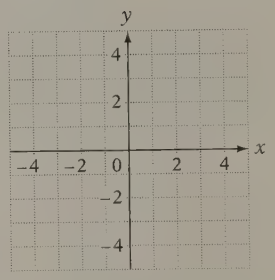
4. $y = -3x$



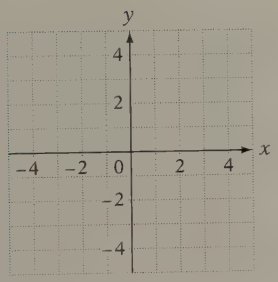
5. $y = \frac{2}{3}x - 1$



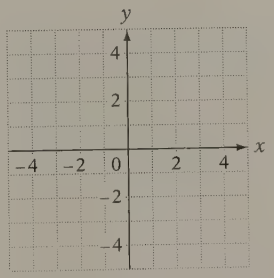
6. $y = \frac{3}{4}x + 2$



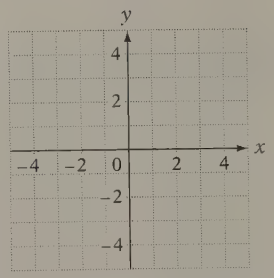
7. $y = -\frac{1}{4}x + 2$



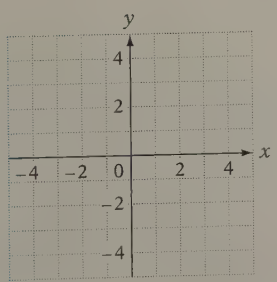
8. $y = -\frac{1}{3}x + 1$



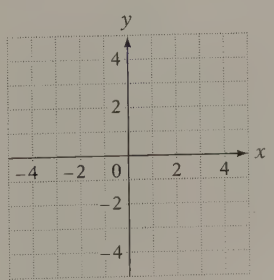
9. $y = -\frac{2}{5}x + 1$



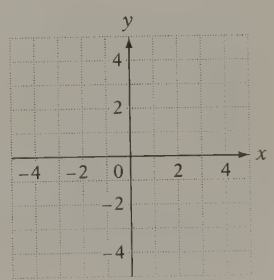
10. $y = -\frac{1}{2}x + 3$



11. $y = 2x - 4$

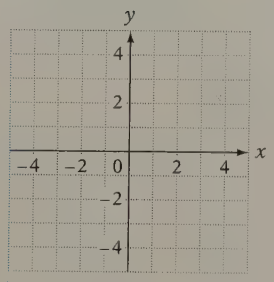


12. $y = 3x - 4$

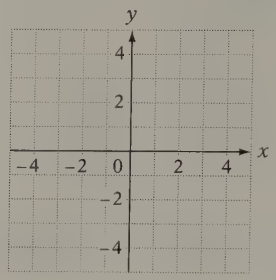


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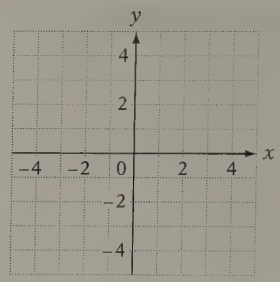
13. $y = x - 3$



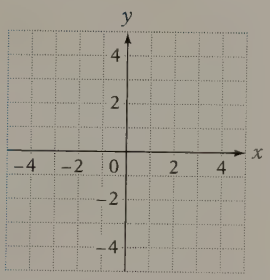
14. $y = x + 2$



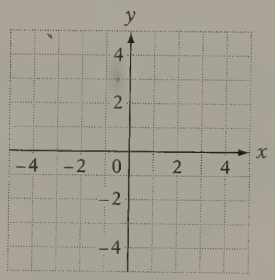
15. $y = -x + 2$



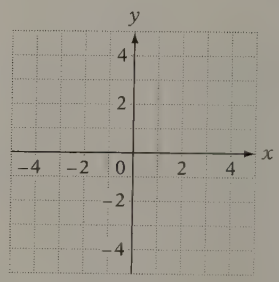
16. $y = -x - 1$



17. $y = -\frac{2}{3}x + 1$



18. $y = 5x - 4$



Objective B

Find the x- and y-intercepts.

19. $x - y = 3$

20. $3x + 4y = 12$

21. $x - 5y = 10$

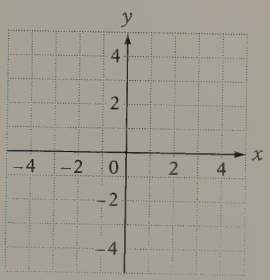
22. $3x + 2y = 12$

23. $2x - 3y = 0$

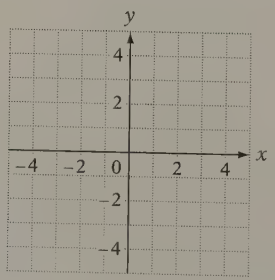
24. $3x + 4y = 0$

Graph.

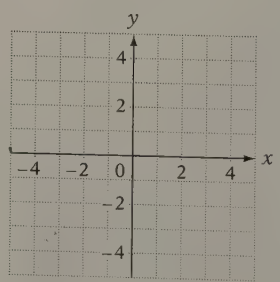
25. $3x + y = 3$



26. $2x + y = 4$

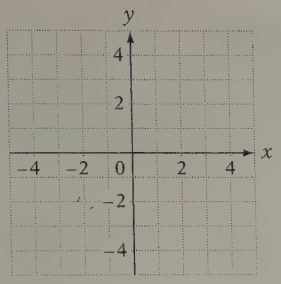


27. $2x + 3y = 6$

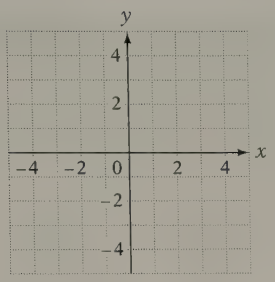


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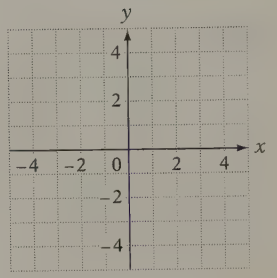
28. $x - 4y = 8$



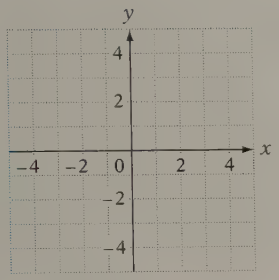
29. $2x - 3y = 12$



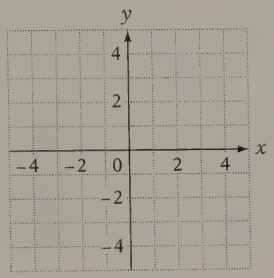
30. $x - 3y = 6$



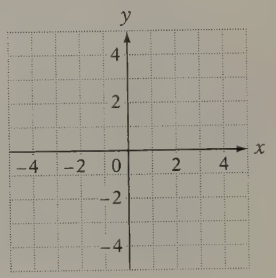
31. $2x - 3y = 6$



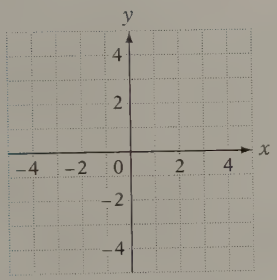
32. $x - 2y = -4$



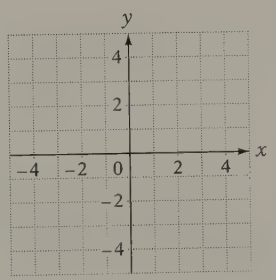
33. $2x + 5y = 10$



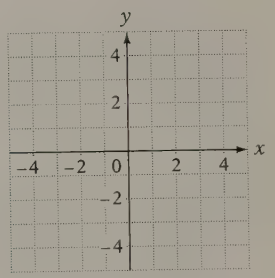
34. $2x - y = 4$



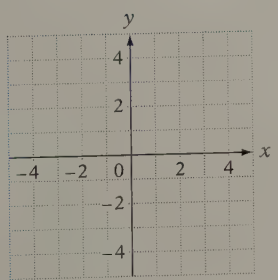
35. $x = 3$



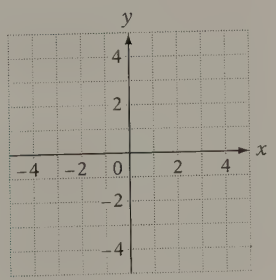
36. $y = -4$



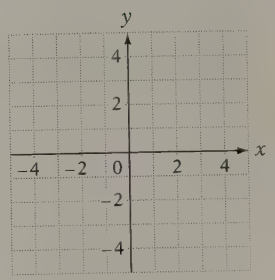
37. $x + 4y = 4$



38. $4x - 3y = 12$



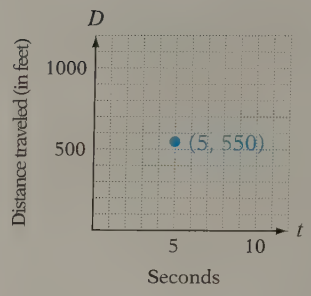
39. $y = 4$



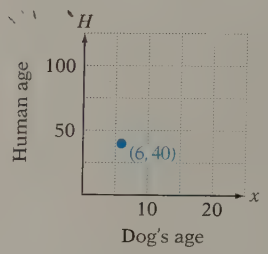
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Objective C Application Problems

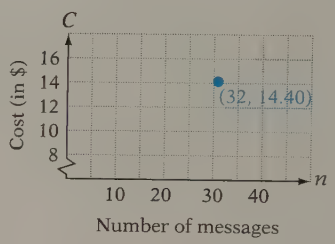
40. The Rattler roller coaster at Fiesta, in San Antonio, Texas, has a maximum speed of approximately 110 ft/s. The equation that describes the total number of feet traveled by the roller coaster in t seconds at this speed is given by $D = 110t$. Graph this equation for $0 \le t \le 10$. The point (5, 550) is on this graph. Write a sentence that describes the meaning of this ordered pair.



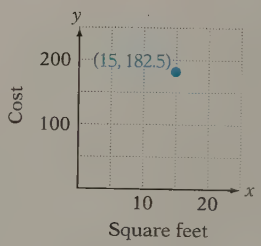
41. According to some veterinarians, the age, x, of a dog can be translated to "human years" by using the equation $H = 4x + 16$, where H is the human equivalent age for the dog. Graph this equation for $2 \le x \le 21$. The point whose coordinates are (6, 40) is on this graph. Write a sentence that explains the meaning of this ordered pair.



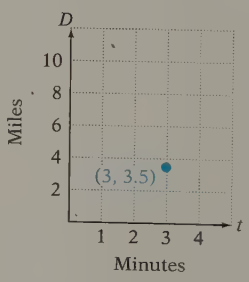
42. The monthly cost for receiving messages from a telephone answering service is \$8.00 plus \$.20 a message. The equation that describes the cost is $C = 0.20n + 8.00$, where n is the number of messages received. Graph this equation for $0 \le n \le 40$. The point (32, 14.40) is on the graph. Write a sentence that describes the meaning of this ordered pair.



43. A custom-illustrated sign or banner can be commissioned for a cost of \$25 for the material and \$10.50 per square foot for the artwork. The equation that represents this cost is given by $y = 10.50x + 25$, where y is the cost and x is the number of square feet in the sign. Graph this equation for $0 \le x \le 20$. The point (15, 182.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



44. A rescue helicopter is rushing at a constant speed of 150 mph to reach several people stranded in the ocean 11 mi away after their boat sank. The rescuers can determine how far they are from the victims using the equation $D = 11 - 2.5t$, where D is the distance in miles and t is the time elapsed in minutes. Graph this equation for $0 \le t \le 4$. The point (3, 3.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



APPLYING THE CONCEPTS

45. Explain what the graph of an equation represents.

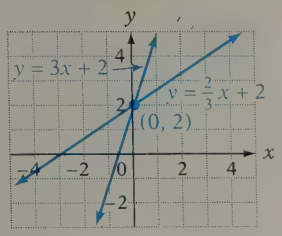
46. Explain why you cannot graph the equation $4x + 3y = 0$ by using just its intercepts.

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4.4 Slopes of Straight Lines

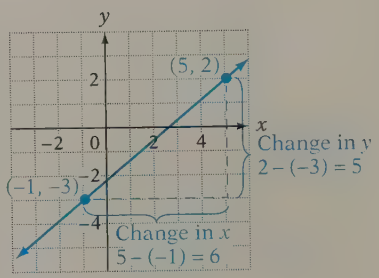
Objective To find the slope of a straight line

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The graphs of $y = 3x + 2$ and $y = \frac{2}{3}x + 2$ are shown at the left. Each graph crosses the y-axis at the point $(0, 2)$, but the graphs have different slants. The **slope** of a line is a measure of the slant of the line. The symbol for slope is m.

The slope of a line containing two points is the ratio of the change in the y values between the two points to the change in the x values. The line containing the points whose coordinates are $(-1, -3)$ and $(5, 2)$ is shown below.



The change in the y values is the difference between the y-coordinates of the two points.

$$\text{Change in } y = 2 - (-3) = 5$$

The change in the x values is the difference between the x-coordinates of the two points.

$$\text{Change in } x = 5 - (-1) = 6$$

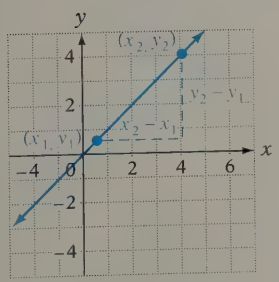
The slope of the line between the two points is the ratio of the change in y to the change in x.

$$\text{Slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{5}{6} \qquad m = \frac{2 - (-3)}{5 - (-1)} = \frac{5}{6}$$

In general, if $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points on a line, then

$$\text{Change in } y = y_2 - y_1 \qquad \text{Change in } x = x_2 - x_1$$

Using these ideas, we can state a formula for slope.



Slope Formula

The slope of the line containing the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Frequently, the Greek letter Δ is used to designate the change in a variable. Using this notation, we can write the equations for the change in y and the change in x as follows:

$$\text{Change in } y = \Delta y = y_2 - y_1 \qquad \text{Change in } x = \Delta x = x_2 - x_1$$

With this notation, the slope formula is written $m = \frac{\Delta y}{\Delta x}$.

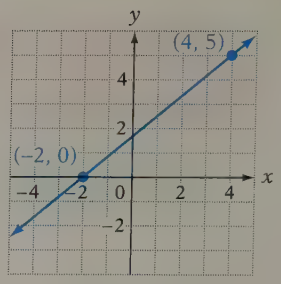
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➡ Find the slope of the line containing the points whose coordinates are $(-2, 0)$ and $(4, 5)$.

Let $P_1 = (-2, 0)$ and $P_2 = (4, 5)$. (It does not matter which point is named P_1 or P_2; the slope will be the same.)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{4 - (-2)} = \frac{5}{6}$$

A line that slants upward to the right always has a **positive slope**.



Positive slope

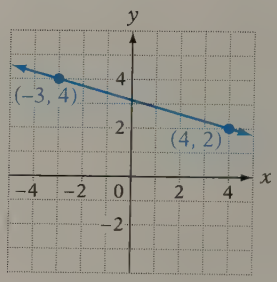
TAKE NOTE
Positive slope means that the value of y increases as the value of x increases.

➡ Find the slope of the line containing the points whose coordinates are $(-3, 4)$ and $(4, 2)$.

Let $P_1 = (-3, 4)$ and $P_2 = (4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{4 - (-3)} = \frac{-2}{7} = -\frac{2}{7}$$

A line that slants downward to the right always has a **negative slope**.



Negative slope

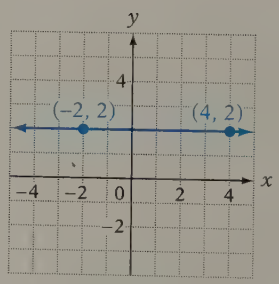
TAKE NOTE
Negative slope means that the value of y decreases as the value of x increases. Compare this to positive slope.

➡ Find the slope of the line containing the points whose coordinates are $(-2, 2)$ and $(4, 2)$.

Let $P_1 = (-2, 2)$ and $P_2 = (4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{4 - (-2)} = \frac{0}{6} = 0$$

A horizontal line has **zero slope**.



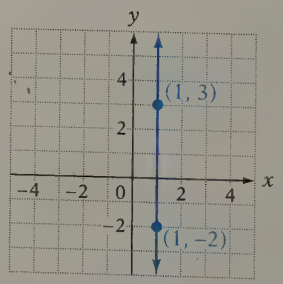
Zero slope

➡ Find the slope of the line containing the points whose coordinates are $(1, -2)$ and $(1, 3)$.

Let $P_1 = (1, -2)$ and $P_2 = (1, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{1 - 1} = \frac{5}{0}$$ Not a real number

The slope of a vertical line is **undefined**.



Undefined

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Point of Interest

One of the motivations for the discovery of calculus was to solve a more complicated version of the distance-rate problem at the right.

You may be familiar with twirling a ball on the end of a string. If you release the string, the ball flies off in a path as shown below.

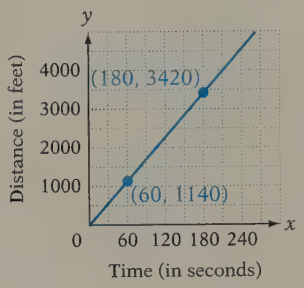


The question that mathematicians tried to answer was essentially "What is the slope of the line represented by the arrow?"

Answering questions similar to that led to the development of one aspect of calculus.

There are many applications of slope. Here are two possibilities.

The first record for the one-mile run was recorded in 1865 in England. Richard Webster ran the mile in 4 min 36.5 s. His average speed was approximately 19 feet per second.



The graph at the right shows the distance Webster ran during that run. From the graph, note that after 60 s (1 min) he had traveled 1140 ft, and that after 180 s (3 min) he had traveled 3420 ft.

Let the point (60, 1140) be (x_1, y_1) and the point (180, 3420) be (x_2, y_2). The slope of the line between these two points is

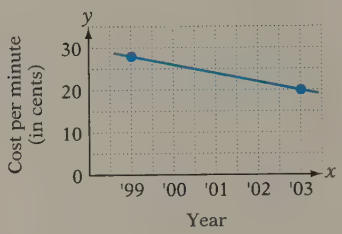
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3420 - 1140}{180 - 60} = \frac{2280}{120} = 19$$

Note that the slope of the line is the same as Webster's average speed, 19 feet per second.

Average speed is related to slope.

Here is another example, this one related to economics.

As a result of cheaper digital networks and more competition, the cost of wireless phone use is decreasing. The average cost per minute in 1999 was 28¢. The projected average cost in 2003 is 20¢ (Source: The Strategic Group).



Let the point (1999, 28) be (x_1, y_1) and the point (2003, 20) be (x_2, y_2). The slope of the line between these two points is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 28}{2003 - 1999} = \frac{-8}{4} = -2$$

Note that if we interpret negative slope as decreasing, then the slope of the line is the same as the rate at which the per-minute cost of phone use is decreasing, 2¢ per year.

In general, any quantity that is expressed by using the word *per* is represented mathematically as slope. In the first example, the slope was 19 feet per second. In the second example, the slope was −2¢ per year.

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Example 1

Find the slope of the line containing the points $(2, -5)$ and $(-4, 2)$.

Solution

Let $P_1 = (2, -5)$ and $P_2 = (-4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{-4 - 2} = \frac{7}{-6}$$

The slope is $-\frac{7}{6}$.

You Try It 1

Find the slope of the line containing the points $(4, -3)$ and $(2, 7)$.

Your solution

Example 2

Find the slope of the line containing the points $(-3, 4)$ and $(5, 4)$.

Solution

Let $P_1 = (-3, 4)$ and $P_2 = (5, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{5 - (-3)} = \frac{0}{8} = 0$$

The slope of the line is zero.

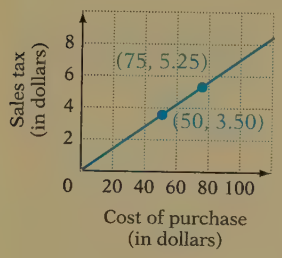
You Try It 2

Find the slope of the line containing the points $(6, -1)$ and $(6, 7)$.

Your solution

Example 3

The graph below shows the relationship between the cost of an item and the sales tax. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



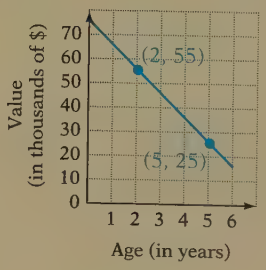
Solution

$$m = \frac{5.25 - 3.50}{75 - 50}$$
$$= \frac{1.75}{25}$$
$$= 0.07$$

A slope of 0.07 means that the sales tax is \$.07 per dollar.

You Try It 3

The graph below shows the decrease in the value of a printing press for a period of 6 years. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



Your solution

Solutions on p. S12

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Objective B

To graph a line using the slope and the y-intercept

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Recall that we can find the y-intercept of a linear equation by letting $x = 0$.

➡ Find the y-intercept of $y = 3x + 4$.

$y = 3x + 4 = 3(0) + 4 = 4$ • Let $x = 0$.

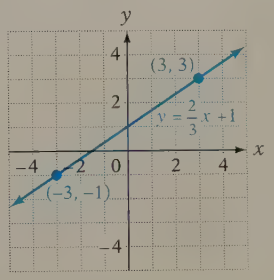
The y-intercept is $(0, 4)$.

For any equation of the form $y = mx + b$, the y-intercept is $(0, b)$.

The graph of the equation $y = \frac{2}{3}x + 1$ is shown at the right. The points $(-3, -1)$ and $(3, 3)$ are on the graph. The slope of the line between the two points is

$$m = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

Observe that the slope of the line is the coefficient of x in the equation $y = \frac{2}{3}x + 1$.



Slope-Intercept Form of a Straight Line

An equation of the form $y = mx + b$ is called the **slope-intercept** form of a straight line. The slope of the line is m, the coefficient of x. The y-intercept is $(0, b)$, where b is the constant term of the equation.

The following equations are written in slope-intercept form.

$y = 2x - 3$	Slope $= 2$, y-intercept $= (0, -3)$
$y = -x + 2$	Slope $= -1$ $(-x = -1x)$, y-intercept $= (0, 2)$
$y = \frac{1}{2}x$	Slope $= \frac{1}{2}$, y-intercept $= (0, 0)$

When the equation of a straight line is in the form $y = mx + b$, the graph can be drawn by using the slope and the y-intercept. First locate the y-intercept. Use the slope to find a second point on the line. Then draw a line through the two points.

➡ Graph $y = \frac{5}{3}x - 4$ by using the slope and y-intercept.

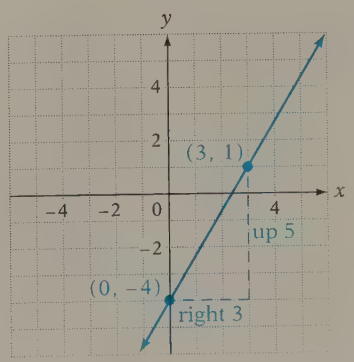
The slope is the coefficient of x:

$$m = \frac{5}{3} = \frac{\text{change in } y}{\text{change in } x}$$

The y-intercept is $(0, -4)$.

Beginning at the y-intercept $(0, -4)$, move right 3 units (change in x) and then up 5 units (change in y).

The point whose coordinates are $(3, 1)$ is a second point on the graph. Draw a line through the points $(0, -4)$ and $(3, 1)$.



TAKE NOTE

When graphing a line by using its slope and y-intercept, *always* start at the y-intercept.

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Graph $y = 2x - 3$.

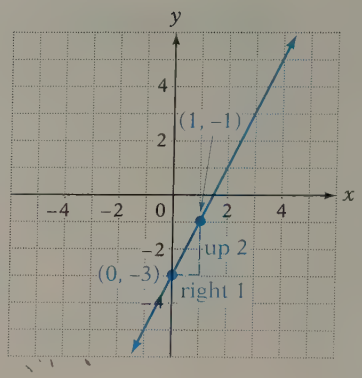
y-intercept $= (0, b) = (0, -3)$

$m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$

Beginning at the y-intercept, move right 1 unit (change in x) and then up 2 units (change in y).

$(1, -1)$ is a second point on the graph.

Draw a line through the two points $(0, -3)$ and $(1, -1)$.

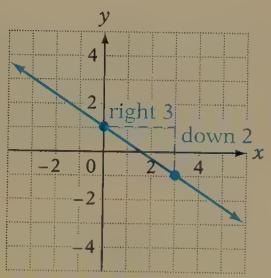


Example 4

Graph $y = -\frac{2}{3}x + 1$ by using the slope and y-intercept.

Solution y-intercept $= (0, b) = (0, 1)$

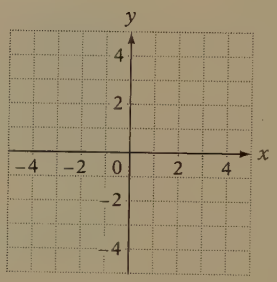
$m = -\frac{2}{3} = \frac{-2}{3}$



You Try It 4

Graph $y = -\frac{1}{4}x - 1$ by using the slope and y-intercept.

Your solution



Example 5 Graph $2x - 3y = 6$ by using the slope and y-intercept.

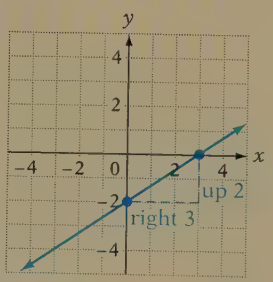
Solution Solve the equation for y.

$2x - 3y = 6$

$-3y = -2x + 6$

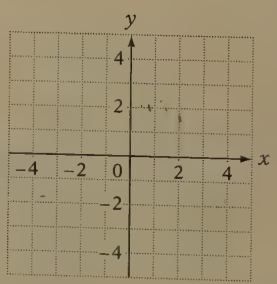
$y = \frac{2}{3}x - 2$

y-intercept $= (0, -2)$; $m = \frac{2}{3}$



You Try It 5 Graph $x - 2y = 4$ by using the slope and y-intercept.

Your solution



Solutions on p. S12

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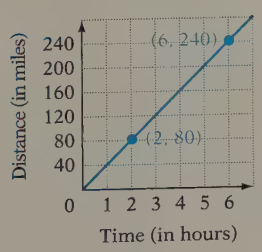
4.4 Exercises

Objective A

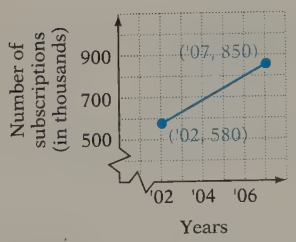
Find the slope of the line containing the points.

1. $P_1(1, 3), P_2(3, 1)$
2. $P_1(2, 3), P_2(5, 1)$
3. $P_1(-1, 4), P_2(2, 5)$
4. $P_1(3, -2), P_2(1, 4)$
5. $P_1(-1, 3), P_2(-4, 5)$
6. $P_1(-1, -2), P_2(-3, 2)$
7. $P_1(0, 3), P_2(4, 0)$
8. $P_1(-2, 0), P_2(0, 3)$
9. $P_1(2, 4), P_2(2, -2)$
10. $P_1(4, 1), P_2(4, -3)$
11. $P_1(2, 5), P_2(-3, -2)$
12. $P_1(4, 1), P_2(-1, -2)$
13. $P_1(2, 3), P_2(-1, 3)$
14. $P_1(3, 4), P_2(0, 4)$
15. $P_1(0, 4), P_2(-2, 5)$
16. $P_1(-2, 3), P_2(-2, 5)$
17. $P_1(-3, -1), P_2(-3, 4)$
18. $P_1(-2, -5), P_2(-4, -1)$

19. The graph below shows the relationship between the distance traveled by a motorist and the time of travel. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

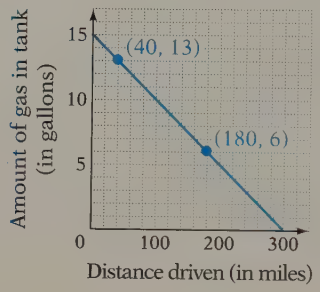


20. The graph below shows the number of people subscribing to a sports magazine whose popularity is increasing. Find the slope of the line between the two points shown on the graph. What is the meaning of the slope?

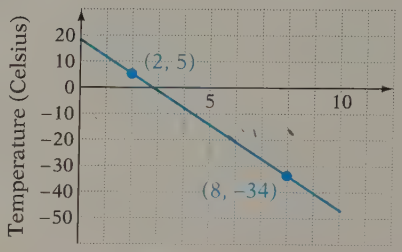


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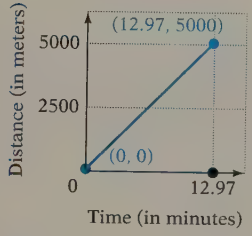
21. The graph below shows how the amount of gas in the tank of a car decreases as the car is driven. Find the slope of the line. Write a sentence that states the meaning of the slope.



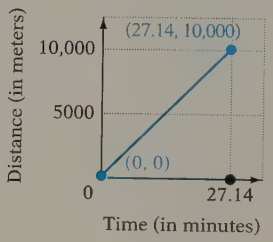
22. The troposphere extends from Earth's surface to an elevation of about 11 km. The graph below shows the decrease in its temperature as altitude increases. Find the slope of the line. Write a sentence that states the meaning of the slope.



23. The graph below shows the relationship between the distance and the time for the 5000-meter run for the world record by Said Aouita in 1987. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



24. The graph below shows the relationship between the distance and the time for the 10,000-meter run for the world record by Arturo Barrios in 1989. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



The American National Standards Institute (ANSI) states that the slope for a wheelchair ramp must not exceed $\frac{1}{12}$.



25. Does a ramp that is 6 in. high and spans a distance of 5 ft along the ground meet the requirements of ANSI?

26. Does a ramp that is 12 in. high and spans a distance of 170 in. along the ground meet the requirements of ANSI?

Objective B

27. Explain why $y = mx + b$ is called the slope-intercept form of the equation of a straight line.

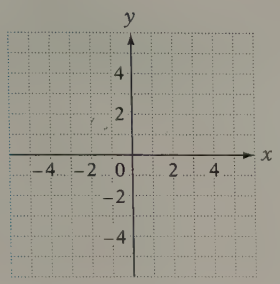
28. Do all straight lines have a y-intercept? If not, give an example of one that does not.

29. If two lines have the same slope and the same y-intercept, must the graphs of the lines be the same? If not, give an example.

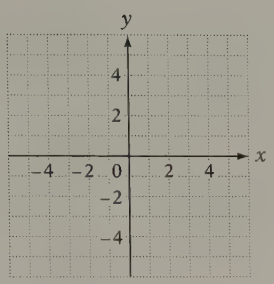
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Graph by using the slope and y-intercept.

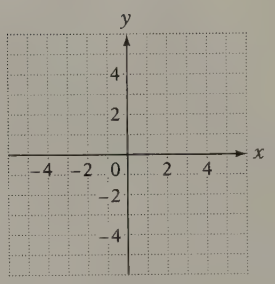
30. $y = 3x + 1$



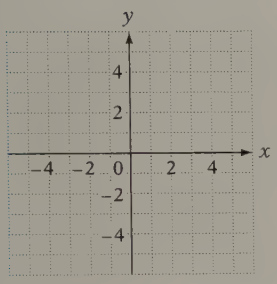
31. $y = -2x - 1$



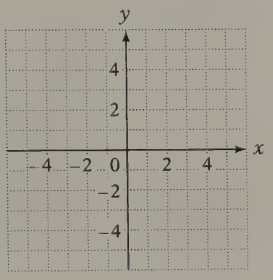
32. $y = \frac{2}{5}x - 2$



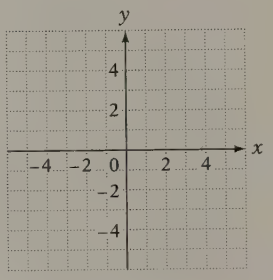
33. $y = \frac{3}{4}x + 1$



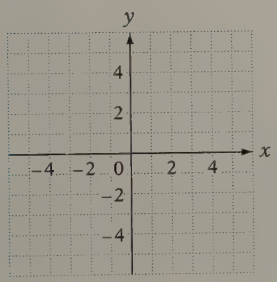
34. $2x + y = 3$



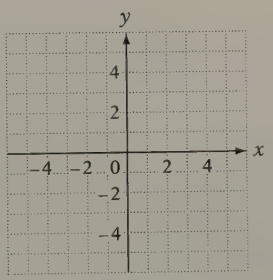
35. $3x - y = 1$



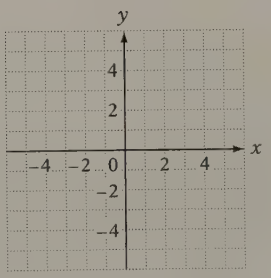
36. $x - 2y = 4$



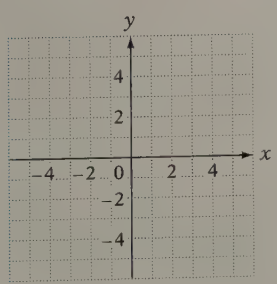
37. $x + 3y = 6$



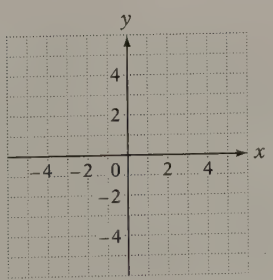
38. $y = \frac{2}{3}x$



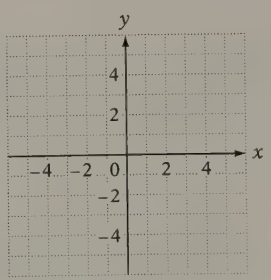
39. $y = \frac{1}{2}x$



40. $y = -x + 1$

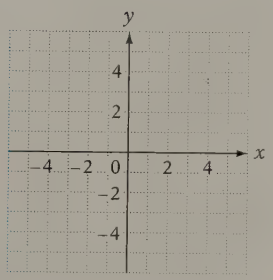


41. $y = -x - 3$

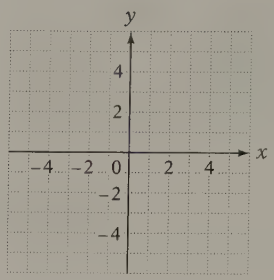


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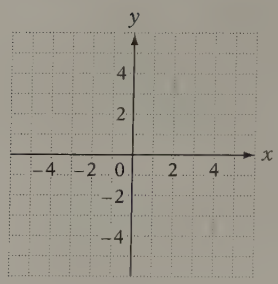
42. $3x - 4y = 12$



43. $5x - 2y = 10$



44. $y = -4x + 2$



APPLYING THE CONCEPTS

45. Match each equation with its graph.

i. $y = -2x + 4$

ii. $y = 2x - 4$

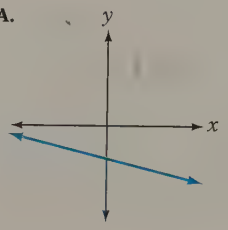
iii. $y = 2$

iv. $2x + 4y = 0$

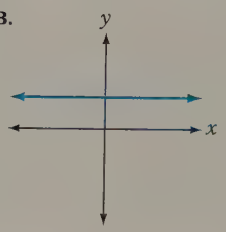
v. $y = \frac{1}{2}x + 4$

vi. $y = -\frac{1}{4}x - 2$

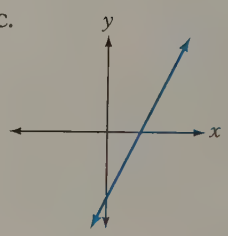
A.



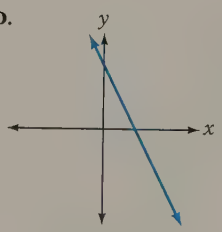
B.



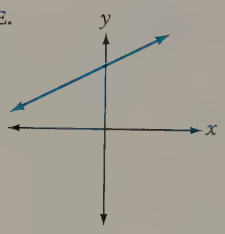
C.



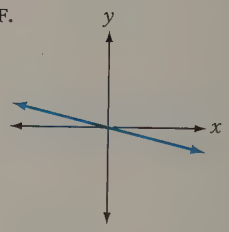
D.



E.



F.



46. Complete the sentences.

a. If a line has a slope of 2, then the value of y *increases/decreases* by _____ as the value of x increases by 1.

b. If a line has a slope of -3, then the value of y *increases/decreases* by _____ as the value of x increases by 1.

c. If a line has a slope of -2, then the value of y *increases/decreases* by _____ as the value of x decreases by 1.

d. If a line has a slope of 3, then the value of y *increases/decreases* by _____ as the value of x decreases by 1.

Determine the value of k such that the points whose coordinates are given below lie on the same line.

47. $(3, 2), (4, 6), (5, k)$

48. $(-2, 3), (1, 0), (k, 2)$

49. $(k, 1), (0, -1), (2, -2)$

50. $(4, -1), (3, -4), (k, k)$

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4.5 Finding Equations of Lines

Objective A **To find the equation of a line given a point and the slope**

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When the slope of a line and a point on the line are known, the equation of the line can be determined. If the particular point is the y-intercept, use the slope-intercept form, $y = mx + b$, to find the equation.

➡ Find the equation of the line that contains the point $(0, 3)$ and has slope $\frac{1}{2}$.

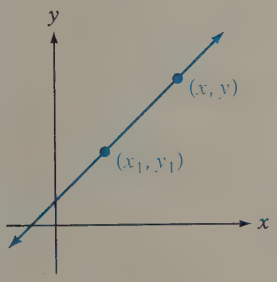
The known point is the y-intercept, $(0, 3)$.

$y = mx + b$ • Use the slope-intercept form.

$y = \frac{1}{2}x + 3$ • Replace m with $\frac{1}{2}$, the given slope. Replace b with 3, the y-coordinate of the y-intercept.

The equation of the line is $y = \frac{1}{2}x + 3$.

One method of finding the equation of a line when the slope and *any* point on the line are known involves using the *point-slope* formula. This formula is derived from the formula for the slope of a line as follows.



Let (x_1, y_1) be the given point on the line, and let (x, y) be any other point on the line. See the graph at the left.

$\frac{y - y_1}{x - x_1} = m$ • Use the formula for the slope of a line.

$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$ • Multiply each side by $(x - x_1)$.

$y - y_1 = m(x - x_1)$ • Simplify.

Point-Slope Formula

Let m be the slope of a line, and let (x_1, y_1) be the coordinates of a point on the line. The equation of the line can be found from the **point-slope formula:**

$$y - y_1 = m(x - x_1)$$

➡ Find the equation of the line that contains the point whose coordinates are $(4, -1)$ and has slope $-\frac{3}{4}$.

$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

$y - (-1) = \left(-\frac{3}{4}\right)(x - 4)$ • $m = -\frac{3}{4}$, $(x_1, y_1) = (4, -1)$

$y + 1 = -\frac{3}{4}x + 3$ • Simplify.

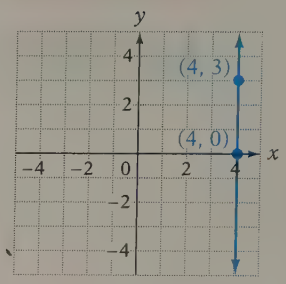
$y = -\frac{3}{4}x + 2$ • Write the equation in the form $y = mx + b$.

The equation of the line is $y = -\frac{3}{4}x + 2$.

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Find the equation of the line that passes through the point whose coordinates are (4, 3) and whose slope is undefined.

Because the slope is undefined, the point-slope formula cannot be used to find the equation. Instead, recall that when the slope is undefined, the line is vertical and that the equation of a vertical line is $x = a$, where a is the x-coordinate of the x-intercept. Because the line is vertical and passes through (4, 3), the x-intercept is (4, 0). The equation of the line is $x = 4$.



Example 1

Find the equation of the line that contains the point (3, 0) and has slope −4.

Solution

$m = -4 \qquad (x_1, y_1) = (3, 0)$

$y - y_1 = m(x - x_1)$
$y - 0 = -4(x - 3)$
$y = -4x + 12$

The equation of the line is $y = -4x + 12$.

You Try It 1

Find the equation of the line that contains the point $(-3, -2)$ and has slope $-\frac{1}{3}$.

Your solution

Example 2

Find the equation of the line that contains the point (−2, 4) and has slope 2.

Solution

$m = 2 \qquad (x_1, y_1) = (-2, 4)$

$y - y_1 = m(x - x_1)$
$y - 4 = 2[x - (-2)]$
$y - 4 = 2(x + 2)$
$y - 4 = 2x + 4$
$y = 2x + 8$

The equation of the line is $y = 2x + 8$.

You Try It 2

Find the equation of the line that contains the point (4, −3) and has slope −3.

Your solution

Solutions on p. S12

Objective B **To find the equation of a line given two points**

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The point-slope formula and the formula for slope are used to find the equation of a line when two points are known.

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Find the equation of the line containing the points (3, 2) and (−5, 6).

To use the point-slope formula, we must know the slope. Use the formula for slope to determine the slope of the line between the two given points. Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (-5, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{-5 - 3} = \frac{4}{-8} = -\frac{1}{2}$$

Now use the point-slope formula with $m = -\frac{1}{2}$ and $(x_1, y_1) = (3, 2)$.

$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

$y - 2 = \left(-\frac{1}{2}\right)(x - 3)$ • $m = -\frac{1}{2}$, $(x_1, y_1) = (3, 2)$

$y - 2 = -\frac{1}{2}x + \frac{3}{2}$ • Simplify.

$y = -\frac{1}{2}x + \frac{7}{2}$

The equation of the line is $y = -\frac{1}{2}x + \frac{7}{2}$.

Example 3

Find the equation of the line containing the points (2, 3) and (4, 1).

Solution

Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 2} = \frac{-2}{2} = -1$$

$y - y_1 = m(x - x_1)$
$y - 3 = -1(x - 2)$
$y - 3 = -x + 2$
$y = -x + 5$

The equation of the line is $y = -x + 5$.

You Try It 3

Find the equation of the line containing the points (2, 0) and (5, 3).

Your solution

Example 4

Find the equation of the line containing the points (2, −3) and (2, 5).

Solution

Let $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (2, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{2 - 2} = \frac{8}{0}$$

The slope is undefined, so the graph of the line is vertical.

The equation of the line is $x = 2$.

You Try It 4

Find the equation of the line containing the points (2, 3) and (−5, 3).

Your solution

Solutions on p. S12

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Objective C **To solve application problems** VIDEO & DVD CD TUTOR WEB SSM

Example 5

On the basis of data from the Kelley Blue Book, the value of a certain car decreases approximately $250 per month. The value of the car 2 years after it was purchased was $14,000. Find a linear equation that models the value of the car after x months of ownership. Use the equation to find the value of the car after 3 years of ownership.

Strategy

- To find a linear equation that models the value of the car:
  Let y represent the value of the car after x months.
  Then $y = 14{,}000$ when $x = 24$. (*Note*: x represents *months* of ownership. Convert 2 years to 24 months.)
  The car is decreasing in value at a rate of $250 per month. Therefore, the slope is -250.
  Use the point-slope formula to write the linear equation.
- To find the value of the car after 3 years, substitute 36 for x, and solve for y. (*Note*: 3 years is 36 months.)

Solution

$$y - y_1 = m(x - x_1)$$
$$y - 14{,}000 = -250(x - 24) \quad \bullet\ m = -250,\ (x_1, y_1) = (24, 14{,}000)$$
$$y - 14{,}000 = -250x + 6000$$
$$y = -250x + 20{,}000$$

A linear equation that models the value of the car after x months of ownership is $y = -250x + 20{,}000$.

$$y = -250x + 20{,}000$$
$$y = -250(36) + 20{,}000$$
$$y = -9000 + 20{,}000$$
$$y = 11{,}000$$

The value of the car after 3 years of ownership is $11,000.

You Try It 5

A manufacturer of pickup trucks has determined that 40,000 trucks per month can be sold at a price of $18,000. At a price of $17,500, the number of trucks that can be sold per month increases to 44,000. Find a linear equation that models the number of trucks that can be sold at a given price. Use the equation to predict the number of trucks that can be sold at a price of $16,500.

Your strategy

Your solution

Solution on p. S13

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4.5 Exercises

Objective A

1. Explain how to find the equation of a line given its slope and its y-intercept.

2. What is the point-slope formula and how is it used?

Find the equation of the line that contains the given point and has the given slope.

3. Point $(0, 5)$, $m = 2$

4. Point $(0, 3)$, $m = 1$

5. Point $(2, 3)$, $m = \frac{1}{2}$

6. Point $(-1, 2)$, $m = -3$

7. Point $(-2, 3)$, $m = \frac{1}{2}$

8. Point $(-2, 1)$, $m = \frac{3}{2}$

9. Point $(3, 0)$, $m = -\frac{5}{3}$

10. Point $(-2, 0)$, $m = \frac{3}{2}$

11. Point $(2, 3)$, $m = -3$

12. Point $(4, -2)$, $m = \frac{3}{4}$

13. Point $(-1, 7)$, $m = -3$

14. Point $(-2, 4)$, $m = -4$

15. Point $(3, 1)$, $m = \frac{1}{3}$

16. Point $(5, -3)$, $m = -\frac{3}{5}$

17. Point $(0, 0)$, $m = \frac{1}{2}$

18. Point $(0, 0)$, $m = \frac{3}{4}$

19. Point $(2, -3)$, $m = 3$

20. Point $(4, -5)$, $m = 2$

21. Point $(3, 5)$, $m = -\frac{2}{3}$

22. Point $(5, 1)$, $m = -\frac{4}{5}$

23. Point $(0, -3)$, $m = -1$

24. Point $(2, 3)$, $m = -\frac{1}{2}$

25. Point $(1, -4)$, $m = \frac{7}{5}$

26. Point $(3, 5)$, $m = -\frac{3}{7}$

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27. Point $(4, -1)$, $m = -\frac{2}{5}$

28. Point $(-3, 5)$, $m = -\frac{1}{4}$

29. Point $(3, -4)$, slope is undefined

30. Point $(-2, 5)$, slope is undefined

31. Point $(5, -1)$, $m = \frac{1}{5}$

32. Point $(-3, -2)$, $m = -\frac{2}{3}$

33. Point $(-2, -3)$, $m = 0$

34. Point $(-3, -2)$, $m = 0$

35. Point $(4, -5)$, $m = -2$

36. Point $(-3, 5)$, $m = 3$

37. Point $(-5, -1)$, slope is undefined

38. Point $(0, 4)$, slope is undefined

Objective B

Find the equation of the line that contains the given points.

39. $(0, 2), (3, 5)$

40. $(0, 4), (1, 5)$

41. $(0, -3), (-4, 5)$

42. $(0, -2), (-3, 4)$

43. $(2, 3), (-4, 0)$

44. $(4, 1), (6, 3)$

45. $(2, 3), (3, 2)$

46. $(1, -1), (-2, -7)$

47. $(-2, 1), (1, -5)$

48. $(-3, -1), (2, 4)$

49. $(0, 3), (2, 0)$

50. $(0, 4), (2, 0)$

51. $(-3, -1), (2, -1)$

52. $(-3, -5), (4, -5)$

53. $(-2, -3), (-1, -2)$

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54. (4, 1), (3, −2)

55. (−2, 3), (2, −1)

56. (−1, −3), (2, −12)

57. (2, 3), (5, −5)

58. (7, 2), (4, 4)

59. (2, 0), (0, −1)

60. (0, 4), (−2, 0)

61. (3, −4), (−2, −4)

62. (−3, 3), (−2, 3)

63. (0, 0), (4, 3)

64. (2, −5), (0, 0)

65. (2, −1), (−1, 3)

66. (3, −5), (−2, 1)

67. (−2, 5), (−2, −5)

68. (3, 2), (3, −4)

69. (2, 1), (−2, −3)

70. (−3, −2), (1, −4)

71. (−4, −3), (2, 5)

72. (4, 5), (−4, 3)

73. (0, 3), (3, 0)

74. (−5, 0), (10, −3)

Objective C *Application Problems*

75. The pilot of a Boeing 747 jet takes off from Boston's Logan Airport, which is at sea level, and climbs to a cruising altitude of 32,000 ft at a constant rate of 1200 ft/min. Write a linear equation for the height of the plane in terms of the time after takeoff. Use your equation to find the height of the plane 11 min after takeoff.

76. The price of a new motorcycle this year is $18,000. According to a consumer group, the value of the motorcycle will be $7200 in 6 years. Write a linear equation that gives the value of the motorcycle in terms of its age. Use the equation to find the value of the motorcycle in 4 years.

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77. The gas tank of a certain car contains 16 gal when the driver of the car begins a trip. Each mile driven by the driver decreases the amount of gas in the tank by 0.032 gal. Write a linear equation for the number of gallons of gas in the tank in terms of the number of miles driven. Use your equation to find the number of gallons in the tank after this driver has driven 150 mi.

78. A jogger running at 9 mph burns approximately 14 calories per minute. Write a linear equation for the number of calories burned by the jogger in terms of the number of minutes run. Use your equation to find the number of calories one has burned after jogging for 32 min.

APPLYING THE CONCEPTS

79. Explain the similarities and differences between the point-slope formula and the slope-intercept form of a straight line.

80. Explain why the point-slope formula cannot be used to find the equation of a line that is parallel to the y-axis.

81. Refer to Example 5 in this section for each of the following.
a. Explain the meaning of the slope of the graph of the linear function.
b. Explain the meaning of the y-intercept.
c. Explain the meaning of the x-intercept.

82. For an equation of the form $y = mx + b$, how does the graph of this equation change if the value of b changes and the value of m remains constant?

83. A line contains the points $(-3, 6)$ and $(6, 0)$. Find the coordinates of three other points that are on this line.

84. A line contains the points $(4, -1)$ and $(2, 1)$. Find the coordinates of three other points that are on this line.

85. Given that f is a linear function for which $f(1) = 3$ and $f(-1) = 5$, determine $f(4)$.

86. Given that f is a linear function for which $f(-3) = 2$ and $f(2) = 7$, determine $f(0)$.

87. Find the equation of the line that passes through the midpoint of the line segment between $P_1(2, 5)$ and $P_2(-4, 1)$ and has slope -2.

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4.6 Parallel and Perpendicular Lines

Objective A

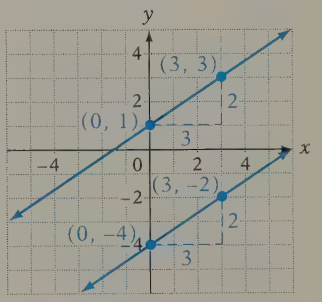
To find parallel and perpendicular lines

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Two lines that have the same slope do not intersect and are called **parallel lines**.

The slope of each of the lines at the right is $\frac{2}{3}$.

The lines are parallel.



Slopes of Parallel Lines

Two nonvertical lines with slopes of m_1 and m_2 are parallel if and only if $m_1 = m_2$. Any two vertical lines are parallel.

➡ Is the line containing the points $(-2, 1)$ and $(-5, -1)$ parallel to the line that contains the points $(1, 0)$ and $(4, 2)$?

$$m_1 = \frac{-1 - 1}{-5 - (-2)} = \frac{-2}{-3} = \frac{2}{3}$$

- Find the slope of the line through $(-2, 1)$ and $(-5, -1)$.

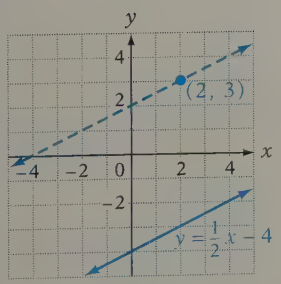
$$m_2 = \frac{2 - 0}{4 - 1} = \frac{2}{3}$$

- Find the slope of the line through $(1, 0)$ and $(4, 2)$.

Because $m_1 = m_2$, the lines are parallel.

➡ Find the equation of the line that contains the point $(2, 3)$ and is parallel to the line $y = \frac{1}{2}x - 4$.

The slope of the given line is $\frac{1}{2}$. Because parallel lines have the same slope, the slope of the unknown line is also $\frac{1}{2}$.



$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

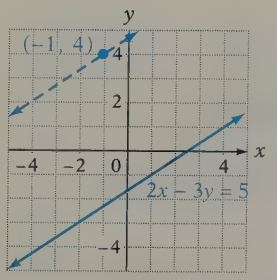
$y - 3 = \frac{1}{2}(x - 2)$ • $m = \frac{1}{2}$, $(x_1, y_1) = (2, 3)$

$y - 3 = \frac{1}{2}x - 1$ • Simplify.

$y = \frac{1}{2}x + 2$ • Write the equation in the form $y = mx + b$.

The equation of the line is $y = \frac{1}{2}x + 2$.

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Find the equation of the line that contains the point $(-1, 4)$ and is parallel to the line $2x - 3y = 5$.

Because the lines are parallel, the slope of the unknown line is the same as the slope of the given line. Solve $2x - 3y = 5$ for y and determine its slope.

$$2x - 3y = 5$$
$$-3y = -2x + 5$$
$$y = \frac{2}{3}x - \frac{5}{3}$$

The slope of the given line is $\frac{2}{3}$. Because the lines are parallel, this is the slope of the unknown line. Use the point-slope formula to determine the equation.

$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

$y - 4 = \frac{2}{3}[x - (-1)]$ • $m = \frac{2}{3}$, $(x_1, y_1) = (-1, 4)$

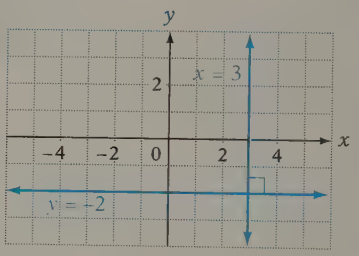
$y - 4 = \frac{2}{3}x + \frac{2}{3}$ • Simplify.

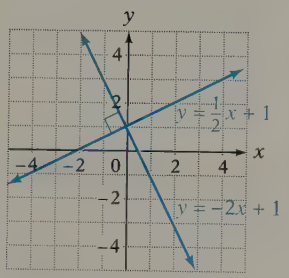
$y = \frac{2}{3}x + \frac{14}{3}$ • Write the equation in the form $y = mx + b$.

The equation of the line is $y = \frac{2}{3}x + \frac{14}{3}$.

Two lines that intersect at right angles are **perpendicular lines**.

Any horizontal line is perpendicular to any vertical line. For example, $x = 3$ is perpendicular to $y = -2$.





Slopes of Perpendicular Lines

If m_1 and m_2 are the slopes of two lines, neither of which is vertical, then the lines are perpendicular if and only if $m_1 \cdot m_2 = -1$.

A vertical line is perpendicular to a horizontal line.

Solving $m_1 \cdot m_2 = -1$ for m_1 gives $m_1 = -\frac{1}{m_2}$. This last equation states that the slopes of perpendicular lines are *negative reciprocals* of each other.

Is the line that contains the points $(4, 2)$ and $(-2, 5)$ perpendicular to the line that contains the points $(-4, 3)$ and $(-3, 5)$?

$m_1 = \frac{5 - 2}{-2 - 4} = \frac{3}{-6} = -\frac{1}{2}$ • Find the slope of the line through (4, 2) and (−2, 5).

$m_2 = \frac{5 - 3}{-3 - (-4)} = \frac{2}{1} = 2$ • Find the slope of the line through (−4, 3) and (−3, 5).

$m_1 \cdot m_2 = -\frac{1}{2}(2) = -1$ • Find the product of the two slopes.

Because $m_1 \cdot m_2 = -1$, the lines are perpendicular.

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Are the graphs of the equations $3x + 4y = 8$ and $8x + 6y = 5$ perpendicular?

To determine whether the lines are perpendicular, solve each equation for y and find the slope of each line. Then use the equation $m_1 \cdot m_2 = -1$.

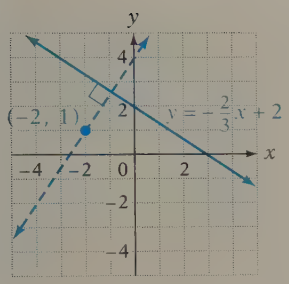
$$\begin{aligned} 3x + 4y &= 8 \\ 4y &= -3x + 8 \\ y &= -\frac{3}{4}x + 2 \end{aligned} \qquad m_1 = -\frac{3}{4}$$

$$\begin{aligned} 8x + 6y &= 5 \\ 6y &= -8x + 5 \\ y &= -\frac{4}{3}x + \frac{5}{6} \end{aligned} \qquad m_2 = -\frac{4}{3}$$

$$m_1 \cdot m_2 = \left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 1$$

Because $m_1 \cdot m_2 = 1 \neq -1$, the lines are not perpendicular.

Find the equation of the line that contains the point $(-2, 1)$ and is perpendicular to the line $y = -\frac{2}{3}x + 2$.



The slope of the given line is $-\frac{2}{3}$. The slope of the line perpendicular to the given line is the negative reciprocal of $-\frac{2}{3}$, which is $\frac{3}{2}$.

$$y - y_1 = m(x - x_1)$$ • Use the point-slope formula.

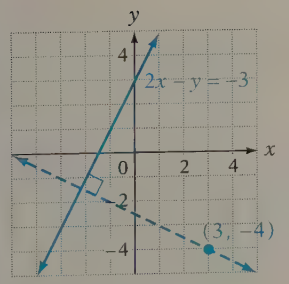
$$y - 1 = \frac{3}{2}[x - (-2)]$$ • $m = \frac{3}{2}$, $(x_1, y_1) = (-2, 1)$

$$y - 1 = \frac{3}{2}x + 3$$ • Write in the form $y = mx + b$.

$$y = \frac{3}{2}x + 4$$

The equation of the perpendicular line is $y = \frac{3}{2}x + 4$.

Find the equation of the line that contains the point $(3, -4)$ and is perpendicular to the line $2x - y = -3$.



$$\begin{aligned} 2x - y &= -3 \\ -y &= -2x - 3 \\ y &= 2x + 3 \end{aligned}$$ • Find the slope of the given line.

• The slope is 2. The slope of the line perpendicular to this line is $-\frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$ • Use the point-slope formula.

$$y - (-4) = -\frac{1}{2}(x - 3)$$ • $m = -\frac{1}{2}$, $(x_1, y_1) = (3, -4)$

$$y + 4 = -\frac{1}{2}x + \frac{3}{2}$$ • Write in the form $y = mx + b$.

$$y = -\frac{1}{2}x - \frac{5}{2}$$

The equation of the perpendicular line is $y = -\frac{1}{2}x - \frac{5}{2}$.

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Example 1

Is the line that contains the points $(-4, 2)$ and $(1, 6)$ parallel to the line that contains the points $(2, -4)$ and $(7, 0)$?

Solution

$$m_1 = \frac{6 - 2}{1 - (-4)} = \frac{4}{5}$$

$$m_2 = \frac{0 - (-4)}{7 - 2} = \frac{4}{5}$$

$$m_1 = m_2 = \frac{4}{5}$$

The lines are parallel.

You Try It 1

Is the line that contains the points $(-2, -3)$ and $(7, 1)$ perpendicular to the line that contains the points $(4, 1)$ and $(6, -5)$?

Your solution

Example 2

Are the lines $4x - y = -2$ and $x + 4y = -12$ perpendicular?

Solution

$$4x - y = -2$$
$$-y = -4x - 2$$
$$y = 4x + 2 \quad \bullet\ m_1 = 4$$

$$x + 4y = -12$$
$$4y = -x - 12$$
$$y = -\frac{1}{4}x - 3 \quad \bullet\ m_2 = -\frac{1}{4}$$

$$m_1 \cdot m_2 = 4\left(-\frac{1}{4}\right) = -1$$

The lines are perpendicular.

You Try It 2

Are the lines $5x + 2y = 2$ and $5x + 2y = -6$ parallel?

Your solution

Example 3

Find the equation of the line that contains the point $(3, -1)$ and is parallel to the line $y = \frac{3}{2}x - 2$.

Solution

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{3}{2}(x - 3) \quad \bullet\ m = \frac{3}{2}$$

$$y + 1 = \frac{3}{2}x - \frac{9}{2}$$

$$y = \frac{3}{2}x - \frac{11}{2}$$

The equation of the line is $y = \frac{3}{2}x - \frac{11}{2}$.

You Try It 3

Find the equation of the line that contains the point $(-2, 2)$ and is perpendicular to the line $y = \frac{1}{4}x - 3$.

Your solution

Solutions on p. S13

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4.6 Exercises

Objective A

1. Explain how to determine whether the graphs of two lines are parallel.

2. Explain how to determine whether the graphs of two lines are perpendicular.

3. The slope of a line is -5. What is the slope of any line parallel to this line?

4. The slope of a line is $\frac{3}{2}$. What is the slope of any line parallel to this line?

5. The slope of a line is 4. What is the slope of any line perpendicular to this line?

6. The slope of a line is $-\frac{4}{5}$. What is the slope of any line perpendicular to this line?

7. Is the line $x = -2$ perpendicular to the line $y = 3$?

8. Is the line $y = \frac{1}{2}$ perpendicular to the line $y = -4$?

9. Is the line $x = -3$ parallel to the line $y = \frac{1}{3}$?

10. Is the line $x = 4$ parallel to the line $x = -4$?

11. Is the line $y = \frac{2}{3}x - 4$ parallel to the line $y = -\frac{3}{2}x - 4$?

12. Is the line $y = -2x + \frac{2}{3}$ parallel to the line $y = -2x + 3$?

13. Is the line $y = \frac{4}{3}x - 2$ perpendicular to the line $y = -\frac{3}{4}x + 2$?

14. Is the line $y = \frac{1}{2}x + \frac{3}{2}$ perpendicular to the line $y = -\frac{1}{2}x + \frac{3}{2}$?

15. Are the lines $2x + 3y = 2$ and $2x + 3y = -4$ parallel?

16. Are the lines $2x - 4y = 3$ and $2x + 4y = -3$ parallel?

17. Are the lines $x - 4y = 2$ and $4x + y = 8$ perpendicular?

18. Are the lines $4x - 3y = 2$ and $4x + 3y = -7$ perpendicular?

19. Is the line that contains the points $(3, 2)$ and $(1, 6)$ parallel to the line that contains the points $(-1, 3)$ and $(-1, -1)$?

20. Is the line that contains the points $(4, -3)$ and $(2, 5)$ parallel to the line that contains the points $(-2, -3)$ and $(-4, 1)$?

21. Is the line that contains the points $(-3, 2)$ and $(4, -1)$ perpendicular to the line that contains the points $(1, 3)$ and $(-2, -4)$?

22. Is the line that contains the points $(-1, 2)$ and $(3, 4)$ perpendicular to the line that contains the points $(-1, 3)$ and $(-4, 1)$?

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23. Find the equation of the line containing the point $(-2, -4)$ and parallel to the line $2x - 3y = 2$.

24. Find the equation of the line containing the point $(3, 2)$ and parallel to the line $3x + y = -3$.

25. Find the equation of the line containing the point $(4, 1)$ and perpendicular to the line $y = -3x + 4$.

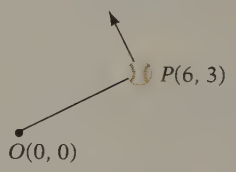
26. Find the equation of the line containing the point $(2, -5)$ and perpendicular to the line $y = \frac{5}{2}x - 4$.

27. Find the equation of the line containing the point $(-1, -3)$ and perpendicular to the line $3x - 5y = 2$.

28. Find the equation of the line containing the point $(-1, 3)$ and perpendicular to the line $2x + 4y = -1$.

APPLYING THE CONCEPTS

For Exercises 29 and 30, suppose a ball is being twirled at the end of a string and the center of rotation is the origin of a coordinate system. If the string breaks, the initial path of the ball is on a line that is perpendicular to the radius of the circle.



29. Suppose the string breaks when the ball is at the point whose coordinates are $P(6, 3)$. Find the equation of the line on which the initial path lies.

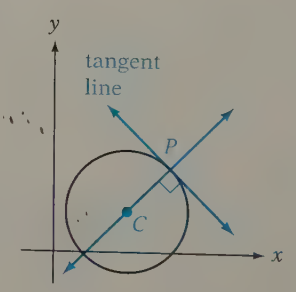
30. Suppose the string breaks when the ball is at the point whose coordinates are $P(2, 8)$. Find the equation of the line on which the initial path lies.

31. If the graphs of $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are perpendicular, express $\frac{A_1}{B_1}$ in terms of A_2 and B_2.

32. If the graphs of $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are parallel, express $\frac{A_1}{B_1}$ in terms of A_2 and B_2.

33. The graphs of $y = -\frac{1}{2}x + 2$ and $y = \frac{2}{3}x - 5$ intersect at the point whose coordinates are $(6, -1)$. Find the equation of a line whose graph intersects the graphs of the given lines to form a right triangle. (*Hint:* There is more than one answer to this question.)

34. A theorem from geometry states that a line passing through the center of a circle and through a point P on the circle is perpendicular to the tangent line at P. (See the figure at the right.) If the coordinates of P are $(5, 4)$ and the coordinates of C are $(3, 2)$, what is the equation of the tangent line?



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4.7 Graphing Linear Inequalities

Objective A

To graph an inequality in two variables

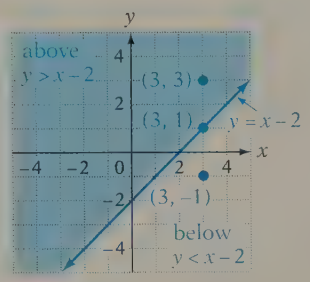
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Point of Interest

Linear inequalities play an important role in applied mathematics. They are used in a branch of mathematics called *linear programming*, which was developed during World War II to solve problems in supplying the Air Force with the machine parts necessary to keep planes flying. Today, its applications have been broadened to many other disciplines.

The graph of the linear equation $y = x - 2$ separates a plane into three sets:

the set of points on the line
the set of points above the line
the set of points below the line



The point (3, 1) is a solution of $y = x - 2$.

$$y = x - 2$$
$$1 \mid 3 - 2$$
$$1 = 1$$

The point (3, 3) is a solution of $y > x - 2$.

$$y > x - 2$$
$$3 \mid 3 - 2$$
$$3 > 1$$

Any point above the line is a solution of $y > x - 2$.

The point (3, −1) is a solution of $y < x - 2$.

$$y < x - 2$$
$$-1 \mid 3 - 2$$
$$-1 < 1$$

Any point below the line is a solution of $y < x - 2$.

The solution set of $y = x - 2$ is all points on the line. The solution set of $y > x - 2$ is all points above the line. The solution set of $y < x - 2$ is all points below the line. The solution set of an inequality in two variables is a **half-plane**.

The following illustrates the procedure for graphing a linear inequality.

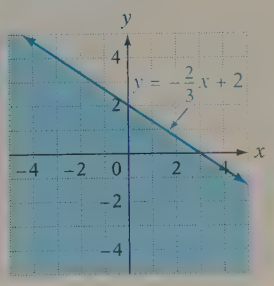
➡ Graph the solution set of $2x + 3y \le 6$.

$$2x + 3y \le 6$$
$$2x - 2x + 3y \le -2x + 6$$
$$3y \le -2x + 6$$
$$\frac{3y}{3} \le \frac{-2x + 6}{3}$$
$$y \le -\frac{2}{3}x + 2$$

- Solve the inequality for y.
- Subtract $2x$ from each side.
- Simplify.
- Divide each side by 3.
- Simplify.

Change the inequality to an equality and graph $y = -\frac{2}{3}x + 2$. If the inequality is **≥ or ≤**, the line is in the solution set and is shown by a **solid line**. If the inequality is **> or <**, the line is not a part of the solution set and is shown by a **dotted line**.

If the inequality is **> or ≥**, shade the **upper half-plane**. If the inequality is **< or ≤**, shade the **lower half-plane**.



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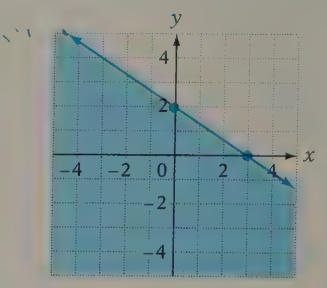
The inequality $2x + 3y \leq 6$ can also be graphed as shown below.

$2x + 3y = 6$ • Change the inequality to an equality.

$2x + 3(0) = 6$
$2x = 6$
$x = 3$ • Find the x- and y-intercepts of the equation. The x-intercept is (3, 0).

$2(0) + 3y = 6$
$3y = 6$
$y = 2$ • The y-intercept is (0, 2).

Graph the ordered pairs (3, 0) and (0, 2). Draw a solid line through the points because the inequality is $\leq$. The point (0, 0) can be used to determine which region to shade. If (0, 0) is a solution of the inequality, then shade the region that includes the point (0, 0). If (0, 0) is not a solution of the inequality, then shade the region that does not include the point (0, 0). For this example, (0, 0) is a solution of the inequality. The region that contains the point (0, 0) is shaded.



TAKE NOTE
Any ordered pair is of the form (x, y). For the point (0, 0), substitute 0 for x and 0 for y in the inequality.

$2x + 3y \leq 6$
$2(0) + 3(0) \leq 6$
$0 \leq 6$ True

If the line passes through point (0, 0), another point must be used to determine which region to shade. For example, use the point (1, 0).

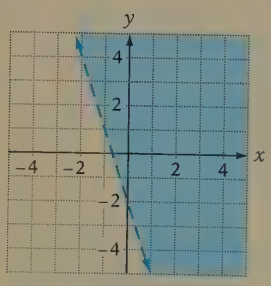
It is important to note that every point in the shaded region is a solution of the inequality and that every solution of the inequality is a point in the shaded region. No point outside the shaded region is a solution of the inequality.

Example 1
Graph the solution set of $3x + y > -2$.

Solution

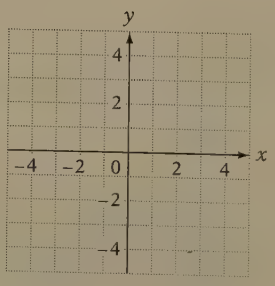
$$3x + y > -2$$
$$3x - 3x + y > -3x - 2$$
$$y > -3x - 2$$

Graph $y = -3x - 2$ as a dashed line.
Shade the upper half-plane.



You Try It 1
Graph the solution set of $x - 3y < 2$.

Your solution



Solution on p. S13

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4.7 Exercises

Objective A

1. What is a half-plane?

2. Explain a method you can use to check that the graph of a linear inequality in two variables has been shaded correctly.

3. Is (0, 0) a solution of $y > 2x - 7$?

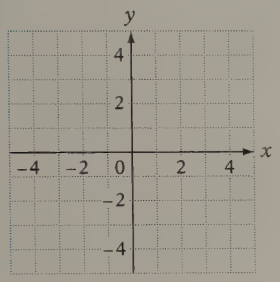
4. Is (0, 0) a solution of $y < 5x + 3$?

5. Is (0, 0) a solution of $y \leq -\frac{2}{3}x - 8$?

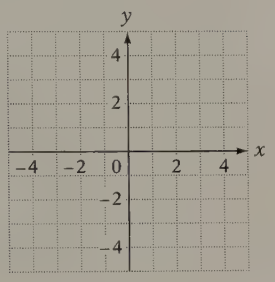
6. Is (0, 0) a solution of $y \geq -\frac{3}{4}x + 9$?

Graph the solution set.

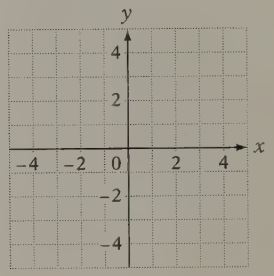
7. $y > -x + 4$



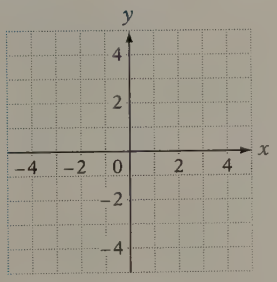
8. $y < x + 3$



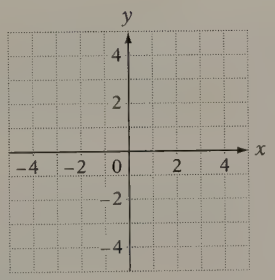
9. $y > 2x + 3$



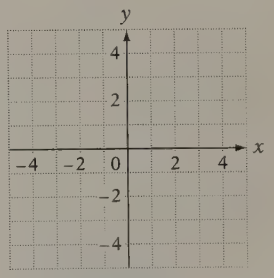
10. $y \geq -2x + 4$



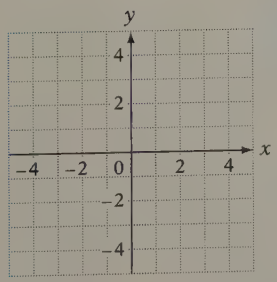
11. $y > \frac{3}{2}x - 4$



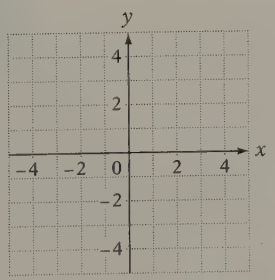
12. $y > -\frac{5}{4}x + 1$



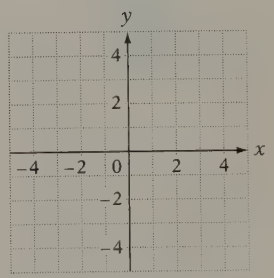
13. $y \leq -\frac{3}{4}x - 1$



14. $y \leq -\frac{5}{2}x - 4$

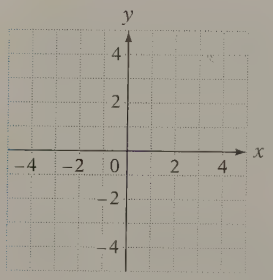


15. $y \leq -2$

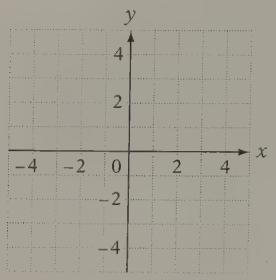


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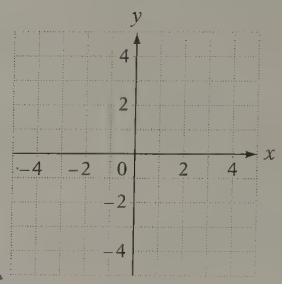
16. $y > 3$



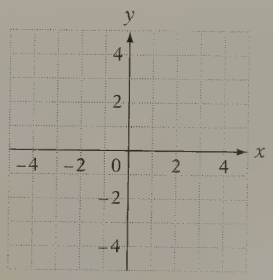
17. $y < \frac{4}{5}x + 3$



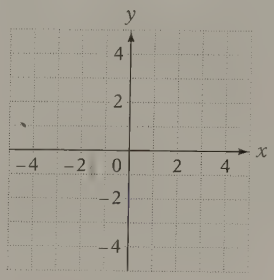
18. $y \leq -\frac{6}{5}x - 2$



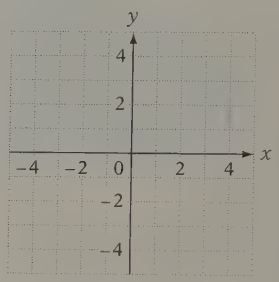
19. $3x - y < 9$



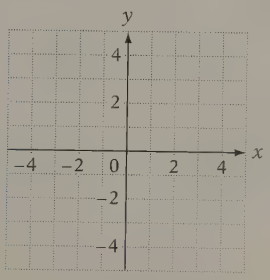
20. $3x + y \geq 6$



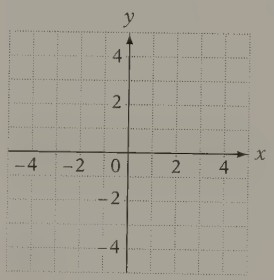
21. $2x + 2y \leq -4$



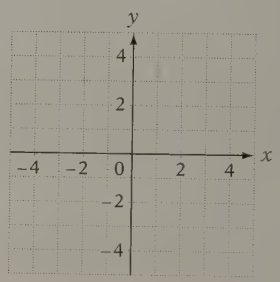
22. $-4x + 3y < -12$



23. $-2x + 3y \leq 6$



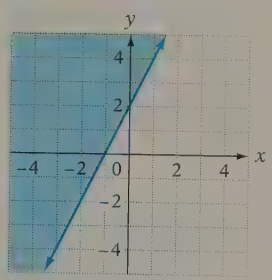
24. $3x - 4y > 12$



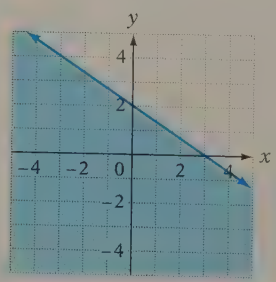
APPLYING THE CONCEPTS

Write the inequality given its graph.

25.



26.



27. Are there any points whose coordinates satisfy both $y \leq x + 3$ and $y \geq -\frac{1}{2}x + 1$? If so, give the coordinates of three such points. If not, explain why not.

28. Are there any points whose coordinates satisfy both $y \leq x - 1$ and $y \geq x + 2$? If so, give the coordinates of three such points. If not, explain why not.

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Focus on Problem Solving

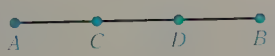
Find a Counterexample

When you are faced with an assertion, it may be that the assertion is false. For instance, consider the statement "Every prime number is an odd number." This assertion is false because the prime number 2 is an even number.

Finding an example that illustrates that an assertion is false is called finding a *counterexample*. The number 2 is a counterexample to the assertion that every prime number is an odd number.

If you are given an unfamiliar problem, one strategy to consider as a means of solving the problem is to try to find a counterexample. For each of the following problems, answer *true* if the assertion is always true. If the assertion is not true, answer *false* and give a counterexample. If there are terms used that you do not understand, consult a reference to find the meaning of the term.

1. If x is a real number, then x^2 is always positive.
2. The product of an odd integer and an even integer is an even integer.
3. If m is a positive integer, then $2m + 1$ is always a positive odd integer.
4. If $x < y$, then $x^2 < y^2$.
5. Given any three positive numbers a, b, and c, it is possible to construct a triangle whose sides have lengths a, b, and c.
6. The product of two irrational numbers is an irrational number.
7. If n is a positive integer greater than 2, then $1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots \cdot n + 1$ is a prime number.
8. Draw a polygon with more than three sides. Select two different points inside the polygon and join the points with a line segment. The line segment always lies completely inside the polygon.
9. Let A, B, and C be three points in the plane that are not collinear. Let d_1 be the distance from A to B, and let d_2 be the distance from A to C. Then the distance between B and C is less than $d_1 + d_2$.
10. Consider the line segment AB shown at the left. Two points, C and D, are randomly selected on the line segment and three new segments are formed: AC, CD, and DB. The three new line segments can always be connected to form a triangle.



It may not be easy to establish that an assertion is true or to find a counterexample to the assertion. For instance, consider the assertion that every positive integer greater than 3 can be written as the sum of two primes. For example, $6 = 3 + 3$, $8 = 3 + 5$, $9 = 2 + 7$. Is this assertion always true? (*Note:* This assertion, called **Goldbach's conjecture,** has never been proved, nor has a counterexample been found!)

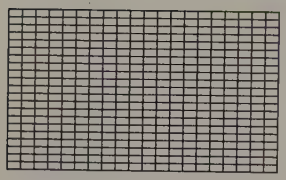
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Projects and Group Activities

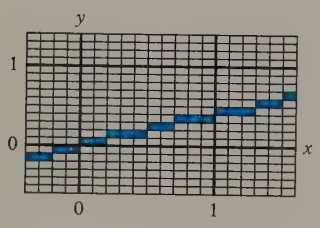
Graphing Linear Equations with a Graphing Utility



A computer or graphing calculator screen is divided into *pixels*. There are approximately 6000 to 790,000 pixels available on the screen (depending on the computer or calculator). The greater the number of pixels, the smoother a graph will appear. A portion of a screen is shown at the left. Each little rectangle represents one pixel.



The graphing utilities that are used by computers or calculators to graph an equation do basically what we have shown in the text: They choose values of x and, for each, calculate the corresponding value of y. The pixel corresponding to the ordered pair is then turned on. The graph is jagged because pixels are much larger than the dots we draw on paper.



The graph of $y = 0.45x$ is shown at the left as the calculator drew it (jagged). The x- and y-axes have been chosen so that each pixel represents $\frac{1}{10}$ of a unit. Consider the region of the graph where $x = 1$, 1.1, and 1.2.

The corresponding values of y are 0.45, 0.495, and 0.54. Because the y-axis is in tenths, the numbers 0.45, 0.495, and 0.54 are rounded to the nearest tenth before plotting. Rounding 0.45, 0.495, and 0.54 to the nearest tenth results in 0.5 for each number. Thus the ordered pairs (1, 0.45), (1.1, 0.495), and (1.2, 0.54) are graphed as (1, 0.5), (1.1, 0.5), and (1.2, 0.5). These points appear as three illuminated horizontal pixels. The graph of the line appears horizontal. However, if you use the TRACE feature of the calculator (see the appendix), the actual y-coordinate for each value of x is displayed.

TAKE NOTE

Xmin and Xmax are the smallest and largest values of x that will be shown on the screen. Ymin and Ymax are the smallest and largest values of y that will be shown on the screen.

Here are the keystrokes to graph $y = \frac{2}{3}x + 1$ on a TI-83. First the equation is entered. Then the domain (Xmin to Xmax) and the range (Ymin to Ymax) are entered. This is called the **viewing window**. By changing the keystrokes 2 [X, T, θ, n] [÷] 3 [+] 1, you can graph different equations.

[Y =] [CLEAR] 2 [X, T, θ, n] [÷] 3
[+] 1 [WINDOW] [(−)] 10
[ENTER] 10 [ENTER] 1 [ENTER]
[(−)] 10 [ENTER] 10 [ENTER] 1
[ENTER] [GRAPH]

1. $y = 2x + 1$

For $2x$, you may enter $2 \times x$ or just $2x$. The times sign $\times$ is not necessary on many graphing calculators.

2. $y = -\frac{1}{2}x - 2$

Use the [(−)] key to enter a negative sign.

3. $3x + 2y = 6$

Solve for y. Then enter the equation.

4. $4x + 3y = 75$

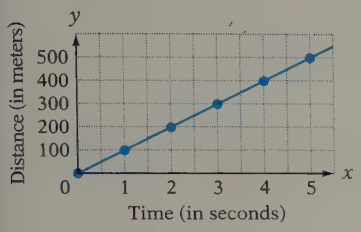
You must adjust the viewing window. *Suggestion:* Xmin = −25, Xmax = 25, Xscl = 5, Ymin = −35, Ymax = 35, Yscl = 5. See the appendix for assistance.

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Graphs of Motion

A graph can be useful in analyzing the motion of a body. For example, consider an airplane in uniform motion traveling at 100 m/s. The table at the right shows the distance, in meters, traveled by the plane at the end of each of five one-second intervals.

Time (in seconds)	*Distance (in meters)*
0	0
1	100
2	200
3	300
4	400
5	500



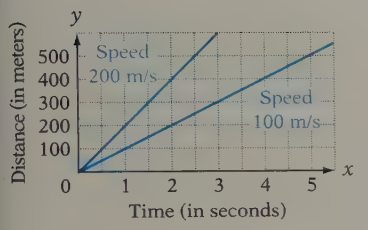
These data can be graphed on a rectangular coordinate system and a straight line drawn through the points plotted. The travel time is shown along the horizontal axis, and the distance traveled by the plane is shown along the vertical axis. (Note that the units along the two axes are not the same length.)

To write the equation for the line just graphed, use the coordinates of any two points on the line to find the slope. The y-intercept is (0, 0).

Let $(x_1, y_1) = (1, 100)$ and $(x_2, y_2) = (2, 200)$.

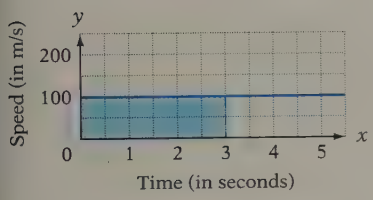
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200 - 100}{2 - 1} = 100$$

$$y = mx + b$$
$$y = 100x + 0$$
$$y = 100x$$



Note that the slope of the line, 100, is equal to the speed, 100 m/s. *The slope of a distance-time graph represents the speed of the object.*

The distance-time graphs for two planes are shown at the left. One plane is traveling at 100 m/s, and the other is traveling at 200 m/s. The slope of the line representing the faster plane is greater than the slope of the line representing the slower plane.



In the speed-time graph at the left, the time a plane has been flying at 100 m/s is shown along the horizontal axis and its speed is shown along the vertical axis. Because the speed is constant, the graph is a horizontal line.

The area between the horizontal line graphed and the horizontal axis is equal to the distance traveled by the plane up to that time. For example, the area of the shaded region on the graph is

$$\text{Length} \cdot \text{width} = (3\text{ s})(100\text{ m/s}) = 300\text{ m}$$

The distance traveled by the plane in 3 s is equal to 300 m.

1. A car in uniform motion is traveling at 20 m/s.
   - **a.** Prepare a distance-time graph for the car for 0 s to 5 s.
   - **b.** Find the slope of the line.
   - **c.** Find the equation of the line.
   - **d.** Prepare a speed-time graph for the car for 0 s to 5 s.
   - **e.** Find the distance traveled by the car after 3 s.

2. One car in uniform motion is traveling at 10 m/s. A second car in uniform motion is traveling at 15 m/s.
   - **a.** Prepare one distance-time graph for both cars for 0 s to 5 s.
   - **b.** Find the slope of each line.
   - **c.** Find the equation of each line graphed.
   - **d.** Assuming that the cars started at the same point at 0 s, find the distance between the cars at the end of 5 s.

3. **a.** In a distance-time graph, is it possible for the graph to be a horizontal line?
   b. What does a horizontal line reveal about the motion of the object during that time period?

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Chapter Summary

Key Words

A *rectangular coordinate system* is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. A rectangular coordinate system divides the plane into four regions called *quadrants*. The number lines that make up a rectangular coordinate system are called the *coordinate axes*, or simply the *axes*. The *origin* is the point of intersection of the two coordinate axes. [p. 201]

Every point in the plane can be identified by an ordered pair (x, y). The first number in an ordered pair is called the *abscissa* or x-coordinate. The second number is called the *ordinate* or y-coordinate. The *coordinates* of a point are the numbers in the ordered pair associated with the point. [p. 201]

A *relation* is any set of ordered pairs. The *domain* of a relation is the set of first coordinates of the ordered pairs. The *range* is the set of second coordinates of the ordered pairs. A *function* is a relation in which no two ordered pairs have the same first coordinate and different second coordinates. A function designated by $f(x)$ is written in *functional notation*. The *value* of the function at x is $f(x)$. [pp. 211–214]

An equation of the form $y = mx + b$, where m is the coefficient of x and b is a constant, is a *linear equation in two variables*. m is the slope of the line, and $(0, b)$ is the y-intercept. An equation of the form $Ax + By = C$ is also a linear equation in two variables. A *solution* of a linear equation in two variables is an ordered pair (x, y) that makes the equation a true statement. The *graph of a linear equation* is a straight line. [pp. 203, 223]

The point at which a graph crosses the x-axis is called the *x-intercept*. The point at which a graph crosses the y-axis is called the *y-intercept*. [p. 227]

The *slope* of a line is a measure of the slant of the line. The symbol for slope is m. A line that slants upward to the right has a *positive slope*. A line that slants downward to the right has a *negative slope*. A horizontal line has *zero slope*. The slope of a vertical line is *undefined*. [pp. 235–236]

Two lines that have the same slope do not intersect and are *parallel lines*. Two lines that intersect at right angles are *perpendicular lines*. [pp. 253–254]

An inequality of the form $y > mx + b$ or $Ax + By > C$ is a *linear inequality in two variables*. The symbol $>$ here could be replaced by $\geq$, $<$, or $\leq$. The solution set of an inequality in two variables is a *half-plane*. [p. 259]

Essential Rules

To find the x-intercept, let $y = 0$
To find the y-intercept, let $x = 0$. [p. 227]

Slope of a Linear Equation [p. 235]
$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$$

Slope-Intercept Form of a Straight Line [p. 239]
$$y = mx + b$$

Point-Slope Formula [p. 245]
$$y - y_1 = m(x - x_1)$$

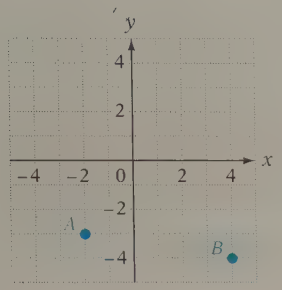
Slopes of Parallel Lines [p. 253]
$$m_1 = m_2$$

Slopes of Perpendicular Lines [p. 254]
$$m_1 \cdot m_2 = -1$$

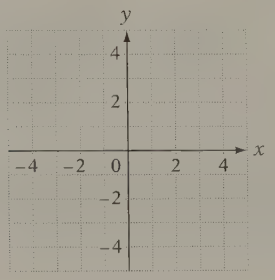
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Chapter Review

1. **a.** Graph the ordered pairs $(-2, 4)$ and $(3, -2)$.
b. Name the abscissa of point A.
c. Name the ordinate of point B.



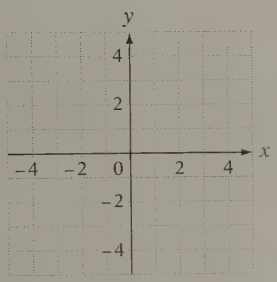
2. Graph the ordered-pair solutions of $y = -\frac{1}{2}x - 2$ when $x = -4, -2, 0$, and 2.



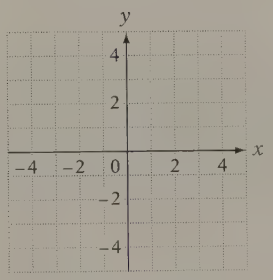
3. Find the x- and y-intercepts of $3x - 2y = 24$.

4. Given $f(x) = x^2 - 2$, find $f(-1)$.

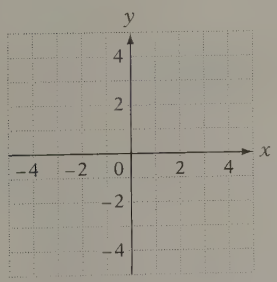
5. Graph $x = -3$.



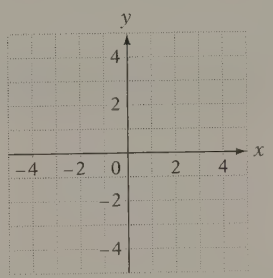
6. Graph the line that has slope $-\frac{2}{3}$ and y-intercept $(0, 2)$.



7. Graph $y = -2x - 1$.



8. Graph $y = \frac{1}{4}x + 3$.



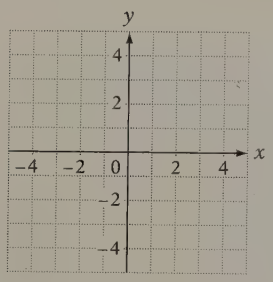
9. Find the slope of the line containing the points $(-2, -3)$ and $(4, -3)$.

10. Find the slope of the line containing the points $(9, 8)$ and $(-2, 1)$.

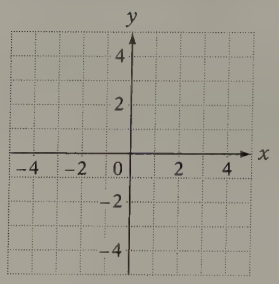
11. What value of x is excluded from the domain of $f(x) = \frac{2x + 1}{x}$?

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12. Graph $3x - 2y = -6$.



13. Graph the solution set of $y < \frac{3}{4}x - 2$.



14. Determine the equation of the line that passes through the points $(-1, 3)$ and $(2, -5)$.

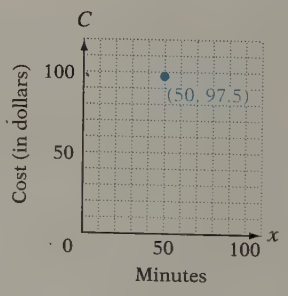
15. Determine the equation of the line that passes through the point $(6, 1)$ and has slope $-\frac{5}{2}$.

16. Find the equation of the line that contains the ordered pair $(-2, -4)$ and is parallel to the graph of $4x - 2y = 7$.

17. Find the equation of the line that contains the point $(-2, -3)$ and is perpendicular to the line $y = -\frac{1}{2}x - 3$.

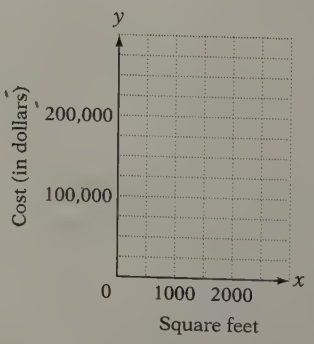
18. A building contractor estimates that the cost to build a new house is \$25,000 plus \$80 for each square foot of floor space. Determine a linear function that will give the cost to build a house that contains a given number of square feet. Use the model to determine the cost to build a house that contains 2000 ft^2.

19. An on-line research service charges a monthly access fee of \$75 plus \$.45 per minute to use the service. An equation that represents the monthly cost to use this service is $C = 0.45x + 75$, where C is the monthly cost and x is the number of minutes. Graph this equation for $0 \le x \le 100$. The point $(50, 97.5)$ is on the graph. Write a sentence that describes the meaning of this ordered pair.



20. The data in the table below show the size of a house in square feet and the cost to build the house. Draw a scatter diagram of these data.

Square feet	1250	1400	1348	2675	2900
Cost	128,000	140,000	136,100	233,450	241,500



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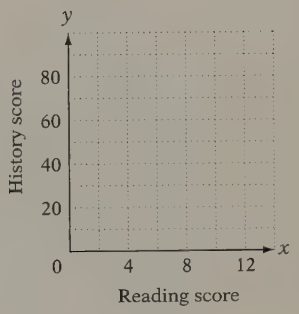
Chapter Test

1. Find the ordered-pair solution of $2x - 3y = 15$ corresponding to $x = 3$.

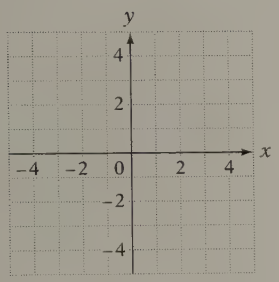
2. Given $f(t) = t^2 + t$, find $f(2)$.

3. The data in the table below show a reading test grade and the final exam grade in a history class. Draw a scatter diagram of these data.

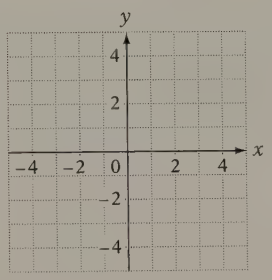
Reading	8.5	9.4	10.0	11.4	12.0
History	64	68	76	87	92



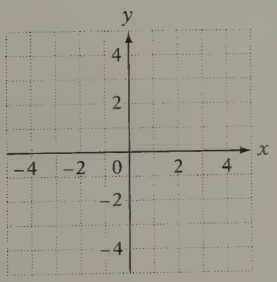
4. Graph the ordered-pair solutions of $y = -\frac{3}{2}x + 1$ when $x = -2$, 0, and 4.



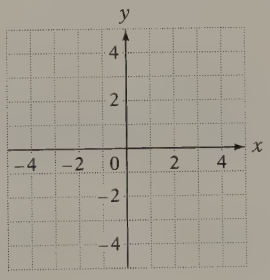
5. Graph $y = -\frac{3}{4}x + 3$.



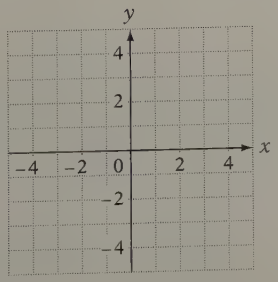
6. Graph $3x - 2y = 6$.



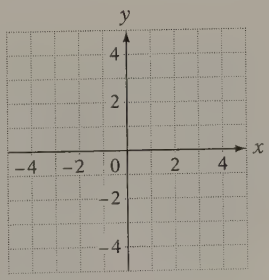
7. Graph the line that has slope $-\frac{2}{3}$ and y-intercept (0, 4).



8. Graph the solution set of $y \geq 2x - 3$.

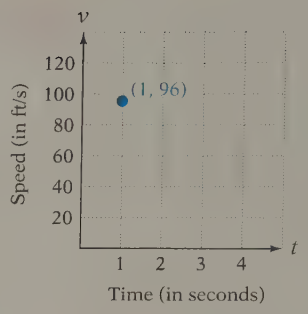


9. Graph the solution set of $3x - 2y \leq 6$.



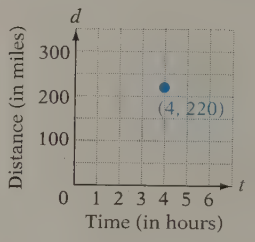
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10. The equation for the speed of a ball that is thrown straight up with an initial speed of 128 ft/s is $v = 128 - 32t$, where v is the speed of the ball after t seconds. Graph this equation for $0 \le t \le 4$. The point whose coordinates are (1, 96) is on the graph at the right. Write a sentence that describes this ordered pair.



11. The director of a baseball school estimates that 100 students will enroll if the tuition is \$250. For each \$20 increase in tuition, 6 fewer students will enroll. Determine a linear function that will predict the number of students who will enroll at a given tuition. Use this model to predict enrollment when the tuition is \$300.

12. A car is traveling at 55 mph. The equation that describes the distance traveled is $d = 55t$. Graph this equation for $0 \le t \le 6$. The point whose coordinates are (4, 220) is on the graph. Write a sentence that explains the meaning of this ordered pair.



13. Find the x- and y-intercepts for $6x - 4y = 12$.

14. Find the slope of the line containing the points (2, −3) and (4, 1).

15. Find the slope of the line containing the points (−5, 2) and (−5, 7).

16. Find the slope of the line whose equation is $2x + 3y = 6$.

17. Find the equation of the line that contains the point (−3, 1) and has slope $\frac{2}{3}$.

18. Find the equation of the line that passes through the points (−2, 0) and (5, −2).

19. Find the equation of the line that contains the ordered pair (3, −2) and is parallel to the graph of $y = -3x + 4$.

20. Find the equation of the line that contains the ordered pair (2, 5) and is perpendicular to the graph of the line $y = -\frac{2}{3}x + 6$.

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Cumulative Review

1. Let $x \in \{-5, -3, -1\}$. For what values of x is the inequality $x \leq -3$ a true statement?

2. Write $\frac{17}{20}$ as a decimal.

3. Simplify: $3\sqrt{45}$

4. Simplify: $12 - 18 \div 3(-2)^2$

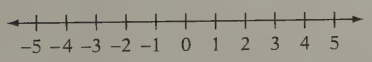
5. Evaluate $\frac{a-b}{a^2-c}$ when $a = -2$, $b = 3$, and $c = -4$.

6. Simplify: $3d - 9 - 7d$

7. Simplify: $4(-8z)$

8. Simplify: $2(x + y) - 5(3x - y)$

9. Graph: $\{x|x < -2\} \cup \{x|x > 0\}$



10. Solve: $2x - \frac{2}{3} = \frac{7}{3}$

11. Solve: $3x - 2(10x - 6) = x - 6$

12. Solve: $4x - 3 < 9x + 2$

13. Solve: $3x - 1 < 4$ and $x - 2 > 2$

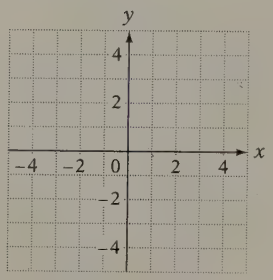
14. Solve: $|3x - 5| < 5$

15. Given $f(t) = t^2 + t$, find $f(2)$.

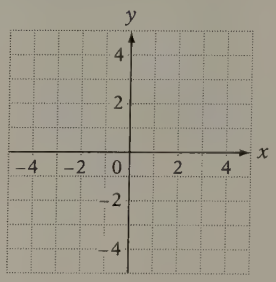
16. Find the slope of the line containing the points $(2, -3)$ and $(4, 1)$.

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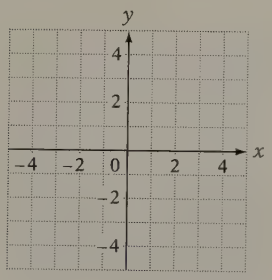
17. Graph $y = 3x + 1$.



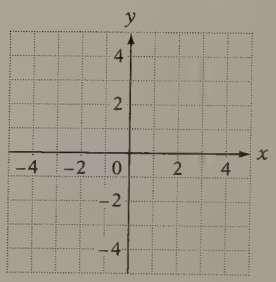
18. Graph $x = -4$.



19. Graph the line that has slope $\frac{1}{2}$ and y-intercept $(0, -1)$.



20. Graph the solution set of $3x - 2y \geq 6$.



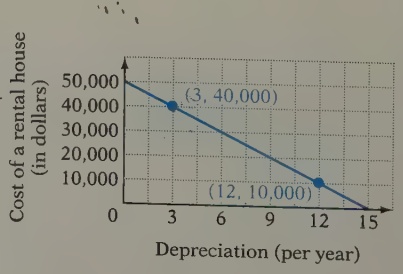
21. Find the equation of the line that passes through the points $(6, -4)$ and $(-3, -1)$.

22. Find the equation of the line that contains the point $(2, 4)$ and is parallel to the line $y = -\frac{3}{2}x + 2$.

23. Two planes are 1800 mi apart and traveling toward each other. The first plane is traveling at twice the speed of the second plane. The planes meet in 3 h. Find the speed of each plane.

24. A grocer combines coffee that costs \$9 per pound with coffee that costs \$6 per pound. How many pounds of each should be used to make 60 lb of a blend that costs \$8 per pound?

25. The graph at the right shows the relationship between the cost of a rental house and the depreciation allowed for income tax purposes. Find the slope of the line between the two points on the graph. Write a sentence that states the meaning of the slope.



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Chapter 5

Systems of Linear Equations and Inequalities



This surveyor is working on a construction site, using the tools necessary to find the area of a plot of land. When a plot is in the shape of a polygon, its vertices can be represented by rectangular coordinates. These coordinates are used in the surveyor's area formula to determine the area of the plot. **Exercise 37 on page 304,** which involves use of the surveyor's area formula, is an application of determinants.

Objectives

Section 5.1

A To solve a system of linear equations by graphing

B To solve a system of linear equations by the substitution method

Section 5.2

A To solve a system of two linear equations in two variables by the addition method

B To solve a system of three linear equations in three variables by the addition method

Section 5.3

A To evaluate a determinant

B To solve a system of equations by using Cramer's Rule

Section 5.4

A To solve rate-of-wind or rate-of-current problems

B To solve application problems using two variables

Section 5.5

A To graph the solution set of a system of linear inequalities



Need help? For online student resources, such as section quizzes, visit this textbook's website at **college.hmco.com/students.**

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Prep Test

1. Simplify: $10\left(\frac{3}{5}x + \frac{1}{2}y\right)$

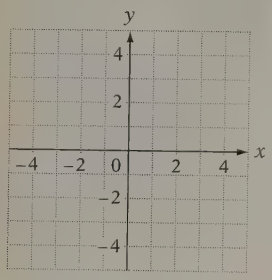
2. Evaluate $3x + 2y - z$ for $x = -1$, $y = 4$, and $z = -2$.

3. Given $3x - 2z = 4$, find the value of x when $z = -2$.

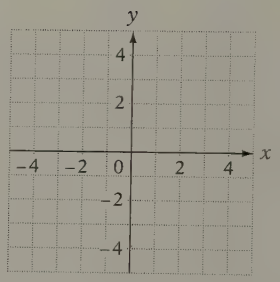
4. Solve: $3x + 4(-2x - 5) = -5$

5. Solve: $0.45x + 0.06(-x + 4000) = 630$

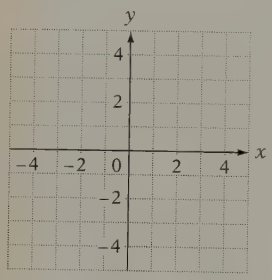
6. Graph: $y = \frac{1}{2}x - 4$



7. Graph: $3x - 2y = 6$



8. Graph: $y > -\frac{3}{5}x + 1$



Go Figure

Two children are on their way from school to home. Carla runs half the time and walks half the time. James runs half the distance and walks half the distance. If Carla and James walk at the same speed and run at the same speed, which child arrives home first?

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5.1 Solving Systems of Linear Equations by Graphing and by the Substitution Method

Objective A

To solve a system of linear equations by graphing

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A **system of equations** is two or more equations considered together. The system at the right is a system of two linear equations in two variables. The graphs of the equations are straight lines.

$$\begin{aligned} 3x + 4y &= 7 \\ 2x - 3y &= 6 \end{aligned}$$

A **solution of a system of equations in two variables** is an ordered pair that is a solution of each equation of the system.

Is $(3, -2)$ a solution of the system $\begin{aligned} 2x - 3y &= 12 \\ 5x + 2y &= 11 \end{aligned}$?

$2x - 3y = 12$	
$2(3) - 3(-2)$	12
$6 - (-6)$	
$12 = 12$ True	

$5x + 2y = 11$	
$5(3) + 2(-2)$	11
$15 + (-4)$	
$11 = 11$ True	

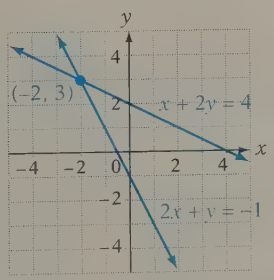
• Replace x by 3 and y by -2.

Yes, because $(3, -2)$ is a solution of each equation, it is a solution of the system of equations.

A solution of a system of linear equations can be found by graphing the lines of the system on the same coordinate axes. Three examples of linear equations in two variables are shown below, along with the graphs of the equations of the system.

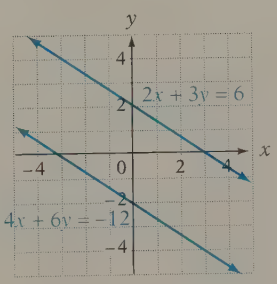
System I

$x + 2y = 4$
$2x + y = -1$



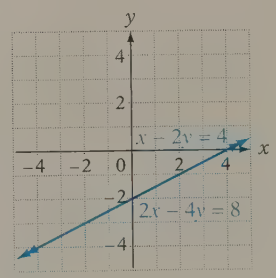
System II

$2x + 3y = 6$
$4x + 6y = -12$



System III

$x - 2y = 4$
$2x - 4y = 8$



$x + 2y = 4$	
$-2 + 2(3)$	4
$-2 + 6$	
$4 = 4$	

$2x + y = -1$	
$2(-2) + 3$	-1
$-4 + 3$	-1
$-1 = -1$	

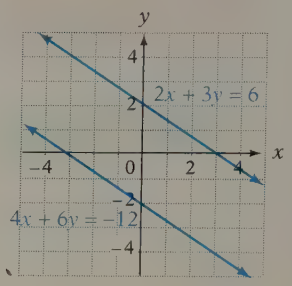
For System I, the two lines intersect at a single point whose coordinates are $(-2, 3)$. Because this point lies on both lines, it is a solution of each equation of the system of equations. We can check this by replacing x by -2 and y by 3. The check is shown at the left. The ordered pair $(-2, 3)$ is a solution of System I.

When the graphs of a system of equations intersect at only one point, the system is called an **independent system of equations.** System I is an independent system of equations.

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System II from the previous page and the graph of the equations of that system are shown again at the right. Note in this case that the graphs of the lines are parallel and do not intersect. Because the graphs do not intersect, there is no point that is on both lines. Therefore, the system of equations has no solution.

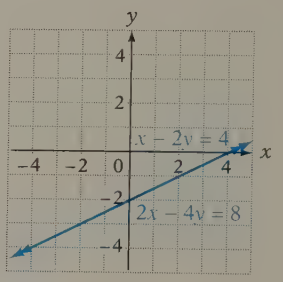
$$2x + 3y = 6$$
$$4x + 6y = -12$$



When a system of equations has no solution, it is called an **inconsistent system of equations.** System II is an inconsistent system of equations.

System III from the previous page and the graph of the equations of that system are shown again at the right. Note that the graph of $x - 2y = 4$ lies directly on top of the graph of $2x - 4y = 8$. Thus the two lines intersect at an infinite number of points, so there are an infinite number of solutions of this system of equations. Because each equation represents the same set of points, the solutions of the system of equations can be stated by using the ordered pairs of either one of the equations. Therefore, we can say, "The solutions are the ordered pairs that satisfy $x - 2y = 4$." Alternatively, we can solve the equation for y and say, "The solutions are the ordered pairs that satisfy $y = \frac{1}{2}x - 2$." We normally state this solution using ordered pairs. In this case, "The solutions are the ordered pairs $\left(x, \frac{1}{2}x - 2\right)$."

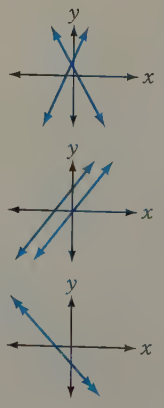
$$x - 2y = 4$$
$$2x - 4y = 8$$



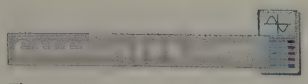
When the two equations in a system of equations represent the same line, the system is called a **dependent system of equations.** System III is a dependent system of equations.

The above systems illustrate the three possibilities for a system of linear equations in two variables:

1. The graphs intersect at one point.
   The solution of the system of equations is the ordered pair (x, y) whose coordinates are the point of intersection.
   The system of equations is independent.
2. The lines are parallel and never intersect.
   There is no solution of the system of equations.
   The system of equations is inconsistent.
3. The graphs are the same line, and they intersect at infinitely many points.
   There are an infinite number of solutions of the system of equations.
   The system of equations is dependent.



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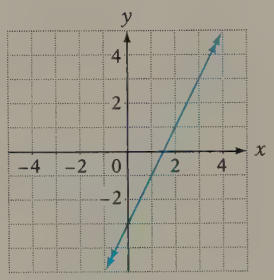
The Projects and Group Activities at the end of this chapter show how to use a graphing calculator to solve a system of equations.

Solve by graphing: $2x - y = 3$
$4x - 2y = 6$

Graph each line.
The system of equations is dependent.
Solve one of the equations for y.

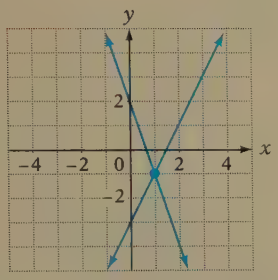
$2x - y = 3$
$-y = -2x + 3$
$y = 2x - 3$

The solutions are the ordered pairs $(x, 2x - 3)$.



Example 1 Solve by graphing:
$2x - y = 3$
$3x + y = 2$

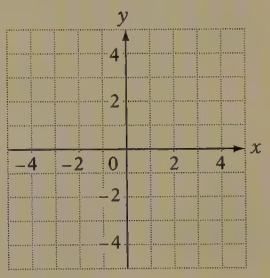
Solution



The solution is $(1, -1)$.

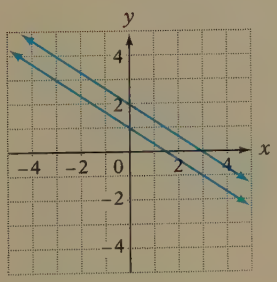
You Try It 1 Solve by graphing:
$x + y = 1$
$2x + y = 0$

Your solution



Example 2 Solve by graphing:
$2x + 3y = 6$
$y = -\frac{2}{3}x + 1$

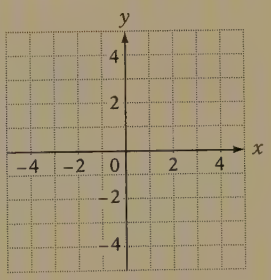
Solution



The lines are parallel and therefore do not intersect. The system of equations has no solution. The system of equations is inconsistent.

You Try It 2 Solve by graphing:
$2x + 5y = 10$
$y = -\frac{2}{5}x - 2$

Your solution



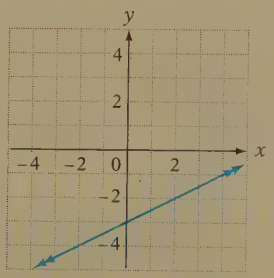
Solutions on p. S14

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Example 3 Solve by graphing:

$$x - 2y = 6$$
$$y = \frac{1}{2}x - 3$$

Solution

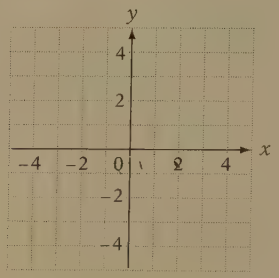


The system of equations is dependent. The solutions are the ordered pairs $\left(x, \frac{1}{2}x - 3\right)$.

You Try It 3 Solve by graphing:

$$3x - 4y = 12$$
$$y = \frac{3}{4}x - 3$$

Your solution



Solution on p. S14

Objective B **To solve a system of linear equations by the substitution method**

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The graphical solution of a system of equations is based on approximating the coordinates of a point of intersection. An algebraic method called the **substitution method** can be used to find an exact solution of a system of equations. In order for us to use the substitution method, one of the equations of the system must be written in terms of x or in terms of y.

Solve by the substitution method: (1) $3x + y = 5$
(2) $4x + 5y = 3$

$$3x + y = 5$$
(3) $$y = -3x + 5$$

- Solve Equation (1) for y.
- This is Equation (3).

(2) $$4x + 5y = 3$$
$$4x + 5(-3x + 5) = 3$$

- This is Equation (2).
- Equation (3) states that $y = -3x + 5$. Substitute $-3x + 5$ for y in Equation (2).

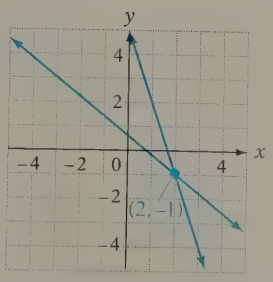
$$4x - 15x + 25 = 3$$
$$-11x + 25 = 3$$
$$-11x = -22$$
$$x = 2$$

- Solve for x.

(3) $y = -3x + 5$
$y = -3(2) + 5$
$y = -6 + 5$
$y = -1$

- Substitute the value of x into Equation (3) and solve for y.

The solution is the ordered pair $(2, -1)$.



The graph of the system of equations is shown at the left. Note that the lines intersect at the point whose coordinates are $(2, -1)$, the solution of the system of equations.

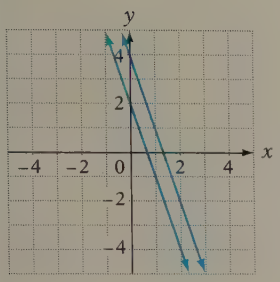
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Solve by the substitution method: (1) $6x + 2y = 8$
(2) $3x + y = 2$

$3x + y = 2$
(3) $y = -3x + 2$ • We will solve Equation (2) for y. • This is Equation (3).

(1) $6x + 2y = 8$ • This is Equation (1).
$6x + 2(-3x + 2) = 8$ • Equation (3) states that $y = -3x + 2$. Substitute $-3x + 2$ for y in Equation (1).

$6x - 6x + 4 = 8$ • Solve for x.
$0x + 4 = 8$
$4 = 8$



This is not a true equation.
The system of equations has no solution.
The system of equations is inconsistent.

The graph of the system of equations is shown at the left. Note that the lines are parallel.

Example 4 Solve by substitution:
(1) $3x - 2y = 4$
(2) $-x + 4y = -3$

Solution

Solve Equation (2) for x.

$-x + 4y = -3$
$-x = -4y - 3$
$x = 4y + 3$ • Equation (3)

Substitute $4y + 3$ for x in Equation (1).

$3x - 2y = 4$ • Equation (1)
$3(4y + 3) - 2y = 4$ • $x = 4y + 3$
$12y + 9 - 2y = 4$
$10y + 9 = 4$
$10y = -5$
$y = -\frac{5}{10} = -\frac{1}{2}$

Substitute the value of y into Equation (3).

$x = 4y + 3$ • Equation (3)
$= 4\left(-\frac{1}{2}\right) + 3$ • $y = -\frac{1}{2}$
$= -2 + 3 = 1$

The solution is $\left(1, -\frac{1}{2}\right)$.

You Try It 4 Solve by substitution:
$3x - y = 3$
$6x + 3y = -4$

Your solution

Solution on p. S14

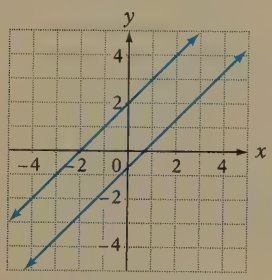
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Example 5 Solve by substitution and graph:

$$3x - 3y = 2$$
$$y = x + 2$$

Solution

$$3x - 3y = 2$$
$$3x - 3(x + 2) = 2$$
$$3x - 3x - 6 = 2$$
$$-6 = 2$$

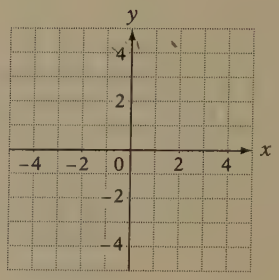


This is not a true equation. The lines are parallel, so the system is inconsistent. The system does not have a solution.

You Try It 5 Solve by substitution and graph:

$$y = 2x - 3$$
$$2x - 4y = 5$$

Your solution



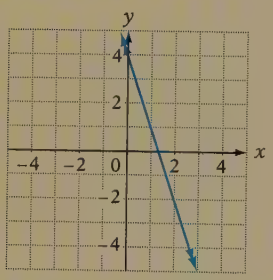
Example 6 Solve by substitution and graph:

$$9x + 3y = 12$$
$$y = -3x + 4$$

Solution

$$9x + 3y = 12$$
$$9x + 3(-3x + 4) = 12$$
$$9x - 9x + 12 = 12$$
$$12 = 12$$

This is a true equation. The system is dependent.

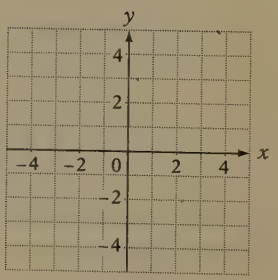


The solutions are the ordered pairs $(x, -3x + 4)$.

You Try It 6 Solve by substitution and graph:

$$6x - 3y = 6$$
$$2x - y = 2$$

Your solution



Solutions on pp. S14–S15

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5.1 Exercises

Objective A

Is the ordered pair a solution of the system of equations?

1. $(0, -1)$;
$3x - 2y = 2$
$x + 2y = 6$

2. $(2, 1)$;
$x + y = 3$
$2x - 3y = 1$

3. $(-3, -5)$
$x + y = -8$
$2x + 5y = -31$

4. $(1, -1)$;
$3x - y = 4$
$7x + 2y = -5$

5. $(4, 3)$;
$5x - 2y = 14$
$x + y = 8$

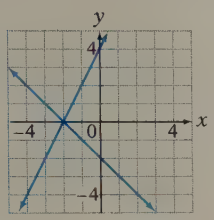
6. $(2, 5)$;
$3x + 2y = 16$
$2x - 3y = 4$

7. $(5, 2)$;
$y = 2x - 8$
$y = 3x - 13$

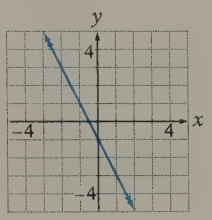
8. $(-4, 3)$;
$y = 2x + 11$
$y = x - 1$

State whether the system of equations is independent, inconsistent, or dependent.

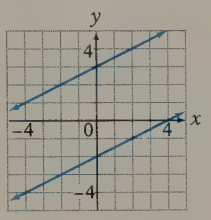
9.



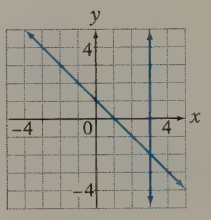
10.



11.

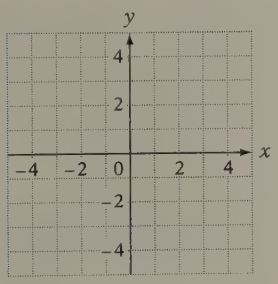


12.

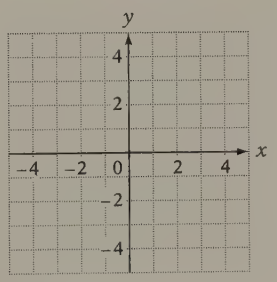


Solve by graphing.

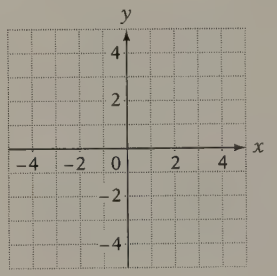
13. $x + y = 2$
$x - y = 4$



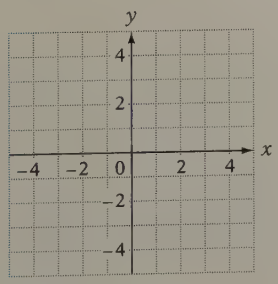
14. $x + y = 1$
$3x - y = -5$



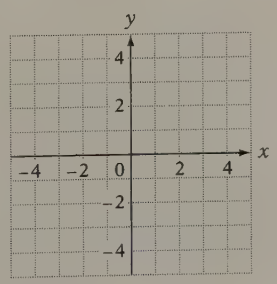
15. $x - y = -2$
$x + 2y = 10$



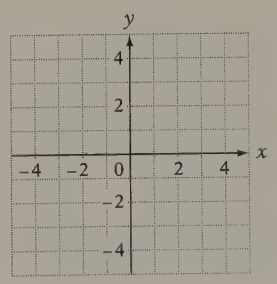
16. $2x - y = 5$
$3x + y = 5$



17. $3x - 2y = 6$
$y = 3$

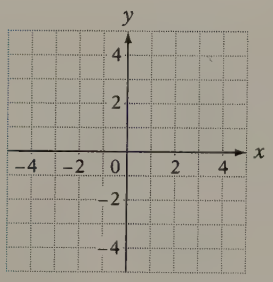


18. $x = 4$
$3x - 2y = 4$

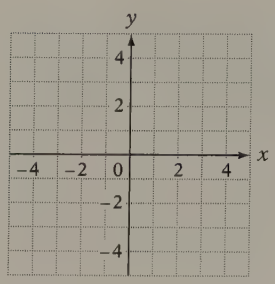


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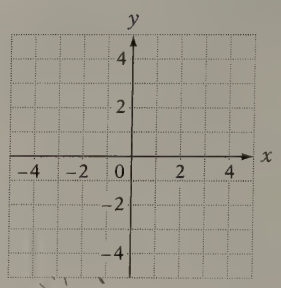
19. $x = 4$
$y = -1$



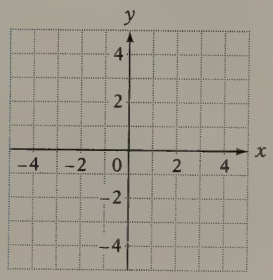
20. $x + 2 = 0$
$y - 1 = 0$



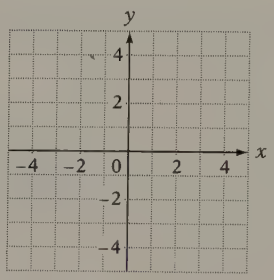
21. $2x + y = 3$
$x - 2 = 0$



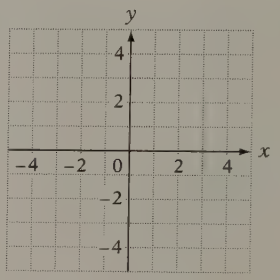
22. $x - 3y = 6$
$y + 3 = 0$



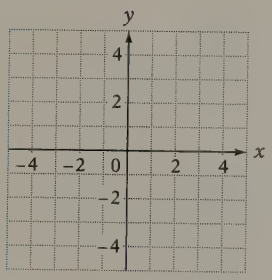
23. $x - y = 6$
$x + y = 2$



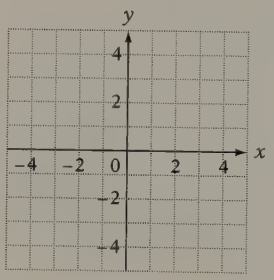
24. $2x + y = 2$
$-x + y = 5$



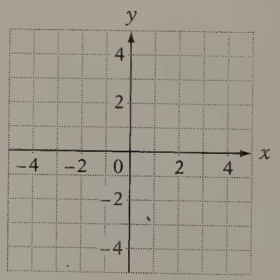
25. $y = x - 5$
$2x + y = 4$



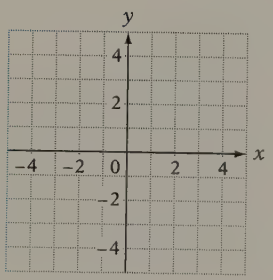
26. $2x - 5y = 4$
$y = x + 1$



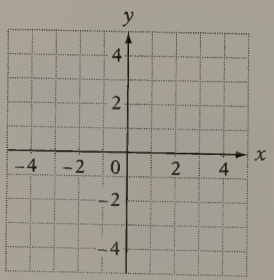
27. $y = \frac{1}{2}x - 2$
$x - 2y = 8$



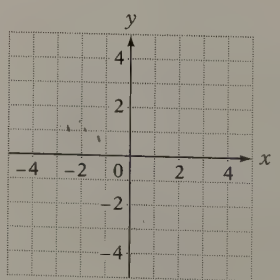
28. $2x + 3y = 6$
$y = -\frac{2}{3}x + 1$



29. $2x - 5y = 10$
$y = \frac{2}{5}x - 2$

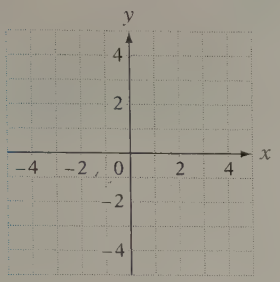


30. $3x - 2y = 6$
$y = \frac{3}{2}x - 3$

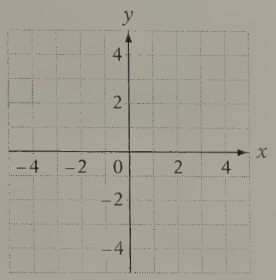


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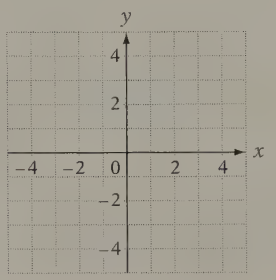
31. $3x - 4y = 12$
$5x + 4y = -12$



32. $2x - 3y = 6$
$2x - 5y = 10$



33. $2x - 3y = 2$
$5x + 4y = 5$



Objective B

Solve by substitution.

34. $2x + 3y = 7$
$x = 2$

35. $y = 3$
$3x - 2y = 6$

36. $y = x - 3$
$x + y = 5$

37. $y = x + 2$
$x + y = 6$

38. $x = y - 2$
$x + 3y = 2$

39. $x = y + 1$
$x + 2y = 7$

40. $y = 4 - 3x$
$3x + y = 5$

41. $y = 2 - 3x$
$6x + 2y = 7$

42. $x = 3y + 3$
$2x - 6y = 12$

43. $x = 2 - y$
$3x + 3y = 6$

44. $3x + 5y = -6$
$x = 5y + 3$

45. $y = 2x + 3$
$4x - 3y = 1$

46. $3x + y = 4$
$4x - 3y = 1$

47. $x - 4y = 9$
$2x - 3y = 11$

48. $3x - y = 6$
$x + 3y = 2$

49. $4x - y = -5$
$2x + 5y = 13$

50. $3x - y = 5$
$2x + 5y = -8$

51. $3x + 4y = 18$
$2x - y = 1$

52. $4x + 3y = 0$
$2x - y = 0$

53. $5x + 2y = 0$
$x - 3y = 0$

54. $2x - y = 2$
$6x - 3y = 6$

55. $3x + y = 4$
$9x + 3y = 12$

56. $x = 3y + 2$
$y = 2x + 6$

57. $x = 4 - 2y$
$y = 2x - 13$

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58. $y = 2x + 11$
$y = 5x - 19$

59. $y = 2x - 8$
$y = 3x - 13$

60. $y = -4x + 2$
$y = -3x - 1$

61. $x = 3y + 7$
$x = 2y - 1$

62. $x = 4y - 2$
$x = 6y + 8$

63. $x = 3 - 2y$
$x = 5y - 10$

APPLYING THE CONCEPTS

For what value of k does the system of equations have no solution?

64. $2x - 3y = 7$
$kx - 3y = 4$

65. $8x - 4y = 1$
$2x - ky = 3$

66. $x = 4y + 4$
$kx - 8y = 4$

67. The following was offered as a solution to the system of equations

(1) $y = \frac{1}{2}x + 2$

(2) $2x + 5y = 10$

$$2x + 5y = 10 \quad \bullet \text{ This is Equation (2).}$$
$$2x + 5\left(\frac{1}{2}x + 2\right) = 10 \quad \bullet \text{ Substitute } \frac{1}{2}x + 2 \text{ for } y.$$
$$2x + \frac{5}{2}x + 10 = 10 \quad \bullet \text{ Solve for } x.$$
$$\frac{9}{2}x = 0$$
$$x = 0$$

At this point the student stated that because $x = 0$, the system of equations has no solution. If this assertion is correct, is the system of equations independent, dependent, or inconsistent? If the assertion is not correct, what is the correct solution?

Use a graphing calculator to solve each of the following systems of equations. Round answers to the nearest hundredth. See the Projects and Group Activities at the end of this chapter for assistance.

68. $y = -\frac{1}{2}x + 2$
$y = 2x - 1$

69. $y = 1.2x + 2$
$y = -1.3x - 3$

70. $y = \sqrt{2}x - 1$
$y = -\sqrt{3}x + 1$

71. $y = \pi x - \frac{2}{3}$
$y = -x + \frac{\pi}{2}$

72. Write three different systems of equations: (a) one that has $(-3, 5)$ as its only solution, (b) one for which there is no solution, and (c) one that is a dependent system of equations.

73. When you solve a system of equations by the substitution method, how do you determine whether the system of equations is dependent? How do you determine whether the system of equations is inconsistent?

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5.2 Solving Systems of Linear Equations by the Addition Method

Objective A

To solve a system of two linear equations in two variables by the addition method



The **addition method** is an alternative method for solving a system of equations. This method is based on the Addition Property of Equations. Use the addition method when it is not convenient to solve one equation for one variable in terms of the other variable.

TAKE NOTE

Equation (1) states that $5x - 3y$ equals 14, and Equation (2) states that $2x + 3y$ equals -7. Thus adding Equations (1) and (2) is like adding

$$\begin{array}{r} 14 = 14 \\ -7 = -7 \\ \hline 7 = 7 \end{array}$$ A true equation

The addition method of solving a system of equations is based on adding the same number to each side of the equation.

Note, for the system of equations at the right, the effect of adding Equation (2) to Equation (1). Because $-3y$ and $3y$ are additive inverses, adding the equations results in an equation with only one variable.

$$\begin{array}{rl} (1) & 5x - 3y = 14 \\ (2) & 2x + 3y = -7 \\ & 7x + 0y = 7 \\ & 7x = 7 \end{array}$$

The solution of the resulting equation is the first component of the ordered-pair solution of the system.

$$\begin{aligned} 7x &= 7 \\ x &= 1 \end{aligned}$$

The second component is found by substituting the value of x into Equation (1) or (2) and then solving for y. Equation (1) is used here.

$$\begin{aligned} (1)\quad 5x - 3y &= 14 \\ 5(1) - 3y &= 14 \\ 5 - 3y &= 14 \\ -3y &= 9 \\ y &= -3 \end{aligned}$$

The solution is $(1, -3)$.

Sometimes each equation of the system of equations must be multiplied by a constant so that the coefficients of one of the variables are opposites.

➡ Solve by the addition method: (1) $3x + 4y = 2$
(2) $2x + 5y = -1$

To eliminate x, multiply Equation (1) by 2 and Equation (2) by -3. Note at the right how the constants are chosen.

$$\begin{aligned} 2(3x + 4y) &= 2 \cdot 2 \\ -3(2x + 5y) &= -3(-1) \end{aligned}$$

- The negative is used so that the coefficients will be opposites.

$$\begin{array}{rl} 6x + 8y = 4 & \bullet \text{ 2 times Equation (1)} \\ -6x - 15y = 3 & \bullet \text{ } -3 \text{ times Equation (2)} \\ \hline -7y = 7 & \bullet \text{ Add the equations.} \\ y = -1 & \bullet \text{ Solve for } y. \end{array}$$

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{array}{rl} (1)\quad 3x + 4y = 2 & \\ 3x + 4(-1) = 2 & \bullet \text{ Substitute } -1 \text{ for } y. \\ 3x - 4 = 2 & \bullet \text{ Solve for } x. \\ 3x = 6 & \\ x = 2 & \end{array}$$

The solution is $(2, -1)$.

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Point of Interest

There are records of Babylonian mathematicians solving systems of equations 3600 years ago. Here is a system of equations from that time (in our modern notation):

$$\frac{2}{3}x = \frac{1}{2}y - 500$$
$$x - y = 1800$$

We say *modern notation* for many reasons. Foremost is the fact that using variables did not become widespread until the 17th century. There are many other reasons, however. The equals sign had not been invented, 2 and 3 did not look like they do today, and zero had not even been considered as a possible number.

➡ Solve by the addition method: (1) $\frac{2}{3}x + \frac{1}{2}y = 4$

(2) $\frac{1}{4}x - \frac{3}{8}y = -\frac{3}{4}$

Clear fractions. Multiply each equation by the LCM of the denominators.

$$6\left(\frac{2}{3}x + \frac{1}{2}y\right) = 6(4)$$
$$8\left(\frac{1}{4}x - \frac{3}{8}y\right) = 8\left(-\frac{3}{4}\right)$$

$$4x + 3y = 24$$
$$2x - 3y = -6$$

$$6x = 18$$ • Eliminate y. Add the equations.

$$x = 3$$ • Solve for x.

Substitute the value of x into Equation (1) and solve for y.

$$\frac{2}{3}x + \frac{1}{2}y = 4$$ • This is Equation (1).

$$\frac{2}{3}(3) + \frac{1}{2}y = 4$$ • $x = 3$

$$2 + \frac{1}{2}y = 4$$

$$\frac{1}{2}y = 2$$

$$y = 4$$

The solution is (3, 4).

TAKE NOTE

The result of adding Equations (3) and (2) is $0 = 0$. It is not $x = 0$, and it is not $y = 0$. There is no variable in the equation $0 = 0$. This does not indicate that the solution is (0, 0). Rather, it indicates a dependent system of equations.

➡ Solve by the addition method: (1) $2x - y = 3$

(2) $4x - 2y = 6$

Eliminate y. Multiply Equation (1) by -2.

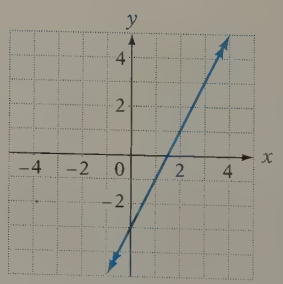
(1) $-2(2x - y) = -2(3)$ • Multiply both sides of Equation (1) by -2.

(3) $-4x + 2y = -6$ • This is Equation (3).

Add Equation (3) to Equation (2).

$$\begin{array}{rl} (2) & 4x - 2y = 6 \\ (3) & -4x + 2y = -6 \\ \hline & 0 = 0 \end{array}$$ • This is a true equation.

The equation $0 = 0$ indicates that the system of equations is dependent. This means that the graphs of the two lines are the same. Therefore, the solutions of the system of equations are the ordered-pair solutions of the equation of the line. Solve Equation (1) for y.



$$2x - y = 3$$
$$-y = -2x + 3$$
$$y = 2x - 3$$

The ordered-pair solutions are $(x, 2x - 3)$.

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Example 1 Solve by the addition method:
(1) $3x - 2y = 2x + 5$
(2) $2x + 3y = -4$

Solution

Write Equation (1) in the form $Ax + By = C$.

$$\begin{aligned} 3x - 2y &= 2x + 5 \\ x - 2y &= 5 \end{aligned}$$

Solve the system:

$$\begin{aligned} x - 2y &= 5 \\ 2x + 3y &= -4 \end{aligned}$$

Eliminate x.

$$\begin{aligned} -2(x - 2y) &= -2(5) \\ 2x + 3y &= -4 \end{aligned}$$

$$\begin{aligned} -2x + 4y &= -10 \\ 2x + 3y &= -4 \end{aligned}$$

Add the equations.

$$\begin{aligned} 7y &= -14 \\ y &= -2 \end{aligned}$$

Replace y in Equation (2).

$$\begin{aligned} 2x + 3y &= -4 \\ 2x + 3(-2) &= -4 \\ 2x - 6 &= -4 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

The solution is $(1, -2)$.

You Try It 1 Solve by the addition method:
$2x + 5y = 6$
$3x - 2y = 6x + 2$

Your solution

Example 2 Solve by the addition method:
(1) $4x - 8y = 36$
(2) $3x - 6y = 27$

Solution

Eliminate x.

$$\begin{aligned} 3(4x - 8y) &= 3(36) \\ -4(3x - 6y) &= -4(27) \end{aligned}$$

$$\begin{aligned} 12x - 24y &= 108 \\ -12x + 24y &= -108 \end{aligned}$$

Add the equations.

$$0 = 0$$

The system of equations is dependent. The solutions are the ordered pairs $\left(x, \frac{1}{2}x - \frac{9}{2}\right)$.

You Try It 2 Solve by the addition method:
$2x + y = 5$
$4x + 2y = 6$

Your solution

Solutions on p. S15

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Objective B

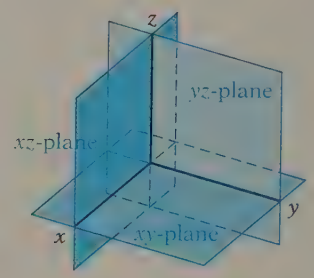
To solve a system of three linear equations in three variables by the addition method

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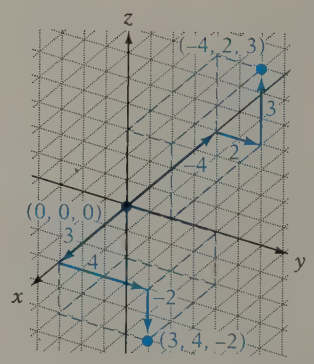
An equation of the form $Ax + By + Cz = D$, where A, B, and C are coefficients of the variables and D is a constant, is a **linear equation in three variables.** Examples of these equations are shown at the right.

$$2x + 4y - 3z = 7$$
$$x - 6y + z = -3$$

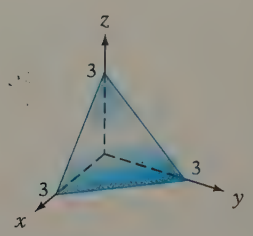
Graphing an equation in three variables requires a third coordinate axis perpendicular to the xy-plane. The third axis is commonly called the z-axis. The result is a three-dimensional coordinate system called the ***xyz*-coordinate system.** To help visualize a three-dimensional coordinate system, think of a corner of a room: the floor is the xy-plane, one wall is the yz-plane, and the other wall is the xz-plane. A three-dimensional coordinate system is shown at the right.



The graph of a point in an xyz-coordinate system is an **ordered triple** (x, y, z). Graphing an ordered triple requires three moves, the first along the x-axis, the second parallel to the y-axis, and the third parallel to the z-axis. The graphs of the points $(-4, 2, 3)$ and $(3, 4, -2)$ are shown at the right.

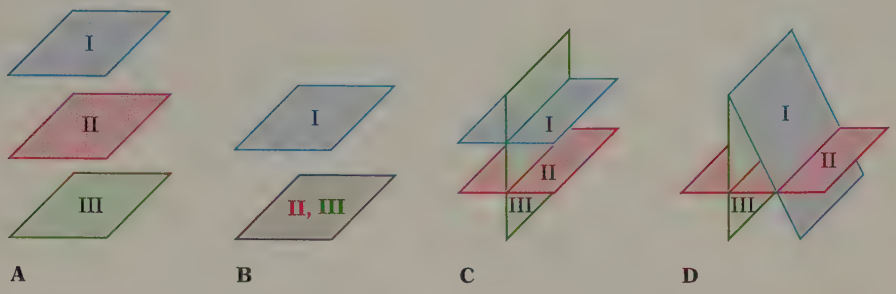


The graph of a linear equation in three variables is a plane. That is, if all the solutions of a linear equation in three variables were plotted in an xyz-coordinate system, the graph would look like a large piece of paper extending infinitely. The graph of $x + y + z = 3$ is shown at the right.



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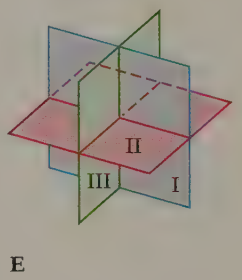
There are different ways in which three planes can be oriented in an xyz-coordinate system. The systems of equations represented by the planes below are inconsistent.



Graphs of Inconsistent Systems of Equations

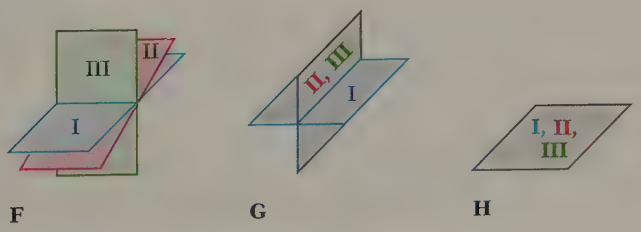
For a system of three equations in three variables to have a solution, the graphs of the planes must intersect at a single point, they must intersect along a common line, or all equations must have a graph that is the same plane. These situations are shown in the figures below.

The three planes shown in Figure E intersect at a point. A system of equations represented by planes that intersect at a point is independent.



An Independent System of Equations

The planes shown in Figures F and G intersect along a common line. The system of equations represented by the planes in Figure H has a graph that is the same plane. The systems of equations represented by these three graphs are dependent.



Dependent Systems of Equations

Just as a solution of an equation in two variables is an ordered pair (x, y), a **solution of an equation in three variables** is an ordered triple (x, y, z). For example, $(2, 1, -3)$ is a solution of the equation $2x - y - 2z = 9$. The ordered triple $(1, 3, 2)$ is not a solution.

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Point of Interest

In the early 1980s, Stephen Hoppe became interested in winning Monopoly strategies. Finding these strategies required solving a system that contained 123 equations in 123 variables!

A **system of linear equations in three variables** is shown at the right. A **solution of a system of equations in three variables** is an ordered triple that is a solution of each equation of the system.

$$\begin{aligned} x - 2y + z &= 6 \\ 3x + y - 2z &= 2 \\ 2x - 3y + 5z &= 1 \end{aligned}$$

A system of linear equations in three variables can be solved by using the addition method. First, eliminate one variable from any two of the given equations. Then eliminate the same variable from any other two equations. The result will be a system of two equations in two variables. Solve this system by the addition method.

Solve: (1) $x + 4y - z = 10$
(2) $3x + 2y + z = 4$
(3) $2x - 3y + 2z = -7$

Eliminate z from Equations (1) and (2) by adding the two equations.

$$\begin{aligned} x + 4y - z &= 10 \\ 3x + 2y + z &= 4 \\ (4) \qquad 4x + 6y &= 14 \end{aligned}$$

- Add the equations. This is Equation (4).

Eliminate z from Equations (1) and (3). Multiply Equation (1) by 2 and add to Equation (3).

$$\begin{aligned} 2x + 8y - 2z &= 20 \\ 2x - 3y + 2z &= -7 \\ (5) \qquad 4x + 5y &= 13 \end{aligned}$$

- 2 times Equation (1)
- This is Equation (3).
- Add the equations. This is Equation (5).

Using Equations (4) and (5), solve the system of two equations in two variables.

$$\begin{aligned} (4) \quad 4x + 6y &= 14 \\ (5) \quad 4x + 5y &= 13 \end{aligned}$$

Eliminate x. Multiply Equation (5) by -1 and add to Equation (4).

$$\begin{aligned} 4x + 6y &= 14 \\ -4x - 5y &= -13 \\ y &= 1 \end{aligned}$$

- This is Equation (4).
- -1 times Equation (5)
- Add the equations.

Substitute the value of y into Equation (4) or Equation (5) and solve for x. Equation (4) is used here.

$$\begin{aligned} 4x + 6y &= 14 \\ 4x + 6(1) &= 14 \\ 4x + 6 &= 14 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

- This is Equation (4).
- $y = 1$
- Solve for x.

Substitute the value of y and the value of x into one of the equations in the original system. Equation (2) is used here.

$$\begin{aligned} 3x + 2y + z &= 4 \\ 3(2) + 2(1) + z &= 4 \\ 6 + 2 + z &= 4 \\ 8 + z &= 4 \\ z &= -4 \end{aligned}$$

- $x = 2, y = 1$

The solution is $(2, 1, -4)$.

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Solve: (1) $2x - 3y - z = 1$
(2) $x + 4y + 3z = 2$
(3) $4x - 6y - 2z = 5$

Eliminate x from Equations (1) and (2).

$$\begin{array}{r} 2x - 3y - z = 1 \\ -2x - 8y - 6z = -4 \\ \hline -11y - 7z = -3 \end{array}$$

- This is Equation (1).
- -2 times Equation (2)
- Add the equations.

Eliminate x from Equations (1) and (3).

$$\begin{array}{r} -4x + 6y + 2z = -2 \\ 4x - 6y - 2z = 5 \\ \hline 0 = 3 \end{array}$$

- -2 times Equation (1)
- This is Equation (3).
- Add the equations.

The equation $0 = 3$ is not a true equation. The system of equations is inconsistent and therefore has no solution.

Solve: (1) $3x - z = -1$
(2) $2y - 3z = 10$
(3) $x + 3y - z = 7$

Eliminate x from Equations (1) and (3). Multiply Equation (3) by -3 and add to Equation (1).

$$\begin{array}{rr} & 3x - z = -1 \\ & -3x - 9y + 3z = -21 \\ \hline (4) & -9y + 2z = -22 \end{array}$$

- This is Equation (1).
- -3 times Equation (3)
- Add the equations.

Use Equations (2) and (4) to form a system of equations in two variables.

$$\begin{array}{rr} (2) & 2y - 3z = 10 \\ (4) & -9y + 2z = -22 \end{array}$$

Eliminate z. Multiply Equation (2) by 2 and Equation (4) by 3.

$$\begin{array}{r} 4y - 6z = 20 \\ -27y + 6z = -66 \\ \hline -23y = -46 \\ y = 2 \end{array}$$

- 2 times Equation (2)
- 3 times Equation (4)
- Add the equations.
- Solve for y.

Substitute the value of y into Equation (2) or Equation (4) and solve for z. Equation (2) is used here.

$$\begin{array}{rr} (2) & 2y - 3z = 10 \\ & 2(2) - 3z = 10 \\ & 4 - 3z = 10 \\ & -3z = 6 \\ & z = -2 \end{array}$$

- This is Equation (2).
- $y = 2$
- Solve for z.

Substitute the value of z into Equation (1) and solve for x.

$$\begin{array}{rr} (1) & 3x - z = -1 \\ & 3x - (-2) = -1 \\ & 3x + 2 = -1 \\ & 3x = -3 \\ & x = -1 \end{array}$$

- This is Equation (1).
- $z = -2$
- Solve for x.

The solution is $(-1, 2, -2)$.

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Example 3 Solve: (1) $3x - y + 2z = 1$
(2) $2x + 3y + 3z = 4$
(3) $x + y - 4z = -9$

Solution

Eliminate y. Add Equations (1) and (3).

$$\begin{aligned} 3x - y + 2z &= 1 \\ x + y - 4z &= -9 \\ 4x - 2z &= -8 \end{aligned}$$

Multiply each side of the equation by $\frac{1}{2}$.

(4) $2x - z = -4$

Multiply Equation (1) by 3 and add to Equation (2).

$$\begin{aligned} 9x - 3y + 6z &= 3 \\ 2x + 3y + 3z &= 4 \\ (5) \qquad 11x + 9z &= 7 \end{aligned}$$

Solve the system of two equations.

$$\begin{aligned} (4) \qquad 2x - z &= -4 \\ (5) \qquad 11x + 9z &= 7 \end{aligned}$$

Multiply Equation (4) by 9 and add to Equation (5).

$$\begin{aligned} 18x - 9z &= -36 \\ 11x + 9z &= 7 \\ 29x &= -29 \\ x &= -1 \end{aligned}$$

Replace x by -1 in Equation (4).

$$\begin{aligned} 2x - z &= -4 \\ 2(-1) - z &= -4 \\ -2 - z &= -4 \\ -z &= -2 \\ z &= 2 \end{aligned}$$

Replace x by -1 and z by 2 in Equation (3).

$$\begin{aligned} x + y - 4z &= -9 \\ -1 + y - 4(2) &= -9 \\ -9 + y &= -9 \\ y &= 0 \end{aligned}$$

The solution is $(-1, 0, 2)$.

You Try It 3 Solve: $x - y + z = 6$
$2x + 3y - z = 1$
$x + 2y + 2z = 5$

Your solution

Solution on p. S15

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5.2 Exercises

Objective A

Solve by the addition method.

1. $x - y = 5$
$x + y = 7$

2. $x + y = 1$
$2x - y = 5$

3. $3x + y = 4$
$x + y = 2$

4. $x - 3y = 4$
$x + 5y = -4$

5. $3x + y = 7$
$x + 2y = 4$

6. $x - 2y = 7$
$3x - 2y = 9$

7. $2x + 3y = -1$
$x + 5y = 3$

8. $x + 5y = 7$
$2x + 7y = 8$

9. $3x - y = 4$
$6x - 2y = 8$

10. $x - 2y = -3$
$-2x + 4y = 6$

11. $2x + 5y = 9$
$4x - 7y = -16$

12. $8x - 3y = 21$
$4x + 5y = -9$

13. $4x - 6y = 5$
$2x - 3y = 7$

14. $3x + 6y = 7$
$2x + 4y = 5$

15. $3x - 5y = 7$
$x - 2y = 3$

16. $3x + 4y = 25$
$2x + y = 10$

17. $x + 3y = 7$
$-2x + 3y = 22$

18. $2x - 3y = 14$
$5x - 6y = 32$

19. $3x + 2y = 16$
$2x - 3y = -11$

20. $2x - 5y = 13$
$5x + 3y = 17$

21. $4x + 4y = 5$
$2x - 8y = -5$

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22. $3x + 7y = 16$
$4x - 3y = 9$

23. $5x + 4y = 0$
$3x + 7y = 0$

24. $3x - 4y = 0$
$4x - 7y = 0$

25. $5x + 2y = 1$
$2x + 3y = 7$

26. $3x + 5y = 16$
$5x - 7y = -4$

27. $3x - 6y = 6$
$9x - 3y = 8$

28. $\frac{2}{3}x - \frac{1}{2}y = 3$
$\frac{1}{3}x - \frac{1}{4}y = \frac{3}{2}$

29. $\frac{3}{4}x + \frac{1}{3}y = -\frac{1}{2}$
$\frac{1}{2}x - \frac{5}{6}y = -\frac{7}{2}$

30. $\frac{2}{5}x - \frac{1}{3}y = 1$
$\frac{3}{5}x + \frac{2}{3}y = 5$

31. $\frac{5x}{6} + \frac{y}{3} = \frac{4}{3}$
$\frac{2x}{3} - \frac{y}{2} = \frac{11}{6}$

32. $\frac{3x}{4} + \frac{2y}{5} = -\frac{3}{20}$
$\frac{3x}{2} - \frac{y}{4} = \frac{3}{4}$

33. $\frac{2x}{5} - \frac{y}{2} = \frac{13}{2}$
$\frac{3x}{4} - \frac{y}{5} = \frac{17}{2}$

34. $\frac{x}{2} + \frac{y}{3} = \frac{5}{12}$
$\frac{x}{2} - \frac{y}{3} = \frac{1}{12}$

35. $\frac{3x}{2} - \frac{y}{4} = -\frac{11}{12}$
$\frac{x}{3} - y = -\frac{5}{6}$

36. $\frac{3x}{4} - \frac{2y}{3} = 0$
$\frac{5x}{4} - \frac{y}{3} = \frac{7}{12}$

37. $4x - 5y = 3y + 4$
$2x + 3y = 2x + 1$

38. $5x - 2y = 8x - 1$
$2x + 7y = 4y + 9$

39. $2x + 5y = 5x + 1$
$3x - 2y = 3y + 3$

40. $4x - 8y = 5$
$8x + 2y = 1$

41. $5x + 2y = 2x + 1$
$2x - 3y = 3x + 2$

42. $3x + 3y = y + 1$
$x + 3y = 9 - x$

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Objective B

Solve by the addition method.

43. $x + 2y - z = 1$
$2x - y + z = 6$
$x + 3y - z = 2$

44. $x + 3y + z = 6$
$3x + y - z = -2$
$2x + 2y - z = 1$

45. $2x - y + 2z = 7$
$x + y + z = 2$
$3x - y + z = 6$

46. $x - 2y + z = 6$
$x + 3y + z = 16$
$3x - y - z = 12$

47. $3x + y = 5$
$3y - z = 2$
$x + z = 5$

48. $2y + z = 7$
$2x - z = 3$
$x - y = 3$

49. $x - y + z = 1$
$2x + 3y - z = 3$
$-x + 2y - 4z = 4$

50. $2x + y - 3z = 7$
$x - 2y + 3z = 1$
$3x + 4y - 3z = 13$

51. $2x + 3z = 5$
$3y + 2z = 3$
$3x + 4y = -10$

52. $3x + 4z = 5$
$2y + 3z = 2$
$2x - 5y = 8$

53. $2x + 4y - 2z = 3$
$x + 3y + 4z = 1$
$x + 2y - z = 4$

54. $x - 3y + 2z = 1$
$x - 2y + 3z = 5$
$2x - 6y + 4z = 3$

55. $2x + y - z = 5$
$x + 3y + z = 14$
$3x - y + 2z = 1$

56. $3x - y - 2z = 11$
$2x + y - 2z = 11$
$x + 3y - z = 8$

57. $3x + y - 2z = 2$
$x + 2y + 3z = 13$
$2x - 2y + 5z = 6$

58. $4x + 5y + z = 6$
$2x - y + 2z = 11$
$x + 2y + 2z = 6$

59. $2x - y + z = 6$
$3x + 2y + z = 4$
$x - 2y + 3z = 12$

60. $3x + 2y - 3z = 8$
$2x + 3y + 2z = 10$
$x + y - z = 2$

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61. $3x - 2y + 3z = -4$
$2x + y - 3z = 2$
$3x + 4y + 5z = 8$

62. $3x - 3y + 4z = 6$
$4x - 5y + 2z = 10$
$x - 2y + 3z = 4$

63. $3x - y + 2z = 2$
$4x + 2y - 7z = 0$
$2x + 3y - 5z = 7$

64. $2x + 2y + 3z = 13$
$-3x + 4y - z = 5$
$5x - 3y + z = 2$

65. $2x - 3y + 7z = 0$
$x + 4y - 4z = -2$
$3x + 2y + 5z = 1$

66. $5x + 3y - z = 5$
$3x - 2y + 4z = 13$
$4x + 3y + 5z = 22$

APPLYING THE CONCEPTS

67. The point of intersection of the graphs of the equations $Ax + 2y = 2$ and $2x + By = 10$ is $(2, -2)$. Find A and B.

68. The point of intersection of the graphs of the equations $Ax - 4y = 9$ and $4x + By = -1$ is $(-1, -3)$. Find A and B.

69. For what value of k is the system of equations dependent?

a. $2x + 3y = 7$
$4x + 6y = k$

b. $y = \frac{2}{3}x - 3$
$y = kx - 3$

c. $x = ky - 1$
$y = 2x + 2$

70. For what values of k is the system of equations independent?

a. $x + y = 7$
$kx + y = 3$

b. $x + 2y = 4$
$kx + 3y = 2$

c. $2x + ky = 1$
$x + 2y = 2$

71. Given that the graphs of the equations $2x - y = 6$, $3x - 4y = 4$, and $Ax - 2y = 0$ all intersect at the same point, find A.

72. Given that the graphs of the equations $3x - 2y = -2$, $2x - y = 0$, and $Ax + y = 8$ all intersect at the same point, find A.

73. Describe in your own words the process of solving a system of two linear equations in two variables by the addition method.

74. Explain, graphically, the following situations when they are related to a system of three linear equations in three variables.

a. The system of equations has no solution.

b. The system of equations has exactly one solution.

c. The system of equations has infinitely many solutions.

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5.3 Solving Systems of Equations by Using Determinants

Objective A To evaluate a determinant

VIDEO & DVD CD TUTOR WEB SSM

Point of Interest

The word *matrix* was first used in a mathematical context in 1850. The root of this word is the Latin *mater*, which means "mother." A matrix was thought of as an object from which something else originates. The idea was that a determinant, discussed below, originated (was born) from a matrix. Today, matrices are one of the most widely used tools of applied mathematics.

A **matrix** is a rectangular array of numbers. Each number in the matrix is called an **element** of the matrix. The matrix at the right, with three rows and four columns, is called a 3×4 (read "3 by 4") matrix.

$$A = \begin{pmatrix} 1 & -3 & 2 & 4 \\ 0 & 4 & -3 & 2 \\ 6 & -5 & 4 & -1 \end{pmatrix}$$

A matrix of m rows and n columns is said to be of **order $m \times n$.** The matrix above has order 3×4. The notation a_{ij} refers to the element of a matrix in the ith row and the jth column. For matrix A, $a_{23} = -3$, $a_{31} = 6$, and $a_{13} = 2$.

A **square matrix** is one that has the same number of rows as columns. A 2×2 matrix and a 3×3 matrix are shown at the right.

$$\begin{pmatrix} -1 & 3 \\ 5 & 2 \end{pmatrix} \qquad \begin{pmatrix} 4 & 0 & 1 \\ 5 & -3 & 7 \\ 2 & 1 & 4 \end{pmatrix}$$

Associated with every square matrix is a number called its **determinant**.

TAKE NOTE

Note that vertical bars are used to represent the determinant and that parentheses are used to represent the matrix.

Determinant of a 2×2 Matrix

The **determinant** of a 2×2 matrix $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is written $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$. The value of this determinant is given by the formula

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

➡ Find the value of the determinant $\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$.

$$\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix} = 3 \cdot 2 - 4(-1) = 6 - (-4) = 10$$

The value of the determinant is 10.

For a square matrix whose order is 3×3 or greater, the value of the determinant of that matrix is found by using 2×2 determinants.

The **minor of an element** in a 3×3 determinant is the 2×2 determinant that is obtained by eliminating the row and column that contain that element.

➡ Find the minor of -3 for the determinant $\begin{vmatrix} 2 & -3 & 4 \\ 0 & 4 & 8 \\ -1 & 3 & 6 \end{vmatrix}$.

The minor of -3 is the 2×2 determinant created by eliminating the row and column that contain -3.

Eliminate the row and column as shown: $\begin{vmatrix} 2 & -3 & 4 \\ 0 & 4 & 8 \\ -1 & 3 & 6 \end{vmatrix}$

The minor of -3 is $\begin{vmatrix} 0 & 8 \\ -1 & 6 \end{vmatrix}$.

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TAKE NOTE

The only difference between the cofactor and the minor is one of sign. The definition at the right can be stated symbolically as follows: If C_{ij} is the cofactor and M_{ij} is the minor of the matrix element a_{ij}, then $C_{ij} = (-1)^{i+j}M_{ij}$. If $i + j$ is an even number, then $(-1)^{i+j} = 1$ and $C_{ij} = M_{ij}$. If $i + j$ is an odd number, then $(-1)^{i+j} = -1$ and $C_{ij} = -M_{ij}$.

Cofactor of an Element of a Matrix

The **cofactor** of an element of a matrix is $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is the column number of the element.

➡ For the determinant $\begin{vmatrix} 3 & -2 & 1 \\ 2 & -5 & -4 \\ 0 & 3 & 1 \end{vmatrix}$, find the cofactor of -2 and of -5.

Because -2 is in the first row and the second column, $i = 1$ and $j = 2$. Thus $(-1)^{i+j} = (-1)^{1+2} = (-1)^3 = -1$. The cofactor of -2 is $(-1)\begin{vmatrix} 2 & -4 \\ 0 & 1 \end{vmatrix}$.

Because -5 is in the second row and the second column, $i = 2$ and $j = 2$. Thus $(-1)^{i+j} = (-1)^{2+2} = (-1)^4 = 1$. The cofactor of -5 is $1 \cdot \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix}$.

Note from this example that the cofactor of an element is -1 times the minor of that element or 1 times the minor of that element, depending on whether the sum $i + j$ is an odd or even integer.

The value of a 3×3 or larger determinant can be found by **expanding by cofactors** of *any* row or *any* column. The result of expanding by cofactors using the first row of a 3×3 matrix is shown below.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

➡ Find the value of the determinant $\begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix}$.

Expand by cofactors of the first row.

$$\begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix} = 2\begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix}$$

$$= 2(6 - 2) - (-3)(2 - 0) + 2(-2 - 0)$$

$$= 2(4) - (-3)(2) + 2(-2) = 8 - (-6) + (-4) = 10$$

To illustrate a statement made earlier, the value of this determinant will now be found by expanding by cofactors using the second column.

$$\begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix} = -3(-1)^{1+2}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 3(-1)^{2+2}\begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + (-2)(-1)^{3+2}\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= -3(-1)\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 3(1)\begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + (-2)(-1)\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 3(2 - 0) + 3(4 - 0) + 2(-2 - 2)$$

$$= 3(2) + 3(4) + 2(-4) = 6 + 12 + (-8) = 10$$

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Note that the value of the determinant is the same whether the first row or the second column is used to expand by cofactors. *Any row or column* can be used to evaluate a determinant by expanding by cofactors.

Example 1

Find the value of $\begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix}$.

Solution

$$\begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix} = 3(-4) - (-2)(6) = -12 + 12 = 0$$

The value of the determinant is 0.

You Try It 1

Find the value of $\begin{vmatrix} -1 & -4 \\ 3 & -5 \end{vmatrix}$.

Your solution

Example 2

Find the value of $\begin{vmatrix} -2 & 3 & 1 \\ 4 & -2 & 0 \\ 1 & -2 & 3 \end{vmatrix}$.

Solution

Expand by cofactors of the first row.

$$\begin{vmatrix} -2 & 3 & 1 \\ 4 & -2 & 0 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= -2\begin{vmatrix} -2 & 0 \\ -2 & 3 \end{vmatrix} - 3\begin{vmatrix} 4 & 0 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 4 & -2 \\ 1 & -2 \end{vmatrix}$$

$$= -2(-6 - 0) - 3(12 - 0) + 1(-8 + 2)$$
$$= -2(-6) - 3(12) + 1(-6)$$
$$= 12 - 36 - 6$$
$$= -30$$

The value of the determinant is -30.

You Try It 2

Find the value of $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$.

Your solution

Example 3

Find the value of $\begin{vmatrix} 0 & -2 & 1 \\ 1 & 4 & 1 \\ 2 & -3 & 4 \end{vmatrix}$.

Solution

$$\begin{vmatrix} 0 & -2 & 1 \\ 1 & 4 & 1 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= 0\begin{vmatrix} 4 & 1 \\ -3 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix}$$

$$= 0 - (-2)(4 - 2) + 1(-3 - 8)$$
$$= 2(2) + 1(-11)$$
$$= 4 - 11$$
$$= -7$$

The value of the determinant is -7.

You Try It 3

Find the value of $\begin{vmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \\ -2 & 1 & 3 \end{vmatrix}$.

Your solution

Solutions on pp. S15–S16

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Objective B To solve a system of equations by using Cramer's Rule

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The connection between determinants and systems of equations can be understood by solving a general system of linear equations.

Solve: (1) $a_1x + b_1y = c_1$
(2) $a_2x + b_2y = c_2$

Eliminate y. Multiply Equation (1) by b_2 and Equation (2) by $-b_1$.

$a_1b_2x + b_1b_2y = c_1b_2$ • b_2 times Equation (1).
$-a_2b_1x - b_1b_2y = -c_2b_1$ • $-b_1$ times Equation (2).

Add the equations.

$a_1b_2x - a_2b_1x = c_1b_2 - c_2b_1$
$(a_1b_2 - a_2b_1)x = c_1b_2 - c_2b_1$ • Solve for x, assuming $a_1b_2 - a_2b_1 \neq 0$.

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

The denominator $a_1b_2 - a_2b_1$ is the determinant of the coefficients of x and y. This is called the **coefficient determinant**.

$$a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

coefficients of x
coefficients of y

The numerator $c_1b_2 - c_2b_1$ is the determinant obtained by replacing the first column in the coefficient determinant by the constants c_1 and c_2. This is called a **numerator determinant**.

$$c_1b_2 - c_2b_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

constants of the equations

Following a similar procedure and eliminating x, it is also possible to express the y-component of the solution in determinant form. These results are summarized in Cramer's Rule.

Point of Interest

Cramer's Rule is named after Gabriel Cramer, who used it in a book he published in 1750. However, this rule was published in 1683 by the Japanese mathematician Seki Kown. That publication occurred seven years before Cramer's birth.

Cramer's Rule

The solution of the system of equations $\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}$ is given by $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$,

where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$, and $D \neq 0$.

➡ Solve by using Cramer's Rule: $\begin{aligned} 3x - 2y &= 1 \\ 2x + 5y &= 3 \end{aligned}$

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix} = 19$$ • Find the value of the coefficient determinant.

$$D_x = \begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix} = 11, \quad D_y = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$ • Find the value of each of the numerator determinants.

$$x = \frac{D_x}{D} = \frac{11}{19}, \quad y = \frac{D_y}{D} = \frac{7}{19}$$ • Use Cramer's Rule to write the solution.

The solution is $\left(\frac{11}{19}, \frac{7}{19}\right)$.

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A procedure similar to that followed for a system of two equations in two variables can be used to extend Cramer's Rule to a system of three equations in three variables.

Cramer's Rule for a System of Three Equations in Three Variables

The solution of the system of equations
$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$
is given by

$$x = \frac{D_x}{D},\ y = \frac{D_y}{D},\ \text{and}\ z = \frac{D_z}{D},\ \text{where}\ D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\ D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},\ \text{and}\ D \neq 0.$$

➡ Solve by using Cramer's Rule:
$$\begin{aligned} 2x - y + z &= 1 \\ x + 3y - 2z &= -2 \\ 3x + y + 3z &= 4 \end{aligned}$$

Find the value of the coefficient determinant.

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$
$$= 2(11) + 1(9) + 1(-8)$$
$$= 23$$

Find the value of each of the numerator determinants.

$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 4 & 1 & 3 \end{vmatrix} = 1\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ 4 & 1 \end{vmatrix}$$
$$= 1(11) + 1(2) + 1(-14)$$
$$= -1$$

$$D_y = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -2 \\ 3 & 4 & 3 \end{vmatrix} = 2\begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$$
$$= 2(2) - 1(9) + 1(10)$$
$$= 5$$

$$D_z = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & 4 \end{vmatrix} = 2\begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$
$$= 2(14) + 1(10) + 1(-8)$$
$$= 30$$

Use Cramer's Rule to write the solution.

$$x = \frac{D_x}{D} = \frac{-1}{23}, \qquad y = \frac{D_y}{D} = \frac{5}{23}, \qquad z = \frac{D_z}{D} = \frac{30}{23}$$

The solution is $\left(-\frac{1}{23}, \frac{5}{23}, \frac{30}{23}\right)$.

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Example 4

Solve by using Cramer's Rule:

$6x - 9y = 5$

$4x - 6y = 4$

Solution

$$D = \begin{vmatrix} 6 & -9 \\ 4 & -6 \end{vmatrix} = -36 + 36 = 0$$

Because $D = 0$, $\frac{D_x}{D}$ is undefined.

Therefore, the system is dependent or inconsistent.

It is not possible to solve this system by using Cramer's Rule.

You Try It 4

Solve by using Cramer's Rule:

$3x - y = 4$

$6x - 2y = 5$

Your solution

Example 5

Solve by using Cramer's Rule:

$3x - y + z = 5$

$x + 2y - 2z = -3$

$2x + 3y + z = 4$

Solution

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 28$$

$$D_x = \begin{vmatrix} 5 & -1 & 1 \\ -3 & 2 & -2 \\ 4 & 3 & 1 \end{vmatrix} = 28$$

$$D_y = \begin{vmatrix} 3 & 5 & 1 \\ 1 & -3 & -2 \\ 2 & 4 & 1 \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} 3 & -1 & 5 \\ 1 & 2 & -3 \\ 2 & 3 & 4 \end{vmatrix} = 56$$

$$x = \frac{D_x}{D} = \frac{28}{28} = 1, \quad y = \frac{D_y}{D} = \frac{0}{28} = 0,$$

$$z = \frac{D_z}{D} = \frac{56}{28} = 2$$

The solution is (1, 0, 2).

You Try It 5

Solve by using Cramer's Rule:

$2x - y + z = -1$

$3x + 2y - z = 3$

$x + 3y + z = -2$

Your solution

Solutions on p. S16

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5.3 Exercises

Objective A

1. How do you find the value of the determinant associated with a 2×2 matrix?

2. What is the cofactor of a given element in a matrix?

Evaluate the determinant.

3. $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$

4. $\begin{vmatrix} 5 & 1 \\ -1 & 2 \end{vmatrix}$

5. $\begin{vmatrix} 6 & -2 \\ -3 & 4 \end{vmatrix}$

6. $\begin{vmatrix} -3 & 5 \\ 1 & 7 \end{vmatrix}$

7. $\begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

8. $\begin{vmatrix} 5 & -10 \\ 1 & -2 \end{vmatrix}$

9. $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 1 & 0 & 4 \end{vmatrix}$

10. $\begin{vmatrix} 4 & 1 & 3 \\ 2 & -2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

11. $\begin{vmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 3 & 2 & -2 \end{vmatrix}$

12. $\begin{vmatrix} 4 & 5 & -2 \\ 3 & -1 & 5 \\ 2 & 1 & 4 \end{vmatrix}$

13. $\begin{vmatrix} 4 & 2 & 6 \\ -2 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

14. $\begin{vmatrix} 3 & 6 & -3 \\ 4 & -1 & 6 \\ -1 & -2 & 3 \end{vmatrix}$

Objective B

Solve by using Cramer's Rule.

15. $2x - 5y = 26$
$5x + 3y = 3$

16. $3x + 7y = 15$
$2x + 5y = 11$

17. $x - 4y = 8$
$3x + 7y = 5$

18. $5x + 2y = -5$
$3x + 4y = 11$

19. $2x + 3y = 4$
$6x - 12y = -5$

20. $5x + 4y = 3$
$15x - 8y = -21$

21. $2x + 5y = 6$
$6x - 2y = 1$

22. $7x + 3y = 4$
$5x - 4y = 9$

23. $-2x + 3y = 7$
$4x - 6y = 9$

24. $9x + 6y = 7$
$3x + 2y = 4$

25. $2x - 5y = -2$
$3x - 7y = -3$

26. $8x + 7y = -3$
$2x + 2y = 5$

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27. $2x - y + 3z = 9$
$x + 4y + 4z = 5$
$3x + 2y + 2z = 5$

28. $3x - 2y + z = 2$
$2x + 3y + 2z = -6$
$3x - y + z = 0$

29. $3x - y + z = 11$
$x + 4y - 2z = -12$
$2x + 2y - z = -3$

30. $x + 2y + 3z = 8$
$2x - 3y + z = 5$
$3x - 4y + 2z = 9$

31. $4x - 2y + 6z = 1$
$3x + 4y + 2z = 1$
$2x - y + 3z = 2$

32. $x - 3y + 2z = 1$
$2x + y - 2z = 3$
$3x - 9y + 6z = -3$

33. $5x - 4y + 2z = 4$
$3x - 5y + 3z = -4$
$3x + y - 5z = 12$

34. $2x + 4y + z = 7$
$x + 3y - z = 1$
$3x + 2y - 2z = 5$

35. $3x - 2y + 2z = 5$
$6x + 3y - 4z = -1$
$3x - y + 2z = 4$

APPLYING THE CONCEPTS

36. Determine whether the following statements are always true, sometimes true, or never true.

a. The determinant of a matrix is a positive number.

b. A determinant can be evaluated by expanding about any row or column of the matrix.

c. Cramer's Rule can be used to solve a system of linear equations in three variables.

37. Surveyors use a formula to find the area of a plot of land. The **surveyor's area formula** states that if the vertices (x_1, y_1), (x_2, y_2), . . . , (x_n, y_n) of a simple polygon are listed counterclockwise around the perimeter, then the area of the polygon is

$$A = \frac{1}{2}\left\{\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} + \begin{bmatrix} x_2 & x_3 \\ y_2 & y_3 \end{bmatrix} + \begin{bmatrix} x_3 & x_4 \\ y_3 & y_4 \end{bmatrix} + \ldots + \begin{bmatrix} x_n & x_1 \\ y_n & y_1 \end{bmatrix}\right\}$$

Use the surveyor's area formula to find the area of the polygon with vertices (9, −3), (26, 6), (18, 21), (16, 10), and (1, 11). Measurements are given in feet.

Complete.

38. **a.** If all the elements in one row or one column of a 2 × 2 matrix are zeros, the value of the determinant of the matrix is _____.

b. If all the elements in one row or one column of a 3 × 3 matrix are zeros, the value of the determinant of the matrix is _____.

39. **a.** The value of the determinant $\begin{vmatrix} x & x & a \\ y & y & b \\ z & z & c \end{vmatrix}$ is _____.

b. If two columns of a 3 × 3 matrix contain identical elements, the value of the determinant is _____.

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5.4 Application Problems in Two Variables

Objective A

To solve rate-of-wind or rate-of-current problems

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Motion problems that involve an object moving with or against a wind or current normally require two variables to solve.

A motorboat traveling with the current can go 24 mi in 2 h. Against the current, it takes 3 h to go the same distance. Find the rate of the motorboat in calm water and the rate of the current.

Strategy for Solving Rate-of-Wind or Rate-of-Current Problems

1. Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $rt = d$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

TAKE NOTE

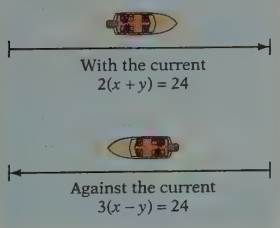
The boat travels faster when it travels with the current than when it travels against the current. Note that the boat's rate is *increased* by the rate of the current when it is traveling with the current, and the boat's rate is *decreased* by the rate of the current when it is traveling against the current.

Rate of the boat in calm water: x
Rate of the current: y

	Rate	·	*Time*	=	*Distance*
With the current	$x + y$	·	2	=	$2(x + y)$
Against the current	$x - y$	·	3	=	$3(x - y)$

2. Determine how the expressions for distance are related.

The distance traveled with the current is 24 mi: $2(x + y) = 24$
The distance traveled against the current is 24 mi: $3(x - y) = 24$



$$\begin{aligned} 2(x + y) &= 24 \\ 3(x - y) &= 24 \end{aligned}$$

• Solve this system of equations.

$$\begin{aligned} x + y &= 12 \\ x - y &= 8 \end{aligned}$$

• Divide each side of the first equation by 2.
• Divide each side of the second equation by 3.

$$\begin{aligned} 2x &= 20 \\ x &= 10 \end{aligned}$$

• Add the equations.
• Solve the equation for x.

$$\begin{aligned} x + y &= 12 \\ 10 + y &= 12 \\ y &= 2 \end{aligned}$$

• Replace x by 10 in the equation $x + y = 12$ and solve for y.

The rate of the boat in calm water is 10 mph.
The rate of the current is 2 mph.

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Example 1

Flying with the wind, a plane flew 1000 mi in 5 h. Flying against the wind, the plane could fly only 500 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

Strategy

- Rate of the plane in calm air: p
  Rate of the wind: w

	Rate	*Time*	*Distance*
With wind	$p + w$	5	$5(p + w)$
Against wind	$p - w$	5	$5(p - w)$

- The distance traveled with the wind is 1000 mi.
  The distance traveled against the wind is 500 mi.

Solution

$5(p + w) = 1000$
$5(p - w) = 500$

$p + w = 200$
$p - w = 100$ • Divide each side of each equation by 5.

$2p = 300$
$p = 150$

$p + w = 200$
$150 + w = 200$
$w = 50$

The rate of the plane in calm air is 150 mph. The rate of the wind is 50 mph.

You Try It 1

A rowing team rowing with the current traveled 18 mi in 2 h. Against the current, the team rowed 10 mi in 2 h. Find the rate of the rowing team in calm water and the rate of the current.

Your strategy

Your solution

Solution on p. S16

Objective 1

To solve application problems using two variables

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The application problems in this section are varieties of problems solved earlier in the text. Each of the strategies for the problems in this section will result in a system of equations.

- A store owner purchased twenty 60-watt light bulbs and 30 fluorescent bulbs for a total cost of $80. A second purchase, at the same prices, included thirty 60-watt light bulbs and 10 fluorescent bulbs for a total cost of $50. Find the cost of a 60-watt bulb and that of a fluorescent bulb.

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Point of Interest

The Babylonians had a method for solving a system of equations. Here is an adaptation of a problem from an ancient (around 1500 B.C.) Babylonian text. "There are two silver blocks. The sum of $\frac{1}{7}$ of the first block and $\frac{1}{11}$ of the second block is one sheqel (a weight). The first block diminished by $\frac{1}{7}$ of its weight equals the second diminished by $\frac{1}{11}$ of its weight. What are the weights of the two blocks?"

Strategy for Solving an Application Problem in Two Variables

1. Choose one variable to represent one of the unknown quantities and a second variable to represent the other unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in tables.

Cost of a 60-watt bulb: b
Cost of a fluorescent bulb: f

First purchase

	Amount	·	*Unit Cost*	=	*Value*
60-watt	20	·	b	=	$20b$
Fluorescent	30	·	f	=	$30f$

Second purchase

	Amount	·	*Unit Cost*	=	*Value*
60-watt	30	·	b	=	$30b$
Fluorescent	10	·	f	=	$10f$

2. Determine a system of equations. The strategies presented in previous chapters can be used to determine the relationships between the expressions in the tables.

The total of the first purchase was \$80: $20b + 30f = 80$
The total of the second purchase was \$50: $30b + 10f = 50$

Solve the system of equations: (1) $20b + 30f = 80$
(2) $30b + 10f = 50$

$$\begin{aligned} 60b + 90f &= 240 \\ -60b - 20f &= -100 \\ 70f &= 140 \\ f &= 2 \end{aligned}$$

- 3 times Equation (1)
- −2 times Equation (2)

Replace f by 2 in Equation (1). Solve for b.

$$\begin{aligned} 20b + 30f &= 80 \\ 20b + 30(2) &= 80 \\ 20b + 60 &= 80 \\ 20b &= 20 \\ b &= 1 \end{aligned}$$

The cost of a 60-watt bulb was \$1.00.
The cost of a fluorescent bulb was \$2.00.

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Example 2

A total of 260 tickets were sold for a softball game. Adult tickets sold for \$6 each, and children's tickets sold for \$2 each. If the total receipts were \$1220, how many adult tickets and how many children's tickets were sold?

Strategy

- Number of adult tickets sold: A
  Number of children's tickets sold: C

Ticket	*Price*	*Number Sold*	*Total Receipts*
Adult	6	A	$6A$
Children	2	C	$2C$

- The total number of tickets sold was 260.
  The total receipts were \$1220.

Solution

$$\begin{aligned} A + C &= 260 \\ 6A + 2C &= 1220 \end{aligned}$$

$$\begin{aligned} -2A - 2C &= -520 \\ 6A + 2C &= 1220 \\ 4A &= 700 \\ A &= 175 \end{aligned}$$

$$\begin{aligned} A + C &= 260 \\ 175 + C &= 260 \\ C &= 85 \end{aligned}$$

There were 175 adult tickets sold.
There were 85 children's tickets sold.

You Try It 2

An investment adviser invested \$15,000 in two accounts. One investment earned 8% annual simple interest. The other investment earned 7% annual simple interest. The total interest in one year was \$1170. How much was invested in each account?

Your strategy

Your solution

Solution on pp. S16–S17

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5.4 Exercises

Objective A *Application Problems*

1. A motorboat traveling with the current went 36 mi in 2 h. Against the current, it took 3 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.

2. A cabin cruiser traveling with the current went 45 mi in 3 h. Against the current, it took 5 h to travel the same distance. Find the rate of the cabin cruiser in calm water and the rate of the current.

3. A jet plane flying with the wind went 2200 mi in 4 h. Against the wind, the plane could fly only 1820 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

4. Flying with the wind, a small plane flew 300 mi in 2 h. Against the wind, the plane could fly only 270 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

5. A rowing team rowing with the current traveled 20 km in 2 h. Rowing against the current, the team rowed 12 km in the same amount of time. Find the rate of the rowing team in calm water and the rate of the current.



6. A motorboat traveling with the current went 72 km in 3 h. Against the current, the boat could go only 48 km in the same amount of time. Find the rate of the boat in calm water and the rate of the current.

7. A turbo-prop plane flying with the wind flew 800 mi in 4 h. Flying against the wind, the plane required 5 h to travel the same distance. Find the rate of the wind and the rate of the plane in calm air.

8. Flying with the wind, a pilot flew 600 mi between two cities in 4 h. The return trip against the wind took 5 h. Find the rate of the plane in calm air and the rate of the wind.

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9. A plane flying with a tailwind flew 600 mi in 5 h. Against the wind, the plane required 6 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

10. Flying with the wind, a plane flew 720 mi in 3 h. Against the wind, the plane required 4 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

11. A motorboat traveling with the current went 48 mi in 3 h. Against the current, it took 4.8 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.

12. A plane traveling with the wind flew 3625 mi in 6.25 h. Against the wind, the plane required 7.25 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

13. A cabin cruiser traveling with the current went 45 mi in 2.5 h. Against the current, the boat could go only 30 mi in the same amount of time. Find the rate of the cabin cruiser and the rate of the current.

14. Flying with the wind, a plane flew 450 mi in 3 h. Against the wind, the plane could fly only 270 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

Objective B *Application Problems*

15. A merchant mixed 10 lb of a cinnamon tea with 5 lb of spice tea. The 15-pound mixture cost $40. A second mixture included 12 lb of the cinnamon tea and 8 lb of the spice tea. The 20-pound mixture cost $54. Find the cost per pound of the cinnamon tea and the spice tea.



16. A carpenter purchased 60 ft of redwood and 80 ft of pine for a total cost of $286. A second purchase, at the same prices, included 100 ft of redwood and 60 ft of pine for a total cost of $396. Find the cost per foot of redwood and the cost per foot of pine.

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17. A contractor buys 16 yd of nylon carpet and 20 yd of wool carpet for $1840. A second purchase, at the same prices, includes 18 yd of nylon carpet and 25 yd of wool carpet for $2200. Find the cost per yard of the wool carpet.

18. During one month, a homeowner used 500 units of electricity and 100 units of gas, for a total cost of $352. The next month, 400 units of electricity and 150 units of gas were used, for a total cost of $304. Find the cost per unit of gas.

19. A company manufactures both mountain bikes and trail bikes. The cost of materials for a mountain bike is $70, and the cost of materials for a trail bike is $50. The cost of labor to manufacture a mountain bike is $80, and the cost of labor to manufacture a trail bike is $40. During a week in which the company has budgeted $2500 for materials and $2600 for labor, how many mountain bikes does the company plan to manufacture?

20. A company manufactures both color and black-and-white monitors. The cost of materials for a black-and-white monitor is $40, and the cost of materials for a color monitor is $160. The cost of labor to manufacture a black-and-white monitor is $60, and the cost of labor to manufacture a color monitor is $100. During a week in which the company has budgeted $8400 for materials and $5600 for labor, how many color monitors does the company plan to manufacture?

21. You have a total of $5000 invested in two simple interest accounts. On one account, the annual simple interest rate is 6%. On the second account, the annual simple interest rate is 8%. The total annual interest earned in one year is $370. How much is invested in each account?

22. Two investments earn total annual interest of $970. One investment is in a 7.5% annual simple interest account. The other investment is in a 9.5% annual simple interest account. The total in the two investments is $12,000. How much is invested in each account?

23. Two angles are complementary. The larger angle is 9° more than eight times the measure of the smaller angle. Find the measure of the two angles. (Complementary angles are two angles whose sum is 90°.)

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24. Two angles are supplementary. The larger angle is 40° more than three times the measure of the smaller angle. Find the measure of the two angles. (Supplementary angles are two angles whose sum is 180°.)

25. On Monday, a computer-manufacturing company sent out three shipments. The first order, which contained a bill for $114,000, was for 4 Model II, 6 Model VI, and 10 Model IX computers. The second shipment, which contained a bill for $72,000, was for 8 Model II, 3 Model VI, and 5 Model IX computers. The third shipment, which contained a bill for $81,000, was for 2 Model II, 9 Model VI, and 5 Model IX computers. What does the manufacturer charge for each Model VI computer?

26. A relief organization supplies blankets, cots, and lanterns to victims of fires, floods, and other natural disasters. One week the organization purchased 15 blankets, 5 cots, and 10 lanterns for a total cost of $1250. The next week, at the same prices, the organization purchased 20 blankets, 10 cots, and 15 lanterns for a total cost of $2000. The next week, at the same prices, the organization purchased 10 blankets, 15 cots, and 5 lanterns for a total cost of $1625. Find the cost of one blanket, the cost of one cot, and the cost of one lantern.

27. A science museum charges $10 for a regular admission ticket, but members receive a discount of $3 and students are admitted for $5. Last Saturday, 750 tickets were sold for a total of $5400. If 20 more student tickets than regular tickets were sold, how many of each type of ticket were sold?

28. An investor has a total of $25,000 deposited in three different accounts, which earn annual interest rates of 8%, 6%, and 4%. The amount deposited in the 8% account is twice the amount in the 6% account. If the three accounts earn total annual interest of $1520, how much money is deposited in each account?

APPLYING THE CONCEPTS

29. The sum of the digits of a two-digit number equals $\frac{1}{7}$ of the number. If the digits of the number are reversed, the new number is equal to 36 less than the original number. Find the original number.

30. The sum of the digits of a two-digit number equals $\frac{1}{5}$ of the number. If the digits of the number are reversed, the new number is equal to 9 more than the original number. Find the original number.

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5.5 Solving Systems of Linear Inequalities

Objective A

To graph the solution set of a system of linear inequalities

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Two or more inequalities considered together are called a **system of inequalities.** The **solution set of a system of inequalities** is the intersection of the solution sets of the individual inequalities. To graph the solution set of a system of inequalities, first graph the solution set of each inequality. The solution set of the system of inequalities is the region of the plane represented by the intersection of the shaded areas.

TAKE NOTE

You can use a test point to check that the correct region has been denoted as the solution set. We can see from the graph that the point (2, 4) is in the solution set, and as shown below, it is a solution of each inequality in the system. This indicates that the solution set as graphed is correct.

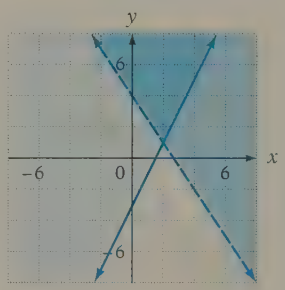
$$\begin{aligned} 2x - y &\le 3 \\ 2(2) - (4) &\le 3 \\ 0 &\le 3 \quad \text{True} \end{aligned}$$

$$\begin{aligned} 3x + 2y &> 8 \\ 3(2) + 2(4) &> 8 \\ 14 &> 8 \quad \text{True} \end{aligned}$$

➡ Graph the solution set: $2x - y \le 3$
$3x + 2y > 8$

Solve each inequality for y.

$$\begin{aligned} 2x - y &\le 3 \\ -y &\le -2x + 3 \\ y &\ge 2x - 3 \end{aligned} \qquad \begin{aligned} 3x + 2y &> 8 \\ 2y &> -3x + 8 \\ y &> -\frac{3}{2}x + 4 \end{aligned}$$



Graph $y = 2x - 3$ as a solid line. Because the inequality is $\ge$, shade above the line.

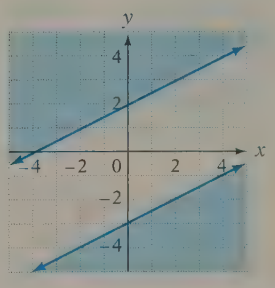
Graph $y = -\frac{3}{2}x + 4$ as a dashed line. Because the inequality is $>$, shade above the line.

The solution set of the system is the region of the plane represented by the intersection of the solution sets of the individual inequalities.

➡ Graph the solution set: $-x + 2y \ge 4$
$x - 2y \ge 6$

Solve each inequality for y.

$$\begin{aligned} -x + 2y &\ge 4 \\ 2y &\ge x + 4 \\ y &\ge \frac{1}{2}x + 2 \end{aligned} \qquad \begin{aligned} x - 2y &\ge 6 \\ -2y &\ge -x + 6 \\ y &\le \frac{1}{2}x - 3 \end{aligned}$$



Shade above the solid line $y = \frac{1}{2}x + 2$.

Shade below the solid line $y = \frac{1}{2}x - 3$.

Because the solution sets of the two inequalities do not intersect, the solution of the system is the empty set.

Point of Interest

Large systems of linear inequalities containing over 100 inequalities have been used to solve application problems in such diverse areas as providing health care and hardening a nuclear missile silo.

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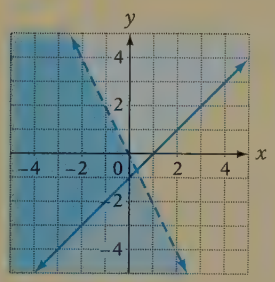
Example 1

Graph the solution set: $y \geq x - 1$
$y < -2x$

Solution

Shade above the solid line $y = x - 1$.
Shade below the dashed line $y = -2x$.

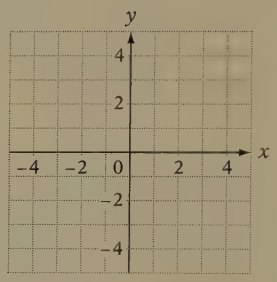
The solution of the system is the intersection of the solution sets of the individual inequalities.



You Try It 1

Graph the solution set: $y \geq 2x - 3$
$y > -3x$

Your solution



Example 2

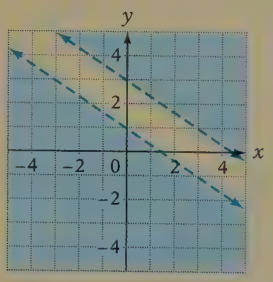
Graph the solution set: $2x + 3y > 9$
$y < -\frac{2}{3}x + 1$

Solution

$2x + 3y > 9$
$3y > -2x + 9$
$y > -\frac{2}{3}x + 3$

Graph above the dashed line $y = -\frac{2}{3}x + 3$.

Graph below the dashed line $y = -\frac{2}{3}x + 1$.

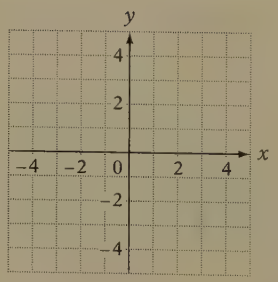


The intersection of the system is the empty set, because the solution sets of the two inequalities do not intersect.

You Try It 2

Graph the solution set: $3x + 4y > 12$
$y < \frac{3}{4}x - 1$

Your solution



Solutions on p. S17

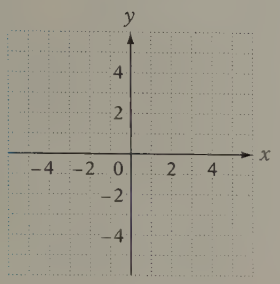
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5.5 Exercises

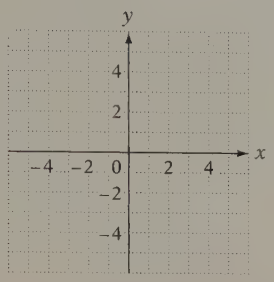
Objective A

Graph the solution set.

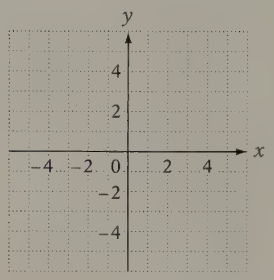
1. $x - y \geq 3$
$x + y \leq 5$



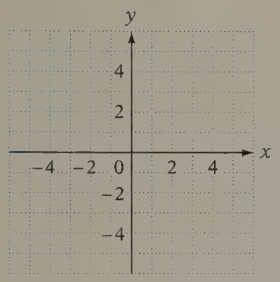
2. $2x - y < 4$
$x + y < 5$



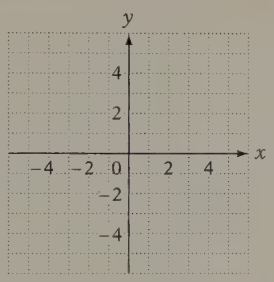
3. $3x - y < 3$
$2x + y \geq 2$



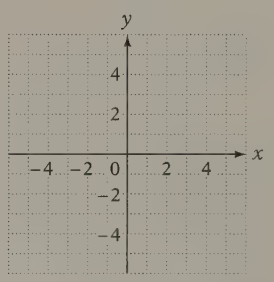
4. $x + 2y \leq 6$
$x - y \leq 3$



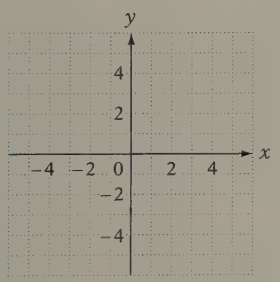
5. $2x + y \geq -2$
$6x + 3y \leq 6$



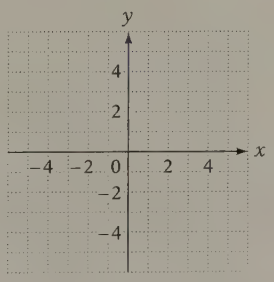
6. $x + y \geq 5$
$3x + 3y \leq 6$



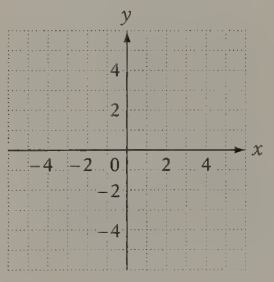
7. $3x - 2y < 6$
$y \leq 3$



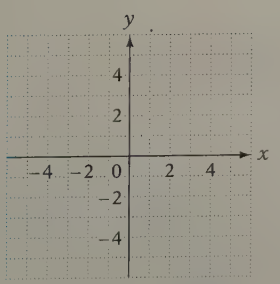
8. $x \leq 2$
$3x + 2y > 4$



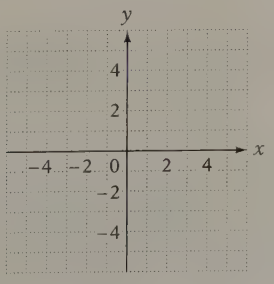
9. $y > 2x - 6$
$x + y < 0$



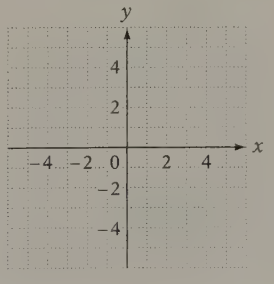
10. $x < 3$
$y < -2$



11. $x + 1 \geq 0$
$y - 3 \leq 0$

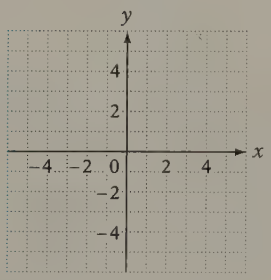


12. $5x - 2y \geq 10$
$3x + 2y \geq 6$

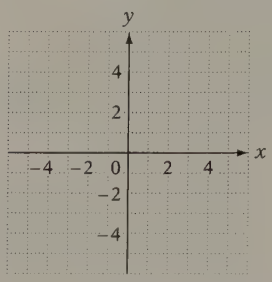


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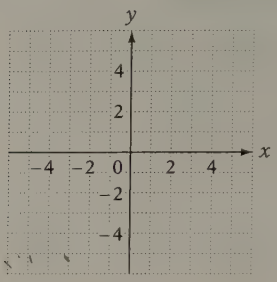
13. $2x + y \geq 4$
$3x - 2y < 6$



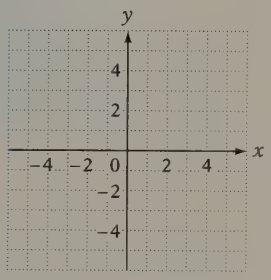
14. $3x - 4y < 12$
$x + 2y < 6$



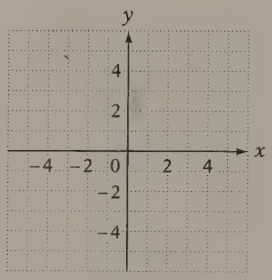
15. $x - 2y \leq 6$
$2x + 3y \leq 6$



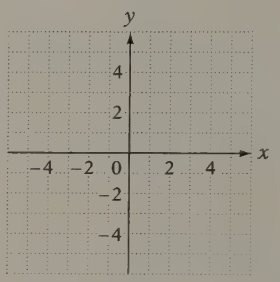
16. $x - 3y > 6$
$2x + y > 5$



17. $x - 2y \leq 4$
$3x + 2y \leq 8$



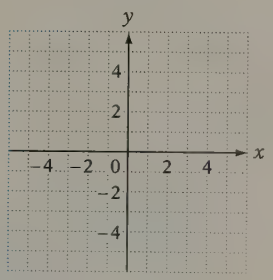
18. $3x - 2y < 0$
$5x + 3y > 9$



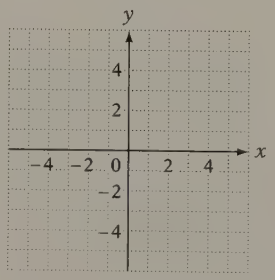
APPLYING THE CONCEPTS

Graph the solution set.

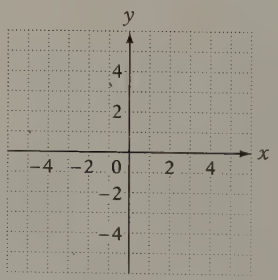
19. $2x + 3y \leq 15$
$3x - y \leq 6$
$y \geq 0$



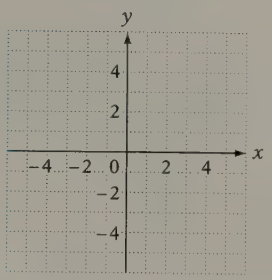
20. $x + y \leq 6$
$x - y \leq 2$
$x \geq 0$



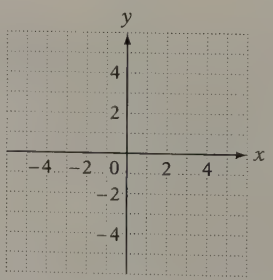
21. $x - y \leq 5$
$2x - y \geq 6$
$y \geq 0$



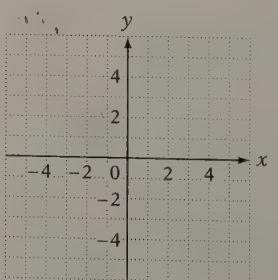
22. $x - 3y \leq 6$
$5x - 2y \geq 4$
$y \geq 0$



23. $2x - y \leq 4$
$3x + y < 1$
$y \leq 0$



24. $x - y \leq 4$
$2x + 3y > 6$
$x \geq 0$

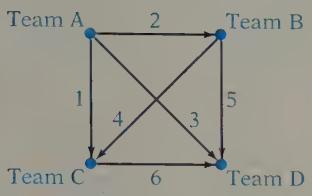


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Focus on Problem Solving

Solve an Easier Problem

Suppose you are in charge of your softball league, which consists of 15 teams. You must devise a schedule in which each team plays every other team once. How many games must be scheduled?



To solve this problem, we will attempt an easier problem first. Suppose that your league contains only a small number of teams. For instance, if there were only 1 team, you would schedule 0 games. If there were 2 teams, you would schedule 1 game. If there were 3 teams, you would schedule 3 games. The diagram at the left shows that 6 games must be scheduled when there are 4 teams in the league.

Here is a table of our results so far. (Remember that making a table is a strategy to be used in problem solving.)

Number of Teams	*Number of Games*	*Possible Pattern*
1	0	0
2	1	1
3	3	1 + 2
4	6	1 + 2 + 3

1. Draw a diagram with 5 dots to represent the teams. Draw lines from each dot to a second dot, and determine the number of games required.
2. What is the apparent pattern for the number of games required?
3. Assuming that the pattern continues, how many games must be scheduled for the 15 teams of the original problem?

After solving a problem, good problem solvers ask whether it is possible to solve the problem in a different manner. Here is a possible alternative method of solving the scheduling problem.

Begin with one of the 15 teams (say team A) and ask, "How many games must this team play?" Because there are 14 teams left to play, you must schedule 14 games. Now move to team B. It is already scheduled to play team A, and it does not play itself, so there are 13 teams left for it to play. Consequently, you must schedule 14 + 13 games.

4. Continue this reasoning for the remaining teams and determine the number of games that must be scheduled. Does this answer correspond to the answer you obtained using the first method?



5. Visit www.sports.com to get lists of professional sports teams, divisions, etc. Find the strategy used to create the schedule of your favorite team.

Projects and Group Activities

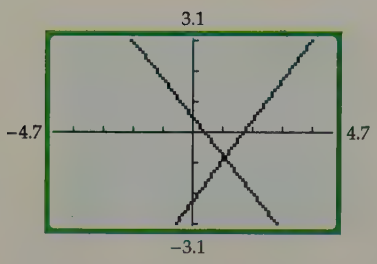
Using a Graphing Calculator to Solve a System of Equations



A graphing calculator can be used to solve a system of equations. For this procedure to work on most calculators, it is necessary that the point of intersection be on the screen. This means that you may have to experiment with Xmin, Xmax, Ymin, and Ymax values until the graphs intersect on the screen.

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To solve a system of equations graphically, solve each equation for y. Then graph the equations of the system. Their point of intersection is the solution.



For instance, to solve the system of equations

$$4x - 3y = 7$$
$$5x + 4y = 2$$

first solve each equation for y.

$$4x - 3y = 7 \Rightarrow y = \frac{4}{3}x - \frac{7}{3}$$

$$5x + 4y = 2 \Rightarrow y = -\frac{5}{4}x + \frac{1}{2}$$

The keystrokes needed to solve this system using a TI-83 are given below. We are using the viewing window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. The approximate solution is $(1.096774, -0.870968)$.

[Y=] [CLEAR] 4 [X, T, θ, n] [÷] 3 [−]
7 [÷] 3 [ENTER] [CLEAR]
[(−)] 5 [X, T, θ, n] [÷] 4 [+] 1 [÷]
2 [GRAPH]

Once the calculator has drawn the graphs, use the TRACE feature and move the cursor to the approximate point of intersection. This will give you an approximate solution of the system of equations. A more accurate solution can be found by using the following keystrokes.

[2nd] [CALC] 5 [ENTER] [ENTER] [ENTER]

Some of the exercises in the first section of this chapter asked you to solve a system of equations by graphing. Try those exercises again, this time using your graphing calculator.

Current models of calculators do not allow you to solve graphically a system of equations in three variables. However, these calculators do have matrix and determinant operations that can be used to solve these systems.

Solving a First-Degree Equation with a Graphing Calculator



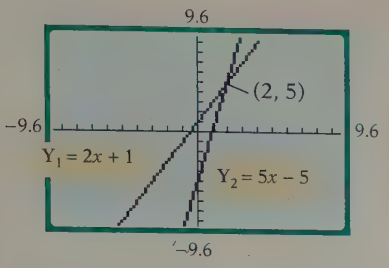
A first-degree equation in one variable can be solved graphically by using a graphing calculator. The idea is to rewrite the equation as a system of equations and then solve the system of equations as illustrated above. For instance, to solve the equation

$$2x + 1 = 5x - 5$$

write the equation as the following system of equations.

$$y = 2x + 1$$
$$y = 5x - 5$$

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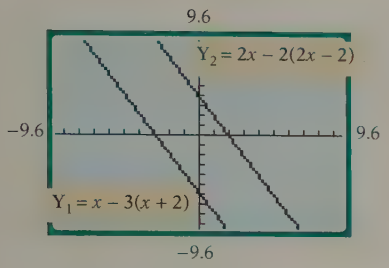


Note that we have used the left and right sides of the original equation to form the system of equations.

Now graph the equations and find the point of intersection. This is shown at the left using a viewing window of $[-9.6, 9.6]$ by $[-9.6, 9.6]$. The solution of the original equation, $2x + 1 = 5x - 5$, is the x-coordinate of the point of intersection. Thus the solution of the equation is 2.

Recall that not all equations have a solution. Consider the equation

$$x - 3(x + 2) = 2x - 2(2x - 2)$$



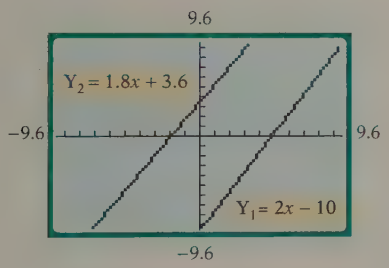
The graphs of the left and right sides of the equation are shown at the left. Note that the lines appear to be parallel and therefore do not intersect. Since the lines do not intersect, there is no solution of the system of equations and thus no solution of the original equation. We algebraically verify this result below.

$$\begin{aligned} x - 3(x + 2) &= 2x - 2(2x - 2) \\ x - 3x - 6 &= 2x - 4x + 4 \\ -2x - 6 &= -2x + 4 \\ -6 &= 4 \end{aligned}$$

- **Use the Distributive Property.**
- **Simplify.**
- **Add $2x$ to each side of the equation.**

Because $-6 = 4$ is not a true equation, there is no solution.

Note that in the third line of the solution we obtained the expressions $-2x - 6$ and $-2x + 4$. Consider the graphs of the equations $y = -2x - 6$ and $y = -2x + 4$; the slopes of the lines are equal (both are -2), and therefore the graphs of the lines are parallel.



Before we leave graphical solutions, a few words of caution. Graphs can be deceiving and appear not to intersect when they do, and appear to intersect when they do not. For instance, an attempt to graphically solve

$$2x - 10 = 1.8x + 3.6$$

is shown at the left.

If you use the viewing window $[-9.6, 9.6]$ by $[-9.6, 9.6]$, the graphs will appear to be parallel. However, the graphs of $y = 2x - 10$ and $y = 1.8x + 3.6$ do not have the same slope and therefore must intersect. For this equation, you need a larger viewing window to see the point of intersection.

Solve the following equations by using a graphing calculator.

1. $3x - 1 = 5x + 1$
2. $3x + 2 = 4$
3. $3 + 2(2x - 4) = 5(x - 3)$
4. $2x - 4 = 5x - 3(x + 2) + 2$
5. Explain how problem 4 relates to an identity as explained in Section 2.1.
6. Find an appropriate viewing window so that you can determine the solution of the equation $2x - 10 = 1.8x + 3.6$ given above. What is the solution?

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Chapter Summary

Key Words

Equations considered together are called a *system of equations.* [p. 275]

A *solution of a system of equations* in two variables is an ordered pair that is a solution of each equation of the system. [p. 275]

When the graphs of a system of equations intersect at only one point, the system is called an *independent system of equations.* When the graphs of a system of equations coincide, the system is called a *dependent system of equations.* When a system of equations has no solution, it is called an *inconsistent system of equations.* [pp. 275–276]

An equation of the form $Ax + By + Cz = D$, where A, B, and C are coefficients of the variables and D is a constant, is a *linear equation in three variables.* A *solution of a system of equations in three variables* is an ordered triple that is a solution of each equation of the system. [pp. 288–290]

A *matrix* is a rectangular array of numbers. Each number in the matrix is called an *element* of the matrix. A matrix of m rows and n columns is said to be of *order* $m \times n$. A *square matrix* has the same number of rows as columns. [p. 297]

A *determinant* is a number associated with a square matrix. [p. 297]

The *minor of an element* in a 3×3 determinant is the 2×2 determinant obtained by eliminating the row and column that contain that element. The *cofactor* of an element of a matrix is $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is its column number. [pp. 297–298]

The evaluation of the determinant of a 3×3 or larger matrix is accomplished by *expanding by cofactors.* [p. 298]

Inequalities considered together are called a *system of inequalities.* The *solution set of a system of inequalities* is the intersection of the solution sets of the individual inequalities. [p. 313]

Essential Rules

A system of equations can be solved by a graphing method, by the substitution method, or by the addition method. [pp. 275, 278, 285]

Cramer's Rule [pp. 300–301]

The solution of the system of equations $\begin{matrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{matrix}$ is given by $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$, where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$, and $D \neq 0$.

The solution of the system of equations

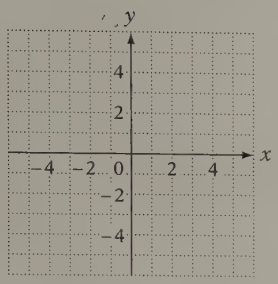
$$\begin{matrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{matrix}$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$, and $D \neq 0$.

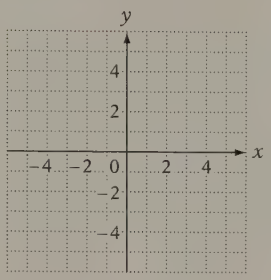
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Chapter Review

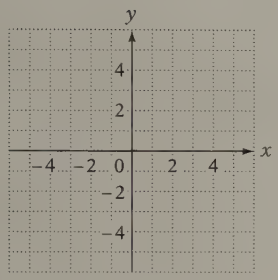
1. Solve by graphing: $2x - 3y = -6$
$2x - y = 2$



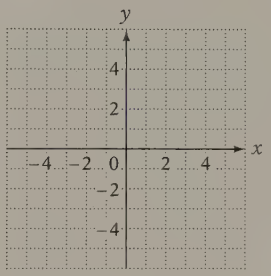
2. Solve by graphing: $x - 2y = -5$
$3x + 4y = -15$



3. Graph the solution set: $2x - y < 3$
$4x + 3y < 11$



4. Graph the solution set: $x + y > 2$
$2x - y < -1$



5. Solve by substitution: $3x + 2y = 4$
$x = 2y - 1$

6. Solve by substitution: $5x + 2y = -23$
$2x + y = -10$

7. Solve by substitution: $y = 3x - 7$
$y = -2x + 3$

8. Solve by the addition method: $3x + 4y = -2$
$2x + 5y = 1$

9. Solve by the addition method:
$4x - 6y = 5$
$6x - 9y = 4$

10. Solve by the addition method:
$3x - y = 2x + y - 1$
$5x + 2y = y + 6$

11. Solve by the addition method:
$2x + 4y - z = 3$
$x + 2y + z = 5$
$4x + 8y - 2z = 7$

12. Solve by the addition method:
$x - y - z = 5$
$2x + z = 2$
$3y - 2z = 1$

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13. Evaluate the determinant: $\begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix}$

14. Evaluate the determinant: $\begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & 1 \\ 2 & -1 & -2 \end{vmatrix}$

15. Solve by using Cramer's Rule:

$$x - y = 3$$
$$2x + y = -4$$

16. Solve by using Cramer's Rule:

$$5x + 2y = 9$$
$$3x + 5y = -7$$

17. Solve by using Cramer's Rule:

$$x - y + z = 2$$
$$2x - y - z = 1$$
$$x + 2y - 3z = -4$$

18. Solve by using Cramer's Rule:

$$3x + 2y + 2z = 2$$
$$x - 2y - z = 1$$
$$2x - 3y - 3z = -3$$

19. A plane flying with the wind went 350 mi in 2 h. The return trip, flying against the wind, took 2.8 h. Find the rate of the plane in calm air and the rate of the wind.

20. A clothing manufacturer purchased 60 yd of cotton and 90 yd of wool for a total cost of \$3600. Another purchase, at the same prices, included 80 yd of cotton and 20 yd of wool for a total cost of \$2000. Find the cost per yard of the cotton and of the wool.

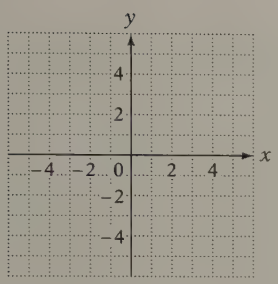
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Chapter Test

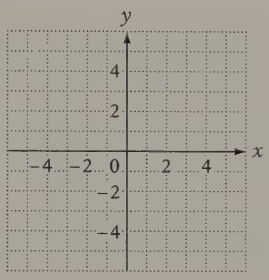
1. Solve by substitution: $2x - 6y = 15$
$x = 4y + 8$

2. Solve by the addition method: $3x + 2y = 2$
$x + y = 3$

3. Solve by graphing: $x + y = 3$
$3x - 2y = -6$



4. Solve by graphing: $2x - y = 4$
$y = 2x - 4$



5. Solve by substitution: $3x + 12y = 18$
$x + 4y = 6$

6. Solve by the addition method: $5x - 15y = 30$
$x - 3y = 6$

7. Solve by the addition method:
$3x - 4y - 2z = 17$
$4x - 3y + 5z = 5$
$5x - 5y + 3z = 14$

8. Solve by the addition method:
$3x + y = 13$
$2y + 3z = 5$
$x + 2z = 11$

9. Evaluate the determinant:

$$\begin{vmatrix} 6 & 1 \\ 2 & 5 \end{vmatrix}$$

10. Evaluate the determinant:

$$\begin{vmatrix} 1 & 5 & -2 \\ -2 & 1 & 4 \\ 4 & 3 & -8 \end{vmatrix}$$

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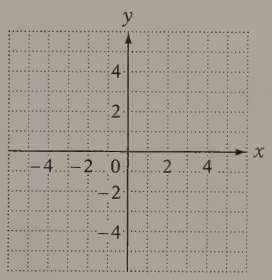
11. Solve by using Cramer's Rule:
$2x - y = 7$
$3x + 2y = 7$

12. Solve by using Cramer's Rule:
$3x - 4y = 10$
$2x + 5y = 15$

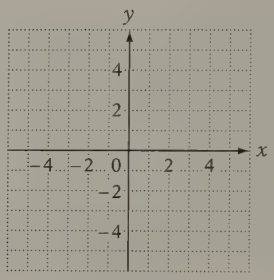
13. Solve by using Cramer's Rule:
$x + y + z = 0$
$x + 2y + 3z = 5$
$2x + y + 2z = 3$

14. Solve by using Cramer's Rule:
$x + 3y + z = 6$
$2x + y - z = 12$
$x + 2y - z = 13$

15. Graph the solution set:
$x + 3y \leq 6$
$2x - y \geq 4$



16. Graph the solution set:
$2x + 4y \geq 8$
$x + y \leq 3$



17. Solve by substitution:
$3x - 2y = 4$
$y = 5x - 9$

18. A cabin cruiser traveling with the current went 60 mi in 3 h. Against the current, it took 5 h to travel the same distance. Find the rate of the cabin cruiser in calm water and the rate of the current.

19. A pilot flying with the wind flew 600 mi in 3 h. Flying against the wind, the pilot required 4 h to travel the same distance. Find the rate of the plane in calm air and the rate of the wind.

20. At a movie theater, admission tickets are \$10 for children and \$16 for adults. The receipts for one Friday evening were \$5000. The next day there were three times as many children as the preceding evening and only half the number of adults as the night before, yet the receipts were still \$5000. Find the number of children who attended on Friday evening.

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Cumulative Review

1. Simplify: $-2\sqrt{90}$

2. Solve: $3(x - 5) = 2x + 7$

3. Simplify: $3[x - 2(5 - 2x) - 4x] + 6$

4. Evaluate $a + bc \div 2$ when $a = 4$, $b = 8$, and $c = -2$.

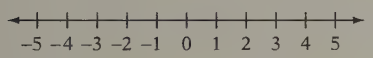
5. Solve: $2x - 3 < 9$ or $5x - 1 < 4$

6. Solve: $|x - 2| - 4 < 2$

7. Solve: $|2x - 3| > 5$

8. Given $F(x) = x^2 - 3$, find $F(2)$.

9. Graph the solution set of $\{x|x \leq 2\} \cap \{x|x > -3\}$.

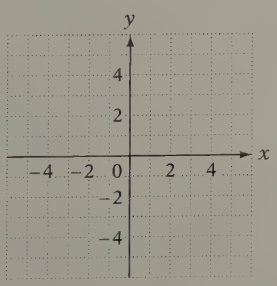


10. Find the equation of the line that contains the point $(-2, 3)$ and has slope $-\frac{2}{3}$.

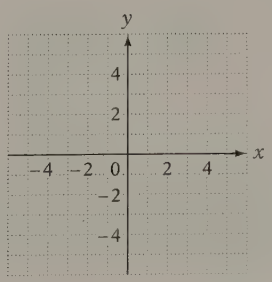
11. Find the equation of the line that contains the points $(2, -1)$ and $(3, 4)$.

12. Find the equation of the line that contains the point $(-2, 2)$ and is perpendicular to the line $2x - 3y = 6$.

13. Graph $2x - 5y = 10$.

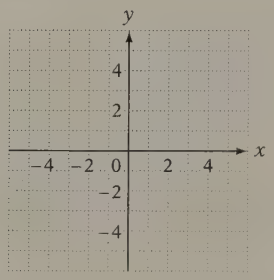


14. Graph the solution set of $3x - 4y \geq 8$.

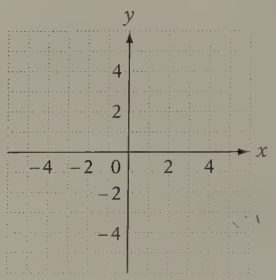


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15. Solve by graphing:
$5x - 2y = 10$
$3x + 2y = 6$



16. Graph the solution set:
$3x - 2y \geq 4$
$x + y < 3$



17. Solve by the addition method:
$3x + 2z = 1$
$2y - z = 1$
$x + 2y = 1$

18. Evaluate the determinant:

$$\begin{vmatrix} 2 & -5 & 1 \\ 3 & 1 & 2 \\ 6 & -1 & 4 \end{vmatrix}$$

19. Solve by using Cramer's Rule:
$4x - 3y = 17$
$3x - 2y = 12$

20. Solve by substitution:
$3x - 2y = 7$
$y = 2x - 1$

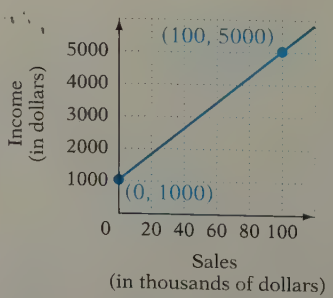
21. How many milliliters of pure water must be added to 100 ml of a 4% salt solution to make a 2.5% salt solution?

22. Flying with the wind, a small plane required 2 h to fly 150 mi. Against the wind, it took 3 h to fly the same distance. Find the rate of the wind.

23. A restaurant manager buys 100 lb of hamburger and 50 lb of steak for a total cost of $980. A second purchase, at the same prices, includes 150 lb of hamburger and 100 lb of steak. The total cost is $1720. Find the price of 1 lb of steak.

24. Find the lower and upper limits of a 12,000-ohm resistor with a 15% tolerance.

25. The graph shows the relationship between the monthly income and the sales of an account executive. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



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Chapter 6 Polynomials



The Pathfinder mission produced the closest look to date at the surface of Mars. This photo of the Pathfinder Sojourner rover was taken at Mermaid Dune on Mars on August 8, 1997. Images such as this, transmitted from Mars to Earth, traveled a distance of over 100 million miles. Distances this great, as well as very small measurements, are generally expressed in scientific notation, as illustrated in **Exercise 119 on page 340.**

Objectives

Section 6.1

- A To multiply monomials
- B To divide monomials and simplify expressions with negative exponents
- C To write a number using scientific notation
- D To solve application problems

Section 6.2

- A To evaluate polynomial functions
- B To add or subtract polynomials

Section 6.3

- A To multiply a polynomial by a monomial
- B To multiply two polynomials
- C To multiply two binomials
- D To multiply binomials that have special products
- E To solve application problems

Section 6.4

- A To divide polynomials
- B To divide polynomials using synthetic division
- C To evaluate a polynomial using synthetic division



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Subtract: $-2 - (-3)$

2. Multiply: $-3(6)$

For Exercises 3 to 8, simplify.

3. $-\dfrac{24}{-36}$

4. $-4(3y)$

5. $(-2)^3$

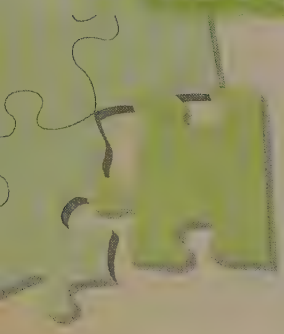
6. $-4a - 8b + 7a$

7. $3x - 2[y - 4(x + 1) + 5]$

8. $-4y + 4y$

9. Are $2x^2$ and $2x$ like terms?

Go Figure



A pair of perpendicular lines are drawn through the interior of a rectangle, dividing it into four smaller rectangles. The area of the smaller rectangles are x, 2, 3, and 6. Find the possible values of x.

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6.1 Exponential Expressions

Objective A: To multiply monomials

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A monomial is a number, a variable, or a product of a number and variables.

Point of Interest

Around A.D. 250, the monomial $3x^2$ shown at the right would have been written $\Delta^{\gamma}3$ or at least approximately like that. In A.D. 250, the symbol for 3 was not the one we use today.

The examples at the right are monomials. The **degree of a monomial** is the sum of the exponents of the variables.

x	degree 1 ($x = x^1$)
$3x^2$	degree 2
$4x^2y$	degree 3
$6x^3y^4z^2$	degree 9

In this chapter, the variable n is considered a positive integer when used as an exponent.

x^n	degree n

The degree of a nonzero constant term is zero.

6	degree 0

The expression $5\sqrt{x}$ is not a monomial because $\sqrt{x}$ cannot be written as a product of variables. The expression $\frac{x}{y}$ is not a monomial because it is a quotient of variables.

The expression x^4 is an exponential expression. The exponent, 4, indicates the number of times the base, x, occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^4 = \underbrace{\overbrace{(x \cdot x \cdot x)}^{3 \text{ factors}} \cdot \overbrace{(x \cdot x \cdot x \cdot x)}^{4 \text{ factors}}}_{7 \text{ factors}}$$
$$= x^7$$

Note that adding the exponents results in the same product.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

Rule for Multiplying Exponential Expressions

If m and n are positive integers, then $x^m \cdot x^n = x^{m+n}$.

➡ Multiply: $(-4x^5y^3)(3xy^2)$

$$(-4x^5y^3)(3xy^2) = (-4 \cdot 3)(x^5 \cdot x)(y^3 \cdot y^2)$$
$$= -12(x^{5+1})(y^{3+2})$$
$$= -12x^6y^5$$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.
- Multiply variables with the same base by adding their exponents. Do these steps mentally.
- Simplify.

TAKE NOTE

The Rule for Multiplying Exponential Expressions requires the bases to be the same. The expression x^6y^5 cannot be simplified.

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Point of Interest

One of the first symbolic representations of powers was given by Diophantus (c. A.D. 250) in his book *Arithmetica*. He used Δ^Y for x^2 and K^Y for x^3. The symbol Δ^Y was the first two letters of the Greek word *dunamis*, meaning "power"; K^Y was from the Greek word *kubos*, meaning "cube." He also combined these symbols to denote higher powers. For instance, ΔK^Y was the symbol for x^5.

The power of a monomial can be simplified by writing the power in factored form and then using the Rule for Multiplying Exponential Expressions.

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{4+4+4} = x^{12} \qquad (a^2b^3)^2 = (a^2b^3)(a^2b^3) = a^{2+2}b^{3+3} = a^4b^6$$

- Write in factored form.
- Use the Rule for Multiplying Exponential Expressions.

Note that multiplying each exponent inside the parentheses by the exponent outside the parentheses results in the same product.

$$(x^4)^3 = x^{4\cdot 3} = x^{12} \qquad (a^2b^3)^2 = a^{2\cdot 2}b^{3\cdot 2} = a^4b^6$$

- Multiply each exponent inside the parentheses by the exponent outside the parentheses.

Rule for Simplifying Powers of Exponential Expressions

If m and n are positive integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m, n, and p are positive integers, then $(x^m y^n)^p = x^{mp}y^{np}$.

➡ Simplify: $(x^4)^5$

$$(x^4)^5 = x^{4\cdot 5} = x^{20}$$

- Use the Rule for Simplifying Powers of Exponential Expressions to multiply the exponents.

➡ Simplify: $(2a^3b^4)^3$

$$(2a^3b^4)^3 = 2^{1\cdot 3}a^{3\cdot 3}b^{4\cdot 3} = 2^3a^9b^{12} = 8a^9b^{12}$$

- Use the Rule for Simplifying Powers of Products to multiply each exponent inside the parentheses by the exponent outside the parentheses. Note that $2 = 2^1$.

Example 1 Simplify: $(2xy^2)(-3xy^4)^3$

Solution

$$(2xy^2)(-3xy^4)^3 = (2xy^2)[(-3)^3x^3y^{12}] = (2xy^2)(-27x^3y^{12}) = -54x^4y^{14}$$

You Try It 1 Simplify: $(-3a^2b^4)(-2ab^3)^4$

Your solution

Example 2 Simplify: $(x^{n+2})^5$

Solution

$$(x^{n+2})^5 = x^{5n+10}$$

You Try It 2 Simplify: $(y^{n-3})^2$

Your solution

Solutions on p. S17

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Example 3 Simplify: $[(2xy^2)^2]^3$

Solution $[(2xy^2)^2]^3 = [2^2x^2y^4]^3$
$= 2^6x^6y^{12} = 64x^6y^{12}$

You Try It 3 Simplify: $[(ab^3)^3]^4$

Your solution

Solution on p. S17

Objective B

To divide monomials and simplify expressions with negative exponents

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The quotient of two exponential expressions with the same base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

$$\frac{x^5}{x^2} = \frac{\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}\cdot x\cdot x\cdot x}{\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}} = x^3$$

Note that subtracting the exponents gives the same result.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

To divide two monomials with the same base, subtract the exponents of the like bases.

➡ Simplify: $\frac{z^8}{z^2}$

$\frac{z^8}{z^2} = z^{8-2}$

$= z^6$

- The bases are the same. Subtract the exponents. This step is often done mentally.

➡ Simplify: $\frac{a^5b^9}{a^4b}$

$\frac{a^5b^9}{a^4b} = a^{5-4}b^{9-1}$

$= ab^8$

- Subtract the exponents of the like bases. This step is often done mentally.

TAKE NOTE
A number divided by itself is 1. Therefore, $\frac{8}{8} = 1$, $\frac{3^4}{3^4} = 1$, and for $x \neq 0$, $\frac{x^4}{x^4} = 1$.

Consider the expression $\frac{x^4}{x^4}$, $x \neq 0$. This expression can be simplified, as shown below, by (1) subtracting exponents and (2) dividing by common factors.

$$\frac{x^4}{x^4} = x^{4-4} = x^0 \qquad \frac{x^4}{x^4} = \frac{\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}} = 1$$

The equations $\frac{x^4}{x^4} = x^0$ and $\frac{x^4}{x^4} = 1$ suggest the following definition of x^0.

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is not defined.

TAKE NOTE
For the example below, we indicated that $z \neq 0$. If we try to evaluate $(16z^5)^0$ when $z = 0$, we have $[16(0)^5]^0 = [16(0)]^0 = 0^0$. However, 0^0 is not defined. Therefore, we must assume that $z \neq 0$. To avoid stating this for every example or exercise, we will assume that variables do not have values that result in the expression 0^0.

➡ Simplify: $(16z^5)^0$, $z \neq 0$

$(16z^5)^0 = 1$

- Any nonzero expression to the zero power is 1.

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Point of Interest

In the 15th century, the expression $12^{2\bar{m}}$ was used to mean $12x^{-2}$. The use of $\bar{m}$ reflects an Italian influence, where m was used for minus and p was used for plus. It was understood that $2\bar{m}$ referred to an unnamed variable. Isaac Newton, in the 17th century, advocated the use of a negative exponent, the symbol we use today.

Simplify: $-(7x^4y^3)^0$, $x \neq 0$, $y \neq 0$

$-(7x^4y^3)^0 = -(1) = -1$ • The negative outside the parentheses is not affected by the exponent.

Consider the expression $\frac{x^4}{x^6}$, $x \neq 0$. This expression can be simplified, as shown below, by (1) subtracting exponents and (2) dividing by common factors.

$$\frac{x^4}{x^6} = x^{4-6} = x^{-2} \qquad \frac{x^4}{x^6} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x} = \frac{1}{x^2}$$

The equations $\frac{x^4}{x^6} = x^{-2}$ and $\frac{x^4}{x^6} = \frac{1}{x^2}$ suggest that $x^{-2} = \frac{1}{x^2}$.

Definition of a Negative Exponent

If $x \neq 0$ and n is a positive integer, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

TAKE NOTE

Note from the example at the right that 2^{-4} is a *positive* number. A negative exponent does not indicate a negative number.

Evaluate: 2^{-4}

$2^{-4} = \frac{1}{2^4}$ • Use the Definition of a Negative Exponent.

$= \frac{1}{16}$ • Evaluate the expression.

A power of the quotient of two exponential expressions can be simplified by multiplying each exponent in the quotient by the exponent outside the parentheses.

Rule for Simplifying Powers of Quotients

If m, n, and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Simplify: $\left(\frac{a^2}{b^3}\right)^{-2}$

$\left(\frac{a^2}{b^3}\right)^{-2} = \frac{a^{2(-2)}}{b^{3(-2)}}$ • Use the Rule for Simplifying Powers of Quotients.

$= \frac{a^{-4}}{b^{-6}} = \frac{b^6}{a^4}$ • Use the Definition of a Negative Exponent.

Simplest Form of an Exponential Expression

An exponential expression is in simplest form when it is written with only positive exponents.

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TAKE NOTE

The exponent on n is -5 (*negative* 5). The n^{-5} is written in the denominator as n^5. The exponent on 3 is 1 (*positive* 1). The 3 remains in the numerator.

Also, we indicated that $n \neq 0$. This is necessary because division by zero is not defined. In this textbook, we will assume that values of the variables are chosen so that division by zero does not occur.

➡ Simplify: $3n^{-5}$, $n \neq 0$

$$3n^{-5} = 3 \cdot \frac{1}{n^5} = \frac{3}{n^5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

➡ Simplify: $\dfrac{2}{5a^{-4}}$

$$\frac{2}{5a^{-4}} = \frac{2}{5} \cdot \frac{1}{a^{-4}} = \frac{2}{5} \cdot a^4 = \frac{2a^4}{5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Now that zero and negative exponents have been defined, a rule for dividing exponential expressions can be stated.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\dfrac{x^m}{x^n} = x^{m-n}$.

➡ Simplify: $\dfrac{x^4}{x^9}$

$$\frac{x^4}{x^9} = \boxed{x^{4-9}}$$

- Use the Rule for Dividing Exponential Expressions. This step is often done mentally.

$$= x^{-5}$$

- Subtract the exponents.

$$= \frac{1}{x^5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integer exponents. These rules are restated here for your convenience.

Rules of Exponents

If m, n, and p are integers, then

$x^m \cdot x^n = x^{m+n}$ $\qquad (x^m)^n = x^{mn}$ $\qquad (x^m y^n)^p = x^{mp} y^{np}$

$\dfrac{x^m}{x^n} = x^{m-n}, x \neq 0$ $\qquad \left(\dfrac{x^m}{y^n}\right)^p = \dfrac{x^{mp}}{y^{np}}, y \neq 0$ $\qquad x^{-n} = \dfrac{1}{x^n}, x \neq 0$

$x^0 = 1, x \neq 0$

➡ Simplify: $(3ab^{-4})(-2a^{-3}b^7)$

$$(3ab^{-4})(-2a^{-3}b^7) = \boxed{[3 \cdot (-2)](a^{1+(-3)}b^{-4+7})}$$

- When multiplying expressions, add the exponents on like bases. Do this step mentally.

$$= -6a^{-2}b^3$$

$$= -\frac{6b^3}{a^2}$$

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Simplify: $\dfrac{4a^{-2}b^5}{6a^5b^2}$

$$\frac{4a^{-2}b^5}{6a^5b^2} = \frac{2 \cdot 2a^{-2}b^5}{2 \cdot 3a^5b^2} = \frac{2a^{-2}b^5}{3a^5b^2}$$
$$= \frac{2a^{-2-5}b^{5-2}}{3}$$
$$= \frac{2a^{-7}b^3}{3} = \frac{2b^3}{3a^7}$$

- Divide the coefficients by their common factor.
- Use the Rule for Dividing Exponential Expressions.
- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Simplify: $\left(\dfrac{6m^2n^3}{8m^7n^2}\right)^{-3}$

$$\left(\frac{6m^2n^3}{8m^7n^2}\right)^{-3} = \left(\frac{3m^{2-7}n^{3-2}}{4}\right)^{-3}$$
$$= \left(\frac{3m^{-5}n}{4}\right)^{-3}$$
$$= \frac{3^{-3}m^{15}n^{-3}}{4^{-3}}$$
$$= \frac{4^3m^{15}}{3^3n^3} = \frac{64m^{15}}{27n^3}$$

- Simplify inside the brackets.
- Subtract the exponents.
- Use the Rule for Simplifying Powers of Quotients.
- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents. Then simplify.

Example 4

Simplify: $\dfrac{-28x^6z^{-3}}{42x^{-1}z^4}$

Solution

$$\frac{-28x^6z^{-3}}{42x^{-1}z^4} = -\frac{14 \cdot 2x^{6-(-1)}z^{-3-4}}{14 \cdot 3}$$
$$= -\frac{2x^7z^{-7}}{3} = -\frac{2x^7}{3z^7}$$

You Try It 4

Simplify: $\dfrac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}}$

Your solution

Example 5

Simplify: $\dfrac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3}$

Solution

$$\frac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3} = \frac{3^{-3}a^3b^{-12}}{6^{-3}a^{-9}b^{-12}}$$
$$= 3^{-3} \cdot 6^3a^{12}b^0$$
$$= \frac{6^3a^{12}}{3^3} = \frac{216a^{12}}{27} = 8a^{12}$$

You Try It 5

Simplify: $\dfrac{(9u^{-6}v^4)^{-1}}{(6u^{-3}v^{-2})^{-2}}$

Your solution

Example 6

Simplify: $\dfrac{x^{4n-2}}{x^{2n-5}}$

Solution

$$\frac{x^{4n-2}}{x^{2n-5}} = x^{4n-2-(2n-5)}$$
$$= x^{4n-2-2n+5} = x^{2n+3}$$

You Try It 6

Simplify: $\dfrac{a^{2n+1}}{a^{n+3}}$

Your solution

Solutions on pp. S17–S18

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Objective C — To write a number using scientific notation

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Point of Interest

Astronomers measure the distance of some stars by using the parsec. One parsec is approximately 1.91×10^{13} mi.

Integer exponents are used to represent the very large and very small numbers encountered in the fields of science and engineering. For example, the mass of the electron is 0.000000000000000000000000000009 g. Numbers such as this are difficult to read and write, so a more convenient system for writing such numbers has been developed. It is called **scientific notation**.

To express a number in scientific notation, write the number as the product of a number between 1 and 10 and a power of 10. The form for scientific notation is $\boldsymbol{a \times 10^n}$, where $1 \le a < 10$.

TAKE NOTE

The numbers 9.65, 3.6, 9.2, 2, 9.74, and 8.6 are all numbers between 1 and 10. Each is multiplied by a power of 10. Note that there are two steps in writing a number in scientific notation: (1) Determine the number between 1 and 10, and (2) determine the exponent on 10.

For numbers greater than 10, move the decimal point to the right of the first digit. The exponent n is positive and equal to the number of places the decimal point has been moved.

$$965{,}000 = 9.65 \times 10^5$$
$$3{,}600{,}000 = 3.6 \times 10^6$$
$$92{,}000{,}000{,}000 = 9.2 \times 10^{10}$$

For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent n is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

$$0.0002 = 2 \times 10^{-4}$$
$$0.0000000974 = 9.74 \times 10^{-8}$$
$$0.000000000086 = 8.6 \times 10^{-11}$$

Converting a number written in scientific notation to decimal notation requires moving the decimal point.

When the exponent is positive, move the decimal point to the right the same number of places as the exponent.

$$1.32 \times 10^4 = 13{,}200$$
$$1.4 \times 10^8 = 140{,}000{,}000$$

When the exponent is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$$1.32 \times 10^{-2} = 0.0132$$
$$1.4 \times 10^{-4} = 0.00014$$

Numerical calculations involving numbers that have more digits than a hand-held calculator is able to handle can be performed using scientific notation.

➡ Simplify: $\dfrac{220{,}000 \times 0.000000092}{0.0000011}$

$$\frac{220{,}000 \times 0.000000092}{0.0000011} = \frac{2.2 \times 10^5 \times 9.2 \times 10^{-8}}{1.1 \times 10^{-6}}$$

• Write the numbers in scientific notation.

$$= \frac{(2.2)(9.2) \times 10^{5+(-8)-(-6)}}{1.1}$$

• Simplify.

$$= 18.4 \times 10^3 = 18{,}400$$

Example 7 Write 0.000041 in scientific notation.

Solution $0.000041 = 4.1 \times 10^{-5}$

You Try It 7 Write 942,000,000 in scientific notation.

Your solution

Solution on p. S18

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Example 8 Write 3.3×10^7 in decimal notation.

Solution $3.3 \times 10^7 = 33{,}000{,}000$

You Try It 8 Write 2.7×10^{-5} in decimal notation.

Your solution

Example 9

Simplify: $\dfrac{2{,}400{,}000{,}000 \times 0.0000063}{0.00009 \times 480}$

Solution

$$\frac{2{,}400{,}000{,}000 \times 0.0000063}{0.00009 \times 480}$$

$$= \frac{2.4 \times 10^9 \times 6.3 \times 10^{-6}}{9 \times 10^{-5} \times 4.8 \times 10^2}$$

$$= \frac{(2.4)(6.3) \times 10^{9+(-6)-(-5)-2}}{(9)(4.8)}$$

$$= 0.35 \times 10^6 = 350{,}000$$

You Try It 9

Simplify: $\dfrac{5{,}600{,}000 \times 0.000000081}{900 \times 0.000000028}$

Your solution

Solutions on p. S18

Objective D To solve application problems

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Example 10

How many miles does light travel in one day? The speed of light is 186,000 mi/s. Write the answer in scientific notation.

Strategy

To find the distance traveled:

- Write the speed of light in scientific notation.
- Write the number of seconds in one day in scientific notation.
- Use the equation $d = rt$, where r is the speed of light and t is the number of seconds in one day.

Solution

$r = 186{,}000 = 1.86 \times 10^5$

$t = 24 \cdot 60 \cdot 60 = 86{,}400 = 8.64 \times 10^4$

$d = rt$

$d = (1.86 \times 10^5)(8.64 \times 10^4)$

$= 1.86 \times 8.64 \times 10^9$

$= 16.0704 \times 10^9$

$= 1.60704 \times 10^{10}$

Light travels 1.60704×10^{10} mi in one day.

You Try It 10

A computer can do an arithmetic operation in 1×10^{-7} s. In scientific notation, how many arithmetic operations can the computer perform in 1 min?

Your strategy

Your solution

Solution on p. S18

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6.1 Exercises

Objective A

1. Which expression is the multiplication of two exponential expressions and which is the power of an exponential expression?
   a. $q^4 \cdot q^{10}$ **b.** $(q^4)^{10}$

2. In the expression $(a^8b^6)^5$, what is the product and what is the power?

Simplify.

3. $(ab^3)(a^3b)$
4. $(-2ab^4)(-3a^2b^4)$
5. $(9xy^2)(-2x^2y^2)$
6. $(x^2y)^2$
7. $(x^2y^4)^4$
8. $(-2ab^2)^3$
9. $(-3x^2y^3)^4$
10. $(2^2a^2b^3)^3$
11. $(3^3a^5b^3)^2$
12. $(xy)(x^2y)^4$
13. $(x^2y^2)(xy^3)^3$
14. $[(2x)^4]^2$
15. $[(3x)^3]^2$
16. $[(x^2y)^4]^5$
17. $[(ab)^3]^6$
18. $[(2ab)^3]^2$
19. $[(2xy)^3]^4$
20. $[(3x^2y^3)^2]^2$
21. $[(2a^4b^3)^3]^2$
22. $y^n \cdot y^{2n}$
23. $x^n \cdot x^{n+1}$
24. $y^{2n} \cdot y^{4n+1}$
25. $y^{3n} \cdot y^{3n-2}$
26. $(a^n)^{2n}$
27. $(a^{n-3})^{2n}$
28. $(y^{2n-1})^3$
29. $(x^{3n+2})^5$
30. $(b^{2n-1})^n$
31. $(2xy)(-3x^2yz)(x^2y^3z^3)$
32. $(x^2z^4)(2xyz^4)(-3x^3y^2)$
33. $(3b^5)(2ab^2)(-2ab^2c^2)$
34. $(-c^3)(-2a^2bc)(3a^2b)$
35. $(-2x^2y^3z)(3x^2yz^4)$
36. $(2a^2b)^3(-3ab^4)^2$
37. $(-3ab^3)^3(-2^2a^2b)^2$
38. $(4ab)^2(-2ab^2c^3)^3$
39. $(-2ab^2)(-3a^4b^5)^3$

Objective B

Simplify.

40. 2^{-3}
41. $\dfrac{1}{3^{-5}}$
42. $\dfrac{1}{x^{-4}}$
43. $\dfrac{1}{y^{-3}}$

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44. $\dfrac{2x^{-2}}{y^4}$ **45.** $\dfrac{a^3}{4b^{-2}}$ **46.** $x^{-3}y$ **47.** xy^{-4}

48. $-5x^0$ **49.** $\dfrac{1}{2x^0}$ **50.** $\dfrac{(2x)^0}{-2^3}$ **51.** $\dfrac{-3^{-2}}{(2y)^0}$

52. $\dfrac{y^{-7}}{y^{-8}}$ **53.** $\dfrac{y^{-2}}{y^6}$ **54.** $(x^2y^{-4})^2$ **55.** $(x^3y^5)^{-2}$

56. $\dfrac{x^{-2}y^{-11}}{xy^{-2}}$ **57.** $\dfrac{x^4y^3}{x^{-1}y^{-2}}$ **58.** $\dfrac{a^{-1}b^{-3}}{a^4b^{-5}}$ **59.** $\dfrac{a^6b^{-4}}{a^{-2}b^5}$

60. $(2a^{-1})^{-2}(2a^{-1})^4$ **61.** $(3a)^{-3}(9a^{-1})^{-2}$ **62.** $(x^{-2}y)^2(xy)^{-2}$ **63.** $(x^{-1}y^2)^{-3}(x^2y^{-4})^{-3}$

64. $\dfrac{50b^{10}}{70b^5}$ **65.** $\dfrac{x^3y^6}{x^6y^2}$ **66.** $\dfrac{x^{17}y^5}{-x^7y^{10}}$ **67.** $\dfrac{-6x^2y}{12x^4y}$

68. $\dfrac{2x^2y^4}{(3xy^2)^3}$ **69.** $\dfrac{-3ab^2}{(9a^2b^4)^3}$ **70.** $\left(\dfrac{-12a^2b^3}{9a^5b^9}\right)^3$ **71.** $\left(\dfrac{12x^3y^2z}{18xy^3z^4}\right)^4$

72. $\dfrac{(4x^2y)^2}{(2xy^3)^3}$ **73.** $\dfrac{(3a^2b)^3}{(-6ab^3)^2}$ **74.** $\dfrac{(-4x^2y^3)^2}{(2xy^2)^3}$ **75.** $\dfrac{(-3a^2b^3)^2}{(-2ab^4)^3}$

76. $\dfrac{(-4xy^3)^3}{(-2x^7y)^4}$ **77.** $\dfrac{(-8x^2y^2)^4}{(16x^3y^7)^2}$ **78.** $\dfrac{a^{5n}}{a^{3n}}$ **79.** $\dfrac{b^{6n}}{b^{10n}}$

80. $\dfrac{-x^{5n}}{x^{2n}}$ **81.** $\dfrac{y^{2n}}{-y^{8n}}$ **82.** $\dfrac{x^{2n-1}}{x^{n-3}}$ **83.** $\dfrac{y^{3n+2}}{y^{2n+4}}$

84. $\dfrac{a^{3n}b^n}{a^nb^{2n}}$ **85.** $\dfrac{x^ny^{3n}}{x^ny^{5n}}$ **86.** $\dfrac{a^{3n-2}b^{n+1}}{a^{2n+1}b^{2n+2}}$ **87.** $\dfrac{x^{2n-1}y^{n-3}}{x^{n+4}y^{n+3}}$

88. $\left(\dfrac{4^{-2}xy^{-3}}{x^{-3}y}\right)^3\left(\dfrac{8^{-1}x^{-2}y}{x^4y^{-1}}\right)^{-2}$ **89.** $\left(\dfrac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\dfrac{3a^{-2}b}{2a^2b^{-2}}\right)^3$ **90.** $\left(\dfrac{2ab^{-1}}{ab}\right)^{-1}\left(\dfrac{3a^{-2}b}{a^2b^2}\right)^{-2}$

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Objective C

Write in scientific notation.

91. 0.00000467 **92.** 0.00000005 **93.** 0.00000000017

94. 4,300,000 **95.** 200,000,000,000 **96.** 9,800,000,000

Write in decimal notation.

97. 1.23×10^{-7} **98.** 6.2×10^{-12} **99.** 8.2×10^{15}

100. 6.34×10^{5} **101.** 3.9×10^{-2} **102.** 4.35×10^{9}

Simplify. Write the answer in decimal notation.

103. $(3 \times 10^{-12})(5 \times 10^{16})$ **104.** $(8.9 \times 10^{-5})(3.2 \times 10^{-6})$

105. (0.0000065)(3,200,000,000,000) **106.** (480,000)(0.0000000096)

107. $\dfrac{9 \times 10^{-3}}{6 \times 10^{5}}$ **108.** $\dfrac{2.7 \times 10^{4}}{3 \times 10^{-6}}$ **109.** $\dfrac{0.0089}{500{,}000{,}000}$

110. $\dfrac{4800}{0.00000024}$ **111.** $\dfrac{0.00056}{0.000000000004}$ **112.** $\dfrac{0.000000346}{0.0000005}$

113. $\dfrac{(3.3 \times 10^{-11})(2.7 \times 10^{15})}{8.1 \times 10^{-3}}$ **114.** $\dfrac{(6.9 \times 10^{27})(8.2 \times 10^{-13})}{4.1 \times 10^{15}}$

115. $\dfrac{(0.00000004)(84{,}000)}{(0.0003)(1{,}400{,}000)}$ **116.** $\dfrac{(720)(0.0000000039)}{(26{,}000{,}000{,}000)(0.18)}$

Objective D *Application Problems*

Solve. Write the answer in scientific notation.

117. How many kilometers does light travel in one day? The speed of light is 300,000 km/s.

118. How many meters does light travel in 8 h? The speed of light is 300,000,000 m/s.

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119. The rover Sojourner landed on Mars in July of 1997. It took 11 min for the commands from a computer on Earth to travel to the Sojourner, a distance of 119 million miles. How fast did the signals from Earth to Mars travel? Write the answer in scientific notation.

120. In the summer of 2001, the federal debt was 3×10^{12} dollars. How much would each American citizen have to contribute in order to pay off the debt? Use 300,000,000 as the number of citizens.

121. A high-speed centrifuge makes 9×10^7 revolutions each minute. Find the time in seconds for the centrifuge to make one revolution.

122. How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7 mi from Earth, and light travels 1.86×10^5 mi/s.

123. The mass of Earth is 5.9×10^{27} g. The mass of the sun is 2×10^{33} g. How many times heavier is the sun than Earth?



124. One astronomical unit (A.U.) is 9.3×10^7 mi. The star Pollux in the constellation Gemini is 1.8228×10^{12} mi from Earth. Find the distance from Pollux to Earth in astronomical units.

125. The weight of 31 million orchid seeds is one ounce. Find the weight of one orchid seed.

126. One light year, an astronomical unit of distance, is the distance that light will travel in one year. Light travels 1.86×10^5 mi/s. Find the measure of one light year in miles. Use a 365-day year.

127. The Coma cluster of galaxies is approximately 2.8×10^8 light years from Earth. Find the distance, in miles, from the Coma cluster to Earth. (See Exercise 126 for the definition of one light year. Use a 365-day year.)

APPLYING THE CONCEPTS

128. Simplify.

a. $1 + (1 + (1 + 2^{-1})^{-1})^{-1}$ **b.** $2 - (2 - (2 - 2^{-1})^{-1})^{-1}$

129. Correct the error in each of the following expressions. Explain which rule or property was used incorrectly.

a. $2x + 3x = 5x^2$ **b.** $a - (b - c) = a - b - c$

c. $x^0 = 0$ **d.** $(x^4)^5 = x^9$

e. $x^2 \cdot x^3 = x^6$ **f.** $b^{m+n} = b^m + b^n$

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6.2 Introduction to Polynomials

Objective A

To evaluate polynomial functions

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TAKE NOTE
An analogy may help you understand these terms. Polynomial is similar to the word *car*; Chevrolet and Ford are types of cars, just as monomials and binomials are types of polynomials.

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of one term is a **monomial**. $5x$

A polynomial of two terms is a **binomial**. $5x^2y + 6x$

A polynomial of three terms is a **trinomial**. $3x^2 + 9xy - 5y$

Polynomials with more than three terms do not have special names.

The **degree of a polynomial** is the greatest of the degrees of any of its terms.

$3x + 2$	degree 1
$3x^2 + 2x - 4$	degree 2
$4x^3y^2 + 6x^4$	degree 5
$3x^{2n} - 5x^n - 2$	degree $2n$

The terms of a polynomial in one variable are usually arranged so that the exponents of the variable decrease from left to right. This is called **descending order**.

$2x^2 - x + 8$

$3y^3 - 3y^2 + y - 12$

For a polynomial in more than one variable, descending order may refer to any one of the variables.

The polynomial at the right is shown first in descending order of the x variable and then in descending order of the y variable.

$2x^2 + 3xy + 5y^2$

$5y^2 + 3xy + 2x^2$

Polynomial functions have many applications in mathematics. In general, a **polynomial function** is an expression whose terms are monomials. The **linear function** given by $f(x) = mx + b$ is an example of a polynomial function. It is a polynomial function of degree one. A second-degree polynomial function, called a **quadratic function**, is given by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. A third-degree polynomial function is called a **cubic function**.

To evaluate a polynomial function, replace the variable by its value and simplify.

Given $P(x) = x^3 - 3x^2 + 4$, evaluate $P(-3)$.

$$\begin{aligned} P(x) &= x^3 - 3x^2 + 4 \\ P(-3) &= (-3)^3 - 3(-3)^2 + 4 \qquad \text{• Substitute } -3 \text{ for } x \text{ and simplify.} \\ &= -27 - 3(9) + 4 \\ &= -27 - 27 + 4 \\ &= -50 \end{aligned}$$

The **leading coefficient** of a polynomial function is the coefficient of the variable with the largest exponent. The constant term is the term without a variable.

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Find the leading coefficient, the constant term, and the degree of the polynomial function $P(x) = 7x^4 - 3x^2 + 2x - 4$.

The leading coefficient is 7, the constant term is -4, and the degree is 4.

The three equations below do not represent polynomial functions.

$f(x) = 3x^2 + 2x^{-1}$ — A polynomial function does not have a variable raised to a negative power.

$g(x) = 2\sqrt{x} - 3$ — A polynomial function does not have a variable expression within a radical.

$h(x) = \dfrac{x}{x - 1}$ — A polynomial function does not have a variable in the denominator of a fraction.

The graph of a linear function is a straight line and can be found by plotting just two points. The graph of a polynomial function of degree greater than one is a curve. Consequently, many points may have to be found before an accurate graph can be drawn.

Evaluating the quadratic function given by the equation $f(x) = x^2 - x - 6$ when $x = -3, -2, -1, 0, 1, 2, 3$, and 4 gives the points shown in Figure 1 below. For instance, $f(-3) = 6$, so $(-3, 6)$ is graphed; $f(2) = -4$, so $(2, -4)$ is graphed; and $f(4) = 6$, so $(4, 6)$ is graphed. Evaluating the function when x is not an integer, such as $x = -\frac{3}{2}$ and $x = \frac{5}{2}$, produces more points to graph, as shown in Figure 2. Connecting the points with a smooth curve results in Figure 3, which is the graph of f.

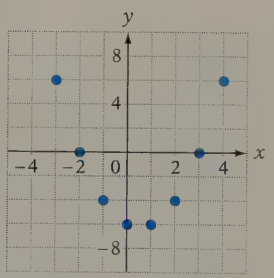


Figure 1

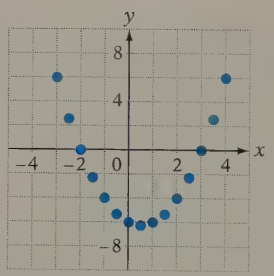


Figure 2

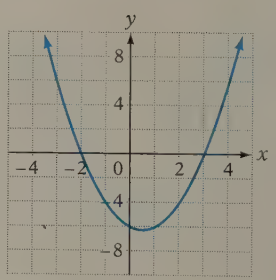


Figure 3



You can verify the graphs of these polynomial functions by using a graphing calculator. See the Appendix: Guidelines for Using a Graphing Calculator for instructions on graphing functions.

Here is an example of graphing a cubic function, $P(x) = x^3 - 2x^2 - 5x + 6$. Evaluating the function when $x = -2, -1, 0, 1, 2, 3$, and 4 gives the graph in Figure 4 below. Evaluating at some noninteger values gives the graph in Figure 5. Finally, connecting the dots with a smooth curve gives the graph in Figure 6.

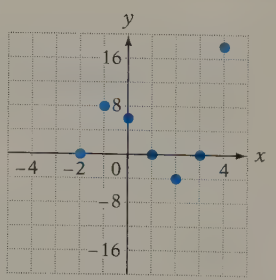


Figure 4

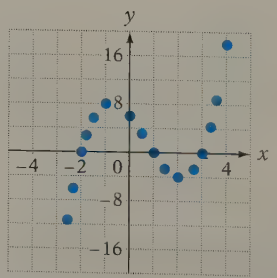


Figure 5

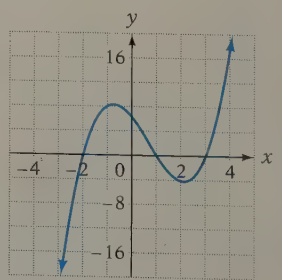


Figure 6

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Example 1

Given $P(x) = x^3 + 3x^2 - 2x + 8$, evaluate $P(-2)$.

Solution

$$\begin{aligned} P(x) &= x^3 + 3x^2 - 2x + 8 \\ P(-2) &= (-2)^3 + 3(-2)^2 - 2(-2) + 8 \\ &= (-8) + 3(4) + 4 + 8 \\ &= -8 + 12 + 4 + 8 \\ &= 16 \end{aligned}$$

You Try It 1

Given $R(x) = -2x^4 - 5x^3 + 2x - 8$, evaluate $R(2)$.

Your solution

Example 2

Find the leading coefficient, the constant term, and the degree of the polynomial.

$P(x) = 5x^6 - 4x^5 - 3x^2 + 7$

Solution

The leading coefficient is 5, the constant term is 7, and the degree is 6.

You Try It 2

Find the leading coefficient, the constant term, and the degree of the polynomial.

$r(x) = -3x^4 + 3x^3 + 3x^2 - 2x - 12$

Your solution

Example 3

Which of the following is a polynomial function?

a. $P(x) = 3x^{\frac{1}{2}} + 21x^2 - 3$

b. $T(x) = 3\sqrt{x} - 2x^2 - 3x + 2$

c. $R(x) = 14x^3 - \pi x^2 + 3x + 2$

Solution

a. This is not a polynomial function. A polynomial function does not have a variable raised to a fractional power.

b. This is not a polynomial function. A polynomial function does not have a variable expression within a radical.

c. This is a polynomial function.

You Try It 3

Which of the following is a polynomial function?

a. $R(x) = 5x^{14} - 5$

b. $V(x) = -x^{-1} + 2x - 7$

c. $P(x) = 2x^4 - 3\sqrt{x} - 3$

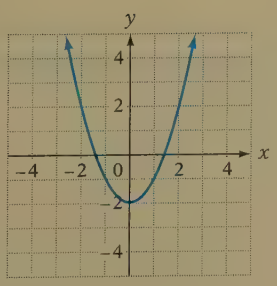
Your solution

Example 4

Graph $f(x) = x^2 - 2$.

Solution

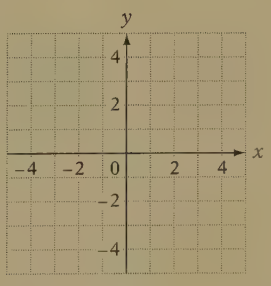
x	$y = f(x)$
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7



You Try It 4

Graph $f(x) = x^2 + 2x - 3$.

Your solution



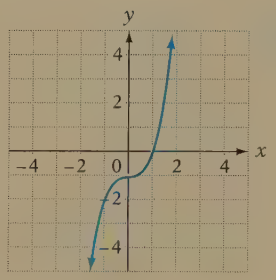
Solutions on p. S18

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Example 5
Graph $f(x) = x^3 - 1$.

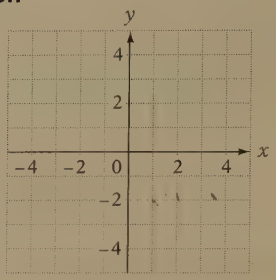
Solution

x	$y = f(x)$
-2	-9
-1	-2
0	-1
1	0
2	7



You Try It 5
Graph $f(x) = -x^3 + 1$.

Your solution



Solution on p. S18

Objective B

To add or subtract polynomials

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Polynomials can be added by combining like terms. Either a vertical or a horizontal format can be used.

Add $(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$. Use a horizontal format.

$$(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$$
$$= 3x^3 + 7x^2 + (-7x + 2x) + (2 - 7)$$
$$= 3x^3 + 7x^2 - 5x - 5$$

- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.
- Combine like terms.

Add $(4x^2 + 5x - 3) + (7x^3 - 7x + 1) + (2x - 3x^2 + 4x^3 + 1)$. Use a vertical format.

$$\begin{array}{r} 4x^2 + 5x - 3 \\ 7x^3 \quad - 7x + 1 \\ 4x^3 - 3x^2 + 2x + 1 \\ \hline 11x^3 + x^2 \quad - 1 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Add the terms in each column.

TAKE NOTE
The opposite of a polynomial is the polynomial with the sign of every term changed.

The additive inverse of the polynomial $x^2 + 5x - 4$ is $-(x^2 + 5x - 4)$.

To simplify the additive inverse of a polynomial, change the sign of every term inside the parentheses.

$$-(x^2 + 5x - 4) = -x^2 - 5x + 4$$

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TAKE NOTE
This is the same definition used for subtraction of integers: Subtraction is addition of the opposite.

To subtract two polynomials, add the additive inverse of the second polynomial to the first.

Subtract $(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2)$. Use a horizontal format.

$$\begin{aligned}(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2) &= (3x^2 - 7xy + y^2) + (4x^2 - 7xy + 3y^2) \\ &= 7x^2 - 14xy + 4y^2\end{aligned}$$

- Rewrite the subtraction as the addition of the additive inverse.
- Combine like terms.

Subtract $(6x^3 - 3x + 7) - (3x^2 - 5x + 12)$. Use a vertical format.

$$\begin{aligned}(6x^3 - 3x + 7) - (3x^2 - 5x + 12) &= (6x^3 - 3x + 7) + (-3x^2 + 5x - 12)\end{aligned}$$

- Rewrite subtraction as the addition of the additive inverse.

$$\begin{array}{r} 6x^3 \quad - 3x + 7 \\ -3x^2 + 5x - 12 \\ \hline 6x^3 - 3x^2 + 2x - 5 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Combine the terms in each column.

Functional notation can be used when adding or subtracting polynomials.

Given $P(x) = 3x^2 - 2x + 4$ and $R(x) = -5x^3 + 4x + 7$, find $P(x) + R(x)$.

$$\begin{aligned}P(x) + R(x) &= (3x^2 - 2x + 4) + (-5x^3 + 4x + 7) \\ &= -5x^3 + 3x^2 + 2x + 11\end{aligned}$$

Given $P(x) = -5x^2 + 8x - 4$ and $R(x) = -3x^2 - 5x + 9$, find $P(x) - R(x)$.

$$\begin{aligned}P(x) - R(x) &= (-5x^2 + 8x - 4) - (-3x^2 - 5x + 9) \\ &= (-5x^2 + 8x - 4) + (3x^2 + 5x - 9) \\ &= -2x^2 + 13x - 13\end{aligned}$$

Given $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$, find $S(x)$, the sum of the two polynomials.

$$\begin{aligned}S(x) = P(x) + R(x) &= (3x^2 - 5x + 6) + (2x^2 - 5x - 7) \\ &= 5x^2 - 10x - 1\end{aligned}$$

Note that evaluating $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$ at, for example, $x = 3$, and then adding the values, is the same as evaluating $S(x) = 5x^2 - 10x - 1$ at 3.

$P(3) = 3(3)^2 - 5(3) + 6 = 27 - 15 + 6 = 18$

$R(3) = 2(3)^2 - 5(3) - 7 = 18 - 15 - 7 = -4$

$P(3) + R(3) = 18 + (-4) = 14$

$S(3) = 5(3)^2 - 10(3) - 1 = 45 - 30 - 1 = 14$

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Example 6

Add
$(4x^2 - 3xy + 7y^2) + (-3x^2 + 7xy + y^2)$.
Use a vertical format.

Solution

$$\begin{array}{r} 4x^2 - 3xy + 7y^2 \\ -3x^2 + 7xy + y^2 \\ \hline x^2 + 4xy + 8y^2 \end{array}$$

You Try It 6

Add
$(-3x^2 - 4x + 9) + (-5x^2 - 7x + 1)$.
Use a vertical format.

Your solution

Example 7

Subtract
$(3x^2 - 2x + 4) - (7x^2 + 3x - 12)$.
Use a vertical format.

Solution

Add the additive inverse of $7x^2 + 3x - 12$ to $3x^2 - 2x + 4$.

$$\begin{array}{r} 3x^2 - 2x + 4 \\ -7x^2 - 3x + 12 \\ \hline -4x^2 - 5x + 16 \end{array}$$

You Try It 7

Subtract
$(-5x^2 + 2x - 3) - (6x^2 + 3x - 7)$.
Use a vertical format.

Your solution

Example 8

Given $P(x) = -3x^2 + 2x - 6$ and $R(x) = 4x^3 - 3x + 4$, find $S(x) = P(x) + R(x)$. Evaluate $S(-2)$.

Solution

$$\begin{aligned} S(x) &= P(x) + R(x) \\ &= (-3x^2 + 2x - 6) + (4x^3 - 3x + 4) \\ &= 4x^3 - 3x^2 - x - 2 \end{aligned}$$

$$\begin{aligned} S(-2) &= 4(-2)^3 - 3(-2)^2 - (-2) - 2 \\ &= 4(-8) - 3(4) - (-2) - 2 \\ &= -32 - 12 + 2 - 2 \\ &= -44 \end{aligned}$$

You Try It 8

Given $P(x) = 4x^3 - 3x^2 + 2$ and $R(x) = -2x^2 + 2x - 3$, find $S(x) = P(x) + R(x)$. Evaluate $S(-1)$.

Your solution

Example 9

Given $P(x) = 2x^{2n} - 3x^n + 7$ and $R(x) = 3x^{2n} + 3x^n + 5$, find $D(x) = P(x) - R(x)$.

Solution

$$\begin{aligned} D(x) &= P(x) - R(x) \\ &= (2x^{2n} - 3x^n + 7) - (3x^{2n} + 3x^n + 5) \\ &= (2x^{2n} - 3x^n + 7) + (-3x^{2n} - 3x^n - 5) \\ &= -x^{2n} - 6x^n + 2 \end{aligned}$$

You Try It 9

Given $P(x) = 5x^{2n} - 3x^n - 7$ and $R(x) = -2x^{2n} - 5x^n + 8$, find $D(x) = P(x) - R(x)$.

Your solution

Solutions on p. S18

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6.2 Exercises

Objective A

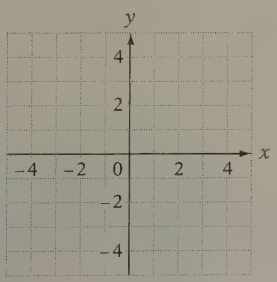
1. Given $P(x) = 3x^2 - 2x - 8$, evaluate $P(3)$.
2. Given $P(x) = -3x^2 - 5x + 8$, evaluate $P(-5)$.
3. Given $R(x) = 2x^3 - 3x^2 + 4x - 2$, evaluate $R(2)$.
4. Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.
5. Given $f(x) = x^4 - 2x^2 - 10$, evaluate $f(-1)$.
6. Given $f(x) = x^5 - 2x^3 + 4x$, evaluate $f(2)$.

Which of the following define a polynomial function? For those that are polynomial functions, identify **a.** the leading coefficient, **b.** the constant term, and **c.** the degree.

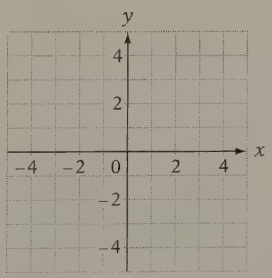
7. $P(x) = -x^2 + 3x + 8$
8. $P(x) = 3x^4 - 3x - 7$
9. $R(x) = \dfrac{x}{x+1}$
10. $R(x) = \dfrac{3x^2 - 2x + 1}{x}$
11. $f(x) = \sqrt{x} - x^2 + 2$
12. $f(x) = x^2 - \sqrt{x+2} - 8$
13. $g(x) = 3x^5 - 2x^2 + \pi$
14. $g(x) = -4x^5 - 3x^2 + x - \sqrt{7}$
15. $P(x) = 3x^2 - 5x^3 + 2$
16. $P(x) = x^2 - 5x^4 - x^6$
17. $R(x) = 14$
18. $R(x) = \dfrac{1}{x} + 2$

Graph.

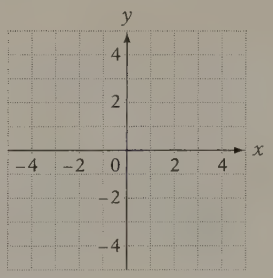
19. $P(x) = x^2 - 1$



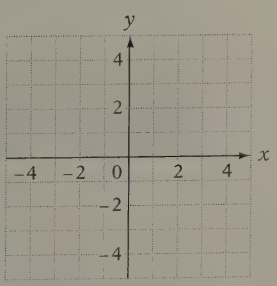
20. $P(x) = 2x^2 + 3$



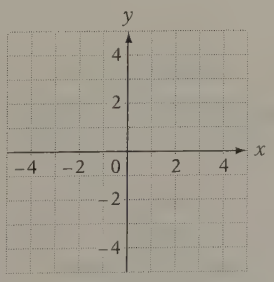
21. $R(x) = x^3 + 2$



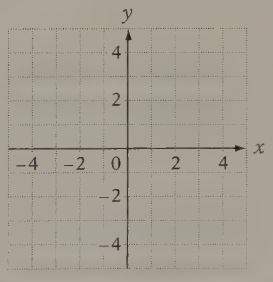
22. $R(x) = x^4 + 1$



23. $f(x) = x^3 - 2x$



24. $f(x) = x^2 - x - 2$



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Objective B

Add or subtract. Use a vertical format.

25. $(5x^2 + 2x - 7) + (x^2 - 8x + 12)$

26. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

27. $(x^2 - 3x + 8) - (2x^2 - 3x + 7)$

28. $(2x^2 + 3x - 7) - (5x^2 - 8x - 1)$

Add or subtract. Use a horizontal format for Exercises 29–32.

29. $(3y^2 - 7y) + (2y^2 - 8y + 2)$

30. $(-2y^2 - 4y - 12) + (5y^2 - 5y)$

31. $(2a^2 - 3a - 7) - (-5a^2 - 2a - 9)$

32. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$

33. Given $P(x) = x^2 - 3x + 1$ and $R(x) = 2x^2 - 3$, find $P(x) + R(x)$.

34. Given $P(x) = x^{2n} + 7x^n - 3$ and $R(x) = -x^{2n} + 2x^n + 8$, find $P(x) + R(x)$.

35. Given $P(x) = 3x^2 + 2$ and $R(x) = -5x^2 + 2x - 3$, find $P(x) - R(x)$.

36. Given $P(x) = 2x^{2n} - x^n - 1$ and $R(x) = 5x^{2n} + 7x^n + 1$, find $P(x) - R(x)$.

37. Given $P(x) = 3x^4 - 3x^3 - x^2$ and $R(x) = 3x^3 - 7x^2 + 2x$, find $S(x) = P(x) + R(x)$. Evaluate $S(2)$.

38. Given $P(x) = 3x^4 - 2x + 1$ and $R(x) = 3x^5 - 5x - 8$, find $S(x) = P(x) + R(x)$. Evaluate $S(-1)$.

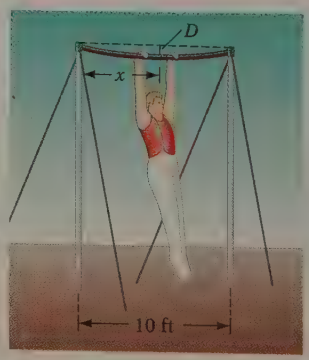
APPLYING THE CONCEPTS

39. For what value of k is the given equation an identity?

a. $(2x^3 + 3x^2 + kx + 5) - (x^3 + 2x^2 + 3x + 7) = x^3 + x^2 + 5x - 2$

b. $(6x^3 + kx^2 - 2x - 1) - (4x^3 - 3x^2 + 1) = 2x^3 - x^2 - 2x - 2$

40. The deflection D (in inches) of a beam that is uniformly loaded is given by the polynomial function $D(x) = 0.005x^4 - 0.1x^3 + 0.5x^2$, where x is the distance in feet from one end of the beam. See the figure at the right. The maximum deflection occurs when x is the midpoint of the beam. Determine the maximum deflection for the beam in the diagram.



41. If $P(x)$ is a third-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) + Q(x)$? Give some examples of polynomials that support your answer.

42. If $P(x)$ is a fifth-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) - Q(x)$? Give some examples of polynomials that support your answer.

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6.3 Multiplication of Polynomials

Objective A — To multiply a polynomial by a monomial



To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

➡ Multiply: $-3a(4a^2 - 5a + 6)$

$$-3a(4a^2 - 5a + 6) = \boxed{-3a(4a^2) - (-3a)(5a) + (-3a)(6)}$$
$$= -12a^3 + 15a^2 - 18a$$

- Use the Distributive Property. This step is frequently done mentally.

Example 1
Multiply: $(5x + 4)(-2x)$

Solution
$(5x + 4)(-2x) = -10x^2 - 8x$

You Try It 1
Multiply: $(-2y + 3)(-4y)$

Your solution

Example 2
Multiply: $2a^2b(4a^2 - 2ab + b^2)$

Solution
$2a^2b(4a^2 - 2ab + b^2)$
$= 8a^4b - 4a^3b^2 + 2a^2b^3$

You Try It 2
Multiply: $-a^2(3a^2 + 2a - 7)$

Your solution

Solutions on p. S18

Objective B — To multiply two polynomials



Multiplication of two polynomials requires the repeated application of the Distributive Property.

$$(y - 2)(y^2 + 3y + 1) = (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$$
$$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$$
$$= y^3 + y^2 - 5y - 2$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

$$\begin{array}{r} y^2 + 3y + 1 \\ \underline{\qquad\quad y - 2} \\ -2y^2 - 6y - 2 \\ \underline{y^3 + 3y^2 + y\quad\;\;} \\ y^3 + y^2 - 5y - 2 \end{array}$$

- Multiply $-2(y^2 + 3y + 1)$.
- Multiply $y(y^2 + 3y + 1)$.
- Add the terms in each column.

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➡ Multiply: $(2a^3 + a - 3)(a + 5)$

$$\begin{array}{r} 2a^3 + a - 3 \\ a + 5 \\ \hline 10a^3 + 5a - 15 \\ 2a^4 + a^2 - 3a \\ \hline 2a^4 + 10a^3 + a^2 + 2a - 15 \end{array}$$

- Note that spaces are provided in each product so that like terms are in the same column.
- Add the terms in each column.

Example 3

Multiply: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{r} 2b^3 - b + 1 \\ 2b + 3 \\ \hline 6b^3 - 3b + 3 \\ 4b^4 - 2b^2 + 2b \\ \hline 4b^4 + 6b^3 - 2b^2 - b + 3 \end{array}$$

You Try It 3

Multiply: $(2y^3 + 2y^2 - 3)(3y - 1)$

Your solution

Solution on p. S19

Objective C — To multiply two binomials



It is frequently necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property. The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast.

TAKE NOTE

FOIL is not really a different way of multiplying. It is based on the Distributive Property.

$(2x + 3)(x + 5)$
$= 2x(x + 5) + 3(x + 5)$
F O I L
$= 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

FOIL is an efficient way to remember how to do binomial multiplication.

➡ Multiply: $(2x + 3)(x + 5)$

Multiply the First terms.	$(2x + 3)(x + 5)$	$2x \cdot x = 2x^2$
Multiply the Outer terms.	$(2x + 3)(x + 5)$	$2x \cdot 5 = 10x$
Multiply the Inner terms.	$(2x + 3)(x + 5)$	$3 \cdot x = 3x$
Multiply the Last terms.	$(2x + 3)(x + 5)$	$3 \cdot 5 = 15$
Add the products.	$(2x + 3)(x + 5)$	F O I L $= 2x^2 + 10x + 3x + 15$
Combine like terms.		$= 2x^2 + 13x + 15$

➡ Multiply: $(4x - 3)(3x - 2)$

$$\begin{aligned} (4x - 3)(3x - 2) &= \boxed{4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2)} \\ &= 12x^2 - 8x - 9x + 6 \\ &= 12x^2 - 17x + 6 \end{aligned}$$

- Do this step mentally.

➡ Multiply: $(3x - 2y)(x + 4y)$

$$\begin{aligned} (3x - 2y)(x + 4y) &= \boxed{3x(x) + 3x(4y) + (-2y)(x) + (-2y)(4y)} \\ &= 3x^2 + 12xy - 2xy - 8y^2 \\ &= 3x^2 + 10xy - 8y^2 \end{aligned}$$

- Do this step mentally.

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Example 4

Multiply: $(2a - 1)(3a - 2)$

Solution

$$(2a - 1)(3a - 2) = 6a^2 - 4a - 3a + 2$$
$$= 6a^2 - 7a + 2$$

You Try It 4

Multiply: $(4y - 5)(2y - 3)$

Your solution

Example 5

Multiply: $(3x - 2)(4x + 3)$

Solution

$$(3x - 2)(4x + 3) = 12x^2 + 9x - 8x - 6$$
$$= 12x^2 + x - 6$$

You Try It 5

Multiply: $(3b + 2)(3b - 5)$

Your solution

Solutions on p. S19

Objective D

To multiply binomials that have special products

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Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

The Sum and Difference of Two Terms

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Square of the first term
Square of the second term

The Square of a Binomial

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Square of the first term
Twice the product of the two terms
Square of the last term

➡ Multiply: $(2x + 3)(2x - 3)$

$(2x + 3)(2x - 3)$ is the sum and difference of two terms.

$$(2x + 3)(2x - 3) = \boxed{(2x)^2 - 3^2}$$
$$= 4x^2 - 9$$

• Do this step mentally.

➡ Multiply: $(3x - 2)^2$

$(3x - 2)^2$ is the square of a binomial.

$$(3x - 2)^2 = \boxed{(3x)^2 + 2(3x)(-2) + (-2)^2}$$
$$= 9x^2 - 12x + 4$$

• Do this step mentally.

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Example 6

Multiply: $(4z - 2w)(4z + 2w)$

Solution

$(4z - 2w)(4z + 2w) = 16z^2 - 4w^2$

You Try It 6

Multiply: $(2a + 5c)(2a - 5c)$

Your solution

Example 7

Simplify: $(2r - 3s)^2$

Solution

$(2r - 3s)^2 = 4r^2 - 12rs + 9s^2$

You Try It 7

Simplify: $(3x + 2y)^2$

Your solution

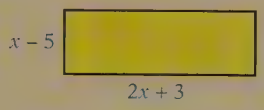
Solutions on p. S19

Objective C To solve application problems

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Example 8

The length of a rectangle is $(2x + 3)$ ft. The width is $(x - 5)$ ft. Find the area of the rectangle in terms of the variable x.



Strategy

To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.

Solution

$A = L \cdot W$

$A = (2x + 3)(x - 5)$

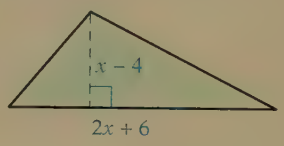
$= 2x^2 - 10x + 3x - 15$

$= 2x^2 - 7x - 15$

The area is $(2x^2 - 7x - 15)$ ft^2.

You Try It 8

The base of a triangle is $(2x + 6)$ ft. The height is $(x - 4)$ ft. Find the area of the triangle in terms of the variable x.



Your strategy

Your solution

Solution on p. S19

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6.3 Exercises

Objective A

Multiply.

1. $x(x - 2)$
2. $y(3 - y)$
3. $-x(x + 7)$
4. $-y(7 - y)$
5. $3a^2(a - 2)$
6. $4b^2(b + 8)$
7. $-5x^2(x^2 - x)$
8. $-6y^2(y + 2y^2)$
9. $-x^3(3x^2 - 7)$
10. $-y^4(2y^2 - y^6)$
11. $2x(6x^2 - 3x)$
12. $3y(4y - y^2)$
13. $(2x - 4)3x$
14. $(3y - 2)y$
15. $(3x + 4)x$
16. $(2x + 1)2x$
17. $-xy(x^2 - y^2)$
18. $-x^2y(2xy - y^2)$
19. $x(2x^3 - 3x + 2)$
20. $y(-3y^2 - 2y + 6)$
21. $-a(-2a^2 - 3a - 2)$
22. $-b(5b^2 + 7b - 35)$
23. $x^2(3x^4 - 3x^2 - 2)$
24. $y^3(-4y^3 - 6y + 7)$
25. $2y^2(-3y^2 - 6y + 7)$
26. $4x^2(3x^2 - 2x + 6)$
27. $(a^2 + 3a - 4)(-2a)$
28. $(b^3 - 2b + 2)(-5b)$
29. $-3y^2(-2y^2 + y - 2)$
30. $-5x^2(3x^2 - 3x - 7)$
31. $xy(x^2 - 3xy + y^2)$
32. $ab(2a^2 - 4ab - 6b^2)$

Objective B

Multiply.

33. $(x^2 + 3x + 2)(x + 1)$
34. $(x^2 - 2x + 7)(x - 2)$
35. $(a^2 - 3a + 4)(a - 3)$

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36. $(x^2 - 3x + 5)(2x - 3)$ **37.** $(-2b^2 - 3b + 4)(b - 5)$ **38.** $(-a^2 + 3a - 2)(2a - 1)$

39. $(-2x^2 + 7x - 2)(3x - 5)$ **40.** $(-a^2 - 2a + 3)(2a - 1)$ **41.** $(x^2 + 5)(x - 3)$

42. $(y^2 - 2y)(2y + 5)$ **43.** $(x^3 - 3x + 2)(x - 4)$ **44.** $(y^3 + 4y^2 - 8)(2y - 1)$

45. $(5y^2 + 8y - 2)(3y - 8)$ **46.** $(3y^2 + 3y - 5)(4y - 3)$ **47.** $(5a^3 - 5a + 2)(a - 4)$

48. $(3b^3 - 5b^2 + 7)(6b - 1)$ **49.** $(y^3 + 2y^2 - 3y + 1)(y + 2)$ **50.** $(2a^3 - 3a^2 + 2a - 1)(2a - 3)$

Objective C

Multiply.

51. $(x + 1)(x + 3)$ **52.** $(y + 2)(y + 5)$ **53.** $(a - 3)(a + 4)$ **54.** $(b - 6)(b + 3)$

55. $(y + 3)(y - 8)$ **56.** $(x + 10)(x - 5)$ **57.** $(y - 7)(y - 3)$ **58.** $(a - 8)(a - 9)$

59. $(2x + 1)(x + 7)$ **60.** $(y + 2)(5y + 1)$ **61.** $(3x - 1)(x + 4)$ **62.** $(7x - 2)(x + 4)$

63. $(4x - 3)(x - 7)$ **64.** $(2x - 3)(4x - 7)$ **65.** $(3y - 8)(y + 2)$ **66.** $(5y - 9)(y + 5)$

67. $(3x + 7)(3x + 11)$ **68.** $(5a + 6)(6a + 5)$ **69.** $(7a - 16)(3a - 5)$ **70.** $(5a - 12)(3a - 7)$

71. $(3a - 2b)(2a - 7b)$ **72.** $(5a - b)(7a - b)$ **73.** $(a - 9b)(2a + 7b)$

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74. $(2a + 5b)(7a - 2b)$ **75.** $(10a - 3b)(10a - 7b)$ **76.** $(12a - 5b)(3a - 4b)$

77. $(5x + 12y)(3x + 4y)$ **78.** $(11x + 2y)(3x + 7y)$ **79.** $(2x - 15y)(7x + 4y)$

80. $(5x + 2y)(2x - 5y)$ **81.** $(8x - 3y)(7x - 5y)$ **82.** $(2x - 9y)(8x - 3y)$

Objective D

83. What does it mean to square a binomial?

84. Why is $(a + b)^2$ not equal to $a^2 + b^2$?

Multiply.

85. $(y - 5)(y + 5)$ **86.** $(y + 6)(y - 6)$ **87.** $(2x + 3)(2x - 3)$ **88.** $(4x - 7)(4x + 7)$

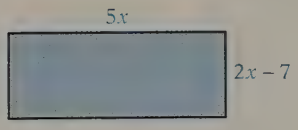
89. $(x + 1)^2$ **90.** $(y - 3)^2$ **91.** $(3a - 5)^2$ **92.** $(6x - 5)^2$

93. $(3x - 7)(3x + 7)$ **94.** $(9x - 2)(9x + 2)$ **95.** $(2a + b)^2$ **96.** $(x + 3y)^2$

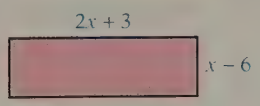
97. $(4 - 3y)(4 + 3y)$ **98.** $(4x - 9y)(4x + 9y)$ **99.** $(5x + 2y)^2$ **100.** $(2a - 9b)^2$

Objective E Application Problems

101. The length of a rectangle is $5x$ ft. The width is $(2x - 7)$ ft. Find the area of the rectangle in terms of the variable x.

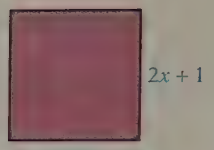


102. The width of a rectangle is $(x - 6)$ m. The length is $(2x + 3)$ m. Find the area of the rectangle in terms of the variable x.



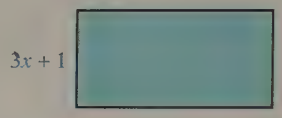
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103. The length of a side of a square is $(2x + 1)$ km. Find the area of the square in terms of the variable x.



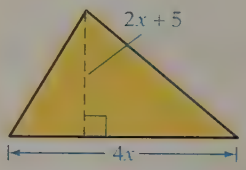
104. The length of a side of a square is $(2x - 3)$ yd. Find the area of the square in terms of the variable x.

105. The width of a rectangle is $(3x + 1)$ in. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

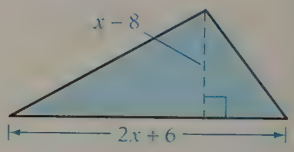


106. The width of a rectangle is $(4x - 3)$ cm. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

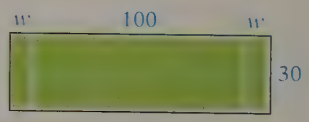
107. The base of a triangle is $4x$ m, and the height is $(2x + 5)$ m. Find the area of the triangle in terms of the variable x.



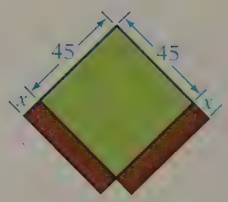
108. The base of a triangle is $(2x + 6)$ in., and the height is $(x - 8)$ in. Find the area of the triangle in terms of the variable x.



109. An athletic field has dimensions 30 yd by 100 yd. An end zone that is w yd wide borders each end of the field. Express the total area of the field and the end zones in terms of the variable w.



110. A softball diamond has dimensions 45 ft by 45 ft. A base path border x ft wide lies on both the first-base side and the third-base side of the diamond. Express the total area of the softball diamond and the base path in terms of the variable x.



APPLYING THE CONCEPTS

Simplify.

111. $(a + b)^2 - (a - b)^2$

112. $(x^2 + x - 3)^2$

113. $(a + 3)^3$

114. **a.** What polynomial has quotient $3x - 4$ when divided by $4x + 5$?
b. What polynomial has quotient $x^2 + 2x - 1$ when divided by $x + 3$?

115. **a.** Add $x^2 + 2x - 3$ to the product of $2x - 5$ and $3x + 1$.
b. Subtract $4x^2 - x - 5$ from the product of $x^2 + x + 3$ and $x - 4$.

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6.4 Division of Polynomials

Objective A

To divide polynomials

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To divide two polynomials, use a method similar to that used for long division of whole numbers. To check division of polynomials, use

Dividend = (quotient × divisor) + remainder

➡ Divide: $(x^2 + 5x - 7) \div (x + 3)$

Step 1

$$x + 3 \overline{)\, x^2 + 5x - 7}$$

$$x^2 + 3x \quad \downarrow$$

$$2x - 7$$

Think: $x \overline{)\, x^2} = \frac{x^2}{x} = x$

Multiply: $x(x + 3) = x^2 + 3x$

Subtract: $(x^2 + 5x) - (x^2 + 3x) = 2x$

Bring down the -7.

Step 2

$$x + 3 \overline{)\, x^2 + 5x - 7}$$

$$\begin{array}{r} x + 2 \\ \hline x + 3 \overline{)\, x^2 + 5x - 7} \\ x^2 + 3x \\ \hline 2x - 7 \\ 2x + 6 \\ \hline -13 \end{array}$$

Think: $x \overline{)\, 2x} = \frac{2x}{x} = 2$

Multiply: $2(x + 3) = 2x + 6$

Subtract: $(2x - 7) - (2x + 6) = -13$

The remainder is -13.

Check: (Quotient)(divisor) + remainder

$$= (x + 2)(x + 3) + (-13) = x^2 + 3x + 2x + 6 - 13 = x^2 + 5x - 7$$

$$(x^2 + 5x - 7) \div (x + 3) = x + 2 - \frac{13}{x + 3}$$

➡ Divide: $\dfrac{6 - 6x^2 + 4x^3}{2x + 3}$

Arrange the terms in descending order. Note that there is no term of x in $4x^3 - 6x^2 + 6$. Insert a $0x$ for the missing term so that like terms will be in the same columns.

$$\begin{array}{r} 2x^2 - 6x + 9 \\ \hline 2x + 3 \overline{)\, 4x^3 - 6x^2 + 0x + 6} \\ 4x^3 + 6x^2 \\ \hline -12x^2 + 0x \\ -12x^2 - 18x \\ \hline 18x + 6 \\ 18x + 27 \\ \hline -21 \end{array}$$

$$\frac{4x^3 - 6x^2 + 6}{2x + 3} = 2x^2 - 6x + 9 - \frac{21}{2x + 3}$$

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Example 1

Divide: $\dfrac{12x^2 - 11x + 10}{4x - 5}$

Solution

$$\begin{array}{r} 3x + 1 \\ 4x - 5\overline{)\,12x^2 - 11x + 10} \\ \underline{12x^2 - 15x} \\ 4x + 10 \\ \underline{4x - 5} \\ 15 \end{array}$$

$$\frac{12x^2 - 11x + 10}{4x - 5} = 3x + 1 + \frac{15}{4x - 5}$$

You Try It 1

Divide: $\dfrac{15x^2 + 17x - 20}{3x + 4}$

Your solution

Example 2

Divide: $\dfrac{x^3 + 1}{x + 1}$

Solution

$$\begin{array}{r} x^2 - x + 1 \\ x + 1\overline{)\,x^3 + 0x^2 + 0x + 1} \\ \underline{x^3 + x^2} \\ -x^2 + 0x \\ \underline{-x^2 - x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

• Insert zeros for the coefficients of the missing terms.

$$\frac{x^3 + 1}{x + 1} = x^2 - x + 1$$

You Try It 2

Divide: $\dfrac{3x^3 + 8x^2 - 6x + 2}{3x - 1}$

Your solution

Example 3

Divide:

$(2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2)$

Solution

$$\begin{array}{r} 2x^2 - 3x + 1 \\ x^2 - 2x - 2\overline{)\,2x^4 - 7x^3 + 3x^2 + 4x - 5} \\ \underline{2x^4 - 4x^3 - 4x^2} \\ -3x^3 + 7x^2 + 4x \\ \underline{-3x^3 + 6x^2 + 6x} \\ x^2 - 2x - 5 \\ \underline{x^2 - 2x - 2} \\ -3 \end{array}$$

$$(2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2)$$

$$= 2x^2 - 3x + 1 - \frac{3}{x^2 - 2x - 2}$$

You Try It 3

Divide:

$(3x^4 - 11x^3 + 16x^2 - 16x + 8) \div (x^2 - 3x + 2)$

Your solution

Solutions on p. S19

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Objective 3 To divide polynomials using synthetic division



Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$.

➡ Divide $(3x^2 - 4x + 6) \div (x - 2)$ by using long division.

$$
\begin{array}{r}
3x + 2 \\
x - 2 \overline{)\,3x^2 - 4x + 6} \\
\underline{3x^2 - 6x} \\
2x + 6 \\
\underline{2x - 4} \\
10
\end{array}
$$

$$(3x^2 - 4x + 6) \div (x - 2) = 3x + 2 + \frac{10}{x - 2}$$

The variables can be omitted because the position of a term indicates the power of the term.

$$
\begin{array}{r}
3 \quad 2 \\
-2 \overline{)\,3 \quad -4 \quad 6} \\
\underline{3 \quad -6} \\
2 \quad 6 \\
\underline{2 \quad -4} \\
10
\end{array}
$$

Each number shown in color above is exactly the same as the number above it. Removing the colored numbers condenses the vertical spacing.

$$
\begin{array}{r}
3 \quad 2 \\
-2 \overline{)\,3 \quad -4 \quad 6} \\
\underline{-6 \quad -4} \\
2 \quad 10
\end{array}
$$

The number in color on the top row is the same as the one in the bottom row. Writing the 3 from the top row in the bottom row allows the spacing to be condensed even further.

$$
\begin{array}{r|rrr}
-2 & 3 & -4 & 6 \\
 & & \underline{-6} & \underline{-4} \\
 & \underbrace{3}_{\text{Terms of the quotient}} & \underbrace{2}_{} & \underbrace{10}_{\text{Remainder}}
\end{array}
$$

Because the degree of the dividend $(3x^2 - 4x + 6)$ is 2 and the degree of the divisor $(x - 2)$ is 1, the degree of the quotient is $2 - 1 = 1$. This means that, using the terms of the quotient given above, the quotient is $3x + 2$. The remainder is 10.

In general, the degree of the quotient of two polynomials is the difference between the degree of the dividend and the degree of the divisor.

By replacing the constant term in the divisor by its additive inverse, we may add rather than subtract terms. This is illustrated in the following example.

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Divide: $(3x^3 + 6x^2 - x - 2) \div (x + 3)$

The additive inverse of the binomial constant

Coefficients of the polynomial

-3	3	6	-1	-2	
	3				• Bring down the 3.
-3	3	6	-1	-2	
		-9			• Multiply $-3(3)$ and add the product to 6.
	3	-3			
-3	3	6	-1	-2	
		-9	9		• Multiply $-3(-3)$ and add the product to -1.
	3	-3	8		
-3	3	6	-1	-2	
		-9	9	-24	• Multiply $-3(8)$ and add the product to -2.
	3	-3	8	-26	

Terms of the quotient — Remainder

TAKE NOTE
Remember that you can check the answer to a synthetic-division problem in the same way that you check the answer to a long-division problem.

The degree of the dividend is 3 and the degree of the divisor is 1. Therefore, the degree of the quotient is $3 - 1 = 2$.

$$(3x^3 + 6x^2 - x - 2) \div (x + 3) = 3x^2 - 3x + 8 - \frac{26}{x + 3}$$

Divide: $(2x^3 - x + 2) \div (x - 2)$

The additive inverse of the binomial constant

Coefficients of the polynomial

2	2	0	-1	2	
	2				• Insert a 0 for the missing term and bring down the 2.
2	2	0	-1	2	
		4			• Multiply $2(2)$ and add the product to 0.
	2	4			
2	2	0	-1	2	
		4	8		• Multiply $2(4)$ and add the product to -1.
	2	4	7		
2	2	0	-1	2	
		4	8	14	• Multiply $2(7)$ and add the product to 2.
	2	4	7	16	

Terms of the quotient — Remainder

$$(2x^3 - x + 2) \div (x - 2) = 2x^2 + 4x + 7 + \frac{16}{x - 2}$$

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Example 4

Divide by using synthetic division:
$(7 - 3x + 5x^2) \div (x - 1)$

Solution

Arrange the coefficients in decreasing powers of x.

$$\begin{array}{r|rrr} 1 & 5 & -3 & 7 \\ & & 5 & 2 \\ \hline & 5 & 2 & 9 \end{array}$$

$$(5x^2 - 3x + 7) \div (x - 1) = 5x + 2 + \frac{9}{x - 1}$$

You Try It 4

Divide by using synthetic division:
$(8x + 6x^2 - 5) \div (x + 2)$

Your solution

Example 5

Divide by using synthetic division:
$(2x^3 + 4x^2 - 3x + 12) \div (x + 4)$

Solution

$$\begin{array}{r|rrrr} -4 & 2 & 4 & -3 & 12 \\ & & -8 & 16 & -52 \\ \hline & 2 & -4 & 13 & -40 \end{array}$$

$$(2x^3 + 4x^2 - 3x + 12) \div (x + 4)$$
$$= 2x^2 - 4x + 13 - \frac{40}{x + 4}$$

You Try It 5

Divide by using synthetic division:
$(5x^3 - 12x^2 - 8x + 16) \div (x - 2)$

Your solution

Example 6

Divide by using synthetic division:
$(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$

Solution

Insert a zero for the missing term.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -8 & 2 & 1 \\ & & -6 & 12 & -8 & 12 \\ \hline & 3 & -6 & 4 & -6 & 13 \end{array}$$

$$(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$$
$$= 3x^3 - 6x^2 + 4x - 6 + \frac{13}{x + 2}$$

You Try It 6

Divide by using synthetic division:
$(2x^4 - 3x^3 - 8x^2 - 2) \div (x - 3)$

Your solution

Solutions on p. S19

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Objective 6

To evaluate a polynomial using synthetic division

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A polynomial can be evaluated by using synthetic division. Consider the polynomial $P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$. One way to evaluate the polynomial when $x = 2$ is to replace x by 2 and then simplify the numerical expression.

$$P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$$

$$\begin{aligned} P(2) &= 2(2)^4 - 3(2)^3 + 4(2)^2 - 5(2) + 1 \\ &= 2(16) - 3(8) + 4(4) - 5(2) + 1 \\ &= 32 - 24 + 16 - 10 + 1 \\ &= 15 \end{aligned}$$

Now use synthetic division to divide $(2x^4 - 3x^3 + 4x^2 - 5x + 1)$ by $(x - 2)$.

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & 4 & -5 & 1 \\ & & 4 & 2 & 12 & 14 \\ \hline & 2 & 1 & 6 & 7 & 15 \end{array}$$

Terms of the quotient — Remainder

Note that the remainder is 15, which is the same value as $P(2)$. This is not a coincidence. The following theorem states that this situation is always true.

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

Use the Remainder Theorem to evaluate $P(x) = x^4 - 3x^2 + 4x - 5$ when $x = -2$.

The value at which the polynomial is evaluated

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -3 & 4 & -5 \\ & & -2 & 4 & -2 & -4 \\ \hline & 1 & -2 & 1 & 2 & -9 \end{array}$$

• A 0 is inserted for the x^3 term.

← The remainder

$P(-2) = -9$

Example 7

Use the Remainder Theorem to evaluate $P(x) = -x^4 + 3x^3 + 2x^2 - x - 5$ when $x = -2$.

Solution

$$\begin{array}{r|rrrrr} -2 & -1 & 3 & 2 & -1 & -5 \\ & & 2 & -10 & 16 & -30 \\ \hline & -1 & 5 & -8 & 15 & -35 \end{array}$$

$P(-2) = -35$

You Try It 7

Use the Remainder Theorem to evaluate $P(x) = 2x^3 - 5x^2 + 7$ when $x = -3$.

Your solution

Solution on p. S19

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6.4 Exercises

Objective A

Divide by using long division.

1. $(x^2 + 3x - 40) \div (x - 5)$

2. $(x^2 - 14x + 24) \div (x - 2)$

3. $(x^3 - 3x^2 + 2) \div (x - 3)$

4. $(x^3 + 4x^2 - 8) \div (x + 4)$

5. $(6x^2 + 13x + 8) \div (2x + 1)$

6. $(12x^2 + 13x - 14) \div (3x - 2)$

7. $(10x^2 + 9x - 5) \div (2x - 1)$

8. $(18x^2 - 3x + 2) \div (3x + 2)$

9. $(8x^3 - 9) \div (2x - 3)$

10. $(64x^3 + 4) \div (4x + 2)$

11. $(6x^4 - 13x^2 - 4) \div (2x^2 - 5)$

12. $(12x^4 - 11x^2 + 10) \div (3x^2 + 1)$

13. $\dfrac{-10 - 33x + 3x^3 - 8x^2}{3x + 1}$

14. $\dfrac{10 - 49x + 38x^2 - 8x^3}{1 - 4x}$

15. $\dfrac{x^3 - 5x^2 + 7x - 4}{x - 3}$

16. $\dfrac{2x^3 - 3x^2 + 6x + 4}{2x + 1}$

17. $\dfrac{16x^2 - 13x^3 + 2x^4 + 20 - 9x}{x - 5}$

18. $\dfrac{x - x^2 + 5x^3 + 3x^4 - 2}{x + 2}$

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19. $\dfrac{2x^3 + 4x^2 - x + 2}{x^2 + 2x - 1}$

20. $\dfrac{3x^3 - 2x^2 + 5x - 4}{x^2 - x + 3}$

21. $\dfrac{x^4 + 2x^3 - 3x^2 - 6x + 2}{x^2 - 2x - 1}$

22. $\dfrac{x^4 - 3x^3 + 4x^2 - x + 1}{x^2 + x - 3}$

23. $\dfrac{x^4 + 3x^2 - 4x + 5}{x^2 + 2x + 3}$

24. $\dfrac{x^4 + 2x^3 - x + 2}{x^2 - x - 1}$

Objective 8

Divide by using synthetic division.

25. $(2x^2 - 6x - 8) \div (x + 1)$

26. $(3x^2 + 19x + 20) \div (x + 5)$

27. $(3x^2 - 14x + 16) \div (x - 2)$

28. $(4x^2 - 23x + 28) \div (x - 4)$

29. $(3x^2 - 4) \div (x - 1)$

30. $(4x^2 - 8) \div (x - 2)$

31. $(2x^3 - x^2 + 6x + 9) \div (x + 1)$

32. $(3x^3 + 10x^2 + 6x - 4) \div (x + 2)$

33. $(18 + x - 4x^3) \div (2 - x)$

34. $(12 - 3x^2 + x^3) \div (x + 3)$

35. $(2x^3 + 5x^2 - 5x + 20) \div (x + 4)$

36. $(5x^3 + 3x^2 - 17x + 6) \div (x + 2)$

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37. $\dfrac{5 + 5x - 8x^2 + 4x^3 - 3x^4}{2 - x}$

38. $\dfrac{3 - 13x - 5x^2 + 9x^3 - 2x^4}{3 - x}$

39. $\dfrac{3x^4 + 3x^3 - x^2 + 3x + 2}{x + 1}$

40. $\dfrac{4x^4 + 12x^3 - x^2 - x + 2}{x + 3}$

41. $\dfrac{2x^4 - x^2 + 2}{x - 3}$

42. $\dfrac{x^4 - 3x^3 - 30}{x + 2}$

Objective C

43. If the polynomial $2x^3 - 3x^2 - 4x - 15$ is divided by $x - 3$ and the remainder is 0, what do we know about $f(3)$ for the function $f(x) = 2x^3 - 3x^2 - 4x - 15$?

44. If the polynomial $3x^4 - 8x^2 + 2x + 1$ is divided by $x + 2$ and the remainder is 13, what do we know about $f(-2)$ for the function $f(x) = 3x^4 - 8x^2 + 2x + 1$?

Use the Remainder Theorem to evaluate the polynomial.

45. $P(x) = 2x^2 - 3x - 1$; $P(3)$

46. $Q(x) = 3x^2 - 5x - 1$; $Q(2)$

47. $R(x) = x^3 - 2x^2 + 3x - 1$; $R(4)$

48. $F(x) = x^3 + 4x^2 - 3x + 2$; $F(3)$

49. $P(z) = 2z^3 - 4z^2 + 3z - 1$; $P(-2)$

50. $R(t) = 3t^3 + t^2 - 4t + 2$; $R(-3)$

51. $Z(p) = 2p^3 - p^2 + 3$; $Z(-3)$

52. $P(y) = 3y^3 + 2y^2 - 5$; $P(-2)$

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53. $Q(x) = x^4 + 3x^3 - 2x^2 + 4x - 9$; $Q(2)$

54. $Y(z) = z^4 - 2z^3 - 3z^2 - z + 7$; $Y(3)$

55. $F(x) = 2x^4 - x^3 + 2x - 5$; $F(-3)$

56. $Q(x) = x^4 - 2x^3 + 4x - 2$; $Q(-2)$

57. $P(x) = x^3 - 3$; $P(5)$

58. $S(t) = 4t^3 + 5$; $S(-4)$

59. $R(t) = 4t^4 - 3t^2 + 5$; $R(-3)$

60. $P(z) = 2z^4 + z^2 - 3$; $P(-4)$

61. $Q(x) = x^5 - 4x^3 - 2x^2 + 5x - 2$; $Q(2)$

62. $T(x) = 2x^5 + 4x^4 - x^2 + 4$; $T(3)$

63. $R(x) = 2x^5 - x^3 + 4x - 1$; $R(-2)$

64. $P(x) = x^5 - x^3 + 4x + 1$; $P(-3)$

APPLYING THE CONCEPTS

65. Divide by using long division.

a. $\dfrac{a^3 + b^3}{a + b}$ **b.** $\dfrac{x^5 + y^5}{x + y}$ **c.** $\dfrac{x^6 - y^6}{x + y}$

66. For what value of k will the remainder be zero?

a. $(x^3 - x^2 - 3x + k) \div (x + 3)$ **b.** $(2x^3 - x + k) \div (x - 1)$

67. Divide.

a. $(2x^3 + 7x^2 + 2x - 8) \div (4x + 8)$ **b.** $(4x^3 + 13x^2 - 22x + 24) \div (6x - 12)$

c. $(2x^4 - 3x^3 + 4x^2 + x - 10) \div (x^2 - x + 1)$ **d.** $(x^4 + 4x^3 + 2x^2 - x + 5) \div (x^2 - 2x - 3)$

68. Show how synthetic division can be modified so that the divisor can be of the form $ax + b$.

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Focus on Problem Solving

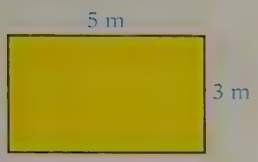
Dimensional Analysis

In solving application problems, it may be useful to include the units in order to organize the problem so that the answer is in the proper units. Using units to organize and check the correctness of an application is called **dimensional analysis**. We use the operations of multiplying units and dividing units in applying dimensional analysis to application problems.

The Rule for Multiplying Exponential Expressions states that we multiply two expressions with the same base by adding the exponents.

$$x^4 \cdot x^6 = x^{4+6} = x^{10}$$

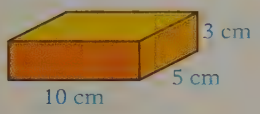
In calculations that involve quantities, the units are operated on algebraically.



➡ A rectangle measures 3 m by 5 m. Find the area of the rectangle.

$A = LW = (3\text{ m})(5\text{ m}) = (3 \cdot 5)(\text{m} \cdot \text{m}) = 15\text{ m}^2$

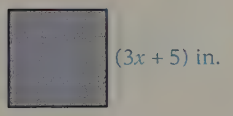
The area of the rectangle is 15 m^2 (square meters).



➡ A box measures 10 cm by 5 cm by 3 cm. Find the volume of the box.

$V = LWH = (10\text{ cm})(5\text{ cm})(3\text{ cm}) = (10 \cdot 5 \cdot 3)(\text{cm} \cdot \text{cm} \cdot \text{cm}) = 150\text{ cm}^3$

The volume of the box is 150 cm^3 (cubic centimeters).

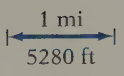


➡ Find the area of a square whose side measures $(3x + 5)$ in.

$A = s^2 = [(3x + 5)\text{ in.}]^2 = (3x + 5)^2\text{ in}^2 = (9x^2 + 30x + 25)\text{ in}^2$

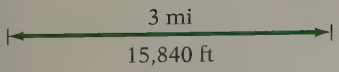
The area of the square is $(9x^2 + 30x + 25)\text{ in}^2$ (square inches).

Dimensional analysis is used in the conversion of units.



The following example converts the unit miles to feet. The equivalent measures $1\text{ mi} = 5280\text{ ft}$ are used to form the following rates, which are called conversion factors: $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$. Because $1\text{ mi} = 5280\text{ ft}$, both of the conversion factors $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$ are equal to 1.

To convert 3 mi to feet, multiply 3 mi by the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$.



$$3\text{ mi} = 3\text{ mi} \cdot 1 = \frac{3\text{ mi}}{1} \cdot \frac{5280\text{ ft}}{1\text{ mi}} = \frac{3\text{ mi} \cdot 5280\text{ ft}}{1\text{ mi}} = 3 \cdot 5280\text{ ft} = 15{,}840\text{ ft}$$

There are two important points in the above illustration. First, you can think of dividing the numerator and denominator by the common unit "mile" just as you would divide the numerator and denominator of a fraction by a common factor. Second, the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$ is equal to 1, and multiplying an expression by 1 does not change the value of the expression.

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In the application problem that follows, the units are kept in the problem while the problem is worked.

In 2000, a horse named Fusaichi Pegasus ran a 1.25-mile race in 2.02 min. Find Fusaichi Pegasus's average speed for that race in miles per hour. Round to the nearest tenth.

Strategy To find the average speed, use the formula $r = \frac{d}{t}$, where r is the speed, d is the distance, and t is the time. Use the conversion factor $\frac{60 \text{ min}}{1 \text{ h}}$.

Solution
$$r = \frac{d}{t} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$
$$= \frac{75 \text{ mi}}{2.02 \text{ h}} \approx 37.1 \text{ mph}$$

Fusaichi Pegasus's average speed was 37.1 mph.

Try each of the following problems. Round to the nearest tenth.

1. Convert 88 ft/s to miles per hour.
2. Convert 8 m/s to kilometers per hour (1 km = 1000 m).
3. A carpet is to be placed in a meeting hall that is 36 ft wide and 80 ft long. At $29.50 per square yard, how much will it cost to carpet the meeting hall?
4. A carpet is to be placed in a room that is 20 ft wide and 30 ft long. At $32.25 per square yard, how much will it cost to carpet the area?
5. Find the number of gallons of water in a fish tank that is 36 in. long and 24 in. wide and is filled to a depth of 16 in. (1 gal = 231 in^3).
6. Find the number of gallons of water in a fish tank that is 24 in. long and 18 in. wide and is filled to a depth of 12 in. (1 gal = 231 in^3).
7. A $\frac{1}{4}$-acre commercial lot is on sale for $2.15 per square foot. Find the sale price of the commercial lot (1 acre = 43,560 ft^2).
8. A 0.75-acre industrial parcel was sold for $98,010. Find the parcel's price per square foot (1 acre = 43,560 ft^2).
9. A new driveway will require 800 ft^3 of concrete. Concrete is ordered by the cubic yard. How much concrete should be ordered?
10. A piston-engined dragster traveled 440 yd in 4.936 s at Ennis, Texas, on October 9, 1988. Find the average speed of the dragster in miles per hour.
11. The Marianas Trench in the Pacific Ocean is the deepest part of the ocean. Its depth is 6.85 mi. The speed of sound under water is 4700 ft/s. Find the time it takes sound to travel from the surface to the bottom of the Marianas Trench and back.

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Projects and Group Activities

Pascal's Triangle

Simplifying the power of a binomial is called *expanding the binomial.* The expansion of the first three powers of a binomial is shown below.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a + b)^3 = (a + b)^2(a + b) = (a^2 + 2ab + b^2)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$$

Point of Interest

Pascal did not invent the triangle of numbers known as Pascal's Triangle. It was known to mathematicians in China probably as early as A.D. 1050. But Pascal's *Traite du triangle arithmetique* (*Treatise Concerning the Arithmetical Triangle*) brought together all the different aspects of the numbers for the first time.

Find $(a + b)^4$. [*Hint*: $(a + b)^4 = (a + b)^3(a + b)$]

Find $(a + b)^5$. [*Hint*: $(a + b)^5 = (a + b)^4(a + b)$]

If we continue in this way, the results for $(a + b)^6$ are

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Now expand $(a + b)^8$. Before you begin, see whether you can find a pattern that will help you write the expansion of $(a + b)^8$ without having to multiply it out. Here are some hints.

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

1. Write out the variable terms of each binomial expansion from $(a + b)^1$ through $(a + b)^6$. Observe how the exponents on the variables change.
2. Write out the coefficients of all the terms without the variable parts. It will be helpful to make a triangular arrangement as shown at the left. Note that each row begins and ends with a 1. Also note (in the two shaded regions, for example) that any number in a row is the sum of the two closest numbers above it. For instance, $1 + 5 = 6$ and $6 + 4 = 10$.

The triangle of numbers shown at the left is called Pascal's Triangle. To find the expansion of $(a + b)^8$, you need to find the eighth row of Pascal's Triangle. First find row seven. Then find row eight and use the patterns you have observed to write the expansion $(a + b)^8$.



Pascal's Triangle has been the subject of extensive analysis, and many patterns have been found. See whether you can find some of them. You might check the Internet, where you will find some web sites with information on Pascal's Triangle.

Chapter Summary

Key Words

A *monomial* is a number, a variable, or a product of a number and variables. The *degree of a monomial* is the sum of the exponents of the variables. [p. 329]

An *exponential expression is in simplest form* when it is written with only positive exponents. [p. 332]

A number written in *scientific notation* is a number written in the form $a \times 10^n$, where $1 \le a < 10$. [p. 335]

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A *polynomial* is a variable expression in which the terms are monomials. A polynomial of two terms is a *binomial.* A polynomial of three terms is a *trinomial.* The *degree of a polynomial* is the greatest of the degrees of any of its terms. A polynomial in one variable is usually written in *descending order,* which means that the terms are arranged so that the exponents of the variable decrease from left to right. [p. 341]

A *polynomial function* is an expression whose terms are monomials. A *linear function* is given by the equation $f(x) = mx + b$. A second-degree polynomial function, called a *quadratic function,* is given by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. A third-degree polynomial function is called a *cubic function*. The *leading coefficient* of a polynomial function is the coefficient of the variable with the largest exponent. [p. 341]

The product of two binomials can be found using the *FOIL* method, which is based on the Distributive Property. The letters of FOIL stand for First, Outer, Inner, and Last. [p. 350]

Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$. This method uses only the coefficients of the variable terms. [p. 359]

Essential Rules

Rule for Multiplying Exponential Expressions [p. 329]
$$x^m \cdot x^n = x^{m+n}$$

Rule for Simplifying Powers of Exponential Expressions [p. 330]
$$(x^m)^n = x^{mn}$$

Rule for Simplifying Powers of Products [p. 330]
$$(x^m y^n)^p = x^{mp} y^{np}$$

Definition of Zero as an Exponent [p. 331]
If $x \neq 0$, then $x^0 = 1$.

Definition of a Negative Exponent [p. 332]
$$x^{-n} = \frac{1}{x^n} \text{ and } \frac{1}{x^{-n}} = x^n, \quad x \neq 0$$

Rule for Dividing Exponential Expressions [p. 333]
$$\frac{x^m}{x^n} = x^{m-n}, \quad x \neq 0$$

Rule for Simplifying Powers of Quotients [p. 332]
$$\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}, \quad y \neq 0$$

The Sum and Difference of Two Terms [p. 351]
$$(a + b)(a - b) = a^2 - b^2$$

The Square of a Binomial [p. 351]
$$(a + b)^2 = a^2 + 2ab + b^2$$

Remainder Theorem [p. 362]
If the polynomial $P(x)$ is divided by $x - a$, then the remainder is $P(a)$.

Dividend = (quotient × divisor) + remainder [p. 357]

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Chapter Review

1. Multiply: $(-2a^2b^4)(3ab^2)$

2. Simplify: $(-3x^2y^3)^2$

3. Simplify: $\dfrac{(2a^4b^{-3}c^2)^3}{(2a^3b^2c^{-1})^4}$

4. Write 2.54×10^{-3} in decimal notation.

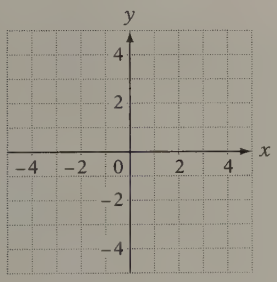
5. Given $P(x) = 2x^3 - x + 7$, find $P(-2)$.

6. Subtract: $(5x^2 - 8xy + 2y^2) - (x^2 - 3y^2)$

7. Simplify: $(2x^{-1}y^2z^5)(-3x^3yz^{-3})^2$

8. Multiply: $(2a^{12}b^3)(-9b^2c^6)(3ac)$

9. Graph: $f(x) = x^2 + 1$



10. Identify (a) the leading coefficient, (b) the constant term, and (c) the degree of the polynomial $P(x) = 3x^5 - 3x^2 + 7x + 8$.

11. Use the Remainder Theorem to evaluate $P(x) = x^3 - 2x^2 + 3x - 5$ when $x = 2$.

12. Multiply: $-2x(4x^2 + 7x - 9)$

13. Multiply: $2ab^3(4a^2 - 2ab + 3b^2)$

14. Multiply: $(3y^2 + 4y - 7)(2y + 3)$

15. Multiply: $(5a - 7)(2a + 9)$

16. Write 0.000000127 in scientific notation.

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17. Simplify: $(5y - 7)^2$

18. Divide: $\dfrac{15x^2 + 2x - 2}{3x - 2}$

19. Divide: $\dfrac{4x^3 + 27x^2 + 10x + 2}{x + 6}$

20. Divide: $\dfrac{x^4 - 4}{x - 4}$

21. Write 8.1×10^9 in decimal notation.

22. Simplify: $\dfrac{-18a^6b}{27a^3b^4}$

23. Multiply: $(5a + 2b)(5a - 2b)$

24. Use the Remainder Theorem to evaluate $P(x) = -2x^3 + 2x^2 - 4$ when $x = -3$.

25. Add: $(12y^2 + 17y - 4) + (9y^2 - 13y + 3)$

26. Multiply: $(6b^3 - 2b^2 - 5)(2b^2 - 1)$

27. Multiply: $(a + 7)(a - 7)$

28. Write 765,000,000,000 in scientific notation.

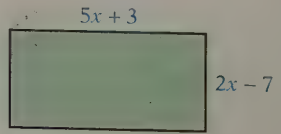
29. The length of a Ping-Pong table is 1 ft less than twice the width of the table. Let w represent the width of the Ping-Pong table. Express the area of the table in terms of the variable w.

30. The most distant object visible from Earth without the aid of a telescope is the Great Galaxy of Andromeda. It takes light from this galaxy 2.2×10^6 years to travel to Earth. Light travels about 6.7×10^8 mph. How far from Earth is the Great Galaxy of Andromeda? Use a 365-day year.

31. Write the number of seconds in one week in scientific notation.

32. The length of a side of a square checkerboard is $(3x - 2)$ in. Express the area of the checkerboard in terms of the variable x.

33. The length of a rectangle is $(5x + 3)$ cm. The width is $(2x - 7)$ cm. Find the area of the rectangle in terms of the variable x.



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Chapter Test

1. Multiply: $2x(2x^2 - 3x)$

2. Use the Remainder Theorem to evaluate $P(x) = -x^3 + 4x - 8$ when $x = -2$.

3. Simplify: $\dfrac{12x^2}{-3x^8}$

4. Simplify: $(-2xy^2)(3x^2y^4)$

5. Divide: $(x^2 + 1) \div (x + 1)$

6. Multiply: $(x - 3)(x^2 - 4x + 5)$

7. Simplify: $(-2a^2b)^3$

8. Simplify: $\dfrac{(3x^{-2}y^3)^3}{3x^4y^{-1}}$

9. Multiply: $(a - 2b)(a + 5b)$

10. Given $P(x) = 3x^2 - 8x + 1$, evaluate $P(2)$.

11. Divide: $(x^2 + 6x - 7) \div (x - 1)$

12. Multiply: $-3y^2(-2y^2 + 3y - 6)$

13. Multiply: $(-2x^3 + x^2 - 7)(2x - 3)$

14. Simplify: $(4y - 3)(4y + 3)$

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15. Simplify: $(ab^2)(a^3b^5)$

16. Simplify: $\dfrac{2a^{-1}b}{2^{-2}a^{-2}b^{-3}}$

17. Simplify: $\dfrac{(2a^{-4}b^2)^3}{4a^{-2}b^{-1}}$

18. Subtract: $(3a^2 - 2a - 7) - (5a^3 + 2a - 10)$

19. Simplify: $(2x - 5)^2$

20. Divide: $\dfrac{x^3 - 2x^2 - 5x + 7}{x + 3}$

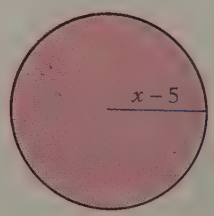
21. Multiply: $(2x - 7y)(5x - 4y)$

22. Add: $(3x^3 - 2x^2 - 4) + (8x^2 - 8x + 7)$

23. Write 0.00000000302 in scientific notation.

24. The mass of the moon is 3.7×10^{-8} times the mass of the sun. The mass of the sun is 2.19×10^{27} tons. Find the mass of the moon. Write the answer in scientific notation.

25. The radius of a circle is $(x - 5)$ m. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of the variable x. Leave the answer in terms of π.



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Cumulative Review

1. Let $x \in \{-8, -3, 3\}$. For what values of x is the inequality $x \geq -3$ a true statement?

2. Find the additive inverse of 83.

3. Simplify: $8 - 2[-3 - (-1)]^2 \div 4$

4. Evaluate $\frac{2a - b}{b - c}$ when $a = 4$, $b = -2$, and $c = 6$.

5. Simplify: $-5\sqrt{300}$

6. Identify the property that justifies the statement $2x + (-2x) = 0$.

7. Simplify: $2x - 4[x - 2(3 - 23x) + 4]$

8. Solve: $\frac{2}{3} - y = \frac{5}{6}$

9. Solve: $8x - 3 - x = -6 + 3x - 8$

10. Solve: $3 - |2 - 3x| = -2$

11. Given $P(x) = 3x^2 - 2x + 2$, find $P(-2)$.

12. Is the relation $\{(-1, 0), (0, 0), (1, 0)\}$ a function?

13. Find the slope of the line containing the points $(-2, 3)$ and $(4, 2)$.

14. Find the equation of the line that contains the point $(-1, 2)$ and has slope $-\frac{3}{2}$.

15. Find the equation of the line that contains the point $(-2, 4)$ and is perpendicular to the line $3x + 2y = 4$.

16. Solve by using Cramer's Rule:

$$2x - 3y = 2$$
$$x + y = -3$$

17. Solve by the addition method:

$$x - y + z = 0$$
$$2x + y - 3z = -7$$
$$-x + 2y + 2z = 5$$

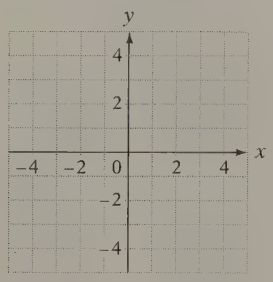
18. Simplify: $-2x - (-xy) + 7x - 4xy$

19. Multiply: $(2x + 3)(2x^2 - 3x + 1)$

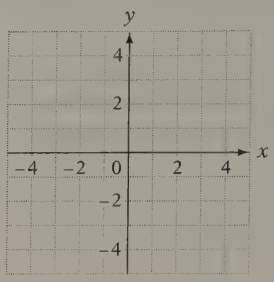
20. Write the number 0.00000501 in scientific notation.

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21. Graph: $3x - 4y = 12$



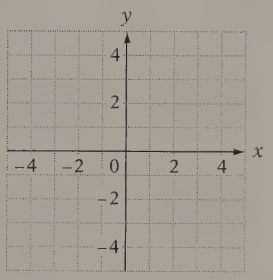
22. Graph: $-3x + 2y < 6$



23. Solve by graphing:

$x - 2y = 3$

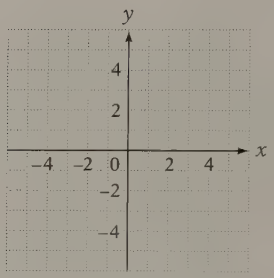
$-2x + y = -3$



24. Graph the solution set:

$2x + y < 2$

$-6x + 3y \geq 6$



25. Simplify: $(4a^{-2}b^3)(2a^3b^{-1})^{-2}$

26. Simplify: $\dfrac{(5x^3y^{-3}z)^{-2}}{y^4z^{-2}}$

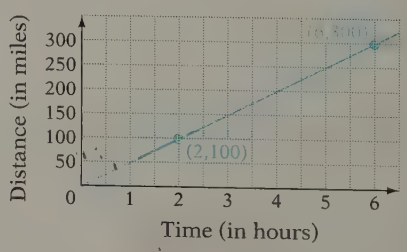
27. The sum of two integers is twenty-four. The difference between four times the smaller integer and nine is three less than twice the larger integer. Find the integers.

28. How many ounces of pure gold that costs \$360 per ounce must be mixed with 80 oz of an alloy that costs \$120 per ounce to make a mixture that costs \$200 per ounce?

29. Two bicycles are 25 mi apart and traveling toward each other. One cyclist is traveling at 1.5 times the rate of the other cyclist. They meet in 2 h. Find the rate of each cyclist.

30. If \$3000 is invested at an annual simple interest rate of 7.5%, how much additional money must be invested at an annual simple interest rate of 10% so that the total interest earned in one year is 9% of the total investment?

31. The graph shows the relationship between the distance traveled and the time of travel. Find the slope of the line between the two points on the graph. Write a sentence that states the meaning of the slope.



32. The width of a rectangle is 40% of the length. The perimeter of the rectangle is 42 m. Find the length and width of the rectangle.

33. The length of a side of a square is $(2x + 3)$ m. Find the area of the square in terms of the variable x.

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Chapter 7 Factoring

Objectives

Section 7.1

A To factor a monomial from a polynomial
B To factor by grouping

Section 7.2

A To factor a trinomial of the form $x^2 + bx + c$
B To factor completely

Section 7.3

A To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors
B To factor a trinomial of the form $ax^2 + bx + c$ by grouping

Section 7.4

A To factor the difference of two perfect squares or a perfect-square trinomial
B To factor the sum or the difference of two cubes
C To factor a trinomial that is quadratic in form
D To factor completely

Section 7.5

A To solve equations by factoring
B To solve application problems



Coordinating the position of each skydiver in a jump involves a lot of careful planning. Position within the "pattern" is determined by when and where each chute is deployed, which is further dependent upon each diver's velocity during free fall. Factors such as the initial altitude of the jump, the size of the parachute, and the weight and body position of the diver affect velocity. The velocity of a falling object can be modeled by a quadratic equation, as shown in **Exercises 72 and 73 on page 417.**



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Write 30 as a product of prime numbers.

2. Simplify: $-3(4y - 5)$

3. Simplify: $-(a - b)$

4. $2(a - b) - 5(a - b)$

5. Solve: $4x = 0$

6. Solve: $2x + 1 = 0$

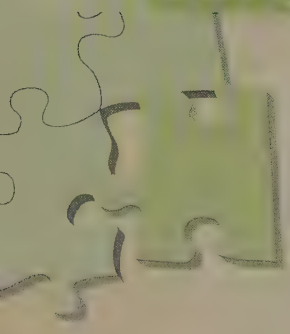
7. Multiply: $(x + 4)(x - 6)$

8. Multiply: $(2x - 5)(3x + 2)$

9. Simplify: $\frac{x^5}{x^2}$

10. Simplify: $\frac{6x^4y^3}{2xy^2}$

Big Figure



Find the values of x and y if $x + y$, xy, and $\frac{x}{y}$ all equal the same number.

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7.1 Common Factors

Objective A

To factor a monomial from a polynomial

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The **greatest common factor (GCF)** of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

$6x^3y = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y$
$8x^2y^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$
$\text{GCF} = 2 \cdot x \cdot x \cdot y = 2x^2y$

Note that the exponent of each variable in the GCF is the same as the *smallest* exponent of that variable in either of the monomials.

The GCF of $6x^3y$ and $8x^2y^2$ is $2x^2y$.

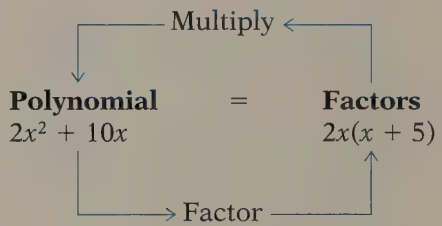
➡ Find the GCF of $12a^4b$ and $18a^2b^2c$.

The common variable factors are a^2 and b; c is not a common variable factor.

$12a^4b = 2 \cdot 2 \cdot 3 \cdot a^4 \cdot b$
$18a^2b^2c = 2 \cdot 3 \cdot 3 \cdot a^2 \cdot b^2 \cdot c$
$\text{GCF} = 2 \cdot 3 \cdot a^2 \cdot b = 6a^2b$

TAKE NOTE
$2x^2 + 10x$ is a sum.
$2x(x + 5)$ is a product.
When we factor a polynomial, we write it as a product.

To **factor** a polynomial means to write the polynomial as a product of other polynomials.



In the example above, $2x$ is the GCF of the terms $2x^2$ and $10x$. It is a **common factor** of the terms.

➡ Factor: $5x^3 - 35x^2 + 10x$

Find the GCF of the terms of the polynomial.

$5x^3 = 5 \cdot x^3$ $\quad 35x^2 = 5 \cdot 7 \cdot x^2$ $\quad 10x = 2 \cdot 5 \cdot x$

The GCF is $5x$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$5x^3 - 35x^2 + 10x = 5x(x^2) + 5x(-7x) + 5x(2)$
$= 5x(x^2 - 7x + 2)$

• **Use the Distributive Property to write the polynomial as a product of factors.**

TAKE NOTE
At the right, the factors in parentheses are determined by dividing each term of the trinomial by the GCF, $5x$.

$\left(\frac{5x^3}{5x}\right) = x^2$,

$\left(\frac{-35x^2}{5x}\right) = -7x$, and

$\left(\frac{10x}{5x}\right) = 2$.

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Example 1
Factor: $8x^2 + 2xy$

Solution
The GCF is $2x$.

$$8x^2 + 2xy = 2x(4x) + 2x(y)$$
$$= 2x(4x + y)$$

You Try It 1
Factor: $14a^2 - 21a^4b$

Your solution

Example 2
Factor: $n^3 - 5n^2 + 2n$

Solution
The GCF is n.

$$n^3 - 5n^2 + 2n = n(n^2) + n(-5n) + n(2)$$
$$= n(n^2 - 5n + 2)$$

You Try It 2
Factor: $27b^2 + 18b + 9$

Your solution

Example 3
Factor: $16x^2y + 8x^4y^2 - 12x^4y^5$

Solution
The GCF is $4x^2y$.

$$16x^2y + 8x^4y^2 - 12x^4y^5$$
$$= 4x^2y(4) + 4x^2y(2x^2y) + 4x^2y(-3x^2y^4)$$
$$= 4x^2y(4 + 2x^2y - 3x^2y^4)$$

You Try It 3
Factor: $6x^4y^2 - 9x^3y^2 + 12x^2y^4$

Your solution

Example 4
Factor: $x^{5n} - 2x^{3n}$

Solution
The GCF is x^{3n}.

$$x^{5n} - 2x^{3n} = x^{3n}(x^{2n}) - x^{3n}(2)$$
$$= x^{3n}(x^{2n} - 2)$$

You Try It 4
Factor: $a^{n+4} + a^2$

Your solution

Solutions on p. S20

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Objective B To factor by grouping



In the examples at the right, the binomials in parentheses are called **binomial factors**.

$2a(a + b)^2$
$3xy(x - y)$

The Distributive Property is used to factor a common binomial factor from an expression.

The common binomial factor of the expression $6x(x - 3) + y^2(x - 3)$ is $(x - 3)$. To factor that expression, use the Distributive Property to write the expression as a product of factors.

$$6x(x - 3) + y^2(x - 3) = (x - 3)(6x + y^2)$$

Consider the following simplification of $-(a - b)$.

$$-(a - b) = -1(a - b) = -a + b = b - a$$

Thus, $$\boldsymbol{b - a = -(a - b)}$$

This equation is sometimes used to factor a common binomial from an expression.

➡ Factor: $2x(x - y) + 5(y - x)$

$$2x(x - y) + 5(y - x) = 2x(x - y) - 5(x - y)$$
$$= (x - y)(2x - 5)$$

- $5(y - x) = 5[(-1)(x - y)]$
  $= -5(x - y)$

Some polynomials can be factored by grouping terms in such a way that a common binomial factor is found.

➡ Factor: $ax + bx - ay - by$

$$ax + bx - ay - by = (ax + bx) - (ay + by)$$

- Group the first two terms and the last two terms. Note that $-ay - by = -(ay + by)$.

$$= x(a + b) - y(a + b)$$
$$= (a + b)(x - y)$$

- Factor the GCF from each group.

➡ Factor: $6x^2 - 9x - 4xy + 6y$

$$6x^2 - 9x - 4xy + 6y = (6x^2 - 9x) - (4xy - 6y)$$

- Group the first two terms and the last two terms. Note that $-4xy + 6y = -(4xy - 6y)$.

$$= 3x(2x - 3) - 2y(2x - 3)$$
$$= (2x - 3)(3x - 2y)$$

- Factor the GCF from each group.

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Example 5

Factor: $4x(3x - 2) - 7(3x - 2)$

Solution

$4x(3x - 2) - 7(3x - 2)$

$= (3x - 2)(4x - 7)$

You Try It 5

Factor: $2y(5x - 2) - 3(2 - 5x)$

Your solution

Example 6

Factor: $9x^2 - 15x - 6xy + 10y$

Solution

$9x^2 - 15x - 6xy + 10y$

$= (9x^2 - 15x) - (6xy - 10y)$

$= 3x(3x - 5) - 2y(3x - 5)$

$= (3x - 5)(3x - 2y)$

You Try It 6

Factor: $a^2 - 3a + 2ab - 6b$

Your solution

Example 7

Factor: $3x^2y - 4x - 15xy + 20$

Solution

$3x^2y - 4x - 15xy + 20$

$= (3x^2y - 4x) - (15xy - 20)$

$= x(3xy - 4) - 5(3xy - 4)$

$= (3xy - 4)(x - 5)$

You Try It 7

Factor: $2mn^2 - n + 8mn - 4$

Your solution

Example 8

Factor: $4ab - 6 + 3b - 2ab^2$

Solution

$4ab - 6 + 3b - 2ab^2$

$= (4ab - 6) + (3b - 2ab^2)$

$= 2(2ab - 3) + b(3 - 2ab)$

$= 2(2ab - 3) - b(2ab - 3)$

$= (2ab - 3)(2 - b)$

You Try It 8

Factor: $3xy - 9y - 12 + 4x$

Your solution

Solutions on p. S20

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7.1 Exercises

Objective A

Factor.

1. $5a + 5$ **2.** $7b - 7$ **3.** $16 - 8a^2$ **4.** $12 + 12y^2$ **5.** $8x + 12$

6. $16a - 24$ **7.** $30a - 6$ **8.** $20b + 5$ **9.** $7x^2 - 3x$ **10.** $12y^2 - 5y$

11. $3a^2 + 5a^5$ **12.** $9x - 5x^2$ **13.** $14y^2 + 11y$ **14.** $6b^3 - 5b^2$ **15.** $2x^4 - 4x$

16. $3y^4 - 9y$ **17.** $10x^4 - 12x^2$ **18.** $12a^5 - 32a^2$ **19.** $8a^8 - 4a^5$ **20.** $16y^4 - 8y^7$

21. $x^2y^2 - xy$ **22.** $a^2b^2 + ab$ **23.** $3x^2y^4 - 6xy$ **24.** $12a^2b^5 - 9ab$ **25.** $x^2y - xy^3$

26. $3x^3 + 6x^2 + 9x$ **27.** $5y^3 - 20y^2 + 10y$ **28.** $2x^4 - 4x^3 + 6x^2$ **29.** $3y^4 - 9y^3 - 6y^2$

30. $2x^3 + 6x^2 - 14x$ **31.** $3y^3 - 9y^2 + 24y$ **32.** $2y^5 - 3y^4 + 7y^3$ **33.** $6a^5 - 3a^3 - 2a^2$

34. $x^3y - 3x^2y^2 + 7xy^3$ **35.** $2a^2b - 5a^2b^2 + 7ab^2$ **36.** $5y^3 + 10y^2 - 25y$

37. $4b^5 + 6b^3 - 12b$ **38.** $3a^2b^2 - 9ab^2 + 15b^2$ **39.** $8x^2y^2 - 4x^2y + x^2$

40. $x^{2n} - x^n$ **41.** $2a^{5n} + a^{2n}$ **42.** $x^{3n} - x^{2n}$

43. $y^{4n} + y^{2n}$ **44.** $a^{2n+2} + a^2$ **45.** $b^{n+5} - b^5$

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Objective B

Factor.

46. $x(b + 4) + 3(b + 4)$

47. $y(a + z) + 7(a + z)$

48. $a(y - x) - b(y - x)$

49. $3r(a - b) + s(a - b)$

50. $x(x - 2) + y(2 - x)$

51. $t(m - 7) + 7(7 - m)$

52. $2x(7 + b) - y(b + 7)$

53. $2y(4a - b) - (b - 4a)$

54. $8c(2m - 3n) + (3n - 2m)$

55. $x^2 + 2x + 2xy + 4y$

56. $x^2 - 3x + 4ax - 12a$

57. $p^2 - 2p - 3rp + 6r$

58. $t^2 + 4t - st - 4s$

59. $ab + 6b - 4a - 24$

60. $xy - 5y - 2x + 10$

61. $2z^2 - z + 2yz - y$

62. $2y^2 - 10y + 7xy - 35x$

63. $8v^2 - 12vy + 14v - 21y$

64. $21x^2 + 6xy - 49x - 14y$

65. $2x^2 - 5x - 6xy + 15y$

66. $4a^2 + 5ab - 10b - 8a$

67. $3y^2 - 6y - ay + 2a$

68. $2ar + a^2 - 2r - a$

69. $3xy - y^2 - y + 3x$

70. $2ab - 3b^2 - 3b + 2a$

71. $3st + t^2 - 2t - 6s$

72. $4x^2 + 3xy - 12y - 16x$

APPLYING THE CONCEPTS

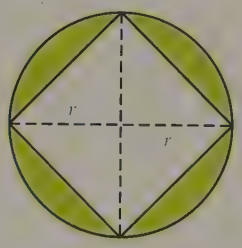
73. A whole number is a perfect number if it is the sum of all of its factors less than itself. For example, 6 is a perfect number because all the factors of 6 that are less than 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$.

a. Find the one perfect number between 20 and 30.

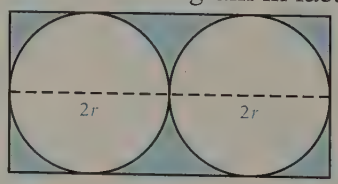
b. Find the one perfect number between 490 and 500.

74. Write the area of the shaded portion of each diagram in factored form.

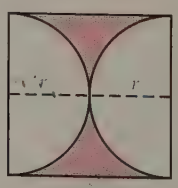
a.



b.



c.



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7.2 Factoring Polynomials of the Form $x^2 + bx + c$

Objective a

To factor a trinomial of the form $x^2 + bx + c$

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Trinomials of the form $x^2 + bx + c$, where b and c are integers, are shown at the right.

$x^2 + 8x + 12$; $b = 8$, $c = 12$
$x^2 - 7x + 12$; $b = -7$, $c = 12$
$x^2 - 2x - 15$; $b = -2$, $c = -15$

To factor a trinomial of this form means to express the trinomial as the product of two binomials.

Trinomials expressed as the product of binomials are shown at the right.

$x^2 + 8x + 12 = (x + 6)(x + 2)$
$x^2 - 7x + 12 = (x - 3)(x - 4)$
$x^2 - 2x - 15 = (x + 3)(x - 5)$

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant terms of the binomials and the terms of the trinomials.

The signs in the binomials are the same.

$(x + 6)(x + 2) = x^2 + 2x + 6x + (6)(2) = x^2 + 8x + 12$
sum of 6 and 2
product of 6 and 2

$(x - 3)(x - 4) = x^2 - 4x - 3x + (-3)(-4) = x^2 - 7x + 12$
sum of -3 and -4
product of -3 and -4

The signs in the binomials are opposite.

$(x + 3)(x - 5) = x^2 - 5x + 3x + (3)(-5) = x^2 - 2x - 15$
sum of 3 and -5
product of 3 and -5

$(x - 4)(x + 6) = x^2 + 6x - 4x + (-4)(6) = x^2 + 2x - 24$
sum of -4 and 6
product of -4 and 6

IMPORTANT RELATIONSHIPS

1. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign. They are both positive when the coefficient of the x term in the trinomial is positive. They are both negative when the coefficient of the x term in the trinomial is negative.
2. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.
3. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.
4. In the trinomial, the constant term is the product of the constant terms of the binomials.

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➡ Factor: $x^2 - 7x + 10$

Because the constant term is positive and the coefficient of x is negative, the binomial constants will be negative. Find two negative factors of 10 whose sum is -7. The results can be recorded in a table.

Negative Factors of 10	*Sum*
$-1, -10$	-11
$-2, -5$	-7

- These are the correct factors.

$x^2 - 7x + 10 = (x - 2)(x - 5)$

- Write the trinomial as a product of its factors.

Check: $(x - 2)(x - 5) = x^2 - 5x - 2x + 10$
$= x^2 - 7x + 10$

- Check the proposed factorization by multiplying the two binomials.

TAKE NOTE
Always check your proposed factorization to ensure its accuracy.

➡ Factor: $x^2 - 9x - 36$

The constant term is negative. The binomial constants will have opposite signs. Find two factors of -36 whose sum is -9.

Factors of −36	*Sum*
$+1, -36$	-35
$-1, +36$	35
$+2, -18$	-16
$-2, +18$	16
$+3, -12$	-9

- Once the correct factors are found, it is not necessary to try the remaining factors.

$x^2 - 9x - 36 = (x + 3)(x - 12)$

- Write the trinomial as a product of its factors.

➡ Factor: $x^2 + 7x + 8$

Because the constant term is positive and the coefficient of x is positive, the binomial constants will be positive. Find two positive factors of 8 whose sum is 7.

Positive Factors of 8	*Sum*
$1, 8$	9
$2, 4$	6

There are no positive integer factors of 8 whose sum is 7. The trinomial $x^2 + 7x + 8$ is said to be **nonfactorable over the integers**. Just as 17 is a prime number, $x^2 + 7x + 8$ is a **prime polynomial**. Binomials of the form $x - a$ and $x + a$ are also prime polynomials.

Example 1
Factor: $x^2 - 8x + 15$

Solution

Factors	*Sum*
$-1, -15$	-16
$-3, -5$	-8

- Find two negative factors of 15 whose sum is -8.

$x^2 - 8x + 15 = (x - 3)(x - 5)$

You Try It 1
Factor: $x^2 + 9x + 20$

Your solution

Solution on p. S20

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Example 2

Factor: $x^2 + 6x - 27$

Solution

Factors	Sum
$+1, -27$	-26
$-1, +27$	26
$+3, -9$	-6

- Find two factors of -27 whose sum is 6.

$x^2 + 6x - 27 = (x - 3)(x + 9)$

You Try It 2

Factor: $x^2 + 7x - 18$

Your solution

Solution on p. S20

Objective B To factor completely

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A polynomial is factored completely when it is written as a product of factors that are nonfactorable over the integers.

TAKE NOTE

The first step in *any* factoring problem is to determine whether the terms of the polynomial have a **common factor.** If they do, factor it out first.

➡ Factor: $4y^3 - 4y^2 - 24y$

$$4y^3 - 4y^2 - 24y = \boxed{4y(y^2) - 4y(y) - 4y(6)}$$
$$= 4y(y^2 - y - 6)$$
$$= 4y(y + 2)(y - 3)$$

- The GCF is $4y$. Do this step mentally.
- Use the Distributive Property to factor out the GCF.
- Factor $y^2 - y - 6$. The two factors of -6 whose sum is -1 are 2 and -3.

It is always possible to check the proposed factorization by multiplying the polynomials. Here is the check for the last example.

Check: $4y(y + 2)(y - 3) = 4y(y^2 - 3y + 2y - 6)$
$= 4y(y^2 - y - 6)$
$= 4y^3 - 4y^2 - 24y$

- This is the original polynomial.

➡ Factor: $5x^2 + 60xy + 100y^2$

$$5x^2 + 60xy + 100y^2 = \boxed{5(x^2) + 5(12xy) + 5(20y^2)}$$
$$= 5(x^2 + 12xy + 20y^2)$$
$$= 5(x + 2y)(x + 10y)$$

- The GCF is 5. Do this step mentally.
- Use the Distributive Property to factor out the GCF.
- Factor $x^2 + 12xy + 20y^2$. The two factors of 20 whose sum is 12 are 2 and 10.

TAKE NOTE

$2y$ and $10y$ are placed in the binomials. This is necessary so that the middle term contains xy and the last term contains y^2.

Note that $2y$ and $10y$ were placed in the binomials. The following check shows that this was necessary.

Check: $5(x + 2y)(x + 10y) = 5(x^2 + 10xy + 2xy + 20y^2)$
$= 5(x^2 + 12xy + 20y^2)$
$= 5x^2 + 60xy + 100y^2$

- The original polynomial

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Factor: $15 - 2x - x^2$

TAKE NOTE
When the coefficient of the highest power in a polynomial is negative, consider factoring out a negative GCF. Example 3 below is another example of this technique.

Because the coefficient of x^2 is -1, factor -1 from the trinomial and then write the resulting trinomial in descending order.

$$15 - 2x - x^2 = -(x^2 + 2x - 15)$$ • $15 - 2x - x^2 = -1(-15 + 2x + x^2) = -(x^2 + 2x - 15)$

$$= -(x + 5)(x - 3)$$ • Factor $x^2 + 2x - 15$. The two factors of -15 whose sum is 2 are 5 and -3.

Check: $-(x + 5)(x - 3) = -(x^2 + 2x - 15)$
$= -x^2 - 2x + 15$
$= 15 - 2x - x^2$ • The original polynomial

Example 3
Factor: $-3x^3 + 9x^2 + 12x$

Solution
The GCF is $-3x$.

$-3x^3 + 9x^2 + 12x = -3x(x^2 - 3x - 4)$

Factor the trinomial $x^2 - 3x - 4$. Find two factors of -4 whose sum is -3.

Factors	*Sum*
$-2, +2$	0
$+1, -4$	-3

$-3x^3 + 9x^2 + 12x = -3x(x + 1)(x - 4)$

You Try It 3
Factor: $-2x^3 + 14x^2 - 12x$

Your solution

Example 4
Factor: $4x^2 - 40xy + 84y^2$

Solution
The GCF is 4.

$4x^2 - 40xy + 84y^2 = 4(x^2 - 10xy + 21y^2)$

Factor the trinomial $x^2 - 10xy + 21y^2$. Find two negative factors of 21 whose sum is -10.

Factors	*Sum*
$-1, -21$	-22
$-3, -7$	-10

$4x^2 - 40xy + 84y^2 = 4(x - 3y)(x - 7y)$

You Try It 4
Factor: $3x^2 - 9xy - 12y^2$

Your solution

Solutions on p. S20

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7.2 Exercises

Objective A

Factor.

1. $x^2 + 3x + 2$

2. $x^2 + 5x + 6$

3. $x^2 - x - 2$

4. $x^2 + x - 6$

5. $a^2 + a - 12$

6. $a^2 - 2a - 35$

7. $a^2 - 3a + 2$

8. $a^2 - 5a + 4$

9. $a^2 + a - 2$

10. $a^2 - 2a - 3$

11. $b^2 - 6b + 9$

12. $b^2 + 8b + 16$

13. $b^2 + 7b - 8$

14. $y^2 - y - 6$

15. $y^2 + 6y - 55$

16. $z^2 - 4z - 45$

17. $y^2 - 5y + 6$

18. $y^2 - 8y + 15$

19. $z^2 - 14z + 45$

20. $z^2 - 14z + 49$

21. $z^2 - 12z - 160$

22. $p^2 + 2p - 35$

23. $p^2 + 12p + 27$

24. $p^2 - 6p + 8$

25. $x^2 + 20x + 100$

26. $x^2 + 18x + 81$

27. $b^2 + 9b + 20$

28. $b^2 + 13b + 40$

29. $x^2 - 11x - 42$

30. $x^2 + 9x - 70$

31. $b^2 - b - 20$

32. $b^2 + 3b - 40$

33. $y^2 - 14y - 51$

34. $y^2 - y - 72$

35. $p^2 - 4p - 21$

36. $p^2 + 16p + 39$

37. $y^2 - 8y + 32$

38. $y^2 - 9y + 81$

39. $x^2 - 20x + 75$

40. $p^2 + 24p + 63$

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41. $x^2 - 15x + 56$ **42.** $x^2 + 21x + 38$ **43.** $x^2 + x - 56$ **44.** $x^2 + 5x - 36$

45. $a^2 - 21a - 72$ **46.** $a^2 - 7a - 44$ **47.** $a^2 - 15a + 36$ **48.** $a^2 - 21a + 54$

49. $z^2 - 9z - 136$ **50.** $z^2 + 14z - 147$ **51.** $c^2 - c - 90$ **52.** $c^2 - 3c - 180$

53. $z^2 + 15z + 44$ **54.** $p^2 + 24p + 135$ **55.** $c^2 + 19c + 34$ **56.** $c^2 + 11c + 18$

57. $x^2 - 4x - 96$ **58.** $x^2 + 10x - 75$ **59.** $x^2 - 22x + 112$ **60.** $x^2 + 21x - 100$

61. $b^2 + 8b - 105$ **62.** $b^2 - 22b + 72$ **63.** $a^2 - 9a - 36$ **64.** $a^2 + 42a - 135$

65. $b^2 - 23b + 102$ **66.** $b^2 - 25b + 126$ **67.** $a^2 + 27a + 72$ **68.** $z^2 + 24z + 144$

69. $x^2 + 25x + 156$ **70.** $x^2 - 29x + 100$ **71.** $x^2 - 10x - 96$ **72.** $x^2 + 9x - 112$

Objective B

Factor.

73. $2x^2 + 6x + 4$ **74.** $3x^2 + 15x + 18$ **75.** $18 + 7x - x^2$

76. $12 - 4x - x^2$ **77.** $ab^2 + 2ab - 15a$ **78.** $ab^2 + 7ab - 8a$

79. $xy^2 - 5xy + 6x$ **80.** $xy^2 + 8xy + 15x$ **81.** $z^3 - 7z^2 + 12z$

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82. $-2a^3 - 6a^2 - 4a$

83. $-3y^3 + 15y^2 - 18y$

84. $4y^3 + 12y^2 - 72y$

85. $3x^2 + 3x - 36$

86. $2x^3 - 2x^2 + 4x$

87. $5z^2 - 15z - 140$

88. $6z^2 + 12z - 90$

89. $2a^3 + 8a^2 - 64a$

90. $3a^3 - 9a^2 - 54a$

91. $x^2 - 5xy + 6y^2$

92. $x^2 + 4xy - 21y^2$

93. $a^2 - 9ab + 20b^2$

94. $a^2 - 15ab + 50b^2$

95. $x^2 - 3xy - 28y^2$

96. $s^2 + 2st - 48t^2$

97. $y^2 - 15yz - 41z^2$

98. $y^2 + 85yz + 36z^2$

99. $z^4 - 12z^3 + 35z^2$

100. $z^4 + 2z^3 - 80z^2$

101. $b^4 - 22b^3 + 120b^2$

102. $b^4 - 3b^3 - 10b^2$

103. $2y^4 - 26y^3 - 96y^2$

104. $3y^4 + 54y^3 + 135y^2$

105. $-x^4 - 7x^3 + 8x^2$

106. $-x^4 + 11x^3 + 12x^2$

107. $4x^2y + 20xy - 56y$

108. $3x^2y - 6xy - 45y$

109. $c^3 + 18c^2 - 40c$

110. $-3x^3 + 36x^2 - 81x$

111. $-4x^3 - 4x^2 + 24x$

112. $x^2 - 8xy + 15y^2$

113. $y^2 - 7xy - 8x^2$

114. $a^2 - 13ab + 42b^2$

115. $y^2 + 4yz - 21z^2$

116. $y^2 + 8yz + 7z^2$

117. $y^2 - 16yz + 15z^2$

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118. $3x^2y + 60xy - 63y$

119. $4x^2y - 68xy - 72y$

120. $3x^3 + 3x^2 - 36x$

121. $4x^3 + 12x^2 - 160x$

122. $4z^3 + 32z^2 - 132z$

123. $5z^3 - 50z^2 - 120z$

124. $4x^3 + 8x^2 - 12x$

125. $5x^3 + 30x^2 + 40x$

126. $5p^2 + 25p - 420$

127. $4p^2 - 28p - 480$

128. $p^4 + 9p^3 - 36p^2$

129. $p^4 + p^3 - 56p^2$

130. $t^2 - 12ts + 35s^2$

131. $a^2 - 10ab + 25b^2$

132. $a^2 - 8ab - 33b^2$

133. $x^2 + 4xy - 60y^2$

134. $5x^4 - 30x^3 + 40x^2$

135. $6x^3 - 6x^2 - 120x$

APPLYING THE CONCEPTS

Factor.

136. $2 + c^2 + 9c$

137. $x^2y - 54y - 3xy$

138. $45a^2 + a^2b^2 - 14a^2b$

Find all integers k such that the trinomial can be factored over the integers.

139. $x^2 + kx + 35$

140. $x^2 + kx + 18$

141. $x^2 + kx + 21$

Determine the positive integer values of k for which the following polynomials are factorable over the integers.

142. $y^2 + 4y + k$

143. $z^2 + 7z + k$

144. $a^2 - 6a + k$

145. $c^2 - 7c + k$

146. $x^2 - 3x + k$

147. $y^2 + 5y + k$

148. In Exercises 142 to 147, there was the stated requirement that $k > 0$. If k is allowed to be any integer, how many different values of k are possible for each polynomial?

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7.3 Factoring Polynomials of the Form $ax^2 + bx + c$

Objective A

To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors



Trinomials of the form $ax^2 + bx + c$, where a, b, and c are integers, are shown at the right.

$3x^2 - x + 4;\ a = 3, b = -1, c = 4$
$6x^2 + 2x - 3;\ a = 6, b = 2, c = -3$

These trinomials differ from those in the previous section in that the coefficient of x^2 is not 1. There are various methods of factoring these trinomials. The method described in this objective is factoring polynomials using trial factors.

To reduce the number of trial factors that must be considered, remember the following:

1. Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials have opposite signs.
2. If the terms of the trinomial do not have a common factor, then the terms of neither of the binomial factors will have a common factor.

➨ Factor: $2x^2 - 7x + 3$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 2 (coefficient of x^2)	*Negative Factors of 3* (constant term)
1, 2	$-1, -3$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-7x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(2x - 3)$	$-3x - 2x = -5x$
$(x - 3)(2x - 1)$	$-x - 6x = -7x$

Write the factors of the trinomial. $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

➨ Factor: $3x^2 - 8x + 4$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 3 (coefficient of x^2)	*Negative Factors of 4* (constant term)
1, 3	$-1, -4$ $-2, -2$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-8x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(3x - 4)$	$-4x - 3x = -7x$
$(x - 4)(3x - 1)$	$-x - 12x = -13x$
$(x - 2)(3x - 2)$	$-2x - 6x = -8x$

Write the factors of the trinomial. $3x^2 - 8x + 4 = (x - 2)(3x - 2)$

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Factor: $6x^3 + 14x^2 - 12x$

Factor the GCF, $2x$, from the terms. $6x^3 + 14x^2 - 12x = 2x(3x^2 + 7x - 6)$

Factor the trinomial $3x^2 + 7x - 6$. The constant term is negative. The binomial constants will have opposite signs.

Positive Factors of 3	*Factors of −6*
1, 3	−1, 6 1, −6 −2, 3 2, −3

TAKE NOTE

The binomial factor $3x + 6$ has a common factor of 3: $3x + 6 = 3(x + 2)$. Because $3x^2 + 7x - 6$ does not have a common factor, one of its binomial factors cannot have a common factor. For the examples and solutions in this text, all trial factors except those that have a common factor will be listed.

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $7x$, of the trinomial.

It is not necessary to test trial factors that have a common factor.

Trial Factors	*Middle Term*
$(x - 1)(3x + 6)$	Common factor
$(x + 6)(3x - 1)$	$-x + 18x = 17x$
$(x + 1)(3x - 6)$	Common factor
$(x - 6)(3x + 1)$	$x - 18x = -17x$
$(x - 2)(3x + 3)$	Common factor
$(x + 3)(3x - 2)$	$-2x + 9x = 7x$
$(x + 2)(3x - 3)$	Common factor
$(x - 3)(3x + 2)$	$2x - 9x = -7x$

Write the factors of the trinomial. $6x^3 + 14x^2 - 12x = 2x(x + 3)(3x - 2)$

For this example, all the trial factors were listed. Once the correct factors have been found, however, the remaining trial factors need not be checked.

Example 1

Factor: $3x^2 + x - 2$

Solution

Positive factors of 3: 1, 3

Factors of −2: 1, −2; −1, 2

Trial Factors	*Middle Term*
$(x + 1)(3x - 2)$	$-2x + 3x = x$
$(x - 2)(3x + 1)$	$x - 6x = -5x$
$(x - 1)(3x + 2)$	$2x - 3x = -x$
$(x + 2)(3x - 1)$	$-x + 6x = 5x$

$3x^2 + x - 2 = (x + 1)(3x - 2)$

You Try It 1

Factor: $2x^2 - x - 3$

Your solution

Example 2

Factor: $-12x^3 - 32x^2 + 12x$

Solution

The GCF is $-4x$.

$-12x^3 - 32x^2 + 12x = -4x(3x^2 + 8x - 3)$

Factor the trinomial.

Positive factors of 3: 1, 3

Factors of −3: 1, −3; −1, 3

Trial Factors	*Middle Term*
$(x - 3)(3x + 1)$	$x - 9x = -8x$
$(x + 3)(3x - 1)$	$-x + 9x = 8x$

$-12x^3 - 32x^2 + 12x = -4x(x + 3)(3x - 1)$

You Try It 2

Factor: $-45y^3 + 12y^2 + 12y$

Your solution

Solutions on pp. S20–S21

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Objective D

To factor a trinomial of the form $ax^2 + bx + c$ by grouping



In the previous objective, trinomials of the form $ax^2 + bx + c$ were factored by using trial factors. In this objective, these trinomials will be factored by grouping.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b. Then use factoring by grouping to write the factorization of the trinomial.

➡ Factor: $2x^2 + 13x + 15$

Find two positive factors of 30 $(2 \cdot 15)$ whose sum is 13.

Positive Factors of 30	*Sum*
1, 30	31
2, 15	17
3, 10	13

- When the required sum has been found, the remaining factors need not be checked.

$$2x^2 + 13x + 15 = 2x^2 + 3x + 10x + 15$$

- Use the factors of 30 whose sum is 13 to write $13x$ as $3x + 10x$.

$$= (2x^2 + 3x) + (10x + 15)$$

- Factor by grouping.

$$= x(2x + 3) + 5(2x + 3)$$
$$= (2x + 3)(x + 5)$$

Check: $(2x + 3)(x + 5) = 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

➡ Factor: $6x^2 - 11x - 10$

Find two factors of -60 $[6(-10)]$ whose sum is -11.

Factors of −60	*Sum*
1, −60	−59
−1, 60	59
2, −30	−28
−2, 30	28
3, −20	−17
−3, 20	17
4, −15	−11

$$6x^2 - 11x - 10 = 6x^2 + 4x - 15x - 10$$

- Use the factors of -60 whose sum is -11 to write $-11x$ as $4x - 15x$.

$$= (6x^2 + 4x) - (15x + 10)$$

- Factor by grouping. Recall that $-15x - 10 = -(15x + 10)$.

$$= 2x(3x + 2) - 5(3x + 2)$$
$$= (3x + 2)(2x - 5)$$

Check: $(3x + 2)(2x - 5) = 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

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Factor: $3x^2 - 2x - 4$

Find two factors of -12 [$3(-4)$] whose sum is -2.

Factors of -12	*Sum*
$1, -12$	-11
$-1, 12$	11
$2, -6$	-4
$-2, 6$	4
$3, -4$	-1
$-3, 4$	1

Because no integer factors of -12 have a sum of -2, $3x^2 - 2x - 4$ is nonfactorable over the integers. $3x^2 - 2x - 4$ is a prime polynomial.

Example 3

Factor: $2x^2 + 19x - 10$

Solution

Factors of -20 [$2(-10)$]	*Sum*
$-1, 20$	19

$$\begin{aligned} 2x^2 + 19x - 10 &= 2x^2 - x + 20x - 10 \\ &= (2x^2 - x) + (20x - 10) \\ &= x(2x - 1) + 10(2x - 1) \\ &= (2x - 1)(x + 10) \end{aligned}$$

You Try It 3

Factor: $2a^2 + 13a - 7$

Your solution

Example 4

Factor: $24x^2y - 76xy + 40y$

Solution

The GCF is $4y$.

$$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$$

Negative Factors of 60 [$6(10)$]	*Sum*
$-1, -60$	-61
$-2, -30$	-32
$-3, -20$	-23
$-4, -15$	-19

$$\begin{aligned} 6x^2 - 19x + 10 &= 6x^2 - 4x - 15x + 10 \\ &= (6x^2 - 4x) - (15x - 10) \\ &= 2x(3x - 2) - 5(3x - 2) \\ &= (3x - 2)(2x - 5) \end{aligned}$$

$$\begin{aligned} 24x^2y - 76xy + 40y &= 4y(6x^2 - 19x + 10) \\ &= 4y(3x - 2)(2x - 5) \end{aligned}$$

You Try It 4

Factor: $15x^3 + 40x^2 - 80x$

Your solution

Solutions on p. S21

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7.3 Exercises

Objective A

Factor by using trial factors.

1. $2x^2 + 3x + 1$

2. $5x^2 + 6x + 1$

3. $2y^2 + 7y + 3$

4. $3y^2 + 7y + 2$

5. $2a^2 - 3a + 1$

6. $3a^2 - 4a + 1$

7. $2b^2 - 11b + 5$

8. $3b^2 - 13b + 4$

9. $2x^2 + x - 1$

10. $4x^2 - 3x - 1$

11. $2x^2 - 5x - 3$

12. $3x^2 + 5x - 2$

13. $2t^2 - t - 10$

14. $2t^2 + 5t - 12$

15. $3p^2 - 16p + 5$

16. $6p^2 + 5p + 1$

17. $12y^2 - 7y + 1$

18. $6y^2 - 5y + 1$

19. $6z^2 - 7z + 3$

20. $9z^2 + 3z + 2$

21. $6t^2 - 11t + 4$

22. $10t^2 + 11t + 3$

23. $8x^2 + 33x + 4$

24. $7x^2 + 50x + 7$

25. $5x^2 - 62x - 7$

26. $9x^2 - 13x - 4$

27. $12y^2 + 19y + 5$

28. $5y^2 - 22y + 8$

29. $7a^2 + 47a - 14$

30. $11a^2 - 54a - 5$

31. $3b^2 - 16b + 16$

32. $6b^2 - 19b + 15$

33. $2z^2 - 27z - 14$

34. $4z^2 + 5z - 6$

35. $3p^2 + 22p - 16$

36. $7p^2 + 19p + 10$

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37. $4x^2 + 6x + 2$

38. $12x^2 + 33x - 9$

39. $15y^2 - 50y + 35$

40. $30y^2 + 10y - 20$

41. $2x^3 - 11x^2 + 5x$

42. $2x^3 - 3x^2 - 5x$

43. $3a^2b - 16ab + 16b$

44. $2a^2b - ab - 21b$

45. $3z^2 + 95z + 10$

46. $8z^2 - 36z + 1$

47. $36x - 3x^2 - 3x^3$

48. $-2x^3 + 2x^2 + 4x$

49. $80y^2 - 36y + 4$

50. $24y^2 - 24y - 18$

51. $8z^3 + 14z^2 + 3z$

52. $6z^3 - 23z^2 + 20z$

53. $6x^2y - 11xy - 10y$

54. $8x^2y - 27xy + 9y$

55. $10t^2 - 5t - 50$

56. $16t^2 + 40t - 96$

57. $3p^3 - 16p^2 + 5p$

58. $6p^3 + 5p^2 + p$

59. $26z^2 + 98z - 24$

60. $30z^2 - 87z + 30$

61. $10y^3 - 44y^2 + 16y$

62. $14y^3 + 94y^2 - 28y$

63. $4yz^3 + 5yz^2 - 6yz$

64. $12a^3 + 14a^2 - 48a$

65. $42a^3 + 45a^2 - 27a$

66. $36p^2 - 9p^3 - p^4$

67. $9x^2y - 30xy^2 + 25y^3$

68. $8x^2y - 38xy^2 + 35y^3$

69. $9x^3y - 24x^2y^2 + 16xy^3$

70. $9x^3y + 12x^2y + 4xy$

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Objective 5

Factor by grouping.

71. $6x^2 - 17x + 12$ **72.** $15x^2 - 19x + 6$ **73.** $5b^2 + 33b - 14$ **74.** $8x^2 - 30x + 25$

75. $6a^2 + 7a - 24$ **76.** $14a^2 + 15a - 9$ **77.** $4z^2 + 11z + 6$ **78.** $6z^2 - 25z + 14$

79. $22p^2 + 51p - 10$ **80.** $14p^2 - 41p + 15$ **81.** $8y^2 + 17y + 9$ **82.** $12y^2 - 145y + 12$

83. $18t^2 - 9t - 5$ **84.** $12t^2 + 28t - 5$ **85.** $6b^2 + 71b - 12$ **86.** $8b^2 + 65b + 8$

87. $9x^2 + 12x + 4$ **88.** $25x^2 - 30x + 9$ **89.** $6b^2 - 13b + 6$ **90.** $20b^2 + 37b + 15$

91. $33b^2 + 34b - 35$ **92.** $15b^2 - 43b + 22$ **93.** $18y^2 - 39y + 20$ **94.** $24y^2 + 41y + 12$

95. $15a^2 + 26a - 21$ **96.** $6a^2 + 23a + 21$ **97.** $8y^2 - 26y + 15$ **98.** $18y^2 - 27y + 4$

99. $8z^2 + 2z - 15$ **100.** $10z^2 + 3z - 4$ **101.** $15x^2 - 82x + 24$ **102.** $13z^2 + 49z - 8$

103. $10z^2 - 29z + 10$ **104.** $15z^2 - 44z + 32$ **105.** $36z^2 + 72z + 35$ **106.** $16z^2 + 8z - 35$

107. $3x^2 + xy - 2y^2$ **108.** $6x^2 + 10xy + 4y^2$ **109.** $3a^2 + 5ab - 2b^2$ **110.** $2a^2 - 9ab + 9b^2$

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111. $4y^2 - 11yz + 6z^2$ **112.** $2y^2 + 7yz + 5z^2$ **113.** $28 + 3z - z^2$ **114.** $15 - 2z - z^2$

115. $8 - 7x - x^2$ **116.** $12 + 11x - x^2$ **117.** $9x^2 + 33x - 60$ **118.** $16x^2 - 16x - 12$

119. $24x^2 - 52x + 24$ **120.** $60x^2 + 95x + 20$ **121.** $35a^4 + 9a^3 - 2a^2$

122. $15a^4 + 26a^3 + 7a^2$ **123.** $15b^2 - 115b + 70$ **124.** $25b^2 + 35b - 30$

125. $3x^2 - 26xy + 35y^2$ **126.** $4x^2 + 16xy + 15y^2$ **127.** $216y^2 - 3y - 3$

128. $360y^2 + 4y - 4$ **129.** $21 - 20x - x^2$ **130.** $18 + 17x - x^2$

131. $15a^2 + 11ab - 14b^2$ **132.** $15a^2 - 31ab + 10b^2$ **133.** $33z - 8z^2 - z^3$

APPLYING THE CONCEPTS

Factor.

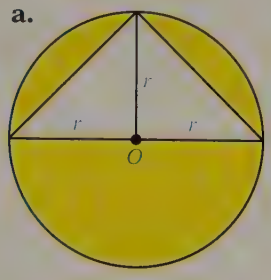
134. $2(y + 2)^2 - (y + 2) - 3$ **135.** $3(a + 2)^2 - (a + 2) - 4$ **136.** $4(y - 1)^2 - 7(y - 1) - 2$

Find all integers k such that the trinomial can be factored over the integers.

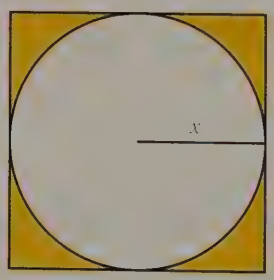
137. $2x^2 + kx + 3$ **138.** $2x^2 + kx - 3$ **139.** $3x^2 + kx + 2$

140. Write the area of the shaded portion of each diagram in factored form.

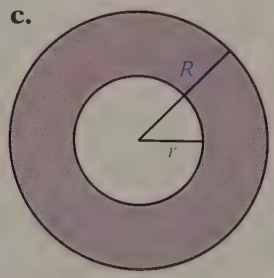
a.



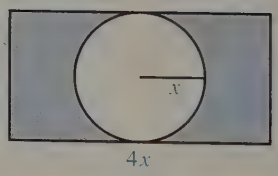
b.



c.



d.



141. In your own words, explain how the signs of the last terms of the two binomial factors of a trinomial are determined.

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7.4 Special Factoring

Objective A

To factor the difference of two perfect squares or a perfect-square trinomial

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TAKE NOTE
Here are some more examples of perfect squares:
$36, 49, x^8, y^{12}$.

The product of a term and itself is called a **perfect square**. The exponents on variables of perfect squares are always even numbers.

Term		*Perfect Square*
5	$5 \cdot 5 =$	25
x	$x \cdot x =$	x^2
$3y^4$	$3y^4 \cdot 3y^4 =$	$9y^8$
x^n	$x^n \cdot x^n =$	x^{2n}

The **square root of a perfect square** is one of the two equal factors of the perfect square. "$\sqrt{\ }$" is the symbol for square root. To find the exponent of the square root of a variable term, divide the exponent by 2.

$\sqrt{25} = 5$
$\sqrt{x^2} = x$
$\sqrt{9y^8} = 3y^4$
$\sqrt{x^{2n}} = x^n$

The difference of two perfect squares is the product of the sum and difference of two terms. The factors of the difference of two perfect squares are the sum and difference of the square roots of the perfect squares.

TAKE NOTE
$(a + b)$ is the sum of the two terms a and b.
$(a - b)$ is the difference of the two terms a and b. a and b are the square roots of a^2 and b^2.

Factors of the Difference of Two Perfect Squares

$a^2 - b^2 = (a + b)(a - b)$

The sum of two perfect squares, $a^2 + b^2$, is nonfactorable over the integers.

Factor: $4x^2 - 81y^2$

$4x^2 - 81y^2 = (2x)^2 - (9y)^2$

$= (2x + 9y)(2x - 9y)$

- Write the binomial as the difference of two perfect squares.
- The factors are the sum and difference of the square roots of the perfect squares.

A perfect-square trinomial is the square of a binomial.

Factors of a Perfect-Square Trinomial

$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

In factoring a perfect-square trinomial, remember that the terms of the binomial are the square roots of the perfect squares of the trinomial. The sign in the binomial is the sign of the middle term of the trinomial.

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➡ Factor: $4x^2 + 12x + 9$

Because $4x^2$ is a perfect square $[4x^2 = (2x)^2]$ and 9 is a perfect square $(9 = 3^2)$, try factoring $4x^2 + 12x + 9$ as the square of a binomial.

$4x^2 + 12x + 9 \stackrel{?}{=} (2x + 3)^2$

Check: $(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$

The check verifies that $4x^2 + 12x + 9 = (2x + 3)^2$.

It is important to check a proposed factorization as we did above. The next example illustrates the importance of this check.

➡ Factor: $x^2 + 13x + 36$

Because x^2 is a perfect square and 36 is a perfect square, try factoring $x^2 + 13x + 36$ as the square of a binomial.

$x^2 + 13x + 36 \stackrel{?}{=} (x + 6)^2$

Check: $(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$

In this case, the proposed factorization of $x^2 + 13x + 36$ does *not* check. Try another factorization. The numbers 4 and 9 are factors of 36 whose sum is 13.

$x^2 + 13x + 36 = (x + 4)(x + 9)$

Example 1

Factor: $25x^2 - 1$

Solution

$25x^2 - 1 = (5x)^2 - (1)^2$
$= (5x + 1)(5x - 1)$

You Try It 1

Factor: $x^2 - 36y^4$

Your solution

Example 2

Factor: $4x^2 - 20x + 25$

Solution

$4x^2 - 20x + 25 = (2x - 5)^2$

You Try It 2

Factor: $9x^2 + 12x + 4$

Your solution

Example 3

Factor: $(x + y)^2 - 4$

Solution

$(x + y)^2 - 4 = (x + y)^2 - (2)^2$
$= (x + y + 2)(x + y - 2)$

You Try It 3

Factor: $(a + b)^2 - (a - b)^2$

Your solution

Solutions on p. S21

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Objective B

To factor the sum or the difference of two cubes

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TAKE NOTE
Here are some more examples of perfect cubes:
64, 1000, b^9, z^{15}

The product of the same three factors is called a **perfect cube**. The exponents on variables of perfect cubes are always divisible by 3.

Term		*Perfect Cube*
2	$2 \cdot 2 \cdot 2 = 2^3 =$	8
$3y$	$3y \cdot 3y \cdot 3y = (3y)^3 =$	$27y^3$
y^2	$y^2 \cdot y^2 \cdot y^2 = (y^2)^3 =$	y^6

The **cube root** of a perfect cube is one of the three equal factors of the perfect cube. "$\sqrt[3]{\ }$" is the symbol for cube root. To find the exponent of the cube root of a variable term, divide the exponent by 3.

$\sqrt[3]{8} = 2$
$\sqrt[3]{27y^3} = 3y$
$\sqrt[3]{y^6} = y^2$

The following rules are used to factor the sum or difference of two perfect cubes.

Factors of the Sum or Difference of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

➨ Factor: $8x^3 - 27$

Write the binomial as the difference of two perfect cubes.

$8x^3 - 27 = (2x)^3 - 3^3$

The terms of the binomial factor are the cube roots of the perfect cubes. The sign of the binomial factor is the same sign as in the given binomial. The trinomial factor is obtained from the binomial factor.

$= (2x - 3)(4x^2 + 6x + 9)$

Square of the first term
Opposite of the product of the two terms
Square of the last term

TAKE NOTE
You can always check a proposed factorization by multiplying the factors.

Check:

$$\begin{array}{r} 4x^2 + 6x + 9 \\ \underline{\qquad\qquad 2x - 3} \\ -12x^2 - 18x - 27 \\ \underline{8x^3 + 12x^2 + 18x\qquad\quad} \\ 8x^3 \qquad\qquad - 27 \end{array}$$

• The original polynomial

➨ Factor: $a^3 + 64y^3$

$a^3 + 64y^3 = a^3 + (4y)^3$ • Write the binomial as the sum of two perfect cubes.

$= (a + 4y)(a^2 - 4ay + 16y^2)$ • Factor.

➨ Factor: $64y^4 - 125y$

$64y^4 - 125y = y(64y^3 - 125)$ • Factor out y, the GCF.

$= y[(4y)^3 - 5^3]$ • Write the binomial as the difference of two cubes.

$= y(4y - 5)(16y^2 + 20y + 25)$ • Factor.

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Example 4
Factor: $x^3y^3 - 1$

Solution
$x^3y^3 - 1 = (xy)^3 - 1^3$
$= (xy - 1)(x^2y^2 + xy + 1)$

You Try It 4
Factor: $8x^3 + y^3z^3$

Your solution

Example 5
Factor: $(x + y)^3 - x^3$

Solution
$(x + y)^3 - x^3$
$= [(x + y) - x][(x + y)^2 + x(x + y) + x^2]$
$= y(x^2 + 2xy + y^2 + x^2 + xy + x^2)$
$= y(3x^2 + 3xy + y^2)$

You Try It 5
Factor: $(x - y)^3 + (x + y)^3$

Your solution

Solutions on p. S21

Objective C To factor a trinomial that is quadratic in form

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Certain trinomials that are not quadratic can be expressed in the form $ax^2 + bx + c$ by making suitable variable substitutions. A trinomial is **quadratic in form** if it can be written as $au^2 + bu + c$.

As shown below, the trinomials $x^4 + 5x^2 + 6$ and $2x^2y^2 + 3xy - 9$ are quadratic in form.

	$x^4 + 5x^2 + 6$		$2x^2y^2 + 3xy - 9$
	$(x^2)^2 + 5(x^2) + 6$		$2(xy)^2 + 3(xy) - 9$
Let $u = x^2$.	$u^2 + 5u + 6$	Let $u = xy$.	$2u^2 + 3u - 9$

When we use this method to factor a trinomial that is quadratic in form, the variable part of the first term in each binomial will be u.

For example, $x^4 + 5x^2 + 6$ is quadratic in form by letting $u = x^2$. Thus,
$x^4 + 5x^2 + 6 = (x^2)^2 + 5(x^2) + 6 = u^2 + 5u + 6$.

Factor: $x^4 + 5x^2 + 6$

$x^4 + 5x^2 + 6 = u^2 + 5u + 6$ • Let $u = x^2$.
$= (u + 3)(u + 2)$ • Factor.
$= (x^2 + 3)(x^2 + 2)$ • Replace u by x^2.

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Example 6

Factor: $6x^2y^2 - xy - 12$

Solution

Let $u = xy$.

$$\begin{aligned} 6x^2y^2 - xy - 12 &= 6u^2 - u - 12 \\ &= (3u + 4)(2u - 3) \\ &= (3xy + 4)(2xy - 3) \end{aligned}$$

You Try It 6

Factor: $6x^2y^2 - 19xy + 10$

Your solution

Example 7

Factor: $2x^4 + 5x^2 - 12$

Solution

Let $u = x^2$.

$$\begin{aligned} 2x^4 + 5x^2 - 12 &= 2u^2 + 5u - 12 \\ &= (2u - 3)(u + 4) \\ &= (2x^2 - 3)(x^2 + 4) \end{aligned}$$

You Try It 7

Factor: $3x^4 + 4x^2 - 4$

Your solution

Example 8

Factor: $x^4y^4 + 3x^2y^2 - 10$

Solution

Let $u = x^2y^2$.

$$\begin{aligned} x^4y^4 + 3x^2y^2 - 10 &= u^2 + 3u - 10 \\ &= (u + 5)(u - 2) \\ &= (x^2y^2 + 5)(x^2y^2 - 2) \end{aligned}$$

You Try It 8

Factor: $a^4b^4 + 6a^2b^2 - 7$

Your solution

Solutions on p. S21

Objective D To factor completely

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When factoring a polynomial completely, ask the following questions about the polynomial.

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two cubes, or the difference of two cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. If the polynomial has four terms, can it be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

TAKE NOTE

Remember that you may have to factor more than once in order to write the polynomial as a product of *prime* factors.

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Example 9

Factor: $6a^3 + 15a^2 - 36a$

Solution

$$6a^3 + 15a^2 - 36a = 3a(2a^2 + 5a - 12)$$
$$= 3a(2a - 3)(a + 4)$$

You Try It 9

Factor: $18x^3 - 6x^2 - 60x$

Your solution

Example 10

Factor: $x^2y + 2x^2 - y - 2$

Solution

$$x^2y + 2x^2 - y - 2 = (x^2y + 2x^2) - (y + 2)$$
$$= x^2(y + 2) - 1(y + 2)$$
$$= (y + 2)(x^2 - 1)$$
$$= (y + 2)(x + 1)(x - 1)$$

You Try It 10

Factor: $4x - 4y - x^3 + x^2y$

Your solution

Example 11

Factor: $x^{4n} - y^{4n}$

Solution

$$x^{4n} - y^{4n} = (x^{2n})^2 - (y^{2n})^2$$
$$= (x^{2n} + y^{2n})(x^{2n} - y^{2n})$$
$$= (x^{2n} + y^{2n})[(x^n)^2 - (y^n)^2]$$
$$= (x^{2n} + y^{2n})(x^n + y^n)(x^n - y^n)$$

You Try It 11

Factor: $x^{4n} - x^{2n}y^{2n}$

Your solution

Example 12

Factor: $x^{n+3} + x^ny^3$

Solution

$$x^{n+3} + x^ny^3 = x^n(x^3 + y^3)$$
$$= x^n(x + y)(x^2 - xy + y^2)$$

You Try It 12

Factor: $ax^5 - ax^2y^6$

Your solution

Solutions on pp. S21–S22

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7.4 Exercises

Objective A

Which of the expressions are perfect squares?

1. $4;\ 8;\ 25x^6;\ 12y^{10};\ 100x^4y^4$

2. $9;\ 18;\ 15a^8;\ 49b^{12};\ 64a^{16}b^2$

Name the square root of the expression.

3. $16z^8$

4. $36d^{10}$

5. $81a^4b^6$

6. $25m^2n^{12}$

Factor.

7. $x^2 - 16$

8. $y^2 - 49$

9. $4x^2 - 1$

10. $81x^2 - 4$

11. $16x^2 - 121$

12. $49y^2 - 36$

13. $1 - 9a^2$

14. $16 - 81y^2$

15. $x^2y^2 - 100$

16. $a^2b^2 - 25$

17. $x^2 + 4$

18. $a^2 + 16$

19. $25 - a^2b^2$

20. $64 - x^2y^2$

21. $a^{2n} - 1$

22. $b^{2n} - 16$

23. $x^2 - 12x + 36$

24. $y^2 - 6y + 9$

25. $b^2 - 2b + 1$

26. $a^2 + 14a + 49$

27. $16x^2 - 40x + 25$

28. $49x^2 + 28x + 4$

29. $4a^2 + 4a - 1$

30. $9x^2 + 12x - 4$

31. $b^2 + 7b + 14$

32. $y^2 - 5y + 25$

33. $x^2 + 6xy + 9y^2$

34. $4x^2y^2 + 12xy + 9$

35. $25a^2 - 40ab + 16b^2$

36. $4a^2 - 36ab + 81b^2$

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37. $x^{2n} + 6x^n + 9$

38. $y^{2n} - 16y^n + 64$

39. $(x - 4)^2 - 9$

40. $16 - (a - 3)^2$

41. $(x - y)^2 - (a + b)^2$

42. $(x - 2y)^2 - (x + y)^2$

Objective B

Which of the expressions are perfect cubes?

43. $4; 8; x^9; a^8b^8; 27c^{15}d^{18}$

44. $9; 27; y^{12}; m^3n^6; 64mn^9$

Name the cube root of the expression.

45. $8x^9$

46. $27y^{15}$

47. $64a^6b^{18}$

48. $125c^{12}d^3$

Factor.

49. $x^3 - 27$

50. $y^3 + 125$

51. $8x^3 - 1$

52. $64a^3 + 27$

53. $x^3 - y^3$

54. $x^3 - 8y^3$

55. $m^3 + n^3$

56. $27a^3 + b^3$

57. $64x^3 + 1$

58. $1 - 125b^3$

59. $27x^3 - 8y^3$

60. $64x^3 + 27y^3$

61. $x^3y^3 + 64$

62. $8x^3y^3 + 27$

63. $16x^3 - y^3$

64. $27x^3 - 8y^2$

65. $8x^3 - 9y^3$

66. $27a^3 - 16$

67. $(a - b)^3 - b^3$

68. $a^3 + (a + b)^3$

69. $x^{6n} + y^{3n}$

70. $x^{3n} + y^{3n}$

71. $x^{3n} + 8$

72. $a^{3n} + 64$

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Objective C

Factor.

73. $x^2y^2 - 8xy + 15$

74. $x^2y^2 - 8xy - 33$

75. $x^2y^2 - 17xy + 60$

76. $a^2b^2 + 10ab + 24$

77. $x^4 - 9x^2 + 18$

78. $y^4 - 6y^2 - 16$

79. $b^4 - 13b^2 - 90$

80. $a^4 + 14a^2 + 45$

81. $x^4y^4 - 8x^2y^2 + 12$

82. $a^4b^4 + 11a^2b^2 - 26$

83. $x^{2n} + 3x^n + 2$

84. $a^{2n} - a^n - 12$

85. $3x^2y^2 - 14xy + 15$

86. $5x^2y^2 - 59xy + 44$

87. $6a^2b^2 - 23ab + 21$

88. $10a^2b^2 + 3ab - 7$

89. $2x^4 - 13x^2 - 15$

90. $3x^4 + 20x^2 + 32$

91. $2x^{2n} - 7x^n + 3$

92. $4x^{2n} + 8x^n - 5$

93. $6a^{2n} + 19a^n + 10$

Objective D

Factor.

94. $5x^2 + 10x + 5$

95. $12x^2 - 36x + 27$

96. $3x^4 - 81x$

97. $27a^4 - a$

98. $7x^2 - 28$

99. $20x^2 - 5$

100. $y^4 - 10y^3 + 21y^2$

101. $y^5 + 6y^4 - 55y^3$

102. $x^4 - 16$

103. $16x^4 - 81$

104. $8x^5 - 98x^3$

105. $16a - 2a^4$

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106. $x^3y^3 - x^3$

107. $a^3b^6 - b^3$

108. $x^6y^6 - x^3y^3$

109. $8x^4 - 40x^3 + 50x^2$

110. $6x^5 + 74x^4 + 24x^3$

111. $x^4 - y^4$

112. $16a^4 - b^4$

113. $x^6 + y^6$

114. $x^4 - 5x^2 - 4$

115. $a^4 - 25a^2 - 144$

116. $3b^5 - 24b^2$

117. $16a^4 - 2a$

118. $16x^3y + 4x^2y^2 - 42xy^3$

119. $24a^2b^2 - 14ab^3 - 90b^4$

120. $x^3 - 2x^2 - x + 2$

121. $x^3 - 2x^2 - 4x + 8$

122. $4x^2y^2 - 4x^2 - 9y^2 + 9$

123. $4x^4 - x^2 - 4x^2y^2 + y^2$

124. $a^{2n+2} - 6a^{n+2} + 9a^2$

125. $x^{2n+1} + 2x^{n+1} + x$

126. $2x^{n+2} - 7x^{n+1} + 3x^n$

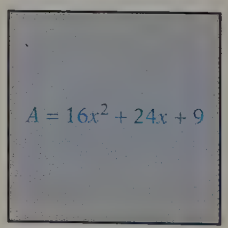
127. $3b^{n+2} + 4b^{n+1} - 4b^n$

APPLYING THE CONCEPTS

128. Find all integers k such that the trinomial is a perfect square.

a. $x^2 + kx + 36$ **b.** $4x^2 - kx + 25$ **c.** $49x^2 + kxy + 64y^2$

129. The area of a square is $(16x^2 + 24x + 9)$ m^2. Find the dimensions of the square in terms of the variable x. Can $x = 0$? What are the possible values of x?



130. Can a third-degree polynomial have factors $(x - 1)$, $(x + 1)$, $(x - 3)$, and $(x + 4)$? Why or why not?

131. Given that $(x - 3)$ and $(x + 4)$ are factors of $x^3 + 6x^2 - 7x - 60$, explain how you can find a third *first-degree* factor of $x^3 + 6x^2 - 7x - 60$. Then find the factor.

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7.5 Solving Equations

Objective A

To solve equations by factoring









The Multiplication Property of Zero states that the product of a number and zero is zero. This property is stated below.

If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Now consider $x \cdot y = 0$. For this to be a true equation, then either $x = 0$ or $y = 0$.

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products is used to solve some equations.

Solve: $(x - 2)(x - 3) = 0$

By the Principle of Zero Products, if $(x - 2)(x - 3) = 0$, then $x - 2 = 0$ or $x - 3 = 0$.

$(x - 2)(x - 3) = 0$

$x - 2 = 0 \qquad x - 3 = 0$ • Let each factor equal zero (the Principle of Zero Products).

$x = 2 \qquad x = 3$ • Solve each equation for x.

Check:

$$\begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (2-2)(2-3) & 0 \\ 0(-1) & 0 \\ 0 = 0 & \text{True} \end{array} \qquad \begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (3-2)(3-3) & 0 \\ (1)(0) & 0 \\ 0 = 0 & \text{True} \end{array}$$

The solutions are 2 and 3.

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a **quadratic equation**. A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. The quadratic equations at the right are in standard form.

$3x^2 + 2x + 1 = 0$

$4x^2 - 3x + 2 = 0$

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TAKE NOTE

Note the steps involved in solving a quadratic equation by factoring:

1. Write in standard form.
2. Factor.
3. Set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

➡ Solve: $2x^2 + x = 6$

$$2x^2 + x = 6$$
$$2x^2 + x - 6 = 0$$ • Write the equation in standard form.
$$(2x - 3)(x + 2) = 0$$ • Factor.

$2x - 3 = 0$ $\qquad$ $x + 2 = 0$ • Use the Principle of Zero Products.

$2x = 3$ $\qquad$ $x = -2$ • Solve each equation for x.

$$x = \frac{3}{2}$$

$\frac{3}{2}$ and -2 check as solutions. The solutions are $\frac{3}{2}$ and -2.

Example 1

Solve: $x(x - 3) = 0$

Solution

$x(x - 3) = 0$

$x = 0$ $\qquad$ $x - 3 = 0$

$x = 3$

The solutions are 0 and 3.

You Try It 1

Solve: $2x(x + 7) = 0$

Your solution

Example 2

Solve: $2x^2 - 50 = 0$

Solution

$$2x^2 - 50 = 0$$
$$2(x^2 - 25) = 0$$
$$2(x + 5)(x - 5) = 0$$

$x + 5 = 0$ $\qquad$ $x - 5 = 0$

$x = -5$ $\qquad$ $x = 5$

The solutions are -5 and 5.

You Try It 2

Solve: $4x^2 - 9 = 0$

Your solution

Example 3

Solve: $(x - 3)(x - 10) = -10$

Solution

$$(x - 3)(x - 10) = -10$$
$$x^2 - 13x + 30 = -10$$ • Multiply $(x - 3)(x - 10)$.
$$x^2 - 13x + 40 = 0$$ • Add 10 to each side of the equation. The equation is now in standard form.
$$(x - 8)(x - 5) = 0$$

$x - 8 = 0$ $\qquad$ $x - 5 = 0$

$x = 8$ $\qquad$ $x = 5$

The solutions are 8 and 5.

You Try It 3

Solve: $(x + 2)(x - 7) = 52$

Your solution

Solutions on p. S22

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Objective B To solve application problems



Recall that the integers are the numbers . . . , −3, −2, −1, 0, 1, 2, 3, ...

An **even integer** is an integer that is divisible by 2. Examples of even integers are −8, 0, and 22. An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are −17, 1, and 39.

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

11, 12, 13
−8, −7, −6
$n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

24, 26, 28
−10, −8, −6
$n, n + 2, n + 4$

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

19, 21, 23
−1, 1, 3
$n, n + 2, n + 4$

Example 4

The sum of the squares of two consecutive positive even integers is equal to 100. Find the two integers.

Strategy

First positive even integer: n
Second positive even integer: $n + 2$

The sum of the square of the first positive even integer and the square of the second positive even integer is 100.

Solution

$$n^2 + (n + 2)^2 = 100$$
$$n^2 + n^2 + 4n + 4 = 100$$
$$2n^2 + 4n + 4 = 100$$
$$2n^2 + 4n - 96 = 0$$
$$2(n^2 + 2n - 48) = 0$$
$$n^2 + 2n - 48 = 0$$

• Divide each side of the equation by 2.

$$(n - 6)(n + 8) = 0$$

$$n - 6 = 0 \qquad n + 8 = 0$$
$$n = 6 \qquad n = -8$$

Because −8 is not a positive even integer, it is not a solution.

$$n = 6$$
$$n + 2 = 6 + 2 = 8$$

The two integers are 6 and 8.

You Try It 4

The sum of the squares of two positive consecutive integers is 61. Find the two integers.

Your strategy

Your solution

Solution on p. S22

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Example 5

A stone is thrown into a well with an initial speed of 4 ft/s. The well is 420 ft deep. How many seconds later will the stone hit the bottom of the well? Use the equation $d = vt + 16t^2$, where d is the distance in feet, v is the initial speed, and t is the time in seconds.

Strategy

To find the time for the stone to drop to the bottom of the well, replace the variables d and v by their given values and solve for t.

Solution

$$d = vt + 16t^2$$
$$420 = 4t + 16t^2$$
$$0 = -420 + 4t + 16t^2$$
$$16t^2 + 4t - 420 = 0$$
$$4(4t^2 + t - 105) = 0$$
$$4t^2 + t - 105 = 0$$

• Divide each side of the equation by 4.

$$(4t + 21)(t - 5) = 0$$

$$4t + 21 = 0 \qquad t - 5 = 0$$
$$4t = -21 \qquad t = 5$$
$$t = -\frac{21}{4}$$

Because the time cannot be a negative number, $-\frac{21}{4}$ is not a solution.

The time is 5 s.

You Try It 5

The length of a rectangle is 4 in. longer than twice the width. The area of the rectangle is 96 in^2. Find the length and width of the rectangle.

Your strategy

Your solution

Solution on p. S22

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7.5 Exercises

Objective A

1. In your own words, explain why it is possible to solve a quadratic equation using the Principle of Zero Products.

Solve.

2. $(y + 3)(y + 2) = 0$ **3.** $(y - 3)(y - 5) = 0$ **4.** $(z - 7)(z - 3) = 0$ **5.** $(z + 8)(z - 9) = 0$

6. $x(x - 5) = 0$ **7.** $x(x + 2) = 0$ **8.** $a(a - 9) = 0$ **9.** $a(a + 12) = 0$

10. $y(2y + 3) = 0$ **11.** $t(4t - 7) = 0$ **12.** $2a(3a - 2) = 0$ **13.** $4b(2b + 5) = 0$

14. $(b + 2)(b - 5) = 0$ **15.** $(b - 8)(b + 3) = 0$ **16.** $x^2 - 81 = 0$ **17.** $x^2 - 121 = 0$

18. $4x^2 - 49 = 0$ **19.** $16x^2 - 1 = 0$ **20.** $9x^2 - 1 = 0$ **21.** $16x^2 - 49 = 0$

22. $x^2 + 6x + 8 = 0$ **23.** $x^2 - 8x + 15 = 0$ **24.** $z^2 + 5z - 14 = 0$ **25.** $z^2 + z - 72 = 0$

26. $2a^2 - 9a - 5 = 0$ **27.** $3a^2 + 14a + 8 = 0$ **28.** $6z^2 + 5z + 1 = 0$ **29.** $6y^2 - 19y + 15 = 0$

30. $x^2 - 3x = 0$ **31.** $a^2 - 5a = 0$ **32.** $x^2 - 7x = 0$ **33.** $2a^2 - 8a = 0$

34. $a^2 + 5a = -4$ **35.** $a^2 - 5a = 24$ **36.** $y^2 - 5y = -6$ **37.** $y^2 - 7y = 8$

38. $2t^2 + 7t = 4$ **39.** $3t^2 + t = 10$ **40.** $3t^2 - 13t = -4$ **41.** $5t^2 - 16t = -12$

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42. $x(x - 12) = -27$ **43.** $x(x - 11) = 12$ **44.** $y(y - 7) = 18$ **45.** $y(y + 8) = -15$

46. $p(p + 3) = -2$ **47.** $p(p - 1) = 20$ **48.** $y(y + 4) = 45$ **49.** $y(y - 8) = -15$

50. $x(x + 3) = 28$ **51.** $p(p - 14) = 15$ **52.** $(x + 8)(x - 3) = -30$ **53.** $(x + 4)(x - 1) = 14$

54. $(z - 5)(z + 4) = 52$ **55.** $(z - 8)(z + 4) = -35$ **56.** $(z - 6)(z + 1) = -10$

57. $(a + 3)(a + 4) = 72$ **58.** $(a - 4)(a + 7) = -18$ **59.** $(2x + 5)(x + 1) = -1$

Objective B Application Problems

60. The square of a positive number is six more than five times the positive number. Find the number.

61. The square of a negative number is fifteen more than twice the negative number. Find the number.

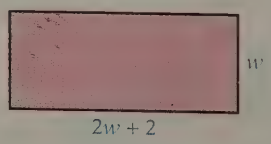
62. The sum of two numbers is six. The sum of the squares of the two numbers is twenty. Find the two numbers.

63. The sum of two numbers is eight. The sum of the squares of the two numbers is thirty-four. Find the two numbers.

64. The sum of the squares of two consecutive positive integers is forty-one. Find the two integers.

65. The sum of the squares of two consecutive positive even integers is one hundred sixty-four. Find the two integers.

66. The length of a rectangle is 2 ft more than twice the width. The area of the rectangle is 84 ft^2. Find the length and width of the rectangle.



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67. The length of a rectangle is 8 cm more than three times the width. The area of the rectangle is 380 cm^2. Find the length and width of the rectangle.

The formula $S = \frac{n^2 + n}{2}$ gives the sum, S, of the first n natural numbers. Use this formula for Exercises 68 and 69.

68. How many consecutive natural numbers beginning with 1 will give a sum of 78?

69. How many consecutive natural numbers beginning with 1 will give a sum of 171?

The formula $N = \frac{t^2 - t}{2}$ gives the number, N, of football games that must be scheduled in a league with t teams if each team is to play every other team once. Use this formula for Exercises 70 and 71.

70. How many teams are in a league that schedules 15 games in such a way that each team plays every other team once?

71. How many teams are in a league that schedules 45 games in such a way that each team plays every other team once?

The distance, s, in feet, that an object will fall (neglecting air resistance) in t seconds is given by $s = vt + 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 72 and 73.

72. An object is released from the top of a building 192 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

73. An object is released from the top of a building 320 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

The height, h, in feet, an object will attain (neglecting air resistance) in t seconds is given by $h = vt - 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 74 and 75.

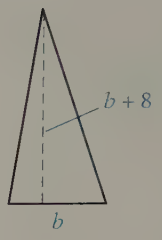


74. A golf ball is thrown onto a cement surface and rebounds straight up. The initial velocity of the rebound is 60 ft/s. How many seconds later will the golf ball return to the ground?

75. A foul ball leaves a bat and travels straight up with an initial velocity of 64 ft/s. How many seconds later will the ball be 64 ft above the ground?

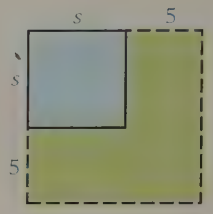
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76. The height of a triangle is 8 cm more than the length of the base. The area of the triangle is 64 cm^2. Find the base and height of the triangle.



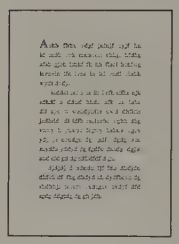
77. The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 m^2. Find the height of the triangle.

78. The length of each side of a square is extended 5 in. The area of the resulting square is 64 in^2. Find the length of a side of the original square.



79. A small garden measures 8 ft by 10 ft. A uniform border around the garden increases the total area to 143 ft^2. What is the width of the border?

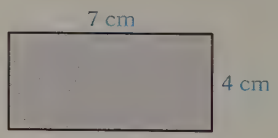
80. The page of a book measures 6 in. by 9 in. A uniform border around the page leaves 28 in^2 for type. What are the dimensions of the type area?



81. The radius of a circle is increased by 3 in.; this increases the area by 100 in^2. Find the radius of the original circle. Round to the nearest hundredth.

APPLYING THE CONCEPTS

82. The length of a rectangle is 7 cm, and the width is 4 cm. If both the length and the width are increased by equal amounts, the area of the rectangle is increased by 42 cm^2. Find the length and width of the larger rectangle.



83. A rectangular piece of cardboard is 10 in. longer than it is wide. Squares 2 in. on a side are to be cut from each corner, and then the sides will be folded up to make an open box with a volume of 192 in^3. Find the length and width of the piece of cardboard.



84. Solve: $p^3 = 9p^2$

85. Solve: $(x + 3)(2x - 1) = (3 - x)(5 - 3x)$

86. Find $3n^2$ if $n(n + 5) = -4$.

87. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$(x + 2)(x - 3) = 6$$
$$x + 2 = 6 \quad x - 3 = 6$$
$$x = 4 \qquad x = 9$$

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Focus on Problem Solving

Polya's Four-Step Process

Your success in mathematics and your success in the workplace are heavily dependent on your ability to solve problems. One of the foremost mathematicians to study problem solving was George Polya (1887–1985). The basic structure that Polya advocated for problem solving has four steps, as outlined below.



(For more information on Polya and his work, conduct a search on the Internet for "Polya." You can also search the Math Forum at **www.forum.swarthmore.edu**.)

Point of Interest

George Polya was born in Hungary and moved to the United States in 1940. He lived in Providence, Rhode Island, where he taught at Brown University until 1942, when he moved to California. There he taught at Stanford University until his retirement. While at Stanford, he published 10 books and a number of articles for mathematics journals. Of the books Polya published, *How To Solve It* (1945) is one of the best known. In this book, Polya outlines a strategy for solving problems. This strategy, although frequently applied to mathematics, can be used to solve problems from virtually any discipline.

1. Understand the Problem

You must have a clear understanding of the problem. To help you focus on understanding the problem, here are some questions to think about.

- Can you restate the problem in your own words?
- Can you determine what is known about these types of problems?
- Is there missing information that you need in order to solve the problem?
- Is there information given that is not needed?
- What is the goal?

2. Devise a Plan

Successful problem solvers use a variety of techniques when they attempt to solve a problem. Here are some frequently used strategies.

- Make a list of the known information.
- Make a list of information that is needed to solve the problem.
- Make a table or draw a diagram.
- Work backwards.
- Try to solve a similar but simpler problem.
- Research the problem to determine whether there are known techniques for solving problems of its kind.
- Try to determine whether some pattern exists.
- Write an equation.

3. Carry Out the Plan

Once you have devised a plan, you must carry it out.

- Work carefully.
- Keep an accurate and neat record of all your attempts.
- Realize that some of your initial plans will not work and that you may have to return to Step 2 and devise another plan or modify your existing plan.

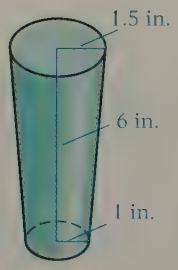
4. Review Your Solution

Once you have found a solution, check the solution against the known facts.

- Ensure that the solution is consistent with the facts of the problem.
- Interpret the solution in the context of the problem.
- Ask yourself whether there are generalizations of the solution that could apply to other problems.
- Determine the strengths and weaknesses of your solution. For instance, is your solution only an approximation to the actual solution?
- Consider the possibility of alternative solutions.

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We will use Polya's four-step process to solve the following problem.



A large soft drink costs \$1.25 at a college cafeteria. The dimensions of the cup are shown at the left. Suppose you don't put any ice in the cup. Determine the cost per ounce for the soft drink.

1. *Understand the problem.* We must determine the cost per ounce for the soda. To do this, we need the dimensions of the cup (which are given), the cost of the drink (given), and a formula for the volume of the cup (unknown). Also, because the dimensions are given in inches, the volume will be in cubic inches; we need a conversion factor that will convert cubic inches to fluid ounces.
2. *Devise a plan.* Consult a resource book that gives the volume of the figure, which is called a **frustrum**. The formula for the volume is

$$V = \frac{\pi h}{3}(r^2 + rR + R^2)$$

where h is the height, r is the radius of the base, and R is the radius of the top. Also from a reference book, $1 \text{ in}^3 \approx 0.55$ fl oz. The general plan is to calculate the volume, convert the answer to fluid ounces, and then divide the cost by the number of fluid ounces.
3. *Carry out the plan.* Using the information from the drawing, evaluate the formula for the volume.

$$V = \frac{6\pi}{3}[1^2 + 1(1.5) + 1.5^2] = 9.5\pi \approx 29.8451 \text{ in}^3$$

$$V \approx 29.8451(0.55) \approx 16.4148 \text{ fl oz} \quad \bullet \text{ Convert to fluid ounces.}$$

$$\text{Cost per ounce} \approx \frac{1.25}{16.4148} \approx 0.07615 \quad \bullet \text{ Divide the cost by the volume.}$$

The cost of the soft drink is approximately 7.62 cents per ounce.

4. *Review the solution.* The cost of a 12-ounce can of soda from a vending machine is generally about 75¢. Therefore, the cost of canned soda is $75¢ \div 12 = 6.25¢$ per ounce. This is consistent with our solution. This does not mean our solution is correct, but it does indicate that it is at least reasonable. Why might soda from a cafeteria be more expensive per ounce than soda from a vending machine?

   Is there an alternative way to obtain the solution? There are probably many, but one possibility is to get a measuring cup, pour the soft drink into it, and read the number of ounces. Name an advantage and a disadvantage of this method.

Use the four-step solution process for the following problems.

1. A cup dispenser next to a water cooler holds cups that have the shape of a right circular cone. The height of the cone is 4 in., and the radius of the circular top is 1.5 in. How many ounces of water can the cup hold?
2. Soft drink manufacturers do research into the preferences of consumers with regard to the look and feel and size of a soft drink can. Suppose that a manufacturer has determined that people want to have their hand reach around approximately 75% of the can. If this preference is to be achieved, how tall should the can be if it contains 12 oz of fluid? Assume the can is a right circular cylinder.

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Projects and Group Activities

Water Displacement

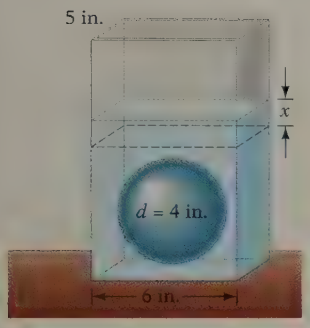
When an object is placed in water, the object displaces an amount of water that is equal to the volume of the object.

Example A sphere with a diameter of 4 in. is placed in a rectangular tank of water that is 6 in. long and 5 in. wide. How much does the water level rise? Round to the nearest hundredth.

$V = \frac{4}{3}\pi r^3$ • Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi(2^3) = \frac{32}{3}\pi$ • $r = \frac{1}{2}d = \frac{1}{2}(4) = 2$

Let x represent the amount of the rise in water level. The volume of the sphere will equal the volume displaced by the water. As shown at the left, this volume is the rectangular solid with width 5 in., length 6 in., and height x in.



$V = LWH$ • Use the formula for the volume of a rectangular solid.

$\frac{32}{3}\pi = (6)(5)x$ • Substitute $\frac{32}{3}\pi$ for V, 5 for W, and 6 for L.

$\frac{32}{90}\pi = x$ • The exact height that the water will fill is $\frac{32}{90}\pi$.

$1.12 \approx x$ • Use a calculator to find an approximation.

The water will rise approximately 1.12 in.

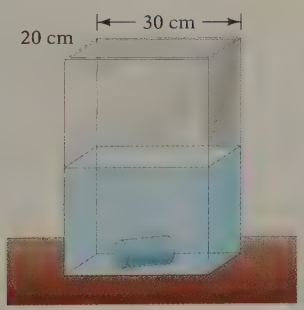


Figure 1

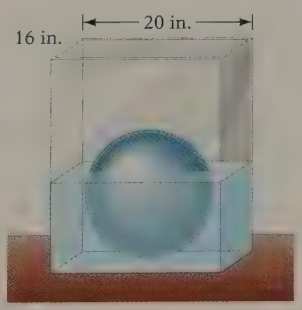


Figure 2

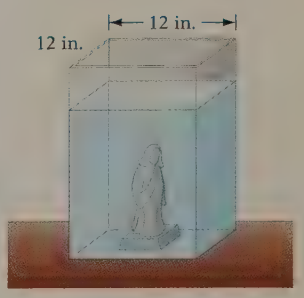


Figure 3

1. A cylinder with a 2-centimeter radius and a height of 10 cm is submerged in a tank of water that is 20 cm wide and 30 cm long (see Figure 1). How much does the water level rise? Round to the nearest hundredth.
2. A sphere with a radius of 6 in. is placed in a rectangular tank of water that is 16 in. wide and 20 in. long (see Figure 2). The sphere displaces water until two-thirds of the sphere is submerged. How much does the water level rise? Round to the nearest hundredth.
3. A chemist wants to know the density of a statue that weighs 15 lb. The statue is placed in a rectangular tank of water that is 12 in. long and 12 in. wide (see Figure 3). The water level rises 0.42 in. Find the density of the statue. Round to the nearest hundredth. (*Hint*: Density = weight ÷ volume)

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Chapter Summary

Key Words

The *greatest common factor* (GCF) of two or more monomials is the product of the GCF of the coefficients and the common variable factors. [p. 379]

To *factor* a polynomial means to write the polynomial as a product of other polynomials. [p. 379]

To *factor* a trinomial of the form $ax^2 + bx + c$ means to express the trinomial as the product of two binomials. [p. 385]

A polynomial is *nonfactorable over the integers* if it does not factor using only integers. Such a polynomial is called a *prime polynomial.* [p. 386]

A product of a term and itself is a *perfect square.* [p. 401]

The product of the same three factors is a *perfect cube.* [p. 403]

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a *quadratic equation.* [p. 411]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. The quadratic equation $ax^2 + bx + c = 0$ is in standard form. [p. 411]

Essential Rules

Factors of the Difference of Two Perfect Squares [p. 401]

$$a^2 - b^2 = (a + b)(a - b)$$

Factors of a Perfect-Square Trinomial [p. 401]

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Factors of the Sum or Difference of Two Cubes [p. 403]

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Principle of Zero Products [p. 411]

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

General Factoring Strategy [p. 405]

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two cubes, or the difference of two cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. If the polynomial has four terms, can it be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

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Chapter Review

1. Factor: $5x^3 + 10x^2 + 35x$

2. Factor: $12a^2b + 3ab^2$

3. Factor: $14y^9 - 49y^6 + 7y^3$

4. Factor: $4x(x - 3) - 5(3 - x)$

5. Factor: $10x^2 + 25x + 4xy + 10y$

6. Factor: $21ax - 35bx - 10by + 6ay$

7. Factor: $b^2 - 13b + 30$

8. Factor: $c^2 + 8c + 12$

9. Factor: $y^2 + 5y - 36$

10. Factor: $3a^2 - 15a - 42$

11. Factor: $4x^3 - 20x^2 - 24x$

12. Factor: $n^4 - 2n^3 - 3n^2$

13. Factor $6x^2 - 29x + 28$ by using trial factors.

14. Factor $12y^2 + 16y - 3$ by using trial factors.

15. Factor $2x^2 - 5x + 6$ by using trial factors.

16. Factor $3x^2 - 17x + 10$ by grouping.

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17. Factor $2a^2 - 19a - 60$ by grouping.

18. Factor $18a^2 - 3a - 10$ by grouping.

19. Factor: $x^2y^2 - 9$

20. Factor: $4x^2 + 12xy + 9y^2$

21. Factor: $x^{2n} - 12x^n + 36$

22. Factor: $64a^3 - 27b^3$

23. Factor: $15x^4 + x^2 - 6$

24. Factor: $21x^4y^4 + 23x^2y^2 + 6$

25. Factor: $3a^6 - 15a^4 - 18a^2$

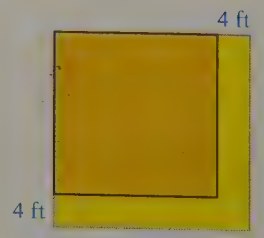
26. Solve: $4x^2 + 27x = 7$

27. Solve: $(x + 1)(x - 5) = 16$

28. The length of a hockey field is 20 yd less than twice the width of the hockey field. The area of the hockey field is 6000 yd^2. Find the length and width of the hockey field.

29. A rectangular photograph has dimensions 15 in. by 12 in. A picture frame around the photograph increases the total area to 270 in^2. What is the width of the frame?

30. The lengths of two sides of a square garden plot are extended 4 ft, as shown in the figure at the right. The area of the resulting square is 576 ft^2. Find the length of a side of the original garden plot.



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Chapter Test

1. Factor: $ab + 6a - 3b - 18$

2. Factor: $2y^4 - 14y^3 - 16y^2$

3. Factor $8x^2 + 20x - 48$ by grouping.

4. Factor $6x^2 + 19x + 8$ by using trial factors.

5. Factor: $a^2 - 19a + 48$

6. Factor: $6x^3 - 8x^2 + 10x$

7. Factor: $x^2 + 2x - 15$

8. Solve: $4x^2 - 1 = 0$

9. Factor: $5x^2 - 45x - 15$

10. Factor: $p^2 + 12p + 36$

11. Solve: $x(x - 8) = -15$

12. Factor: $3x^2 + 12xy + 12y^2$

13. Factor: $27x^3 - 8$

14. Factor $6x^2y^2 + 9xy^2 + 3y^2$ by grouping.

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15. Factor: $6a^4 - 13a^2 - 5$

16. Factor: $a(x - 2) + b(x - 2)$

17. Factor: $x(p + 1) - (p + 1)$

18. Factor: $3a^2 - 75$

19. Factor $2x^2 + 4x - 5$ by using trial factors.

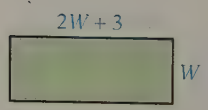
20. Factor: $x^2 - 9x - 36$

21. Factor: $4a^2 - 12ab + 9b^2$

22. Factor: $4x^2 - 49y^2$

23. Solve: $(2a - 3)(a + 7) = 0$

24. The length of a rectangle is 3 cm longer than twice its width. The area of the rectangle is 90 cm^2. Find the length and width of the rectangle.



25. The sum of two numbers is ten. The sum of the squares of the two numbers is fifty-eight. Find the two numbers.

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Cumulative Review

1. Subtract: $-2 - (-3) - 5 - (-11)$

2. Simplify: $(3 - 7)^2 \div (-2) - 3(-4)$

3. Evaluate $-2a^2 \div (2b) - c$ when $a = -4$, $b = 2$, and $c = -1$.

4. Multiply: $-\frac{3}{4}(-20x^2)$

5. Simplify: $-2[4x - 2(3 - 2x) - 8x]$

6. Solve: $-\frac{5}{7}x = -\frac{10}{21}$

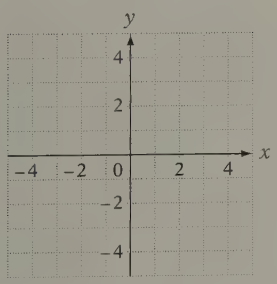
7. Solve: $3x - 2 = 12 - 5x$

8. Solve: $-2 + 4[3x - 2(4 - x) - 3] = 4x + 2$

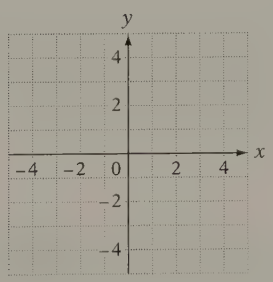
9. 120% of what number is 54?

10. Given $f(x) = -x^2 + 3x - 1$, find $f(2)$.

11. Graph $y = \frac{1}{4}x + 3$.



12. Graph $5x + 3y = 15$.



13. Find the equation of the line that contains the point $(-3, 4)$ and has slope $\frac{2}{3}$.

14. Solve by substitution: $8x - y = 2$
$y = 5x + 1$

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15. Solve by the addition method:
$5x + 2y = -9$
$12x - 7y = 2$

16. Simplify: $(-3a^3b^2)^2$

17. Multiply: $(x + 2)(x^2 - 5x + 4)$

18. Divide: $(8x^2 + 4x - 3) \div (2x - 3)$

19. Simplify: $(x^{-4}y^3)^2$

20. Factor: $3a - 3b - ax + bx$

21. Factor: $15xy^2 - 20xy^4$

22. Factor: $x^2 - 5xy - 14y^2$

23. Solve: $3x^2 + 19x - 14 = 0$

24. Solve: $6x^2 + 60 = 39x$

25. A triangle has a 31° angle and a right angle. Find the measure of the third angle.

26. A rectangular flower garden has a perimeter of 86 ft. The length of the garden is 28 ft. What is the width of the garden?

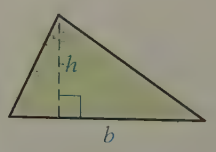


27. A board 10 ft long is cut into two pieces. Four times the length of the shorter piece is 2 ft less than three times the length of the longer piece. Find the length of each piece.

28. An investment of $4000 was made at an annual simple interest rate of 8%. How much more money was invested at an annual simple interest rate of 11% if the total interest earned in 1 year was $1035?

29. A family drove to a resort at an average speed of 42 mph and later returned over the same road at an average speed of 56 mph. Find the distance to the resort if the total driving time was 7 h.

30. The length of the base of a triangle is three times the height. The area of the triangle is 24 in^2. Find the length of the base of the triangle.



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Chapter 8

Rational Expressions



In order to monitor species that are or are becoming endangered, scientists need to determine the present population of that species. Scientists catch and tag a certain number of the animals and then release them. Later, a group of the animals from that same habitat is caught and the number tagged is counted. A proportion is used to estimate the total population size in that region, as shown in **Exercise 70 on page 465.** Tracking the tagged animals also assists scientists in learning more about the habits of that species.

Objectives

Section 8.1

- **A** To simplify a rational expression
- **B** To multiply rational expressions
- **C** To divide rational expressions

Section 8.2

- **A** To find the least common multiple (LCM) of two or more polynomials
- **B** To express two fractions in terms of the LCM of their denominators
- **C** To add or subtract rational expressions with the same denominator
- **D** To add or subtract rational expressions with different denominators

Section 8.3

- **A** To simplify a complex fraction

Section 8.4

- **A** To solve rational equations
- **B** To solve proportions
- **C** To solve problems involving similar triangles
- **D** To solve application problems

Section 8.5

- **A** To solve a literal equation for one of the variables

Section 8.6

- **A** To solve work problems
- **B** To solve uniform motion problems

Section 8.7

- **A** To solve variation problems



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Find the LCM of 10 and 25.

For Exercises 2 to 5, add, subtract, multiply, or divide.

2. $-\frac{3}{8} \cdot \frac{4}{9}$

3. $-\frac{4}{5} \div \frac{8}{15}$

4. $-\frac{5}{6} + \frac{7}{8}$

5. $-\frac{3}{8} - \left(-\frac{7}{12}\right)$

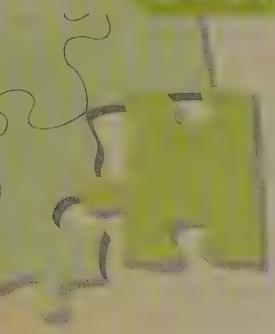
6. Evaluate $\frac{2x - 3}{x^2 - x + 1}$ for $x = 2$.

7. Solve: $4(2x + 1) = 3(x - 2)$

8. Solve: $10\left(\frac{t}{2} + \frac{t}{5}\right) = 10(1)$

9. Two planes start from the same point and fly in opposite directions. The first plane is flying 20 mph slower than the second plane. In 2 h, the planes are 480 mi apart. Find the rate of each plane.

Go Figure



If 6 machines can fill 12 boxes of cereal in 7 min, how many boxes of cereal can 14 machines fill in 12 min?

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8.1 Multiplication and Division of Rational Expressions

Objective A

To simplify a rational expression

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A fraction in which the numerator and denominator are polynomials is called a **rational expression**. Examples of rational expressions are shown at the right.

$$\frac{5}{z}, \quad \frac{x^2+1}{2x-1}, \quad \frac{y^2+y-1}{4y^2+1}$$

Care must be exercised with a rational expression to ensure that when the variables are replaced with numbers, the resulting denominator is not zero.

Consider the rational expression at the right. The value of x cannot be 3, because the denominator would then be zero.

$$\frac{4x^2-9}{2x-6}$$

$$\frac{4(3)^2-9}{2(3)-6}=\frac{27}{0}$$ Not a real number

A rational expression is in simplest form when the numerator and denominator have no common factors. The Multiplication Property of One is used to write a rational expression in simplest form.

Simplify: $\dfrac{x^2-4}{x^2-2x-8}$

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)(x+2)}{(x-4)(x+2)}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}\cdot\boxed{\frac{x+2}{x+2}}=\frac{x-2}{x-4}\cdot 1$$

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- The restrictions $x \neq -2, 4$ are necessary to prevent division by zero.

This simplification is usually shown with slashes through the common factors. The last simplification would be shown as follows:

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)\overset{1}{\cancel{(x+2)}}}{(x-4)\underset{1}{\cancel{(x+2)}}}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- Divide by the common factors.

Simplify: $\dfrac{10+3x-x^2}{x^2-4x-5}$

$$\frac{10+3x-x^2}{x^2-4x-5}=\frac{-(x^2-3x-10)}{x^2-4x-5}$$

- Factor -1 from the trinomial in the numerator.

$$=\frac{-(x+2)\overset{1}{\cancel{(x-5)}}}{(x+1)\underset{1}{\cancel{(x-5)}}}$$

- Factor the numerator and denominator. Divide by the common factors.

$$=-\frac{x+2}{x+1}, x\neq -1, 5$$

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For the remaining examples, we will omit the restrictions on the variables that prevent division by zero and assume that the values of the variables are such that division by zero is not possible.

Example 1

Simplify: $\dfrac{4x^3y^4}{6x^4y}$

Solution

$$\frac{4x^3y^4}{6x^4y} = \frac{2y^3}{3x}$$ • Use rules of exponents.

You Try It 1

Simplify: $\dfrac{6x^5y}{12x^2y^3}$

Your solution

Example 2

Simplify: $\dfrac{9 - x^2}{x^2 + x - 12}$

Solution

$$\frac{9 - x^2}{x^2 + x - 12} = \frac{-(x^2 - 9)}{x^2 + x - 12}$$

$$= \frac{-(x + 3)(\cancel{x - 3}^{1})}{(\cancel{x - 3}_{1})(x + 4)} = -\frac{x + 3}{x + 4}$$

You Try It 2

Simplify: $\dfrac{x^2 + 2x - 24}{16 - x^2}$

Your solution

Example 3

Simplify: $\dfrac{x^2 + 2x - 15}{x^2 - 7x + 12}$

Solution

$$\frac{x^2 + 2x - 15}{x^2 - 7x + 12} = \frac{(x + 5)(\cancel{x - 3}^{1})}{(\cancel{x - 3}_{1})(x - 4)} = \frac{x + 5}{x - 4}$$

You Try It 3

Simplify: $\dfrac{x^2 + 4x - 12}{x^2 - 3x + 2}$

Your solution

Solutions on pp. S22–S23

Objective B

To multiply rational expressions

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The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of the denominators of the two fractions.

Rule for Multiplying Fractions

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are fractions and $b \neq 0$, $d \neq 0$, then $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$.

$$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15} \qquad \frac{3x}{y} \cdot \frac{2}{z} = \frac{6x}{yz} \qquad \frac{x + 2}{x} \cdot \frac{3}{x - 2} = \frac{3x + 6}{x^2 - 2x}$$

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Multiply: $\dfrac{x^2+3x}{x^2-3x-4}\cdot\dfrac{x^2-5x+4}{x^2+2x-3}$

$$\frac{x^2+3x}{x^2-3x-4}\cdot\frac{x^2-5x+4}{x^2+2x-3}$$

$$=\frac{x(x+3)}{(x-4)(x+1)}\cdot\frac{(x-4)(x-1)}{(x+3)(x-1)}$$

- Factor the numerator and denominator of each fraction.

$$=\frac{\overset{1}{\cancel{x}}(\overset{1}{\cancel{x+3}})(\overset{1}{\cancel{x-4}})(\overset{1}{\cancel{x-1}})}{(\underset{1}{\cancel{x-4}})(x+1)(\underset{1}{\cancel{x+3}})(\underset{1}{\cancel{x-1}})}$$

- Multiply.

$$=\frac{x}{x+1}$$

- Write the answer in simplest form.

Example 4

Multiply: $\dfrac{10x^2-15x}{12x-8}\cdot\dfrac{3x-2}{20x-25}$

Solution

$$\frac{10x^2-15x}{12x-8}\cdot\frac{3x-2}{20x-25}$$

$$=\frac{5x(2x-3)}{4(3x-2)}\cdot\frac{(3x-2)}{5(4x-5)}$$

$$=\frac{\overset{1}{5}x(2x-3)(\overset{1}{\cancel{3x-2}})}{4(\underset{1}{\cancel{3x-2}})\underset{1}{5}(4x-5)}=\frac{x(2x-3)}{4(4x-5)}$$

You Try It 4

Multiply: $\dfrac{12x^2+3x}{10x-15}\cdot\dfrac{8x-12}{9x+18}$

Your solution

Example 5

Multiply: $\dfrac{x^2+x-6}{x^2+7x+12}\cdot\dfrac{x^2+3x-4}{4-x^2}$

Solution

$$\frac{x^2+x-6}{x^2+7x+12}\cdot\frac{x^2+3x-4}{4-x^2}$$

$$=\frac{(x+3)(x-2)}{(x+3)(x+4)}\cdot\frac{(x+4)(x-1)}{(2-x)(2+x)}$$

$$=\frac{\overset{1}{\cancel{(x+3)}}(\overset{-1}{\cancel{x-2}})(\overset{1}{\cancel{x+4}})(x-1)}{(\underset{1}{\cancel{x+3}})(\underset{1}{\cancel{x+4}})(\underset{1}{\cancel{2-x}})(2+x)}=-\frac{x-1}{x+2}$$

You Try It 5

Multiply: $\dfrac{x^2+2x-15}{9-x^2}\cdot\dfrac{x^2-3x-18}{x^2-7x+6}$

Your solution

Solutions on p. S23

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Objective 5 To divide rational expressions

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The reciprocal of a fraction is a fraction with the numerator and denominator interchanged.

Fraction	Reciprocal
$\frac{a}{b}$	$\frac{b}{a}$
$x^2 = \frac{x^2}{1}$	$\frac{1}{x^2}$
$\frac{x+2}{x}$	$\frac{x}{x+2}$

Rule for Dividing Fractions

Divide fractions by multiplying the dividend by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{4}{x} \div \frac{y}{5} = \frac{4}{x} \cdot \frac{5}{y} = \frac{20}{xy}$$

$$\frac{x+4}{x} \div \frac{x-2}{4} = \frac{x+4}{x} \cdot \frac{4}{x-2} = \frac{4(x+4)}{x(x-2)}$$

The basis for the division rule is shown at the right.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 6

Divide: $\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$

Solution

$$\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$$

$$= \frac{xy^2 - 3x^2y}{z^2} \cdot \frac{z^3}{6x^2 - 2xy}$$

$$= \frac{xy(y - 3x) \cdot z^3}{z^2 \cdot 2x(3x - y)} = -\frac{yz}{2}$$

(The -1 and 1 indicate the factors that remain after dividing out common factors.)

You Try It 6

Divide: $\frac{a^2}{4bc^2 - 2b^2c} \div \frac{a}{6bc - 3b^2}$

Your solution

Example 7

Divide: $\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$

Solution

$$\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$$

$$= \frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \cdot \frac{2x^2 + 7x - 4}{3x^2 + 13x + 4}$$

$$= \frac{(2x + 1)(x + 2)(2x - 1)(x + 4)}{(2x - 1)(x + 2)(3x + 1)(x + 4)} = \frac{2x + 1}{3x + 1}$$

(The 1 indicates the factors that remain after dividing out common factors.)

You Try It 7

Divide: $\frac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \div \frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$

Your solution

Solutions on p. S23

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8.1 Exercises

Objective A

1. When is a rational expression in simplest form?

2. Are the rational expressions $\frac{x(x-2)}{2(x-2)}$ and $\frac{x}{2}$ equal for all values of x? Why or why not?

Simplify.

3. $\frac{9x^3}{12x^4}$

4. $\frac{16x^2y}{24xy^3}$

5. $\frac{(x+3)^2}{(x+3)^3}$

6. $\frac{(2x-1)^5}{(2x-1)^4}$

7. $\frac{3n-4}{4-3n}$

8. $\frac{5-2x}{2x-5}$

9. $\frac{6y(y+2)}{9y^2(y+2)}$

10. $\frac{12x^2(3-x)}{18x(3-x)}$

11. $\frac{6x(x-5)}{8x^2(5-x)}$

12. $\frac{14x^3(7-3x)}{21x(3x-7)}$

13. $\frac{a^2+4a}{ab+4b}$

14. $\frac{x^2-3x}{2x-6}$

15. $\frac{4-6x}{3x^2-2x}$

16. $\frac{5xy-3y}{9-15x}$

17. $\frac{y^2-3y+2}{y^2-4y+3}$

18. $\frac{x^2+5x+6}{x^2+8x+15}$

19. $\frac{x^2+3x-10}{x^2+2x-8}$

20. $\frac{a^2+7a-8}{a^2+6a-7}$

21. $\frac{x^2+x-12}{x^2-6x+9}$

22. $\frac{x^2+8x+16}{x^2-2x-24}$

23. $\frac{x^2-3x-10}{25-x^2}$

24. $\frac{4-y^2}{y^2-3y-10}$

25. $\frac{2x^3+2x^2-4x}{x^3+2x^2-3x}$

26. $\frac{3x^3-12x}{6x^3-24x^2+24x}$

27. $\frac{6x^2-7x+2}{6x^2+5x-6}$

28. $\frac{2n^2-9n+4}{2n^2-5n-12}$

29. $\frac{x^2+3x-28}{24-2x-x^2}$

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Objective B

Multiply.

30. $\frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^3}$

31. $\frac{14a^2b^3}{15x^5y^2} \cdot \frac{25x^3y}{16ab}$

32. $\frac{12x^3y^4}{7a^2b^3} \cdot \frac{14a^3b^4}{9x^2y^2}$

33. $\frac{18a^4b^2}{25x^2y^3} \cdot \frac{50x^5y^6}{27a^6b^2}$

34. $\frac{3x - 6}{5x - 20} \cdot \frac{10x - 40}{27x - 54}$

35. $\frac{8x - 12}{14x + 7} \cdot \frac{42x + 21}{32x - 48}$

36. $\frac{3x^2 + 2x}{2xy - 3y} \cdot \frac{2xy^3 - 3y^3}{3x^3 + 2x^2}$

37. $\frac{4a^2x - 3a^2}{2by + 5b} \cdot \frac{2b^3y + 5b^3}{4ax - 3a}$

38. $\frac{x^2 + 5x + 4}{x^3y^2} \cdot \frac{x^2y^3}{x^2 + 2x + 1}$

39. $\frac{x^2 + x - 2}{xy^2} \cdot \frac{x^3y}{x^2 + 5x + 6}$

40. $\frac{x^4y^2}{x^2 + 3x - 28} \cdot \frac{x^2 - 49}{xy^4}$

41. $\frac{x^5y^3}{x^2 + 13x + 30} \cdot \frac{x^2 + 2x - 3}{x^7y^2}$

42. $\frac{2x^2 - 5x}{2xy + y} \cdot \frac{2xy^2 + y^2}{5x^2 - 2x^3}$

43. $\frac{3a^3 + 4a^2}{5ab - 3b} \cdot \frac{3b^3 - 5ab^3}{3a^2 + 4a}$

44. $\frac{x^2 - 2x - 24}{x^2 - 5x - 6} \cdot \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$

45. $\frac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \frac{x^2 + 3x - 10}{x^2 - 9x + 14}$

46. $\frac{x^2 + 2x - 35}{x^2 + 4x - 21} \cdot \frac{x^2 + 3x - 18}{x^2 + 9x + 18}$

47. $\frac{y^2 + y - 20}{y^2 + 2y - 15} \cdot \frac{y^2 + 4y - 21}{y^2 + 3y - 28}$

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48. $\dfrac{x^2 - 3x - 4}{x^2 + 6x + 5} \cdot \dfrac{x^2 + 5x + 6}{8 + 2x - x^2}$

49. $\dfrac{25 - n^2}{n^2 - 2n - 35} \cdot \dfrac{n^2 - 8n - 20}{n^2 - 3n - 10}$

50. $\dfrac{12x^2 - 6x}{x^2 + 6x + 5} \cdot \dfrac{2x^4 + 10x^3}{4x^2 - 1}$

51. $\dfrac{8x^3 + 4x^2}{x^2 - 3x + 2} \cdot \dfrac{x^2 - 4}{16x^2 + 8x}$

52. $\dfrac{16 + 6x - x^2}{x^2 - 10x - 24} \cdot \dfrac{x^2 - 6x - 27}{x^2 - 17x + 72}$

53. $\dfrac{x^2 - 11x + 28}{x^2 - 13x + 42} \cdot \dfrac{x^2 + 7x + 10}{20 - x - x^2}$

54. $\dfrac{2x^2 + 5x + 2}{2x^2 + 7x + 3} \cdot \dfrac{x^2 - 7x - 30}{x^2 - 6x - 40}$

55. $\dfrac{x^2 - 4x - 32}{x^2 - 8x - 48} \cdot \dfrac{3x^2 + 17x + 10}{3x^2 - 22x - 16}$

Objective C

Divide.

56. $\dfrac{4x^2y^3}{15a^2b^3} \div \dfrac{6xy}{5a^3b^5}$

57. $\dfrac{9x^3y^4}{16a^4b^2} \div \dfrac{45x^4y^2}{14a^7b}$

58. $\dfrac{6x - 12}{8x + 32} \div \dfrac{18x - 36}{10x + 40}$

59. $\dfrac{28x + 14}{45x - 30} \div \dfrac{14x + 7}{30x - 20}$

60. $\dfrac{6x^3 + 7x^2}{12x - 3} \div \dfrac{6x^2 + 7x}{36x - 9}$

61. $\dfrac{5a^2y + 3a^2}{2x^3 + 5x^2} \div \dfrac{10ay + 6a}{6x^3 + 15x^2}$

62. $\dfrac{x^2 + 4x + 3}{x^2y} \div \dfrac{x^2 + 2x + 1}{xy^2}$

63. $\dfrac{x^3y^2}{x^2 - 3x - 10} \div \dfrac{xy^4}{x^2 - x - 20}$

64. $\dfrac{x^2 - 49}{x^4y^3} \div \dfrac{x^2 - 14x + 49}{x^4y^3}$

65. $\dfrac{x^2y^5}{x^2 - 11x + 30} \div \dfrac{xy^6}{x^2 - 7x + 10}$

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66. $\dfrac{4ax - 8a}{c^2} \div \dfrac{2y - xy}{c^3}$

67. $\dfrac{3x^2y - 9xy}{a^2b} \div \dfrac{3x^2 - x^3}{ab^2}$

68. $\dfrac{x^2 - 5x + 6}{x^2 - 9x + 18} \div \dfrac{x^2 - 6x + 8}{x^2 - 9x + 20}$

69. $\dfrac{x^2 + 3x - 40}{x^2 + 2x - 35} \div \dfrac{x^2 + 2x - 48}{x^2 + 3x - 18}$

70. $\dfrac{x^2 + 2x - 15}{x^2 - 4x - 45} \div \dfrac{x^2 + x - 12}{x^2 - 5x - 36}$

71. $\dfrac{y^2 - y - 56}{y^2 + 8y + 7} \div \dfrac{y^2 - 13y + 40}{y^2 - 4y - 5}$

72. $\dfrac{8 + 2x - x^2}{x^2 + 7x + 10} \div \dfrac{x^2 - 11x + 28}{x^2 - x - 42}$

73. $\dfrac{x^2 - x - 2}{x^2 - 7x + 10} \div \dfrac{x^2 - 3x - 4}{40 - 3x - x^2}$

74. $\dfrac{2x^2 - 3x - 20}{2x^2 - 7x - 30} \div \dfrac{2x^2 - 5x - 12}{4x^2 + 12x + 9}$

75. $\dfrac{6n^2 + 13n + 6}{4n^2 - 9} \div \dfrac{6n^2 + n - 2}{4n^2 - 1}$

APPLYING THE CONCEPTS

76. Given the expression $\dfrac{9}{x^2 + 1}$, choose some values of x and evaluate the expression for those values. Is it possible to choose a value of x for which the value of the expression is greater than 10? If so, give such a value. If not, explain why it is not possible.

77. Given the expression $\dfrac{1}{y - 3}$, choose some values of y and evaluate the expression for those values. Is it possible to choose a value of y for which the value of the expression is greater than 10,000,000? If so, give such a value. If not, explain why it is not possible.

Complete each simplification.

78. $\dfrac{8x}{9y} \div \underline{\ ?\ } = \dfrac{10y}{9}$

79. $\dfrac{n}{n + 3} \div \underline{\ ?\ } = \dfrac{n}{n - 2}$

Simplify.

80. $\dfrac{xy}{3} \cdot \dfrac{x}{y^2} \div \dfrac{x}{4}$

81. $\left(\dfrac{y}{3}\right) \div \left(\dfrac{y}{2} \cdot \dfrac{y}{4}\right)$

82. $\left(\dfrac{x - 4}{y^2}\right)^3 \cdot \left(\dfrac{y}{4 - x}\right)^3$

83. $\dfrac{x - 2}{x + 5} \div \dfrac{x - 3}{x + 5} \cdot \dfrac{x - 3}{x - 2}$

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8.2 Addition and Subtraction of Rational Expressions

Objective A

To find the least common multiple (LCM) of two or more polynomials

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The **least common multiple (LCM)** of two or more numbers is the smallest number that contains the prime factorization of each number.

TAKE NOTE
The LCM of 12 and 18 is 36 because 36 is the smallest number that both 12 and 18 divide evenly into.

The LCM of 12 and 18 is 36 because 36 contains the prime factors of 12 and the prime factors of 18.

$$12 = 2 \cdot 2 \cdot 3$$
$$18 = 2 \cdot 3 \cdot 3$$

$$\text{LCM} = 36 = \overbrace{2 \cdot 2}^{\text{Factors of 12}} \cdot \underbrace{3 \cdot 3}_{\text{Factors of 18}}$$

The least common multiple of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial.

To find the LCM of two or more polynomials, first factor each polynomial completely. The LCM is the product of each factor the greatest number of times it occurs in any one factorization.

➡ Find the LCM of $4x^2 + 4x$ and $x^2 + 2x + 1$.

TAKE NOTE
The LCM must contain the factors of each polynomial. As shown with the braces at the right, the LCM contains the factors of $4x^2 + 4x$ and the factors of $x^2 + 2x + 1$.

The LCM of the polynomials is the product of the LCM of the numerical coefficients and each variable factor the greatest number of times it occurs in any one factorization.

$$4x^2 + 4x = 4x(x + 1) = 2 \cdot 2 \cdot x(x + 1)$$
$$x^2 + 2x + 1 = (x + 1)(x + 1)$$

$$\text{LCM} = \overbrace{2 \cdot 2 \cdot x}^{\text{Factors of } 4x^2 + 4x}\underbrace{(x + 1)(x + 1)}_{\text{Factors of } x^2 + 2x + 1} = 4x(x + 1)(x + 1)$$

Example 1
Find the LCM of $4x^2y$ and $6xy^2$.

Solution
$4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y$
$6xy^2 = 2 \cdot 3 \cdot x \cdot y \cdot y$
$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$

You Try It 1
Find the LCM of $8uv^2$ and $12uw$.

Your solution

Example 2
Find the LCM of $x^2 - x - 6$ and $9 - x^2$.

Solution
$x^2 - x - 6 = (x - 3)(x + 2)$
$9 - x^2 = -(x^2 - 9) = -(x + 3)(x - 3)$
$\text{LCM} = (x - 3)(x + 2)(x + 3)$

You Try It 2
Find the LCM of $m^2 - 6m + 9$ and $m^2 - 2m - 3$.

Your solution

Solutions on p. S23

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Objective B

To express two fractions in terms of the LCM of their denominators

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When adding and subtracting fractions, it is frequently necessary to express two or more fractions in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

Write the fractions $\frac{x+1}{4x^2}$ and $\frac{x-3}{6x^2-12x}$ in terms of the LCM of the denominators.

Find the LCM of the denominators.

The LCM is $12x^2(x-2)$.

For each fraction, multiply the numerator and denominator by the factors whose product with the denominator is the LCM.

$$\frac{x+1}{4x^2} = \frac{x+1}{4x^2}\cdot\frac{3(x-2)}{3(x-2)} = \frac{3x^2-3x-6}{12x^2(x-2)}$$

$$\frac{x-3}{6x^2-12x} = \frac{x-3}{6x(x-2)}\cdot\frac{2x}{2x} = \frac{2x^2-6x}{12x^2(x-2)}$$

LCM

TAKE NOTE

$\frac{3(x-2)}{3(x-2)} = 1$ and $\frac{2x}{2x} = 1$. We are multiplying each fraction by 1, so we are not changing the value of either fraction.

Example 3

Write the fractions $\frac{x+2}{3x^2}$ and $\frac{x-1}{8xy}$ in terms of the LCM of the denominators.

Solution

The LCM is $24x^2y$.

$$\frac{x+2}{3x^2} = \frac{x+2}{3x^2}\cdot\frac{8y}{8y} = \frac{8xy+16y}{24x^2y}$$

$$\frac{x-1}{8xy} = \frac{x-1}{8xy}\cdot\frac{3x}{3x} = \frac{3x^2-3x}{24x^2y}$$

You Try It 3

Write the fractions $\frac{x-3}{4xy^2}$ and $\frac{2x+1}{9y^2z}$ in terms of the LCM of the denominators.

Your solution

Example 4

Write the fractions $\frac{2x-1}{2x-x^2}$ and $\frac{x}{x^2+x-6}$ in terms of the LCM of the denominators.

Solution

$$\frac{2x-1}{2x-x^2} = \frac{2x-1}{-(x^2-2x)} = -\frac{2x-1}{x^2-2x}$$

The LCM is $x(x-2)(x+3)$.

$$\frac{2x-1}{2x-x^2} = -\frac{2x-1}{x(x-2)}\cdot\frac{x+3}{x+3} = -\frac{2x^2+5x-3}{x(x-2)(x+3)}$$

$$\frac{x}{x^2+x-6} = \frac{x}{(x-2)(x+3)}\cdot\frac{x}{x} = \frac{x^2}{x(x-2)(x+3)}$$

You Try It 4

Write the fractions $\frac{x+4}{x^2-3x-10}$ and $\frac{2x}{25-x^2}$ in terms of the LCM of the denominators.

Your solution

Solutions on p. S23

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Objective C

To add or subtract rational expressions with the same denominator

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When adding rational expressions in which the denominators are the same, add the numerators. The denominator of the sum is the common denominator.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$

$$\frac{5x}{18}+\frac{7x}{18}=\frac{5x+7x}{18}=\frac{12x}{18}=\frac{2x}{3}$$

$$\frac{x}{x^2-1}+\frac{1}{x^2-1}=\frac{x+1}{x^2-1}=\frac{\cancel{(x+1)}^{1}}{(x-1)\cancel{(x+1)}_{1}}=\frac{1}{x-1}$$

Note that the sum is written in simplest form.

When subtracting rational expressions in which the denominators are the same, subtract the numerators. The denominator of the difference is the common denominator. Write the answer in simplest form.

TAKE NOTE

Be careful with signs when subtracting algebraic fractions. Note that we must subtract the *entire* numerator $2x + 3$.
$(3x - 1) - (2x + 3) = 3x - 1 - 2x - 3$.

$$\frac{2x}{x-2}-\frac{4}{x-2}=\frac{2x-4}{x-2}=\frac{2\cancel{(x-2)}^{1}}{\cancel{x-2}_{1}}=2$$

$$\frac{3x-1}{x^2-5x+4}-\frac{2x+3}{x^2-5x+4}=\frac{(3x-1)-(2x+3)}{x^2-5x+4}=\frac{3x-1-2x-3}{x^2-5x+4}$$

$$=\frac{x-4}{x^2-5x+4}=\frac{\cancel{(x-4)}^{1}}{\cancel{(x-4)}_{1}(x-1)}=\frac{1}{x-1}$$

Example 5

Add: $\frac{7}{x^2}+\frac{9}{x^2}$

Solution

$$\frac{7}{x^2}+\frac{9}{x^2}=\frac{7+9}{x^2}=\frac{16}{x^2}$$

You Try It 5

Add: $\frac{3}{xy}+\frac{12}{xy}$

Your solution

Example 6

Subtract: $\frac{3x^2}{x^2-1}-\frac{x+4}{x^2-1}$

Solution

$$\frac{3x^2}{x^2-1}-\frac{x+4}{x^2-1}=\frac{3x^2-(x+4)}{x^2-1}$$

$$=\frac{3x^2-x-4}{x^2-1}$$

$$=\frac{(3x-4)\cancel{(x+1)}^{1}}{(x-1)\cancel{(x+1)}_{1}}=\frac{3x-4}{x-1}$$

You Try It 6

Subtract: $\frac{2x^2}{x^2-x-12}-\frac{7x+4}{x^2-x-12}$

Your solution

Solutions on p. S23

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Example 7

Simplify:

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

Solution

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

$$=\frac{(2x^2+5)-(x^2-3x)+(x-2)}{x^2+2x-3}$$

$$=\frac{2x^2+5-x^2+3x+x-2}{x^2+2x-3}$$

$$=\frac{x^2+4x+3}{x^2+2x-3}=\frac{\cancel{(x+3)}^{1}(x+1)}{\cancel{(x+3)}_{1}(x-1)}=\frac{x+1}{x-1}$$

You Try It 7

Simplify:

$$\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}$$

Your solution

Solution on p. S23

Objective B

To add or subtract rational expressions with different denominators



TAKE NOTE

This objective requires using the skills learned in Objective 8.2B (to express two fractions in terms of the LCM of their denominators) and 8.2C (to add or subtract algebraic fractions with the same denominator).

Note the steps involved in adding or subtracting rational expressions:

1. Find the LCM of the denominators.
2. Rewrite each fraction in terms of the common denominator.
3. Add or subtract the rational expressions.
4. Simplify the resulting sum or difference.

Before two fractions with unlike denominators can be added or subtracted, each fraction must be expressed in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

Add: $\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}$

$x^2-2x=x(x-2)$; $x^2-4=(x+2)(x-2)$ The LCM is $x(x-2)(x+2)$.

$$\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}=\frac{x-3}{x(x-2)}\cdot\frac{x+2}{x+2}+\frac{6}{(x-2)(x+2)}\cdot\frac{x}{x}$$

- Rewrite each fraction with $x(x-2)(x+2)$ as the denominator.

$$=\frac{x^2-x-6}{x(x-2)(x+2)}+\frac{6x}{x(x-2)(x+2)}$$

$$=\frac{(x^2-x-6)+6x}{x(x-2)(x+2)}$$

- Add the fractions.

$$=\frac{x^2+5x-6}{x(x-2)(x+2)}$$

$$=\frac{(x+6)(x-1)}{x(x-2)(x+2)}$$

The last step is to factor the numerator to determine whether there are common factors in the numerator and denominator. For this example there are no common factors, so the answer is in simplest form.

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Example 8

Simplify: $\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x}$

Solution

The LCM of the denominators is $12x$.

$$\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x} = \frac{y}{x} \cdot \frac{12}{12} - \frac{4y}{3x} \cdot \frac{4}{4} + \frac{3y}{4x} \cdot \frac{3}{3}$$

$$= \frac{12y}{12x} - \frac{16y}{12x} + \frac{9y}{12x}$$

$$= \frac{12y - 16y + 9y}{12x} = \frac{5y}{12x}$$

You Try It 8

Simplify: $\frac{z}{8y} - \frac{4z}{3y} + \frac{5z}{4y}$

Your solution

Example 9

Subtract: $\frac{2x}{x-3} - \frac{5}{3-x}$

Solution

Remember: $3 - x = -(x - 3)$.

Therefore, $\frac{5}{3-x} = \frac{5}{-(x-3)} = \frac{-5}{x-3}$.

$$\frac{2x}{x-3} - \frac{5}{3-x} = \frac{2x}{x-3} - \frac{-5}{x-3}$$

$$= \frac{2x - (-5)}{x-3} = \frac{2x+5}{x-3}$$

You Try It 9

Subtract: $\frac{5x}{x-2} - \frac{3}{2-x}$

Your solution

Example 10

Subtract: $\frac{2x}{2x-3} - \frac{1}{x+1}$

Solution

The LCM is $(2x - 3)(x + 1)$.

$$\frac{2x}{2x-3} - \frac{1}{x+1}$$

$$= \frac{2x}{2x-3} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{2x-3}{2x-3}$$

$$= \frac{2x^2 + 2x}{(2x-3)(x+1)} - \frac{2x-3}{(2x-3)(x+1)}$$

$$= \frac{(2x^2 + 2x) - (2x-3)}{(2x-3)(x+1)} = \frac{2x^2+3}{(2x-3)(x+1)}$$

You Try It 10

Subtract: $\frac{4x}{3x-1} - \frac{9}{x+4}$

Your solution

Solutions on pp. S23–S24

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Example 11

Add: $1 + \frac{3}{x^2}$

Solution

The LCM is x^2.

$$1 + \frac{3}{x^2} = 1 \cdot \frac{x^2}{x^2} + \frac{3}{x^2}$$

$$= \frac{x^2}{x^2} + \frac{3}{x^2} = \frac{x^2 + 3}{x^2}$$

You Try It 11

Subtract: $2 - \frac{1}{x - 3}$

Your solution

Example 12

Subtract: $\frac{x}{2x - 4} - \frac{4 - x}{x^2 - 2x}$

Solution

$2x - 4 = 2(x - 2)$; $x^2 - 2x = x(x - 2)$

The LCM is $2x(x - 2)$.

$$\frac{x}{2x - 4} - \frac{4 - x}{x^2 - 2x} = \frac{x}{2(x - 2)} \cdot \frac{x}{x} - \frac{4 - x}{x(x - 2)} \cdot \frac{2}{2}$$

$$= \frac{x^2 - (4 - x)2}{2x(x - 2)}$$

$$= \frac{x^2 - (8 - 2x)}{2x(x - 2)} = \frac{x^2 + 2x - 8}{2x(x - 2)}$$

$$= \frac{(x + 4)(\cancel{x - 2})}{2x(\cancel{x - 2})} = \frac{x + 4}{2x}$$

You Try It 12

Add: $\frac{a - 3}{a^2 - 5a} + \frac{a - 9}{a^2 - 25}$

Your solution

Example 13

Simplify: $\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$

Solution

The LCM is $(2x + 1)(x - 1)$.

$$\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3}{2x + 1} \cdot \frac{x - 1}{x - 1} + \frac{4}{x - 1} \cdot \frac{2x + 1}{2x + 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3x - 3}{(2x + 1)(x - 1)} + \frac{8x + 4}{(2x + 1)(x - 1)}$$

$$= \frac{(3x + 2) - (3x - 3) + (8x + 4)}{(2x + 1)(x - 1)}$$

$$= \frac{3x + 2 - 3x + 3 + 8x + 4}{(2x + 1)(x - 1)} = \frac{8x + 9}{(2x + 1)(x - 1)}$$

You Try It 13

Simplify: $\frac{2x - 3}{3x^2 - x - 2} + \frac{5}{3x + 2} - \frac{1}{x - 1}$

Your solution

Solutions on p. S24

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8.2 Exercises

Objective A

Find the LCM of the expressions.

1. $8x^3y$, $12xy^2$

2. $6ab^2$, $18ab^3$

3. $10x^4y^2$, $15x^3y$

4. $12a^2b$, $18ab^3$

5. $8x^2$, $4x^2 + 8x$

6. $6y^2$, $4y + 12$

7. $2x^2y$, $3x^2 + 12x$

8. $4xy^2$, $6xy^2 + 12y^2$

9. $9x(x + 2)$, $12(x + 2)^2$

10. $8x^2(x - 1)^2$, $10x^3(x - 1)$

11. $3x + 3$, $2x^2 + 4x + 2$

12. $4x - 12$, $2x^2 - 12x + 18$

13. $(x - 1)(x + 2)$, $(x - 1)(x + 3)$

14. $(2x - 1)(x + 4)$, $(2x + 1)(x + 4)$

15. $(2x + 3)^2$, $(2x + 3)(x - 5)$

16. $(x - 7)(x + 2)$, $(x - 7)^2$

17. $x - 1$, $x - 2$, $(x - 1)(x - 2)$

18. $(x + 4)(x - 3)$, $x + 4$, $x - 3$

19. $x^2 - x - 6$, $x^2 + x - 12$

20. $x^2 + 3x - 10$, $x^2 + 5x - 14$

21. $x^2 + 5x + 4$, $x^2 - 3x - 28$

22. $x^2 - 10x + 21$, $x^2 - 8x + 15$

23. $x^2 - 2x - 24$, $x^2 - 36$

24. $x^2 + 7x + 10$, $x^2 - 25$

25. $x^2 - 7x - 30$, $x^2 - 5x - 24$

26. $2x^2 - 7x + 3$, $2x^2 + x - 1$

27. $3x^2 - 11x + 6$, $3x^2 + 4x - 4$

28. $2x^2 - 9x + 10$, $2x^2 + x - 15$

29. $6 + x - x^2$, $x + 2$, $x - 3$

30. $15 + 2x - x^2$, $x - 5$, $x + 3$

31. $5 + 4x - x^2$, $x - 5$, $x + 1$

32. $x^2 + 3x - 18$, $3 - x$, $x + 6$

33. $x^2 - 5x + 6$, $1 - x$, $x - 6$

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Objective B

Write each fraction in terms of the LCM of the denominators.

34. $\frac{4}{x}, \frac{3}{x^2}$

35. $\frac{5}{ab^2}, \frac{6}{ab}$

36. $\frac{x}{3y^2}, \frac{z}{4y}$

37. $\frac{5y}{6x^2}, \frac{7}{9xy}$

38. $\frac{y}{x(x-3)}, \frac{6}{x^2}$

39. $\frac{a}{y^2}, \frac{6}{y(y+5)}$

40. $\frac{9}{(x-1)^2}, \frac{6}{x(x-1)}$

41. $\frac{a^2}{y(y+7)}, \frac{a}{(y+7)^2}$

42. $\frac{3}{x-3}, \frac{5}{x(3-x)}$

43. $\frac{b}{y(y-4)}, \frac{b^2}{4-y}$

44. $\frac{3}{(x-5)^2}, \frac{2}{5-x}$

45. $\frac{3}{7-y}, \frac{2}{(y-7)^2}$

46. $\frac{3}{x^2+2x}, \frac{4}{x^2}$

47. $\frac{2}{y-3}, \frac{3}{y^3-3y^2}$

48. $\frac{x-2}{x+3}, \frac{x}{x-4}$

49. $\frac{x^2}{2x-1}, \frac{x+1}{x+4}$

50. $\frac{3}{x^2+x-2}, \frac{x}{x+2}$

51. $\frac{3x}{x-5}, \frac{4}{x^2-25}$

52. $\frac{5}{2x^2-9x+10}, \frac{x-1}{2x-5}$

53. $\frac{x-3}{3x^2+4x-4}, \frac{2}{x+2}$

54. $\frac{x}{x^2+x-6}, \frac{2x}{x^2-9}$

55. $\frac{x-1}{x^2+2x-15}, \frac{x}{x^2+6x+5}$

Objective C

56. Explain the procedure for subtracting rational expressions with the same denominator.

Add or subtract.

57. $\frac{3}{y^2}+\frac{8}{y^2}$

58. $\frac{6}{ab}-\frac{2}{ab}$

59. $\frac{3}{x+4}-\frac{10}{x+4}$

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60. $\frac{x}{x+6} - \frac{2}{x+6}$

61. $\frac{3x}{2x+3} + \frac{5x}{2x+3}$

62. $\frac{6y}{4y+1} - \frac{11y}{4y+1}$

63. $\frac{2x+1}{x-3} + \frac{3x+6}{x-3}$

64. $\frac{4x+3}{2x-7} + \frac{3x-8}{2x-7}$

65. $\frac{5x-1}{x+9} - \frac{3x+4}{x+9}$

66. $\frac{6x-5}{x-10} - \frac{3x-4}{x-10}$

67. $\frac{x-7}{2x+7} - \frac{4x-3}{2x+7}$

68. $\frac{2n}{3n+4} - \frac{5n-3}{3n+4}$

69. $\frac{x}{x^2+2x-15} - \frac{3}{x^2+2x-15}$

70. $\frac{3x}{x^2+3x-10} - \frac{6}{x^2+3x-10}$

71. $\frac{2x+3}{x^2-x-30} - \frac{x-2}{x^2-x-30}$

72. $\frac{3x-1}{x^2+5x-6} - \frac{2x-7}{x^2+5x-6}$

73. $\frac{4y+7}{2y^2+7y-4} - \frac{y-5}{2y^2+7y-4}$

74. $\frac{x+1}{2x^2-5x-12} + \frac{x+2}{2x^2-5x-12}$

75. $\frac{2x^2+3x}{x^2-9x+20} + \frac{2x^2-3}{x^2-9x+20} - \frac{4x^2+2x+1}{x^2-9x+20}$

76. $\frac{2x^2+3x}{x^2-2x-63} - \frac{x^2-3x+21}{x^2-2x-63} - \frac{x-7}{x^2-2x-63}$

Objective B

77. Explain the procedure for adding rational expressions with different denominators.

Add or subtract.

78. $\frac{4}{x} + \frac{5}{y}$

79. $\frac{7}{a} + \frac{5}{b}$

80. $\frac{12}{x} - \frac{5}{2x}$

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81. $\frac{5}{3a} - \frac{3}{4a}$

82. $\frac{1}{2x} - \frac{5}{4x} + \frac{7}{6x}$

83. $\frac{7}{4y} + \frac{11}{6y} - \frac{8}{3y}$

84. $\frac{5}{3x} - \frac{2}{x^2} + \frac{3}{2x}$

85. $\frac{6}{y^2} + \frac{3}{4y} - \frac{2}{5y}$

86. $\frac{2}{x} - \frac{3}{2y} + \frac{3}{5x} - \frac{1}{4y}$

87. $\frac{5}{2a} + \frac{7}{3b} - \frac{2}{b} - \frac{3}{4a}$

88. $\frac{2x+1}{3x} + \frac{x-1}{5x}$

89. $\frac{4x-3}{6x} + \frac{2x+3}{4x}$

90. $\frac{x-3}{6x} + \frac{x+4}{8x}$

91. $\frac{2x-3}{2x} + \frac{x+3}{3x}$

92. $\frac{2x+9}{9x} - \frac{x-5}{5x}$

93. $\frac{3y-2}{12y} - \frac{y-3}{18y}$

94. $\frac{x+4}{2x} - \frac{x-1}{x^2}$

95. $\frac{x-2}{3x^2} - \frac{x+4}{x}$

96. $\frac{x-10}{4x^2} + \frac{x+1}{2x}$

97. $\frac{x+5}{3x^2} + \frac{2x+1}{2x}$

98. $\frac{4}{x+4} - x$

99. $2x + \frac{1}{x}$

100. $5 - \frac{x-2}{x+1}$

101. $3 + \frac{x-1}{x+1}$

102. $\frac{x+3}{6x} - \frac{x-3}{8x^2}$

103. $\frac{x+2}{xy} - \frac{3x-2}{x^2y}$

104. $\frac{3x-1}{xy^2} - \frac{2x+3}{xy}$

105. $\frac{4x-3}{3x^2y} + \frac{2x+1}{4xy^2}$

106. $\frac{5x+7}{6xy^2} - \frac{4x-3}{8x^2y}$

107. $\frac{x-2}{8x^2} - \frac{x+7}{12xy}$

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108. $\frac{3x-1}{6y^2} - \frac{x+5}{9xy}$

109. $\frac{4}{x-2} + \frac{5}{x+3}$

110. $\frac{2}{x-3} + \frac{5}{x-4}$

111. $\frac{6}{x-7} - \frac{4}{x+3}$

112. $\frac{3}{y+6} - \frac{4}{y-3}$

113. $\frac{2x}{x+1} + \frac{1}{x-3}$

114. $\frac{3x}{x-4} + \frac{2}{x+6}$

115. $\frac{4x}{2x-1} - \frac{5}{x-6}$

116. $\frac{6x}{x+5} - \frac{3}{2x+3}$

117. $\frac{2a}{a-7} + \frac{5}{7-a}$

118. $\frac{4x}{6-x} + \frac{5}{x-6}$

119. $\frac{x}{x^2-9} + \frac{3}{x-3}$

120. $\frac{y}{y^2-16} + \frac{1}{y-4}$

121. $\frac{2x}{x^2-x-6} - \frac{3}{x+2}$

122. $\frac{(x-1)^2}{(x+1)^2} - 1$

123. $1 - \frac{(y-2)^2}{(y+2)^2}$

124. $\frac{x}{1-x^2} - 1 + \frac{x}{1+x}$

125. $\frac{y}{x-y} + 2 - \frac{x}{y-x}$

126. $\frac{3x-1}{x^2-10x+25} - \frac{3}{x-5}$

127. $\frac{2a+3}{a^2-7a+12} - \frac{2}{a-3}$

128. $\frac{x+4}{x^2-x-42} + \frac{3}{7-x}$

129. $\frac{x+3}{x^2-3x-10} + \frac{2}{5-x}$

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130. $\dfrac{1}{x+1}+\dfrac{x}{x-6}-\dfrac{5x-2}{x^2-5x-6}$

131. $\dfrac{x}{x-4}+\dfrac{5}{x+5}-\dfrac{11x-8}{x^2+x-20}$

132. $\dfrac{3x+1}{x-1}-\dfrac{x-1}{x-3}+\dfrac{x+1}{x^2-4x+3}$

133. $\dfrac{4x+1}{x-8}-\dfrac{3x+2}{x+4}-\dfrac{49x+4}{x^2-4x-32}$

134. $\dfrac{2x+9}{3-x}+\dfrac{x+5}{x+7}-\dfrac{2x^2+3x-3}{x^2+4x-21}$

135. $\dfrac{3x+5}{x+5}-\dfrac{x+1}{2-x}-\dfrac{4x^2-3x-1}{x^2+3x-10}$

APPLYING THE CONCEPTS

136. Simplify.

a. $\left(\dfrac{b}{6}-\dfrac{6}{b}\right)\div\left(\dfrac{6}{b}-4+\dfrac{b}{2}\right)$

b. $\dfrac{x^2+x-6}{x^2+2x-8}\cdot\dfrac{x^2+5x+4}{x^2+2x-3}-\dfrac{2}{x-1}$

137. Rewrite the fraction as the sum of two fractions in simplest form.

a. $\dfrac{3x+6y}{xy}$

b. $\dfrac{4a^2+3ab}{a^2b^2}$

c. $\dfrac{3m^2n+2mn^2}{12m^3n^2}$

138. Let $f(x)=\dfrac{x}{x+2}$, $g(x)=\dfrac{4}{x-3}$, and $S(x)=\dfrac{x^2+x+8}{x^2-x-6}$. Evaluate $f(4)$, $g(4)$, and $S(4)$. Does $f(4)+g(4)=S(4)$? Let a be a real number ($a\neq -2, a\neq 3$). Express $S(a)$ in terms of $f(a)$ and $g(a)$.

139. Suppose you drive about 12,000 mi per year and that the cost of gasoline averages \$1.60 per gallon.

a. Let x represent the number of miles per gallon your car gets. Write a variable expression for the amount you spend on gasoline for 1 year.

b. Write and simplify a variable expression for the amount of money you will save each year if you can increase your gas mileage by 5 mi per gallon.

c. If you currently get 25 mi per gallon and you increase your gas mileage by 5 mi per gallon, how much will you save in 1 year?

140. When is the LCM of two expressions equal to their product?

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8.3 Complex Fractions

Objective A

To simplify a complex fraction

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Point of Interest

There are many instances of complex fractions in application problems. The fraction $\dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$ is used to determine the total resistance in certain electric circuits.

A **complex fraction** is a fraction whose numerator or denominator contains one or more fractions. Examples of complex fractions are shown at the right.

$$\frac{3}{2-\frac{1}{2}}, \quad \frac{4+\frac{1}{x}}{3+\frac{2}{x}}, \quad \frac{\frac{1}{x-1}+x+3}{x-3+\frac{1}{x+4}}$$

➨ Simplify: $\dfrac{1-\frac{4}{x^2}}{1+\frac{2}{x}}$

TAKE NOTE

First of all, we are multiplying the complex fraction by $\frac{x^2}{x^2}$, which equals 1, so we are not changing the value of the fraction.

Second, we are using the Distributive Property to multiply $\left(1-\frac{4}{x^2}\right)x^2$ and $\left(1+\frac{2}{x}\right)x^2$.

Find the LCM of the denominators of the fractions in the numerator and denominator. The LCM of x and x^2 is x^2.

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} \cdot \frac{x^2}{x^2}$$

• Multiply the numerator and denominator by the LCM.

$$= \frac{1 \cdot x^2 - \frac{4}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{2}{x} \cdot x^2}$$

• Simplify.

$$= \frac{x^2-4}{x^2+2x} = \frac{(x-2)\cancel{(x+2)}^{1}}{x\cancel{(x+2)}_{1}}$$

$$= \frac{x-2}{x}$$

The method shown above of simplifying a complex fraction by multiplying the numerator and denominator by the LCM of the denominators is used in the examples on the next page. However, a different approach is to rewrite the numerator and denominator of the complex fraction as single fractions and then divide the numerator by the denominator. The example shown above is simplified below using this alternative method.

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1 \cdot \frac{x^2}{x^2} - \frac{4}{x^2}}{1 \cdot \frac{x}{x} + \frac{2}{x}} = \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x}{x} + \frac{2}{x}} = \frac{\frac{x^2-4}{x^2}}{\frac{x+2}{x}}$$

• Rewrite the numerator and denominator of the complex fraction as single fractions.

TAKE NOTE

Recall that the fraction bar can be read "divided by."

$$= \frac{x^2-4}{x^2} \div \frac{x+2}{x} = \frac{x^2-4}{x^2} \cdot \frac{x}{x+2}$$

• Divide the numerator of the complex fraction by the denominator.

$$= \frac{(x^2-4)x}{x^2(x+2)} = \frac{(x+2)(x-2)x}{x^2(x+2)}$$

• Multiply the fractions. Factor the numerator.

$$= \frac{x-2}{x}$$

• Simplify.

Note that this is the same result as shown above.

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Example 1

Simplify: $\dfrac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}}$

Solution

The LCM of x, 2, x^2, and 4 is $4x^2$.

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} = \frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} \cdot \frac{4x^2}{4x^2} = \frac{\frac{1}{x}\cdot 4x^2+\frac{1}{2}\cdot 4x^2}{\frac{1}{x^2}\cdot 4x^2-\frac{1}{4}\cdot 4x^2}$$

$$= \frac{4x+2x^2}{4-x^2} = \frac{2x\cancel{(2+x)}^{1}}{(2-x)\cancel{(2+x)}_{1}} = \frac{2x}{2-x}$$

You Try It 1

Simplify: $\dfrac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}$

Your solution

Example 2

Simplify: $\dfrac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$

Solution

The LCM is $x+4$.

$$\frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$$

$$= \frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}} \cdot \frac{x+4}{x+4}$$

$$= \frac{x(x+4)-8(x+4)+\frac{20}{x+4}\cdot(x+4)}{x(x+4)-10(x+4)+\frac{24}{x+4}\cdot(x+4)}$$

$$= \frac{x^2+4x-8x-32+20}{x^2+4x-10x-40+24} = \frac{x^2-4x-12}{x^2-6x-16}$$

$$= \frac{(x-6)\cancel{(x+2)}^{1}}{(x-8)\cancel{(x+2)}_{1}} = \frac{x-6}{x-8}$$

You Try It 2

Simplify: $\dfrac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}$

Your solution

Solutions on p. S24

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8.3 Exercises

Objective A

Simplify.

1. $\dfrac{1+\frac{3}{x}}{1-\frac{9}{x^2}}$

2. $\dfrac{1+\frac{4}{x}}{1-\frac{16}{x^2}}$

3. $\dfrac{2-\frac{8}{x+4}}{3-\frac{12}{x+4}}$

4. $\dfrac{5-\frac{25}{x+5}}{1-\frac{3}{x+5}}$

5. $\dfrac{1+\frac{5}{y-2}}{1-\frac{2}{y-2}}$

6. $\dfrac{2-\frac{11}{2x-1}}{3-\frac{17}{2x-1}}$

7. $\dfrac{4-\frac{2}{x+7}}{5+\frac{1}{x+7}}$

8. $\dfrac{5+\frac{3}{x-8}}{2-\frac{1}{x-8}}$

9. $\dfrac{1-\frac{1}{x}-\frac{6}{x^2}}{1-\frac{9}{x^2}}$

10. $\dfrac{1+\frac{4}{x}+\frac{4}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}}$

11. $\dfrac{1-\frac{5}{x}-\frac{6}{x^2}}{1+\frac{6}{x}+\frac{5}{x^2}}$

12. $\dfrac{1-\frac{7}{a}+\frac{12}{a^2}}{1+\frac{1}{a}-\frac{20}{a^2}}$

13. $\dfrac{1-\frac{6}{x}+\frac{8}{x^2}}{\frac{4}{x^2}+\frac{3}{x}-1}$

14. $\dfrac{1+\frac{3}{x}-\frac{18}{x^2}}{\frac{21}{x^2}-\frac{4}{x}-1}$

15. $\dfrac{x-\frac{4}{x+3}}{1+\frac{1}{x+3}}$

16. $\dfrac{y+\frac{1}{y-2}}{1+\frac{1}{y-2}}$

17. $\dfrac{1-\frac{x}{2x+1}}{x-\frac{1}{2x+1}}$

18. $\dfrac{1-\frac{2x-2}{3x-1}}{x-\frac{4}{3x-1}}$

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19. $\dfrac{x-5+\dfrac{14}{x+4}}{x+3-\dfrac{2}{x+4}}$

20. $\dfrac{a+4+\dfrac{5}{a-2}}{a+6+\dfrac{15}{a-2}}$

21. $\dfrac{x+3-\dfrac{10}{x-6}}{x+2-\dfrac{20}{x-6}}$

22. $\dfrac{x-7+\dfrac{5}{x-1}}{x-3+\dfrac{1}{x-1}}$

23. $\dfrac{y-6+\dfrac{22}{2y+3}}{y-5+\dfrac{11}{2y+3}}$

24. $\dfrac{x+2-\dfrac{12}{2x-1}}{x+1-\dfrac{9}{2x-1}}$

25. $\dfrac{x-\dfrac{2}{2x-3}}{2x-1-\dfrac{8}{2x-3}}$

26. $\dfrac{x+3-\dfrac{18}{2x+1}}{x-\dfrac{6}{2x+1}}$

27. $\dfrac{\dfrac{1}{x}-\dfrac{2}{x-1}}{\dfrac{3}{x}+\dfrac{1}{x-1}}$

28. $\dfrac{\dfrac{3}{n+1}+\dfrac{1}{n}}{\dfrac{2}{n+1}+\dfrac{3}{n}}$

29. $\dfrac{\dfrac{3}{2x-1}-\dfrac{1}{x}}{\dfrac{4}{x}+\dfrac{2}{2x-1}}$

30. $\dfrac{\dfrac{4}{3x+1}+\dfrac{3}{x}}{\dfrac{6}{x}-\dfrac{2}{3x+1}}$

APPLYING THE CONCEPTS

Simplify.

31. $1+\dfrac{1}{1+\dfrac{1}{2}}$

32. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}$

33. $1-\dfrac{1}{1-\dfrac{1}{x}}$

34. $\dfrac{a^{-1}-b^{-1}}{a^{-2}-b^{-2}}$

35. $\left(\dfrac{y}{4}-\dfrac{4}{y}\right)\div\left(\dfrac{4}{y}-3+\dfrac{y}{2}\right)$

36. $\dfrac{1+x^{-1}}{1-x^{-1}}$

37. How would you explain to a classmate why we multiply the numerator and denominator of a complex fraction by the LCM of the denominators of the fractions in the numerator and denominator?

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8.4 Rational Equations

To solve rational equations









To solve an equation containing fractions, **clear denominators** by multiplying each side of the equation by the LCM of the denominators. Then solve for the variable.

TAKE NOTE
Note that we are now solving *equations*, not operating on *expressions*. We are not writing each fraction in terms of the LCM of the denominators; we are multiplying both sides of the equation by the LCM of the denominators.

Solve: $\frac{3x-1}{4x}+\frac{2}{3x}=\frac{7}{6x}$

$$\frac{3x-1}{4x}+\frac{2}{3x}=\frac{7}{6x}$$

- The LCM of $4x$, $3x$, and $6x$ is $12x$.

$$12x\left(\frac{3x-1}{4x}+\frac{2}{3x}\right)=12x\left(\frac{7}{6x}\right)$$

- Multiply each side of the equation by the LCM of the denominators.

$$12x\left(\frac{3x-1}{4x}\right)+12x\left(\frac{2}{3x}\right)=12x\left(\frac{7}{6x}\right)$$

- Simplify using the Distributive Property.

$$\frac{12x}{1}\left(\frac{3x-1}{4x}\right)+\frac{12x}{1}\left(\frac{2}{3x}\right)=\frac{12x}{1}\left(\frac{7}{6x}\right)$$

$$3(3x-1)+4(2)=2(7)$$

- Solve for x.

$$9x-3+8=14$$
$$9x+5=14$$
$$9x=9$$
$$x=1$$

1 checks as a solution. The solution is 1.

Occasionally, a value of the variable that appears to be a solution of an equation will make one of the denominators zero. In this case, the equation has no solution for that value of the variable.

Solve: $\frac{2x}{x-2}=1+\frac{4}{x-2}$

$$\frac{2x}{x-2}=1+\frac{4}{x-2}$$

$$(x-2)\frac{2x}{x-2}=(x-2)\left(1+\frac{4}{x-2}\right)$$

- The LCM is $x-2$. Multiply each side of the equation by the LCM.

$$(x-2)\frac{2x}{x-2}=(x-2)\cdot 1+(x-2)\frac{4}{x-2}$$

- Simplify using the Distributive Property and the Properties of Fractions.

$$\frac{\cancel{(x-2)}^{1}}{1}\cdot\frac{2x}{\cancel{x-2}_{1}}=(x-2)\cdot 1+\frac{\cancel{(x-2)}^{1}}{1}\cdot\frac{4}{\cancel{x-2}_{1}}$$

$$2x=x-2+4$$

- Solve for x.

$$2x=x+2$$
$$x=2$$

When x is replaced by 2, the denominators of $\frac{2x}{x-2}$ and $\frac{4}{x-2}$ are zero. Therefore, the equation has no solution.

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Example 1

Solve: $\frac{x}{x+4} = \frac{2}{x}$

Solution

The LCM is $x(x + 4)$.

$$\frac{x}{x+4} = \frac{2}{x}$$

$$x(x+4)\left(\frac{x}{x+4}\right) = x(x+4)\left(\frac{2}{x}\right)$$

$$\frac{\overset{1}{\cancel{x(x+4)}}}{1} \cdot \frac{x}{\underset{1}{\cancel{x+4}}} = \frac{\overset{1}{\cancel{x}}(x+4)}{1} \cdot \frac{2}{\underset{1}{\cancel{x}}}$$

$$x^2 = (x+4)2$$

$$x^2 = 2x + 8$$

Solve the quadratic equation by factoring.

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x - 4 = 0 \qquad x + 2 = 0$$

$$x = 4 \qquad x = -2$$

Both 4 and -2 check as solutions.

The solutions are 4 and -2.

You Try It 1

Solve: $\frac{x}{x+6} = \frac{3}{x}$

Your solution

Example 2

Solve: $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

Solution

The LCM is $x - 4$.

$$\frac{3x}{x-4} = 5 + \frac{12}{x-4}$$

$$(x-4)\left(\frac{3x}{x-4}\right) = (x-4)\left(5 + \frac{12}{x-4}\right)$$

$$\frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{3x}{\underset{1}{\cancel{x-4}}} = (x-4)5 + \frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{12}{\underset{1}{\cancel{x-4}}}$$

$$3x = (x-4)5 + 12$$

$$3x = 5x - 20 + 12$$

$$3x = 5x - 8$$

$$-2x = -8$$

$$x = 4$$

4 does not check as a solution.

The equation has no solution.

You Try It 2

Solve: $\frac{5x}{x+2} = 3 - \frac{10}{x+2}$

Your solution

Solutions on p. S25

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To solve proportions



Quantities such as 4 meters, 15 seconds, and 8 gallons are number quantities written with units. In these examples, the units are meters, seconds, and gallons.

A **ratio** is the quotient of two quantities that have the same unit.

The length of a living room is 16 ft and the width is 12 ft. The ratio of the length to the width is written

$$\frac{16 \text{ ft}}{12 \text{ ft}} = \frac{16}{12} = \frac{4}{3}$$

A ratio is in simplest form when the two numbers do not have a common factor. Note that the units are not written.

A **rate** is the quotient of two quantities that have different units.

There are 2 lb of salt in 8 gal of water. The salt-to-water rate is

$$\frac{2 \text{ lb}}{8 \text{ gal}} = \frac{1 \text{ lb}}{4 \text{ gal}}$$

A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

A **proportion** is an equation that states the equality of two ratios or rates. Examples of proportions are shown at the right.

$$\frac{30 \text{ mi}}{4 \text{ h}} = \frac{15 \text{ mi}}{2 \text{ h}}$$

$$\frac{4}{6} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{x}{8}$$

Solve: $\frac{4}{x} = \frac{2}{3}$

$$\frac{4}{x} = \frac{2}{3}$$ • The LCM of x and 3 is $3x$.

$$3x\left(\frac{4}{x}\right) = 3x\left(\frac{2}{3}\right)$$ • Multiply each side of the proportion by $3x$.

$$12 = 2x$$ • Solve the equation.

$$6 = x$$

The solution is 6.

Example 3

Solve: $\frac{8}{x+3} = \frac{4}{x}$

Solution

$$\frac{8}{x+3} = \frac{4}{x}$$

$$x(x+3)\frac{8}{x+3} = x(x+3)\frac{4}{x}$$

$$8x = 4(x+3)$$

$$8x = 4x + 12$$

$$4x = 12$$

$$x = 3$$

The solution is 3.

You Try It 3

Solve: $\frac{2}{x+3} = \frac{6}{5x+5}$

Your solution

Solution on p. S25

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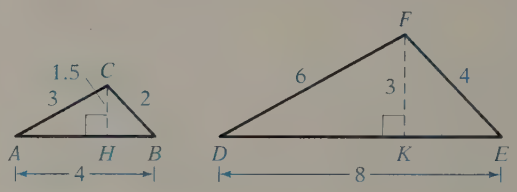
To solve problems involving similar triangles

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Similar objects have the same shape but not necessarily the same size. A tennis ball is similar to a basketball. A model ship is similar to an actual ship.

Similar objects have corresponding parts; for example, the rudder on the model ship corresponds to the rudder on the actual ship. The relationship between the sizes of each of the corresponding parts can be written as a ratio, and each ratio will be the same. If the rudder on the model ship is $\frac{1}{100}$ the size of the rudder on the actual ship, then the model wheelhouse is $\frac{1}{100}$ the size of the actual wheelhouse, the width of the model is $\frac{1}{100}$ the width of the actual ship, and so on.

The two triangles ABC and DEF shown at the right are similar. Side $\overline{AB}$ corresponds to $\overline{DE}$, side $\overline{BC}$ corresponds to $\overline{EF}$, and side $\overline{AC}$ corresponds to $\overline{DF}$. The height $\overline{CH}$ corresponds to the height $\overline{FK}$. The ratios of corresponding parts are equal.



$$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}, \quad \text{and} \quad \frac{CH}{FK} = \frac{1.5}{3} = \frac{1}{2}$$

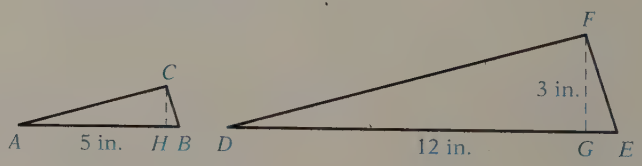
Since the ratios of corresponding parts are equal, three proportions can be formed using the sides of the triangles.

$$\frac{AB}{DE} = \frac{AC}{DF}, \quad \frac{AB}{DE} = \frac{BC}{EF}, \quad \text{and} \quad \frac{AC}{DF} = \frac{BC}{EF}$$

Three proportions can also be formed by using the sides and height of the triangles.

$$\frac{AB}{DE} = \frac{CH}{FK}, \quad \frac{AC}{DF} = \frac{CH}{FK}, \quad \text{and} \quad \frac{BC}{EF} = \frac{CH}{FK}$$

Triangles ABC and DEF at the right are similar. Find the height of triangle ABC.



• Solve a proportion to find the height of triangle ABC.

$$\frac{AB}{DE} = \frac{CH}{FG}$$

$$\frac{5}{12} = \frac{CH}{3}$$

$$12 \cdot \frac{5}{12} = 12 \cdot \frac{CH}{3}$$

$$5 = 4(CH)$$

$$1.25 = CH$$

The height of triangle ABC is 1.25 in.

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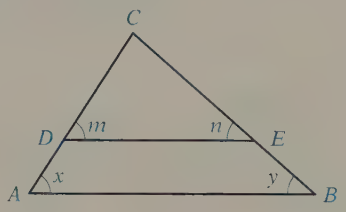
The corresponding angles in similar triangles are equal. Therefore, for the triangles in the last example,

$$\angle A = \angle D, \quad \angle B = \angle E, \quad \text{and} \quad \angle C = \angle F$$

It is also true that if the three angles of one triangle are equal respectively to the three angles of another triangle, then the two triangles are similar.

TAKE NOTE
Vertical angles of intersecting lines, parallel lines, and angles of a triangle are discussed in the chapter titled "Geometry."

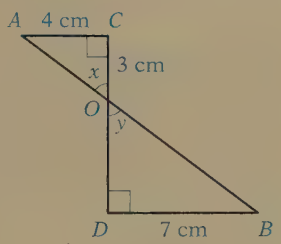
A line segment $\overline{DE}$ is drawn parallel to the base AB in the triangle at the right. $\angle x = \angle m$ and $\angle y = \angle n$ because corresponding angles are equal. $\angle C = \angle C$, and thus the three angles of triangle DEC are equal respectively to the three angles of triangle ABC. The triangle DEC is similar to the triangle ABC.



The sum of the three angles of a triangle is 180°. If two angles of one triangle are equal to two angles of another triangle, then the third angles must be equal. Thus we can say that if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar. This fact is used in Example 4.

Example 4

In the figure at the right, $\overline{AC}$ is parallel to $\overline{BD}$ and angles C and D are right angles. Find the length of DO.



Strategy

$\angle C = \angle D$ because they are right angles.
$\angle x = \angle y$ because they are vertical angles.
Therefore, triangle AOC is similar to triangle BOD because two angles of one triangle are equal to two angles of the other. Use a proportion to find the length of DO.

Solution

$$\frac{AC}{DB} = \frac{CO}{DO}$$

$$\frac{4}{7} = \frac{3}{DO}$$

$$7(DO)\frac{4}{7} = 7(DO)\frac{3}{DO}$$

$$4(DO) = 7(3)$$

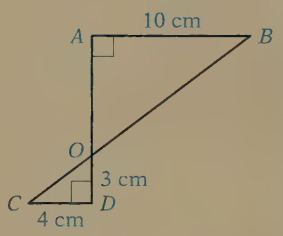
$$4(DO) = 21$$

$$DO = 5.25$$

The length of DO is 5.25 cm.

You Try It 4

In the figure at the right, $\overline{AB}$ is parallel to $\overline{DC}$ and angles A and D are right angles. Find the area of triangle AOB.



Your strategy

Your solution

Solution on p. S25

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Objective 3 To solve application problems

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Example 5

The monthly loan payment for a car is $28.35 for each $1000 borrowed. At this rate, find the monthly payment for a $6000 car loan.

Strategy

To find the monthly payment, write and solve a proportion, using P to represent the monthly car payment.

Solution

$$\frac{28.35}{1000} = \frac{P}{6000}$$

$$6000\left(\frac{28.35}{1000}\right) = 6000\left(\frac{P}{6000}\right)$$

$$170.10 = P$$

The monthly payment is $170.10.

You Try It 5

Sixteen ceramic tiles are needed to tile an area of 9 ft^2. At this rate, how many square feet can be tiled using 256 ceramic tiles?

Your strategy

Your solution

Example 6

An investment of $500 earns $60 each year. At the same rate, how much additional money must be invested to earn $90 each year?

Strategy

To find the additional amount of money that must be invested, write and solve a proportion, using x to represent the additional money. Then $500 + x$ is the total amount invested.

Solution

$$\frac{60}{500} = \frac{90}{500 + x}$$

$$\frac{3}{25} = \frac{90}{500 + x}$$

$$25(500 + x)\left(\frac{3}{25}\right) = 25(500 + x)\left(\frac{90}{500 + x}\right)$$

$$(500 + x)3 = 25(90)$$

$$1500 + 3x = 2250$$

$$3x = 750$$

$$x = 250$$

An additional $250 must be invested.

You Try It 6

Three ounces of a certain medication are required for a 150-pound adult. At the same rate, how many additional ounces of this medication are required for a 200-pound adult?

Your strategy

Your solution

Solutions on pp. S25–S26

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8.4 Exercises

Objective A

Solve.

1. $\frac{2x}{3} - \frac{5}{2} = -\frac{1}{2}$

2. $\frac{x}{3} - \frac{1}{4} = \frac{1}{12}$

3. $\frac{x}{3} - \frac{1}{4} = \frac{x}{4} - \frac{1}{6}$

4. $\frac{2y}{9} - \frac{1}{6} = \frac{y}{9} + \frac{1}{6}$

5. $\frac{2x - 5}{8} + \frac{1}{4} = \frac{x}{8} + \frac{3}{4}$

6. $\frac{3x + 4}{12} - \frac{1}{3} = \frac{5x + 2}{12} - \frac{1}{2}$

7. $\frac{6}{2a + 1} = 2$

8. $\frac{12}{3x - 2} = 3$

9. $\frac{9}{2x - 5} = -2$

10. $\frac{6}{4 - 3x} = 3$

11. $2 + \frac{5}{x} = 7$

12. $3 + \frac{8}{n} = 5$

13. $1 - \frac{9}{x} = 4$

14. $3 - \frac{12}{x} = 7$

15. $\frac{2}{y} + 5 = 9$

16. $\frac{6}{x} + 3 = 11$

17. $\frac{3}{x - 2} = \frac{4}{x}$

18. $\frac{5}{x + 3} = \frac{3}{x - 1}$

19. $\frac{2}{3x - 1} = \frac{3}{4x + 1}$

20. $\frac{5}{3x - 4} = \frac{-3}{1 - 2x}$

21. $\frac{-3}{2x + 5} = \frac{2}{x - 1}$

22. $\frac{4}{5y - 1} = \frac{2}{2y - 1}$

23. $\frac{4x}{x - 4} + 5 = \frac{5x}{x - 4}$

24. $\frac{2x}{x + 2} - 5 = \frac{7x}{x + 2}$

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25. $2 + \frac{3}{a-3} = \frac{a}{a-3}$

26. $\frac{x}{x+4} = 3 - \frac{4}{x+4}$

27. $\frac{x}{x-1} = \frac{8}{x+2}$

28. $\frac{x}{x+12} = \frac{1}{x+5}$

29. $\frac{2x}{x+4} = \frac{3}{x-1}$

30. $\frac{5}{3n-8} = \frac{n}{n+2}$

31. $x + \frac{6}{x-2} = \frac{3x}{x-2}$

32. $x - \frac{6}{x-3} = \frac{2x}{x-3}$

33. $\frac{8}{y} = \frac{2}{y-2} + 1$

Objective B

34. How does a ratio differ from a rate?

35. What is a proportion?

Solve.

36. $\frac{x}{12} = \frac{3}{4}$

37. $\frac{6}{x} = \frac{2}{3}$

38. $\frac{4}{9} = \frac{x}{27}$

39. $\frac{16}{9} = \frac{64}{x}$

40. $\frac{x+3}{12} = \frac{5}{6}$

41. $\frac{3}{5} = \frac{x-4}{10}$

42. $\frac{18}{x+4} = \frac{9}{5}$

43. $\frac{2}{11} = \frac{20}{x-3}$

44. $\frac{2}{x} = \frac{4}{x+1}$

45. $\frac{16}{x-2} = \frac{8}{x}$

46. $\frac{x+3}{4} = \frac{x}{8}$

47. $\frac{x-6}{3} = \frac{x}{5}$

48. $\frac{2}{x-1} = \frac{6}{2x+1}$

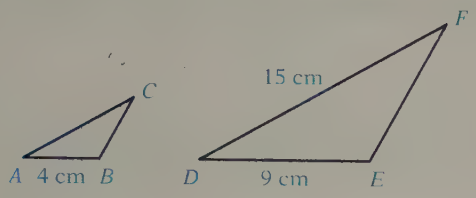
49. $\frac{9}{x+2} = \frac{3}{x-2}$

50. $\frac{2x}{7} = \frac{x-2}{14}$

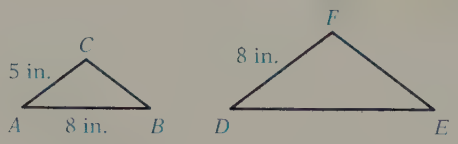
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Triangles *ABC* and *DEF* in Exercises 51 to 58 are similar. Round answers to the nearest tenth.

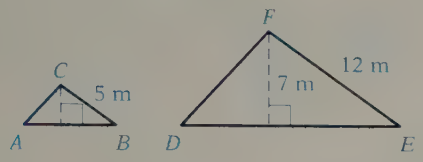
51. Find side *AC*.



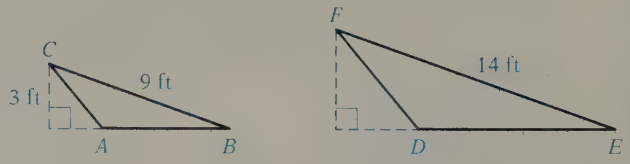
52. Find side *DE*.



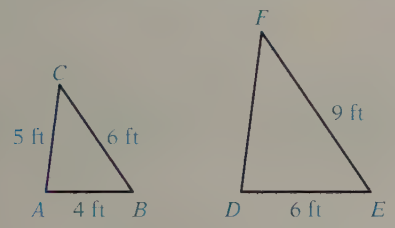
53. Find the height of triangle *ABC*.



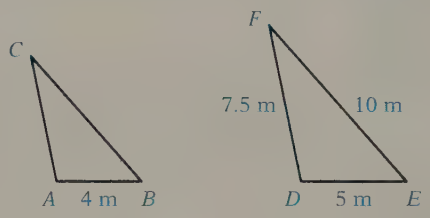
54. Find the height of triangle *DEF*.



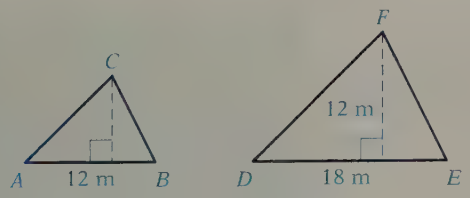
55. Find the perimeter of triangle *DEF*.



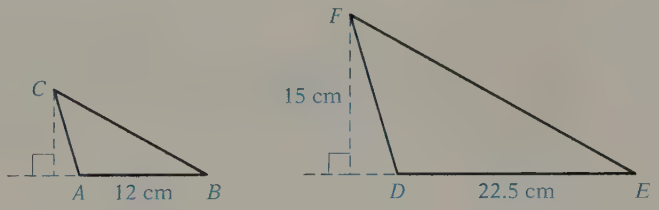
56. Find the perimeter of triangle *ABC*.



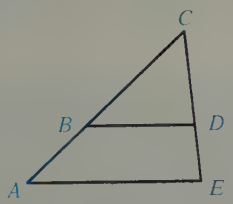
57. Find the area of triangle *ABC*.



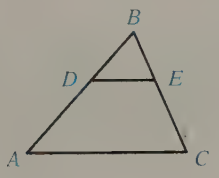
58. Find the area of triangle *ABC*.



59. Given $\overline{BD} \parallel \overline{AE}$, *BD* measures 5 cm, *AE* measures 8 cm, and *AC* measures 10 cm, find the length of *BC*.

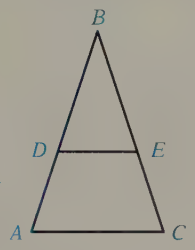


60. Given $\overline{AC} \parallel \overline{DE}$, *BD* measures 8 m, *AD* measures 12 m, and *BE* measures 6 m, find the length of *BC*.

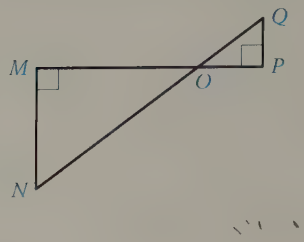


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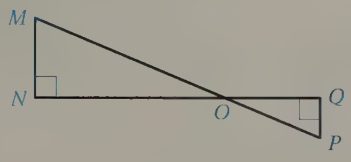
61. Given $\overline{DE} \parallel \overline{AC}$, DE measures 6 in., AC measures 10 in., and AB measures 15 in., find the length of DA.



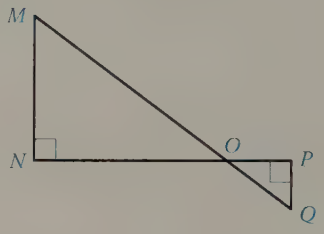
62. Given $\overline{MP}$ and $\overline{NQ}$ intersect at O, NO measures 25 ft, MO measures 20 ft, and PO measures 8 ft, find the length of QO.



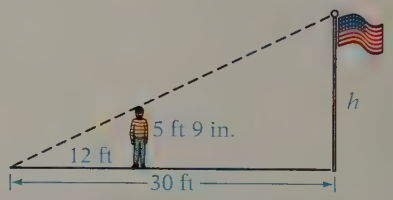
63. Given $\overline{MP}$ and $\overline{NQ}$ intersect at O, NO measures 24 cm, MN measures 10 cm, MP measures 39 cm, and QO measures 12 cm, find the length of OP.



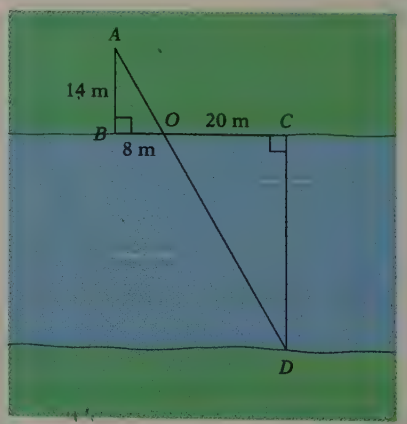
64. Given $\overline{MQ}$ and $\overline{NP}$ intersect at O, NO measures 12 m, MN measures 9 m, PQ measures 3 m, and MQ measures 20 m, find the perimeter of triangle OPQ.



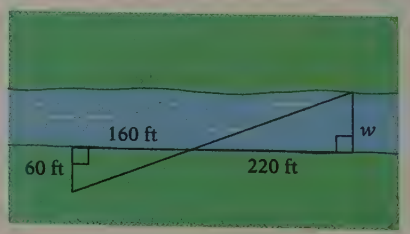
65. The sun's rays cast a shadow as shown in the diagram at the right. Find the height of the flagpole. Write the answer in terms of feet.



66. Similar triangles can be used as an indirect way to measure inaccessible distances. The diagram at the right represents a river of width DC. The triangles AOB and DOC are similar. The distances AB, BO, and OC can be measured. Find the width of the river.

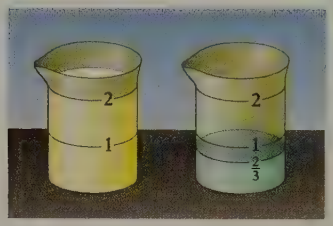


67. The diagram at the right shows how surveyors laid out similar triangles along a ravine. Find the width, w, of the ravine.



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Objective b *Application Problems*



68. Simple syrup used in making some desserts requires 2 cups (c) of sugar for every $\frac{2}{3}$ c of boiling water. At this rate, how many cups of sugar are required for 2 c boiling water?

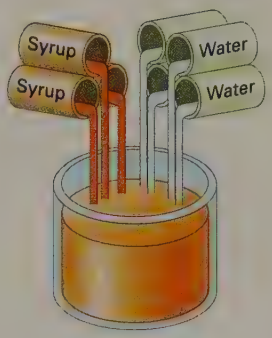
69. An exit poll survey showed that 4 out of every 7 voters cast a ballot in favor of an amendment to a city charter. At this rate, how many voters voted in favor of the amendment if 35,000 people voted?

70. As part of a conservation effort for a lake, 40 fish are caught, tagged, and then released. Later 80 fish are caught. Four of the 80 fish are found to have tags. Estimate the number of fish in the lake.



71. On a map, two cities are $2\frac{5}{8}$ in. apart. If $\frac{3}{8}$ in. on the map represents 25 mi, find the number of miles between the two cities.

72. The lighting for some billboards is provided by using solar energy. If 3 small solar energy panels can generate 10 W of power, how many panels are necessary to provide 600 W of power?



73. A soft drink is made by mixing 4 parts carbonated water with every 3 parts syrup. How many milliliters of water are in 280 ml of soft drink?

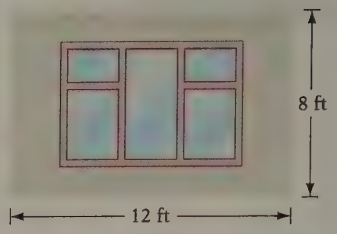
74. An air conditioning specialist recommends 2 air vents for each 300 ft^2 of floor space. At this rate, how many air vents are required for a 21,000-square-foot office building?

75. A laser printer is rated by the number of pages per minute it can print. An inexpensive laser printer can print 5 pages every 2 min. At this rate, how long would it take to print a document 45 pages long?

76. A company decides to accept a large shipment of 10,000 computer chips if there are 2 or fewer defects in a sample of 100 randomly chosen chips. Assuming that there are 300 defective chips in the shipment and that the rate of defective chips in the sample is the same as the rate in the shipment, will the shipment be accepted?

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77. To conserve energy and still allow for as much natural lighting as possible, an architect suggests that the ratio of the area of a window to the area of the total wall surface be 5 to 12. Using this ratio, determine the recommended area of a window to be installed in a wall that measures 8 ft by 12 ft.

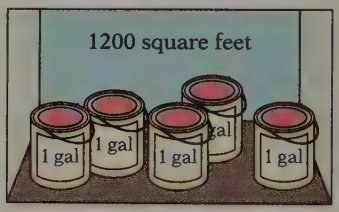


78. Leonardo da Vinci measured various distances on the human body in order to make accurate drawings. He determined that generally the ratio of the kneeling height of a person to the standing height of that person is $\frac{3}{4}$. Using this ratio, determine the height of a person who has a kneeling height of 48 in.



79. In one of Leonardo da Vinci's notebooks he wrote that ". . . from the top to the bottom of the chin is the sixth part of a face, and it is the fifty-fourth part of the man." Suppose the distance from the top to the bottom of the chin of a person is 1.25 in. Using da Vinci's measurements, determine the height of this person.

80. A painter estimates that 5 gal of paint will cover 1200 ft^2 of wall space. At this rate, how many additional gallons will be necessary to cover 1680 ft^2?



APPLYING THE CONCEPTS

81. Three people put their money together to buy lottery tickets. The first person put in \$25, the second person put in \$30, and the third person put in \$35. One of their tickets was a winning ticket. If they won \$4.5 million, what was the first person's share of the winnings?

82. No one belongs to both the Math Club and the Photography Club, but the two clubs join to hold a car wash. Ten members of the Math Club and 6 members of the Photography Club participate. The profits from the car wash are \$120. If each club's profits are proportional to the number of members participating, what share of the profits does the Math Club receive?



83. A basketball player has made 5 out of every 6 foul shots attempted in one year of play. If 42 foul shots were missed that year, how many shots did the basketball player make?

84. Explain the procedure for solving an equation that contains fractions. Include in your discussion how the LCM of the denominators is used to eliminate fractions in the equation.

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8.5 Literal Equations

Objective A

To solve a literal equation for one of the variables

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A **literal equation** is an equation that contains more than one variable. Examples of literal equations are shown at the right.

$2x + 3y = 6$
$4w - 2x + z = 0$

Formulas are used to express a relationship among physical quantities. A **formula** is a literal equation that states rules about measurements. Examples of formulas are shown at the right.

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$ (Physics)
$s = a + (n - 1)d$ (Mathematics)
$A = P + Prt$ (Business)

The Addition and Multiplication Properties can be used to solve a literal equation for one of the variables. The goal is to rewrite the equation so that the variable being solved for is alone on one side of the equation and all the other numbers and variables are on the other side.

Solve $A = P(1 + i)$ for i.

The goal is to rewrite the equation so that i is on one side of the equation and all other variables are on the other side.

$A = P(1 + i)$
$A = P + Pi$ • Use the Distributive Property to remove parentheses.
$A - P = P - P + Pi$ • Subtract P from each side of the equation.
$A - P = Pi$
$\frac{A - P}{P} = \frac{Pi}{P}$ • Divide each side of the equation by P.
$\frac{A - P}{P} = i$

Example 1

Solve $s = a + (n - 1)d$ for d.

Solution

$s = a + (n - 1)d$
$s - a = a - a + (n - 1)d$
$s - a = (n - 1)d$
$\frac{s - a}{n - 1} = \frac{(n - 1)d}{n - 1}$
$\frac{s - a}{n - 1} = d$

You Try It 1

Solve $A = P + Prt$ for r.

Your solution

Solution on p. S26

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Example 2

Solve $I = \frac{E}{R + r}$ for R.

Solution

$$I = \frac{E}{R + r}$$

$$(R + r)I = (R + r)\frac{E}{R + r}$$

$$RI + rI = E$$

$$RI + rI - rI = E - rI$$

$$RI = E - rI$$

$$\frac{RI}{I} = \frac{E - rI}{I}$$

$$R = \frac{E - rI}{I}$$

You Try It 2

Solve $s = \frac{A + L}{2}$ for L.

Your solution

Example 3

Solve $L = a(1 + ct)$ for c.

Solution

$$L = a(1 + ct)$$

$$L = a + act$$

$$L - a = a - a + act$$

$$L - a = act$$

$$\frac{L - a}{at} = \frac{act}{at}$$

$$\frac{L - a}{at} = c$$

You Try It 3

Solve $s = a + (n - 1)d$ for n.

Your solution

Example 4

Solve $M = C - rC$ for C.

Solution

$$M = C - rC$$

$$M = (1 - r)C$$

$$\frac{M}{1 - r} = \frac{(1 - r)C}{1 - r}$$

$$\frac{M}{1 - r} = C$$

You Try It 4

Solve $S = C + rC$ for C.

Your solution

Solutions on p. S26

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8.5 Exercises

Objective A

Solve the formula for the given variable.

1. $A = \frac{1}{2}bh; h$ (Geometry)
2. $P = a + b + c; b$ (Geometry)
3. $d = rt; t$ (Physics)
4. $E = IR; R$ (Physics)
5. $PV = nRT; T$ (Chemistry)
6. $A = bh; h$ (Geometry)
7. $P = 2L + 2W; L$ (Geometry)
8. $F = \frac{9}{5}C + 32; C$ (Temperature conversion)
9. $A = \frac{1}{2}h(b_1 + b_2); b_1$ (Geometry)
10. $C = \frac{5}{9}(F - 32); F$ (Temperature conversion)
11. $V = \frac{1}{3}Ah; h$ (Geometry)
12. $P = R - C; C$ (Business)
13. $R = \frac{C - S}{t}; S$ (Business)
14. $P = \frac{R - C}{n}; R$ (Business)
15. $A = P + Prt; P$ (Business)
16. $T = fm - gm; m$ (Engineering)
17. $A = Sw + w; w$ (Physics)
18. $a = S - Sr; S$ (Mathematics)

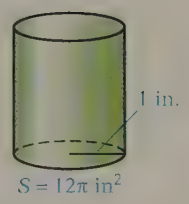
APPLYING THE CONCEPTS

The surface area of a right circular cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$, where r is the radius of the base, and h is the height of the cylinder.

19. **a.** Solve the formula $S = 2\pi rh + 2\pi r^2$ for h.

    b. Use your answer to part **a** to find the height of a right circular cylinder when the surface area is 12π in^2 and the radius is 1 in.

    c. Use your answer to part **a** to find the height of a right circular cylinder when the surface area is 24π in^2 and the radius is 2 in.



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When markup is based on selling price, the selling price of a product is given by the formula $S = C + rC$, where C is the cost of the product, and r is the markup rate.

20. **a.** Solve the formula $S = C + rC$ for r.

b. Use your answer to part **a** to find the markup rate on a tennis racket when the cost is \$112 and the selling price is \$180.

c. Use your answer to part **a** to find the markup rate on a radio when the cost is \$120 and the selling price is \$172.

Break-even analysis is a method used to determine the sales volume required for a company to break even, or experience neither a profit nor a loss on the sale of a product. The **break-even point** represents the number of units that must be made and sold for income from sales to equal the cost of the product. The break-even point can be calculated using the formula $B = \dfrac{F}{S - V}$, where F is the fixed costs, S is the selling price per unit, and V is the variable costs per unit.

21. **a.** Solve the formula $B = \dfrac{F}{S - V}$ for S.

b. Use your answer to part **a** to find the required selling price per desk for a company to break even. The fixed costs are \$20,000, the variable costs per desk are \$80, and the company plans to make and sell 200 desks.

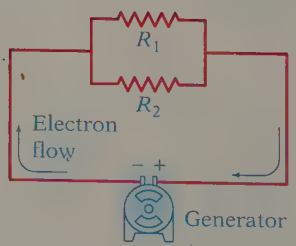
c. Use your answer to part **a** to find the required selling price per camera for a company to break even. The fixed costs are \$15,000, the variable costs per camera are \$50, and the company plans to make and sell 600 cameras.

Resistors are used to control the flow of current. The total resistance of two resistors in a circuit can be given by the formula $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$, where R_1 and R_2 are the two resistors in the circuit. Resistance is measured in ohms.

22. **a.** Solve the formula $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$ for R_1.

b. Use your answer to part **a** to find the resistance in R_1 if the resistance in R_2 is 30 ohms and the total resistance is 12 ohms.

c. Use your answer to part **a** to find the resistance in R_1 if the resistance in R_2 is 15 ohms and the total resistance is 6 ohms.



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8.6 Work and Uniform Motion Problems

To solve work problems

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Point of Interest

The following problem was recorded in the *Jiuzhang*, a Chinese text that dates to the Han dynasty (about 200 B.C. to A.D. 200). "A reservoir has 5 channels bringing water to it. The first can fill the reservoir in $\frac{1}{3}$ day, the second in 1 day, the third in $2\frac{1}{2}$ days, the fourth in 3 days, and the fifth in 5 days. If all channels are open, how long does it take to fill the reservoir?" This problem is the earliest known work problem.

If a painter can paint a room in 4 h, then in 1 h the painter can paint $\frac{1}{4}$ of the room. The painter's rate of work is $\frac{1}{4}$ of the room each hour. The **rate of work** is the part of a task that is completed in one unit of time.

A pipe can fill a tank in 30 min. This pipe can fill $\frac{1}{30}$ of the tank in 1 min. The rate of work is $\frac{1}{30}$ of the tank each minute. If a second pipe can fill the tank in x min, the rate of work for the second pipe is $\frac{1}{x}$ of the tank each minute.

In solving a work problem, the goal is to determine the time it takes to complete a task. The basic equation that is used to solve work problems is

Rate of work × time worked = part of task completed

For example, if a faucet can fill a sink in 6 min, then in 5 min the faucet can fill $\frac{1}{6} \times 5 = \frac{5}{6}$ of the sink. In 5 min the faucet completes $\frac{5}{6}$ of the task.

Example A painter can paint a wall in 20 min. The painter's apprentice can paint the same wall in 30 min. How long will it take them to paint the wall when they work together?

Strategy for Solving a Work Problem

1. For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

TAKE NOTE

Use the information given in the problem to fill in the "Rate" and "Time" columns of the table. Fill in the "Part Completed" column by multiplying the two expressions you wrote in each row.

Unknown time to paint the wall working together: t

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Painter	$\frac{1}{20}$	·	t	=	$\frac{t}{20}$
Apprentice	$\frac{1}{30}$	·	t	=	$\frac{t}{30}$

2. Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1; the complete task.

$$\frac{t}{20} + \frac{t}{30} = 1$$

- The sum of the part of the task completed by the painter and the part of the task completed by the apprentice is 1.

$$60\left(\frac{t}{20} + \frac{t}{30}\right) = 60 \cdot 1$$

- Multiply by the LCM of 20 and 30.

$$3t + 2t = 60$$
$$5t = 60$$
$$t = 12$$

Working together, they will paint the wall in 12 min.

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Example 1

A small water pipe takes three times longer to fill a tank than does a large water pipe. With both pipes open, it takes 4 h to fill the tank. Find the time it would take the small pipe, working alone, to fill the tank.

Strategy

- Time for large pipe to fill the tank: t
  Time for small pipe to fill the tank: $3t$

Fills tank in $3t$ hours



Fills $\frac{4}{3t}$ of the tank in 4 hours

Fills tank in t hours



Fills $\frac{4}{t}$ of the tank in 4 hours

	Rate	*Time*	*Part*
Small pipe	$\frac{1}{3t}$	4	$\frac{4}{3t}$
Large pipe	$\frac{1}{t}$	4	$\frac{4}{t}$

- The sum of the parts of the task completed by each pipe must equal 1.

Solution

$$\frac{4}{3t} + \frac{4}{t} = 1$$

$$3t\left(\frac{4}{3t} + \frac{4}{t}\right) = 3t \cdot 1$$

$$4 + 12 = 3t$$

$$16 = 3t$$

$$\frac{16}{3} = t$$

$$3t = 3\left(\frac{16}{3}\right) = 16$$

The small pipe working alone takes 16 h to fill the tank.

You Try It 1

Two computer printers that work at the same rate are working together to print the payroll checks for a large corporation. After they work together for 2 h, one of the printers quits. The second requires 3 h more to complete the payroll checks. Find the time it would take one printer, working alone, to print the payroll.

Your strategy

Your solution

Solution on p. S26

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Objective 1 **To solve uniform motion problems**



A car that travels constantly in a straight line at 30 mph is in uniform motion. **Uniform motion** means that the speed or direction of an object does not change.

The basic equation used to solve uniform motion problems is

$$\textbf{Distance} = \textbf{rate} \times \textbf{time}$$

An alternative form of this equation can be written by solving the equation for time.

$$\frac{\textbf{Distance}}{\textbf{Rate}} = \textbf{time}$$

This form of the equation is useful when the total time of travel for two objects or the time of travel between two points is known.

The speed of a boat in still water is 20 mph. The boat traveled 75 mi down a river in the same amount of time it took to travel 45 mi up the river. Find the rate of the river's current.

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

TAKE NOTE
Use the information given in the problem to fill in the "Distance" and "Rate" columns of the table. Fill in the "Time" column by dividing the two expressions you wrote in each row.

The unknown rate of the river's current: r

	Distance	÷	*Rate*	=	*Time*
Down river	75	÷	$20 + r$	=	$\frac{75}{20 + r}$
Up river	45	÷	$20 - r$	=	$\frac{45}{20 - r}$

2. Determine how the times traveled by each object are related. For example, it may be known that the times are equal, or the total time may be known.

$$\frac{75}{20 + r} = \frac{45}{20 - r}$$

- The time down the river is equal to the time up the river.

$$(20 + r)(20 - r)\frac{75}{20 + r} = (20 + r)(20 - r)\frac{45}{20 - r}$$

- Multiply by the LCM of the denominators.

$$\begin{aligned} (20 - r)75 &= (20 + r)45 \\ 1500 - 75r &= 900 + 45r \\ -120r &= -600 \\ r &= 5 \end{aligned}$$

The rate of the river's current is 5 mph.

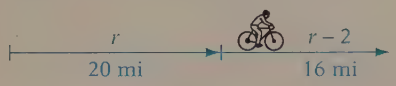
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Example 2

A cyclist rode the first 20 mi of a trip at a constant rate. For the next 16 mi, the cyclist reduced the speed by 2 mph. The total time for the 36 mi was 4 h. Find the rate of the cyclist for each leg of the trip.

Strategy

- Rate for the first 20 mi: r
  Rate for the next 16 mi: $r - 2$



	Distance	*Rate*	*Time*
First 20 mi	20	r	$\frac{20}{r}$
Next 16 mi	16	$r - 2$	$\frac{16}{r-2}$

- The total time for the trip was 4 h.

Solution

$$\frac{20}{r} + \frac{16}{r-2} = 4$$

$$r(r-2)\left[\frac{20}{r} + \frac{16}{r-2}\right] = r(r-2) \cdot 4$$

$$(r-2)20 + 16r = (r^2 - 2r) \cdot 4$$

$$20r - 40 + 16r = 4r^2 - 8r$$

$$36r - 40 = 4r^2 - 8r$$

Solve the quadratic equation by factoring.

$$0 = 4r^2 - 44r + 40$$

$$0 = 4(r^2 - 11r + 10)$$

$$0 = 4(r-10)(r-1)$$

$r - 10 = 0$ $\qquad$ $r - 1 = 0$

$r = 10$ $\qquad$ $r = 1$

The solution $r = 1$ mph is not possible, because the rate on the last 16 mi would then be -1 mph.

10 mph was the rate for the first 20 mi.
8 mph was the rate for the next 16 mi.

You Try It 2

The total time it took for a sailboat to sail back and forth across a lake 6 km wide was 2 h. The rate sailing back was three times the rate sailing across. Find the rate sailing out across the lake.

Your strategy

Your solution

Solution on p. S26

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8.6 Exercises

Objective A Application Problems

1. A park has two sprinklers that are used to fill a fountain. One sprinkler can fill the fountain in 3 h, whereas the second sprinkler can fill the fountain in 6 h. How long will it take to fill the fountain with both sprinklers operating?

	Rate	*Time*	*Part*
First	$\frac{1}{3}$	t	$\frac{t}{3}$
Second	$\frac{1}{6}$	t	$\frac{t}{6}$

2. One grocery clerk can stock a shelf in 20 min, whereas a second clerk requires 30 min to stock the same shelf. How long would it take to stock the shelf if the two clerks worked together?

3. One person with a skiploader requires 12 h to remove a large quantity of earth. A second, larger skiploader can remove the same amount of earth in 4 h. How long would it take to remove the earth with both skiploaders working together?

4. An experienced painter can paint a fence twice as fast as an inexperienced painter. Working together, the painters require 4 h to paint the fence. How long would it take the experienced painter working alone to paint the fence?

5. One computer can solve a complex prime factorization problem in 75 h. A second computer can solve the same problem in 50 h. How long would it take both computers, working together, to solve the problem?

6. A new machine can make 10,000 aluminum cans three times as fast as an older machine. With both machines working, 10,000 cans can be made in 9 h. How long would it take the new machine, working alone, to make the 10,000 cans?

7. A small air conditioner can cool a room 5° in 75 min. A larger air conditioner can cool the room 5° in 50 min. How long would it take to cool the room 5° with both air conditioners working?

8. One printing press can print the first edition of a book in 55 min, whereas a second printing press requires 66 min to print the same number of copies. How long would it take to print the first edition with both presses operating?

9. Two oil pipelines can fill a small tank in 30 min. Using one of the pipelines would require 45 min to fill the tank. How long would it take the second pipeline alone to fill the tank?

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10. Working together, two dock workers can load a crate in 6 min. One dock worker, working alone, can load the crate in 15 min. How long would it take the second dock worker, working alone, to load the crate?

11. A mason can construct a retaining wall in 10 h. With the mason's apprentice assisting, the task would take 6 h. How long would it take the apprentice working alone to construct the wall?

12. A mechanic requires 2 h to repair a transmission, whereas an apprentice requires 6 h to make the same repairs. The mechanic worked alone for 1 h and then stopped. How long will it take the apprentice, working alone, to complete the repairs?

13. One computer technician can wire a modem in 4 h, whereas it takes 6 h for a second technician to do the same job. After working alone for 2 h, the first technician quit. How long will it take the second technician to complete the wiring?

14. A wallpaper hanger requires 2 h to hang the wallpaper on one wall of a room. A second wallpaper hanger requires 4 h to hang the same amount of paper. The first wallpaper hanger worked alone for 1 h and then quit. How long will it take the second wallpaper hanger, working alone, to complete the wall?



15. Two welders who work at the same rate are welding the girders of a building. After they work together for 10 h, one of the welders quits. The second welder requires 20 more hours to complete the welds. Find the time it would have taken one of the welders, working alone, to complete the welds.

16. A large and a small heating unit are being used to heat the water of a pool. The larger unit, working alone, requires 8 h to heat the pool. After both units have been operating for 2 h, the larger unit is turned off. The small unit requires 9 h more to heat the pool. How long would it take the small unit, working alone, to heat the pool?

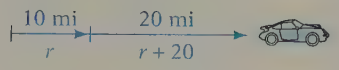
17. Two machines that fill cereal boxes work at the same rate. After they work together for 7 h, one machine breaks down. The second machine requires 14 h more to finish filling the boxes. How long would it have taken one of the machines, working alone, to fill the boxes?

18. A large and a small drain are opened to drain a pool. The large drain can empty the pool in 6 h. After both drains have been open for 1 h, the large drain becomes clogged and is closed. The smaller drain remains open and requires 9 h more to empty the pool. How long would it have taken the small drain, working alone, to empty the pool?

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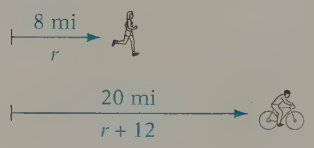
Objective B Application Problems

19. Commuting from work to home, a lab technician traveled 10 mi at a constant rate through congested traffic. On reaching the expressway, the technician increased the speed by 20 mph. An additional 20 mi was traveled at the increased speed. The total time for the trip was 1 h. Find the rate of travel through the congested traffic.



20. The president of a company traveled 1800 mi by jet and 300 mi on a prop plane. The rate of the jet was four times the rate of the prop plane. The entire trip took a total of 5 h. Find the rate of the jet plane.

21. As part of a conditioning program, a jogger ran 8 mi in the same time a cyclist rode 20 mi. The rate of the cyclist was 12 mph faster than the rate of the jogger. Find the rate of the jogger and that of the cyclist.



22. An express train travels 600 mi in the same amount of time it takes a freight train to travel 360 mi. The rate of the express train is 20 mph faster than that of the freight train. Find the rate of each train.

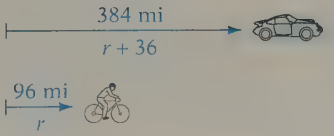
23. To assess the damage done by a fire, a forest ranger traveled 1080 mi by jet and then an additional 180 mi by helicopter. The rate of the jet was 4 times the rate of the helicopter. The entire trip took a total of 5 h. Find the rate of the jet.

24. A twin-engine plane can fly 800 mi in the same time that it takes a single-engine plane to fly 600 mi. The rate of the twin-engine plane is 50 mph faster than that of the single-engine plane. Find the rate of the twin-engine plane.

25. The rate of a bicyclist is 7 mph greater than the rate of a long-distance runner. The bicyclist travels 30 mi in the same amount of time as it takes the runner to travel 16 mi. Find the rate of the runner.

26. A car and a bus leave a town at 1 P.M. and head for a town 300 mi away. The rate of the car is twice the rate of the bus. The car arrives 5 h ahead of the bus. Find the rate of the car.

27. A car is traveling at a rate that is 36 mph greater than the rate of a cyclist. The car travels 384 mi in the same time it takes the cyclist to travel 96 mi. Find the rate of the car.

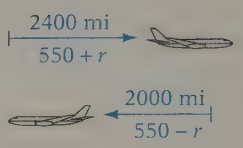


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28. A backpacker hiking into a wilderness area walked 9 mi at a constant rate and then reduced this rate by 1 mph. Another 4 mi was hiked at this reduced rate. The time required to hike the 4 mi was 1 h less than the time required to walk the 9 mi. Find the rate at which the hiker walked the first 9 mi.

29. A plane can fly 180 mph in calm air. Flying with the wind, the plane can fly 600 mi in the same amount of time it takes to fly 480 mi against the wind. Find the rate of the wind.

30. A commercial jet can fly 550 mph in calm air. Traveling with the jet stream, the plane flew 2400 mi in the same amount of time it takes to fly 2000 mi against the jet stream. Find the rate of the jet stream.



31. A cruise ship can sail at 28 mph in calm water. Sailing with the gulf current, the ship can sail 170 mi in the same amount of time that it can sail 110 mi against the gulf current. Find the rate of the gulf current.

32. Rowing with the current of a river, a rowing team can row 25 mi in the same amount of time it takes to row 15 mi against the current. The rate of the rowing team in calm water is 20 mph. Find the rate of the current.

33. On a recent trip, a trucker traveled 330 mi at a constant rate. Because of road construction, the trucker then had to reduce the speed by 25 mph. An additional 30 mi was traveled at the reduced rate. The total time for the entire trip was 7 h. Find the rate of the trucker for the first 330 mi.

APPLYING THE CONCEPTS

34. One pipe can fill a tank in 2 h, a second pipe can fill the tank in 4 h, and a third pipe can fill the tank in 5 h. How long will it take to fill the tank with all three pipes working?

35. A mason can construct a retaining wall in 10 h. The mason's more-experienced apprentice can do the same job in 15 h. How long would it take the mason's less-experienced apprentice to do the job if, working together, all three can complete the job in 5 h?

36. A surveyor traveled 32 mi by canoe and then hiked 4 mi. The rate by boat was four times the rate on foot. If the time spent walking was 1 h less than the time spent canoeing, find the amount of time spent traveling by canoe.

37. Because of bad weather, a bus driver reduced the usual speed along a 150-mile bus route by 10 mph. The bus arrived only 30 min later than its usual arrival time. How fast does the bus usually travel?

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8.7 Variation

To solve variation problems

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Direct variation is a special function that can be expressed as the equation $y = kx$, where k is a constant. The equation $y = kx$ is read "y varies directly as x" or "y is directly proportional to x." The constant k is called the **constant of variation** or the **constant of proportionality**.

The circumference (C) of a circle varies directly as the diameter (d). The direct variation equation is written $C = \pi d$. The constant of variation is π.

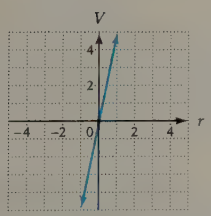
A nurse makes \$25 per hour. The total wage (w) of the nurse is directly proportional to the number of hours (h) worked. The equation of variation is $w = 25h$. The constant of proportionality is 25.

A direct variation equation can be written in the form $y = kx^n$, where n is a positive number. For example, the equation $y = kx^2$ is read "y varies directly as the square of x."

The area (A) of a circle varies directly as the square of the radius (r) of the circle. The direct variation equation is $A = \pi r^2$. The constant of variation is π.

➡ Given that V varies directly as r and that $V = 20$ when $r = 4$, find the constant of variation and the equation of variation.

$V = kr$	• Write the basic direct variation equation.
$20 = k \cdot 4$	• Replace V and r by the given values. Then solve for k.
$5 = k$	• This is the constant of variation.
$V = 5r$	• Write the direct variation equation by substituting the value of k into the basic direct variation equation.

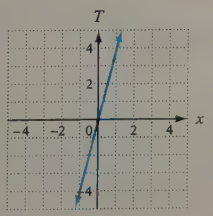


The Graph of $V = 5r$

➡ The tension (T) in a spring varies directly as the distance (x) it is stretched. If $T = 8$ lb when $x = 2$ in., find T when $x = 4$ in.

$T = kx$	• Write the basic direct variation equation.
$8 = k \cdot 2$	• Replace T and x by the given values.
$4 = k$	• Solve for the constant of variation.
$T = 4x$	• Write the direct variation equation.
$T = 4x = 4 \cdot 4 = 16$	• To find T when $x = 4$, substitute 4 for x in the equation and solve for T.

The tension is 16 lb.

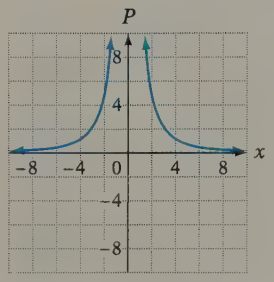


The Graph of $T = 4x$

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Inverse variation is a function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. The equation $y = \frac{k}{x}$ is read "y varies inversely as x" or "y is inversely proportional to x."

In general, an inverse variation equation can be written $y = \frac{k}{x^n}$, where n is a positive number. For example, the equation $y = \frac{k}{x^2}$ is read "y varies inversely as the square of x."



The Graph of $P = \frac{20}{x^2}$

Given that P varies inversely as the square of x and that $P = 5$ when $x = 2$, find the constant of variation and the equation of variation.

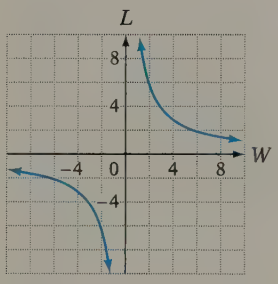
$P = \frac{k}{x^2}$ • Write the basic inverse variation equation.

$5 = \frac{k}{2^2}$ • Replace P and x by the given values. Then solve for k.

$5 = \frac{k}{4}$

$20 = k$ • This is the constant of variation.

$P = \frac{20}{x^2}$ • Write the inverse variation equation by substituting the value of k into the basic inverse variation equation.



The Graph of $L = \frac{12}{W}$

The length (L) of a rectangle with fixed area is inversely proportional to the width (W). If $L = 6$ ft when $W = 2$ ft, find L when $W = 3$ ft.

$L = \frac{k}{W}$ • Write the basic inverse variation equation.

$6 = \frac{k}{2}$ • Replace L and W by the given values.

$12 = k$ • Solve for the constant of variation.

$L = \frac{12}{W}$ • Write the inverse variation equation.

$L = \frac{12}{W} = \frac{12}{3} = 4$ • To find L when $W = 3$ ft, substitute 3 for W in the equation and solve for L.

The length is 4 ft.

Joint variation is a variation in which a variable varies directly as the product of two or more other variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. The equation $z = kxy$ is read "z varies jointly as x and y."

The area (A) of a triangle varies jointly as the base (b) and the height (h). The joint variation equation is written $A = \frac{1}{2}bh$. The constant of variation is $\frac{1}{2}$.

A **combined variation** is a variation in which two or more types of variation occur at the same time. For example, in physics, the volume (V) of a gas varies directly as the temperature (T) and inversely as the pressure (P). This combined variation is written $V = \frac{kT}{P}$.

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A ball is being twirled on the end of a string. The tension (T) in the string is directly proportional to the square of the speed (v) of the ball and inversely proportional to the length (r) of the string. The tension is 96 lb when the length of the string is 0.5 ft and the speed is 4 ft/s. Find the tension when the length of the string is 1 ft and the speed is 5 ft/s.

$$T = \frac{kv^2}{r}$$ • Write the basic combined variation equation.

$$96 = \frac{k \cdot 4^2}{0.5}$$ • Replace T, v, and r by the given values.

$$96 = \frac{k \cdot 16}{0.5}$$

$$96 = k \cdot 32$$ • Solve for the constant of variation.

$$3 = k$$

$$T = \frac{3v^2}{r}$$ • Write the combined variation equation.

$$T = \frac{3v^2}{r} = \frac{3 \cdot 5^2}{1} = 3 \cdot 25 = 75$$ • To find T when $r = 1$ ft and $v = 5$ ft/s, substitute 1 for r and 5 for v and solve for T.

The tension is 75 lb.

Example 1

The amount (A) of medication prescribed for a person is directly related to the person's weight (W). For a 50-kilogram person, 2 ml of medication are prescribed. How many milliliters of medication are required for a person who weighs 75 kg?

Strategy

To find the required amount of medication:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 75 for W and solve for A.

Solution

$$A = kW$$

$$2 = k \cdot 50$$

$$\frac{1}{25} = k$$

$$A = \frac{1}{25}W = \frac{1}{25} \cdot 75 = 3$$

The required amount of medication is 3 ml.

You Try It 1

The distance (s) a body falls from rest varies directly as the square of the time (t) of the fall. An object falls 64 ft in 2 s. How far will it fall in 5 s?

Your strategy

Your solution

Solution on pp. S26–S27

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Example 2

A company that produces personal computers has determined that the number of computers it can sell (s) is inversely proportional to the price (P) of the computer. Two thousand computers can be sold when the price is \$2500. How many computers can be sold when the price of a computer is \$2000?

Strategy

To find the number of computers:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 2000 for P and solve for s.

Solution

$$s = \frac{k}{P}$$

$$2000 = \frac{k}{2500}$$

$$5{,}000{,}000 = k$$

$$s = \frac{5{,}000{,}000}{P} = \frac{5{,}000{,}000}{2000} = 2500$$

At \$2000 each, 2500 computers can be sold.

You Try It 2

The resistance (R) to the flow of electric current in a wire of fixed length is inversely proportional to the square of the diameter (d) of a wire. If a wire of diameter 0.01 cm has a resistance of 0.5 ohm, what is the resistance in a wire that is 0.02 cm in diameter?

Your strategy

Your solution

Example 3

The pressure (P) of a gas varies directly as the temperature (T) and inversely as the volume (V). When $T = 50°$ and $V = 275$ in^3, $P = 20$ lb/in^2. Find the pressure of a gas when $T = 60°$ and $V = 250$ in^3.

Strategy

To find the pressure:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 60 for T and 250 for V, and solve for P.

Solution

$$P = \frac{kT}{V}$$

$$20 = \frac{k \cdot 50}{275}$$

$$110 = k$$

$$P = \frac{110T}{V} = \frac{110 \cdot 60}{250} = 26.4$$

The pressure is 26.4 lb/in^2.

You Try It 3

The strength (s) of a rectangular beam varies jointly as its width (w) and the square of its depth (d) and inversely as its length (L). If the strength of a beam 2 in. wide, 12 in. deep, and 12 ft long is 1200 lb, find the strength of a beam that is 4 in. wide, 8 in. deep, and 16 ft long.

Your strategy

Your solution

Solutions on p. S27

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8.7 Exercises

Objective 4

1. Which of the following represent(s) direct variation? Why?

a. $y = kx$ **b.** $y = \frac{k}{x}$ **c.** $y = k + x$ **d.** $y = \frac{k}{x^2}$

2. Which of the following equations represent(s) "d varies directly as t"? Explain your answer.

a. $d = 400t$ **b.** $d = \frac{16}{t}$ **c.** $d = 25t$ **d.** $d = t - 50$

3. The profit (P) realized by a company varies directly as the number of products it sells (s). If a company makes a profit of \$4000 on the sale of 250 products, what is the profit when the company sells 5000 products?

4. The income (I) of a computer analyst varies directly as the number of hours (h) worked. If the analyst earns \$672 for working 8 h, how much will the analyst earn by working 36 h?



5. The pressure (p) on a diver in the water varies directly as the depth (d). If the pressure is 4.5 lb/in^2 when the depth is 10 ft, what is the pressure when the depth is 15 ft?

6. The distance (d) a spring will stretch varies directly as the force (f) applied to the spring. If a force of 6 lb is required to stretch a spring 3 in., what force is required to stretch the spring 4 in.?

7. The distance (d) an object will fall is directly proportional to the square of the time (t) of the fall. If an object falls 144 ft in 3 s, how far will the object fall in 10 s?

8. The period (p) of a pendulum, or the time it takes the pendulum to make one complete swing, varies directly as the square root of the length (L) of the pendulum. If the period of a pendulum is 1.5 s when the length is 2 ft, find the period when the length is 5 ft. Round to the nearest hundredth.

9. The distance (s) a ball will roll down an inclined plane is directly proportional to the square of the time (t). If the ball rolls 6 ft in 1 s, how far will it roll in 3 s?

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10. The stopping distance (s) of a car varies directly as the square of its speed (v). If a car traveling 30 mph requires 63 ft to stop, find the stopping distance for a car traveling 55 mph.

11. The time (t) for a car to travel between two cities is inversely proportional to the rate (r) of travel. If it takes 5 h to travel between the cities at a rate of 55 mph, find the time to travel between the two cities at a rate of 65 mph. Round to the nearest tenth.

12. The speed (v) of a gear varies inversely as the number of teeth (t). If a gear that has 45 teeth makes 24 revolutions per minute, how many revolutions per minute will a gear that has 36 teeth make?

13. The pressure (p) of a liquid varies directly as the product of the depth (d) and the density (D) of the liquid. If the pressure is 150 lb/in^2 when the depth is 100 in. and the density is 1.2, find the pressure when the density remains the same and the depth is 75 in.

14. The current (I) in a wire varies directly as the voltage (V) and inversely as the resistance (R). If the current is 10 amps when the voltage is 110 volts and the resistance is 11 ohms, find the current when the voltage is 180 volts and the resistance is 24 ohms.

15. The repulsive force (f) between the north poles of two magnets is inversely proportional to the square of the distance (d) between them. If the repulsive force is 20 lb when the distance is 4 in., find the repulsive force when the distance is 2 in.

APPLYING THE CONCEPTS

16. In the inverse variation equation $y = \frac{k}{x}$, what is the effect on x if y doubles?

17. In the direct variation equation $y = kx$, what is the effect on y when x doubles?

Complete using the word *directly* or *inversely*.

18. If a varies directly as b and inversely as c, then c varies _____ as b and _____ as a.

19. If a varies _____ as b and c, then abc is constant.

20. If the length of a rectangle is held constant, the area of the rectangle varies _____ as the width.

21. If the area of a rectangle is held constant, the length of the rectangle varies _____ as the width.

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Focus on Problem Solving

Find a Pattern

Polya's four recommended problem-solving steps are stated below.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review your solution.

Point of Interest

Karl Friedrich Gauss (1777–1855) has been called the "Prince of Mathematicians" by some historians. He applied his genius to many areas of mathematics and science. A unit of magnetism, the *gauss*, is named in his honor. Some electronic equipment (the television, for instance) contains a *degausser* that controls magnetic fields.

One of the several ways of devising a plan is first to try to find a pattern. Karl Friedrich Gauss supposedly used this method to solve a problem that was given to his math class when he was in elementary school. As the story goes, his teacher wanted to grade some papers while the class worked on a math problem. The problem given to the class was to find the sum

$$1 + 2 + 3 + 4 + \cdots + 100$$

Gauss quickly solved the problem by seeing a pattern. Here is what he saw.

$$\underbrace{1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100}$$

101 101 101 101

Note that

$1 + 100 = 101$
$2 + 99 = 101$
$3 + 98 = 101$
$4 + 97 = 101$

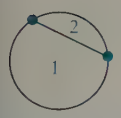
Gauss noted that there were 50 sums of 101. Therefore, the sum of the first 100 natural numbers is

$$1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100 = 50(101) = 5050$$

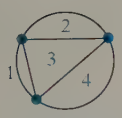
Try to solve the following problems by finding a pattern.

1. Find the sum $2 + 4 + 6 + \cdots + 96 + 98 + 100$.
2. Find the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$.
3. Find another method of finding the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$ given in the previous exercise.
4. Find the sum $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50}$.

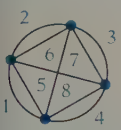
   Hint: $\frac{1}{1 \cdot 2} = \frac{1}{2}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{3}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{4}$



2 points, 2 regions



3 points, 4 regions



4 points, 8 regions


5 points, ? regions

5. The following problem shows that checking a few cases does not always result in a conjecture that is true for *all* cases. Select any two points on a circle and draw a *chord,* the line connecting the points (see the drawing in the left margin). The chord divides the circle into 2 regions. Now select 3 different points and draw chords connecting each of the three points with every other point. The chords divide the circle into 4 regions. Now select 4 points and connect each of the points with every other point. Make a conjecture about the relationship between the number of regions and the number of points on the circle. Does your conjecture work for 5 points? 6 points?
6. A *polygonal number* is a number that can be represented by arranging that number of dots in rows to form a geometric figure such as a triangle, square, pentagon, or hexagon. For instance, the first four *triangular numbers,* 3, 6, 10, and 15, are given (and shown) below. What are the next two triangular numbers?


3 6 10 15

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Projects and Group Activities

Continued Fractions

The following complex fraction is called a **continued fraction**.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

The dots indicate that the pattern continues to repeat forever.

A **convergent** of a continued fraction is an approximation of the repeated pattern. For instance,

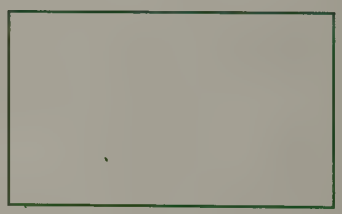
$$c_2 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}} \qquad c_3 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}} \qquad c_4 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}}}$$

1. Calculate c_5 for the continued fraction above.

This particular continued fraction is related to the golden rectangle, which has been used in architectural designs as diverse as the Parthenon in Athens, built around 440 B.C., and the United Nations building. A golden rectangle is one for which

$$\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}$$

An example of a golden rectangle is shown at the right.



Here is another continued fraction that was discovered by Leonhard Euler (1707–1793). Calculating the convergents of this continued fraction yields approximations that are closer and closer to π.

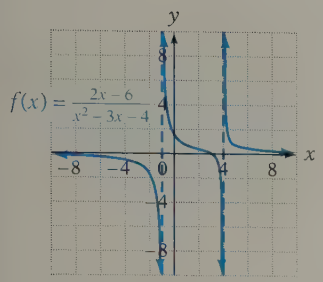
$$\pi = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cdots}}}}$$

2. Calculate $c_5 = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cfrac{9^2}{6 + 11^2}}}}}$

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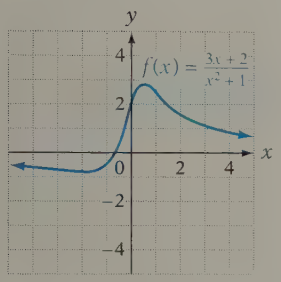
Graphing Rational Functions

The domain of a function is the set of the first coordinates of all the ordered pairs of the function. When a function is given by an equation, the domain of the function is all real numbers for which the function evaluates to a real number. For rational functions, we must exclude from the domain all those values of the variable for which the denominator of the rational function is zero. The graphing calculator is a useful tool to show the graphs of functions with excluded values in the domain of the function.



The graph of the function $f(x) = \dfrac{2x - 6}{x^2 - 3x - 4}$ is shown at the left. Note that the graph never intersects the lines $x = -1$ and $x = 4$ (shown as dashed lines). These are the two values excluded from the domain of f. The graph of the function f gets closer to the dashed line $x = 4$ as x gets closer to 4. The graph of the function f also gets closer to $x = -1$ as x gets closer to -1. The lines that are "approached" by the function are called **asymptotes**.

The domain is $\{x | x \neq -1, x \neq 4\}$.



The graph of the function $f(x) = \dfrac{3x + 2}{x^2 + 1}$ is shown at the left. The domain must exclude values of x for which $x^2 + 1 = 0$. It is not possible that $x^2 + 1 = 0$, because $x^2 \geq 0$, and a positive number added to a number equal to or greater than zero cannot equal zero. Therefore, there are no real numbers that must be excluded from the domain of f.

The domain is $\{x | x \in \text{real numbers}\}$.

Use a graphing calculator to find the domain of the rational function.

1. $h(x) = \dfrac{5x}{3x + 9}$ **2.** $f(x) = \dfrac{-2x}{6 - 2x}$ **3.** $q(x) = \dfrac{2x - 1}{x^2 + x - 6}$

4. $G(x) = \dfrac{3 - 4x}{x^2 + 4x - 5}$ **5.** $H(x) = \dfrac{x + 3}{x^2 + 2}$ **6.** $g(x) = \dfrac{x^2 + 1}{x^2}$

Graphing Variation Equations

1. Graph $y = kx$ when $k = 2$.
What kind of function does the graph represent?

2. Graph $y = kx$ when $k = \dfrac{1}{2}$.
What kind of function does the graph represent?

3. Graph $y = \dfrac{k}{x}$ when $k = 2$ and $x > 0$.
Is this the graph of a function?

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Chapter Summary

Key Words

A *rational expression* is a fraction in which the numerator and denominator are polynomials. A rational expression is in *simplest form* when the numerator and denominator have no common factors. [p. 431]

The *least common multiple* (LCM) of two or more polynomials is the simplest polynomial that contains the factors of each polynomial. [p. 439]

The *reciprocal* of a rational expression is the rational expression with the numerator and denominator interchanged. [p. 434]

A *complex fraction* is a fraction whose numerator or denominator contains one or more fractions. [p. 451]

A *ratio* is the quotient of two quantities that have the same unit. [p. 457]

A *rate* is the quotient of two quantities that have different units. [p. 457]

A *proportion* is an equation that states the equality of two ratios or rates. [p. 457]

A *literal equation* is an equation that contains more than one variable. [p. 467]

Direct variation is a special function that can be expressed as the equation $y = kx$, where k is a constant called the *constant of variation* or the *constant of proportionality*. [p. 479]

Inverse variation is a function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. [p. 480]

Joint variation is a variation in which a variable varies directly as the product of two or more variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. [p. 480]

Combined variation is a variation in which two or more types of variation occur at the same time. [p. 480]

Essential Rules

To Multiply Fractions [p. 432]

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To Divide Fractions [p. 434]

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

To Add Fractions [p. 441]

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To Subtract Fractions [p. 441]

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Equation for Work Problems [p. 471]

$$\text{Rate of work} \times \text{time worked} = \text{part of task completed}$$

Uniform Motion Equation [p. 473]

$$\text{Distance} = \text{rate} \times \text{time}$$

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Chapter Review

1. Divide: $\dfrac{6a^2b^7}{25x^3y} \div \dfrac{12a^3b^4}{5x^2y^2}$

2. Add: $\dfrac{x+7}{15x} + \dfrac{x-2}{20x}$

3. Simplify: $\dfrac{x - \dfrac{16}{5x-2}}{3x - 4 - \dfrac{88}{5x-2}}$

4. Simplify: $\dfrac{x^2 + x - 30}{15 + 2x - x^2}$

5. Simplify: $\dfrac{16x^5y^3}{24xy^{10}}$

6. Solve: $\dfrac{20}{x+2} = \dfrac{5}{16}$

7. Divide: $\dfrac{10 - 23y + 12y^2}{6y^2 - y - 5} \div \dfrac{4y^2 - 13y + 10}{18y^2 + 3y - 10}$

8. Multiply: $\dfrac{8ab^2}{15x^3y} \cdot \dfrac{5xy^4}{16a^2b}$

9. Simplify: $\dfrac{1 - \dfrac{1}{x}}{1 - \dfrac{8x-7}{x^2}}$

10. Write each fraction in terms of the LCM of the denominators.

$$\frac{x}{12x^2 + 16x - 3}, \frac{4x^2}{6x^2 + 7x - 3}$$

11. Solve $T = 2(ab + bc + ca)$ for a.

12. Solve: $\dfrac{5}{7} + \dfrac{x}{2} = 2 - \dfrac{x}{7}$

13. Solve $i = \dfrac{100m}{c}$ for c.

14. Solve: $\dfrac{x+8}{x+4} = 1 + \dfrac{5}{x+4}$

15. Divide: $\dfrac{20x^2 - 45x}{6x^3 + 4x^2} \div \dfrac{40x^3 - 90x^2}{12x^2 + 8x}$

16. Add: $\dfrac{2y}{5y-7} + \dfrac{3}{7-5y}$

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17. Subtract: $\dfrac{5x+3}{2x^2+5x-3} - \dfrac{3x+4}{2x^2+5x-3}$

18. Find the LCM of $10x^2 - 11x + 3$ and $20x^2 - 17x + 3$.

19. Solve $4x + 9y = 18$ for y.

20. Multiply: $\dfrac{24x^2-94x+15}{12x^2-49x+15} \cdot \dfrac{24x^2+7x-5}{4-27x+18x^2}$

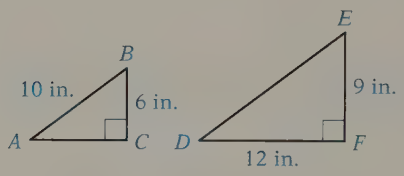
21. Solve: $\dfrac{20}{2x+3} = \dfrac{17x}{2x+3} - 5$

22. Add: $\dfrac{x-1}{x+2} + \dfrac{3x-2}{5-x} + \dfrac{5x^2+15x-11}{x^2-3x-10}$

23. Solve: $\dfrac{6}{x-7} = \dfrac{8}{x-6}$

24. Solve: $\dfrac{3}{20} = \dfrac{x}{80}$

25. Triangles *ABC* and *DEF* are similar. Find the perimeter of triangle *ABC*.



26. One hose can fill a pool in 15 h. The second hose can fill the pool in 10 h. How long would it take to fill the pool using both hoses?

27. A car travels 315 mi in the same amount of time that a bus travels 245 mi. The rate of the car is 10 mph greater than that of the bus. Find the rate of the car.

28. The rate of a jet is 400 mph in calm air. Traveling with the wind, the jet can fly 2100 mi in the same amount of time it takes to fly 1900 mi against the wind. Find the rate of the wind.

29. A pitcher's earned run average (ERA) is the average number of runs allowed in 9 innings of pitching. If a pitcher allows 15 runs in 100 innings, find the pitcher's ERA.

30. The current (I) in an electric circuit varies inversely as the resistance (R). If the current in the circuit is 4 amps when the resistance is 50 ohms, find the current in the circuit when the resistance is 100 ohms.

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Chapter Test

1. Simplify: $\dfrac{16x^5y}{24x^2y^4}$

2. Simplify: $\dfrac{x^2+4x-5}{1-x^2}$

3. Multiply: $\dfrac{x^3y^4}{x^2-4x+4}\cdot\dfrac{x^2-x-2}{x^6y^4}$

4. Multiply: $\dfrac{x^2+2x-3}{x^2+6x+9}\cdot\dfrac{2x^2-11x+5}{2x^2+3x-5}$

5. Divide: $\dfrac{x^2+3x+2}{x^2+5x+4}\div\dfrac{x^2-x-6}{x^2+2x-15}$

6. Find the LCM of $6x-3$ and $2x^2+x-1$.

7. Write each fraction in terms of the LCM of the denominators.

$$\frac{3}{x^2-2x},\ \frac{x}{x^2-4}$$

8. Subtract: $\dfrac{2x}{x^2+3x-10}-\dfrac{4}{x^2+3x-10}$

9. Subtract: $\dfrac{2}{2x-1}-\dfrac{3}{3x+1}$

10. Subtract: $\dfrac{x}{x+3}-\dfrac{2x-5}{x^2+x-6}$

11. Simplify: $\dfrac{1+\dfrac{1}{x}-\dfrac{12}{x^2}}{1+\dfrac{2}{x}-\dfrac{8}{x^2}}$

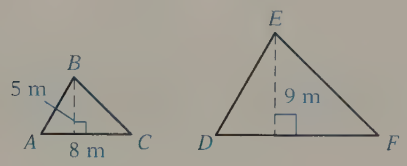
12. Solve: $\dfrac{6}{x}-2=1$

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13. Solve: $\frac{2x}{x+1} - 3 = \frac{-2}{x+1}$

14. Solve: $\frac{3}{x+4} = \frac{5}{x+6}$

15. Triangles *ABC* and *DEF* are similar. Find the area of triangle *DEF*.



16. Solve $d = s + rt$ for t.

17. An interior designer uses 2 rolls of wallpaper for every 45 ft^2 of wall space in an office. At this rate, how many rolls of wallpaper are needed for an office that has 315 ft^2 of wall space?

18. One landscaper can till the soil for a lawn in 30 min, whereas it takes a second landscaper 15 min to do the same job. How long would it take to till the soil for the lawn with both landscapers working together?

19. A cyclist travels 20 mi in the same amount of time as it takes a hiker to walk 6 mi. The rate of the cyclist is 7 mph faster than the rate of the hiker. Find the rate of the cyclist.

20. The electrical resistance (r) of a cable varies directly as its length (l) and inversely as the square of its diameter (d). If a cable 16,000 ft long and $\frac{1}{4}$ in. in diameter has a resistance of 3.2 ohms, what is the resistance of a cable that is 8000 ft long and $\frac{1}{2}$ in. in diameter?

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Cumulative Review

1. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{3}{2} - \frac{2}{3}\right) + \frac{1}{2}$

2. Evaluate $-a^2 + (a - b)^2$ when $a = -2$ and $b = 3$.

3. Simplify: $-2x - (-3y) + 7x - 5y$

4. Simplify: $2[3x - 7(x - 3) - 8]$

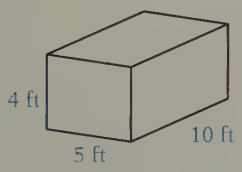
5. Solve: $4 - \frac{2}{3}x = 7$

6. Solve: $3[x - 2(x - 3)] = 2(3 - 2x)$

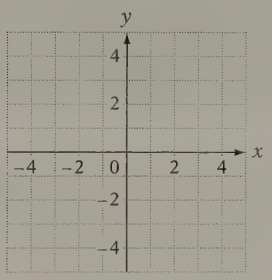
7. Find $16\frac{2}{3}\%$ of 60.

8. Solve: $x - 3(1 - 2x) \geq 1 - 4(3 - 2x)$

9. Find the volume of the rectangular solid shown in the figure.



10. Graph: $x - 2y = 2$



11. Given $P(x) = \frac{x - 1}{2x - 3}$, find $P(-2)$.

12. Find the equation of the line that contains the point $(-2, -1)$ and is parallel to the line $3x - 2y = 6$.

13. Evaluate the determinant:

$$\begin{vmatrix} 6 & 5 \\ 2 & -3 \end{vmatrix}$$

14. Multiply: $(a^2b^5)(ab^2)$

15. Simplify: $\frac{(2a^{-2}b^3)^{-2}}{(4a)^{-1}}$

16. Write 0.000000035 in scientific notation.

17. Multiply: $(2a^2 - 3a + 1)(-2a^2)$

18. Multiply: $(a - 3b)(a + 4b)$

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19. Divide: $(x^3 - 8) \div (x - 2)$

20. Factor: $y^2 - 7y + 6$

21. Factor: $12x^2 - x - 1$

22. Factor: $2a^3 + 7a^2 - 15a$

23. Factor: $4b^2 - 100$

24. Solve: $(x + 3)(2x - 5) = 0$

25. Simplify: $\dfrac{12x^4y^2}{18xy^7}$

26. Simplify: $\dfrac{x^2 - 7x + 10}{25 - x^2}$

27. Divide: $\dfrac{x^2 - x - 56}{x^2 + 8x + 7} \div \dfrac{x^2 - 13x + 40}{x^2 - 4x - 5}$

28. Subtract: $\dfrac{2}{2x - 1} - \dfrac{1}{x + 1}$

29. Simplify: $\dfrac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{25}{x^2}}$

30. Solve: $\dfrac{3x}{x - 3} - 2 = \dfrac{10}{x - 3}$

31. A silversmith mixes 60 g of an alloy that is 40% silver with 120 g of another silver alloy. The resulting alloy is 60% silver. Find the percent of silver in the 120 g of alloy.

32. A life insurance policy costs \$32 for every \$1000 of coverage. At this rate, how much money would a policy of \$5000 cost?

33. One water pipe can fill a tank in 9 min, whereas a second pipe requires 18 min to fill the tank. How long would it take both pipes, working together, to fill the tank?

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Chapter 9

Rational Exponents and Radicals



These floating icebergs stand out dramatically against the horizon. A submarine periscope would magnify and bring an object like this one clearly into focus. The periscope could also be used to determine the distance from the submarine to the iceberg. A simple formula involving a radical expression is used to calculate this distance, given the height of the periscope above the surface of the water. The **Project on page 534** shows how to use a radical equation to calculate the distance.

Objectives

Section 9.1

A To simplify expressions with rational exponents
B To write exponential expressions as radical expressions and to write radical expressions as exponential expressions
C To simplify radical expressions that are roots of perfect powers

Section 9.2

A To simplify radical expressions
B To add or subtract radical expressions
C To multiply radical expressions
D To divide radical expressions

Section 9.3

A To simplify a complex number
B To add or subtract complex numbers
C To multiply complex numbers
D To divide complex numbers

Section 9.4

A To solve a radical equation
B To solve application problems

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Complete: $48 = ? \cdot 3$

For Exercises 2 to 7, simplify.

2. 2^5

3. $6\left(\frac{3}{2}\right)$

4. $\frac{1}{2} - \frac{2}{3} + \frac{1}{4}$

5. $(3 - 7x) - (4 - 2x)$

6. $\frac{3x^5y^6}{12x^4y}$

7. $(3x - 2)^2$

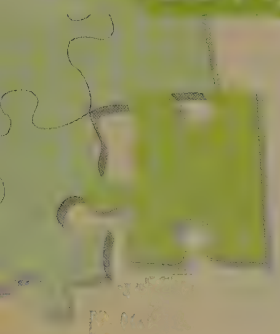
For Exercises 8 and 9, multiply.

8. $(2 + 4x)(5 - 3x)$

9. $(6x - 1)(6x + 1)$

10. Solve: $x^2 - 14x - 15 = 0$

Go Figure



You are planning a large dinner party. If you seat 5 people at each table, you end up with only 2 people at the last table. If you seat 3 people at each table, you have 9 people left over with no place to sit. There are fewer than 10 tables. How many guests are coming to the dinner party?

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9.1 Rational Exponents and Radical Expressions

Objective A

To simplify expressions with rational exponents



Point of Interest

Nicolas Chuquet (c. 1475), a French physician, wrote an algebra text in which he used a notation for expressions with fractional exponents. He wrote $R^2 6$ to mean $6^{1/2}$ and $R^3 15$ to mean $15^{1/3}$. This was an improvement over earlier notations that used words for these expressions.

In this section, the definition of an exponent is extended beyond integers so that any rational number can be used as an exponent. The definition is expressed in such a way that the Rules of Exponents hold true for rational exponents.

Consider the expression $(a^{1/n})^n$ for $a > 0$ and n a positive integer. Now simplify, assuming that the Rule for Simplifying Powers of Exponential Expressions is true.

$$(a^{1/n})^n = a^{\frac{1}{n}\cdot n} = a^1 = a$$

Because $(a^{1/n})^n = a$, the number $a^{1/n}$ is the number whose nth power is a.

If $a > 0$ and n is a positive number, then $a^{1/n}$ is called the nth root of a.

$25^{1/2} = 5$ because $(5)^2 = 25$. $\quad 8^{1/3} = 2$ because $(2)^3 = 8$.

In the expression $a^{1/n}$, if a is a negative number and n is a positive *even* integer, then $a^{1/n}$ is not a real number.

$(-4)^{1/2}$ is not a real number, because there is no real number whose second power is -4.

When n is a positive *odd* integer, a can be a positive or a negative number.

$(-27)^{1/3} = -3$ because $(-3)^3 = -27$.

Using the definition of $a^{1/n}$ and the Rules of Exponents, it is possible to define any exponential expression that contains a rational exponent.

Rule for Rational Exponents

If m and n are positive integers and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m$$

The expression $a^{m/n}$ can also be written $a^{m/n} = a^{m\cdot\frac{1}{n}} = (a^m)^{1/n}$. However, rewriting $a^{m/n}$ as $(a^m)^{1/n}$ is not so useful as rewriting it as $(a^{1/n})^m$. See the Take Note at the top of the next page.

As shown above, expressions that contain rational exponents do not always represent real numbers when the base of the exponential expression is a negative number. For this reason, **all variables in this chapter represent positive numbers unless otherwise stated.**

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TAKE NOTE

Although we can simplify an expression by rewriting $a^{m/n}$ in the form $(a^m)^{1/n}$, it is usually easier to simplify the form $(a^{1/n})^m$. For instance, simplifying $(27^{1/3})^2$ is easier than simplifying $(27^2)^{1/3}$.

➡ Simplify: $27^{2/3}$

$$27^{2/3} = (3^3)^{2/3}$$ • Rewrite 27 as 3^3.

$$= 3^{3(2/3)}$$ • Multiply the exponents.

$$= 3^2$$ • Simplify.

$$= 9$$

TAKE NOTE

Note that $32^{-2/5} = \frac{1}{4}$, a positive number. The negative exponent does not affect the sign of a number.

➡ Simplify: $32^{-2/5}$

$$32^{-2/5} = (2^5)^{-2/5}$$ • Rewrite 32 as 2^5.

$$= 2^{-2}$$ • Multiply the exponents.

$$= \frac{1}{2^2}$$ • Use the Rule of Negative Exponents.

$$= \frac{1}{4}$$ • Simplify.

➡ Simplify: $a^{1/2} \cdot a^{2/3} \cdot a^{-1/4}$

$$a^{1/2} \cdot a^{2/3} \cdot a^{-1/4} = a^{1/2+2/3-1/4}$$ • Use the Rule for Multiplying Exponential Expressions.

$$= a^{6/12+8/12-3/12}$$

$$= a^{11/12}$$ • Simplify.

➡ Simplify: $(x^6y^4)^{3/2}$

$$(x^6y^4)^{3/2} = x^{6(3/2)}y^{4(3/2)}$$ • Use the Rule for Simplifying Powers of Products.

$$= x^9y^6$$ • Simplify.

➡ Simplify: $\left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3}$

$$\left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3} = \left(\frac{2^3a^3b^{-4}}{2^6a^{-9}b^2}\right)^{2/3}$$ • Rewrite 8 as 2^3 and 64 as 2^6.

$$= (2^{-3}a^{12}b^{-6})^{2/3}$$ • Use the Rule for Dividing Exponential Expressions.

$$= 2^{-2}a^8b^{-4}$$ • Use the Rule for Simplifying Powers of Products.

$$= \frac{a^8}{2^2b^4} = \frac{a^8}{4b^4}$$ • Use the Rule of Negative Exponents and simplify.

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Example 1 Simplify: $64^{-2/3}$

Solution
$$64^{-2/3} = (2^6)^{-2/3} = 2^{-4}$$
$$= \frac{1}{2^4} = \frac{1}{16}$$

You Try It 1 Simplify: $16^{-3/4}$

Your solution

Example 2 Simplify: $(-49)^{3/2}$

Solution The base of the exponential expression is a negative number, while the denominator of the exponent is a positive even number.

Therefore, $(-49)^{3/2}$ is not a real number.

You Try It 2 Simplify: $(-81)^{3/4}$

Your solution

Example 3 Simplify: $(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$

Solution
$$(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$$
$$= x^{-3/4}y^{9/4}z^{-3/8}$$
$$= \frac{y^{9/4}}{x^{3/4}z^{3/8}}$$

You Try It 3 Simplify: $(x^{3/4}y^{1/2}z^{-2/3})^{-4/3}$

Your solution

Example 4

Simplify: $\dfrac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$

Solution
$$\frac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$$
$$= x^{3/6 - (-8/6)}y^{-15/12 - 4/12}$$
$$= x^{11/6}y^{-19/12} = \frac{x^{11/6}}{y^{19/12}}$$

You Try It 4

Simplify: $\left(\dfrac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2}$

Your solution

Solutions on p. S27

Objective B

To write exponential expressions as radical expressions and to write radical expressions as exponential expressions

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Point of Interest

The radical sign was introduced in 1525 in a book by Christoff Rudolff called *Coss*. He modified the symbol to indicate square roots, cube roots, and fourth roots. The idea of using an index, as we do in our modern notation, did not occur until some years later.

Recall that $a^{1/n}$ is the nth root of a. The expression $\sqrt[n]{a}$ is another symbol for the nth root of a.

If a is a real number, then $a^{1/n} = \sqrt[n]{a}$.

In the expression $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called a **radical sign**, n is the **index** of the radical, and a is the **radicand**. When $n = 2$, the radical expression represents a square root and the index 2 is usually not written.

An exponential expression with a rational exponent can be written as a radical expression.

Rule for Writing Exponential Expressions as Radical Expressions

If $a^{1/n}$ is a real number, then $a^{m/n} = a^{m \cdot 1/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

The expression $a^{m/n}$ can also be written $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$.

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The exponential expression at the right has been written as a radical expression.

$$y^{2/3} = (y^2)^{1/3} = \sqrt[3]{y^2}$$

The radical expressions at the right have been written as exponential expressions.

$$\sqrt[5]{x^6} = (x^6)^{1/5} = x^{6/5}$$
$$\sqrt{17} = (17)^{1/2} = 17^{1/2}$$

➡ Write $(5x)^{2/5}$ as a radical expression.

$(5x)^{2/5} = \sqrt[5]{(5x)^2}$ • The denominator of the rational exponent is the index of the radical. The numerator is the power of the radicand.

$= \sqrt[5]{25x^2}$ • Simplify.

➡ Write $\sqrt[3]{x^4}$ as an exponential expression with a rational exponent.

$\sqrt[3]{x^4} = (x^4)^{1/3}$ • The index of the radical is the denominator of the rational exponent. The power of the radicand is the numerator of the rational exponent.

$= x^{4/3}$ • Simplify.

➡ Write $\sqrt[3]{a^3 + b^3}$ as an exponential expression with a rational exponent.

$\sqrt[3]{a^3 + b^3} = (a^3 + b^3)^{1/3}$

Note that $(a^3 + b^3)^{1/3} \neq a + b$.

Example 5 Write $(3x)^{5/4}$ as a radical expression.

Solution $(3x)^{5/4} = \sqrt[4]{(3x)^5} = \sqrt[4]{243x^5}$

You Try It 5 Write $(2x^3)^{3/4}$ as a radical expression.

Your solution

Example 6 Write $-2x^{2/3}$ as a radical expression.

Solution $-2x^{2/3} = -2(x^2)^{1/3} = -2\sqrt[3]{x^2}$

You Try It 6 Write $-5a^{5/6}$ as a radical expression.

Your solution

Example 7 Write $\sqrt[4]{3a}$ as an exponential expression.

Solution $\sqrt[4]{3a} = (3a)^{1/4}$

You Try It 7 Write $\sqrt[3]{3ab}$ as an exponential expression.

Your solution

Example 8 Write $\sqrt{a^2 - b^2}$ as an exponential expression.

Solution $\sqrt{a^2 - b^2} = (a^2 - b^2)^{1/2}$

You Try It 8 Write $\sqrt[4]{x^4 + y^4}$ as an exponential expression.

Your solution

Solutions on pp. S27–S28

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Objective C

To simplify radical expressions that are roots of perfect powers



Every positive number has two square roots, one a positive number and one a negative number. For example, because $(5)^2 = 25$ and $(-5)^2 = 25$, there are two square roots of 25: 5 and -5.

The symbol $\sqrt{\ }$ is used to indicate the positive or **principal square root**. To indicate the negative square root of a number, a negative sign is placed in front of the radical.

$\sqrt{25} = 5$

$-\sqrt{25} = -5$

The square root of zero is zero.

$\sqrt{0} = 0$

The square root of a negative number is not a real number, because the square of a real number must be positive.

$\sqrt{-25}$ is not a real number.

Note that

$$\sqrt{(-5)^2} = \sqrt{25} = 5 \text{ and } \sqrt{5^2} = \sqrt{25} = 5$$

This is true for all real numbers and is stated as the following result.

For any real number a, $\sqrt{a^2} = |a|$ and $-\sqrt{a^2} = -|a|$. If a is a positive real number, then $\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$.

Besides square roots, we can also determine cube roots, fourth roots, and so on.

$\sqrt[3]{8} = 2$, because $2^3 = 8$. • The cube root of a positive number is positive.

$\sqrt[3]{-8} = -2$, because $(-2)^3 = -8$. • The cube root of a negative number is negative.

$\sqrt[4]{625} = 5$, because $5^4 = 625$.

$\sqrt[5]{243} = 3$, because $3^5 = 243$.

The following properties hold true for finding the nth root of a real number.

If n is an even integer, then $\sqrt[n]{a^n} = |a|$ and $-\sqrt[n]{a^n} = -|a|$. If n is an odd integer, then $\sqrt[n]{a^n} = a$.

For example,

$$\sqrt[6]{y^6} = |y| \qquad -\sqrt[12]{x^{12}} = -|x| \qquad \sqrt[5]{b^5} = b$$

For the remainder of this chapter, we will assume that variable expressions inside a radical represent positive numbers. Therefore, it is not necessary to use the absolute value signs.

TAKE NOTE

Note that when the index is an even natural number, the nth root requires absolute value symbols.

$\sqrt[6]{y^6} = |y|$ but $\sqrt[5]{y^5} = y$

Because we stated that variables within radicals represent *positive* numbers, we will omit the absolute value symbols when writing an answer.

➡ Simplify: $\sqrt[4]{x^4y^8}$

$\sqrt[4]{x^4y^8} = (x^4y^8)^{1/4}$ • The radicand is a perfect fourth power because the exponents on the variables are divisible by 4. Write the radical expression as an exponential expression.

$= xy^2$ • Use the Rule for Simplifying Powers of Products.

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➡ Simplify: $\sqrt[3]{125c^9d^6}$

$\sqrt[3]{125c^9d^6} = (5^3c^9d^6)^{1/3}$ • The radicand is a perfect cube because 125 is a perfect cube ($125 = 5^3$) and all the exponents on the variables are divisible by 3.

$= 5c^3d^2$ • Use the Rule for Simplifying Powers of Products.

Note that **a variable expression is a perfect power if the exponents on the factors are evenly divisible by the index of the radical.**

The chart below shows roots of perfect powers. Knowledge of these roots is very helpful when simplifying radical expressions.

Square Roots		*Cube Roots*	*Fourth Roots*	*Fifth Roots*
$\sqrt{1} = 1$	$\sqrt{36} = 6$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$	$\sqrt[5]{1} = 1$
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$	$\sqrt[5]{32} = 2$
$\sqrt{9} = 3$	$\sqrt{64} = 8$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$	$\sqrt[5]{243} = 3$
$\sqrt{16} = 4$	$\sqrt{81} = 9$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$	
$\sqrt{25} = 5$	$\sqrt{100} = 10$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$	

TAKE NOTE
From the chart, $\sqrt[5]{243} = 3$, which means that $3^5 = 243$. From this we know that $(-3)^5 = -243$, which means $\sqrt[5]{-243} = -3$.

➡ Simplify: $\sqrt[5]{-243x^5y^{15}}$

$\sqrt[5]{-243x^5y^{15}} = -3xy^3$ • From the chart, 243 is a perfect fifth power, and each exponent is divisible by 5. Therefore, the radicand is a perfect fifth power.

Example 9 Simplify: $\sqrt[3]{-125a^6b^9}$

Solution The radicand is a perfect cube.

$\sqrt[3]{-125a^6b^9} = -5a^2b^3$ • Divide each exponent by 3.

You Try It 9 Simplify: $\sqrt[3]{-8x^{12}y^3}$

Your solution

Example 10 Simplify: $-\sqrt[4]{16a^4b^8}$

Solution The radicand is a perfect fourth power.

$-\sqrt[4]{16a^4b^8} = -2ab^2$ • Divide each exponent by 4.

You Try It 10 Simplify: $-\sqrt[4]{81x^{12}y^8}$

Your solution

Solutions on p. S28

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9.1 Exercises

Objective A

Simplify.

1. $8^{1/3}$
2. $16^{1/2}$
3. $9^{3/2}$
4. $25^{3/2}$
5. $27^{-2/3}$
6. $64^{-1/3}$
7. $32^{2/5}$
8. $16^{3/4}$
9. $(-25)^{5/2}$
10. $(-36)^{1/4}$
11. $\left(\frac{25}{49}\right)^{-3/2}$
12. $\left(\frac{8}{27}\right)^{-2/3}$
13. $x^{1/2}x^{1/2}$
14. $a^{1/3}a^{5/3}$
15. $y^{-1/4}y^{3/4}$
16. $x^{2/5} \cdot x^{-4/5}$
17. $x^{-2/3} \cdot x^{3/4}$
18. $x \cdot x^{-1/2}$
19. $a^{1/3} \cdot a^{3/4} \cdot a^{-1/2}$
20. $y^{-1/6} \cdot y^{2/3} \cdot y^{1/2}$
21. $\frac{a^{1/2}}{a^{3/2}}$
22. $\frac{b^{1/3}}{b^{4/3}}$
23. $\frac{y^{-3/4}}{y^{1/4}}$
24. $\frac{x^{-3/5}}{x^{1/5}}$
25. $\frac{y^{2/3}}{y^{-5/6}}$
26. $\frac{b^{3/4}}{b^{-3/2}}$
27. $(x^2)^{-1/2}$
28. $(a^8)^{-3/4}$
29. $(x^{-2/3})^6$
30. $(y^{-5/6})^{12}$
31. $(a^{-1/2})^{-2}$
32. $(b^{-2/3})^{-6}$
33. $(x^{-3/8})^{-4/5}$
34. $(y^{-3/2})^{-2/9}$
35. $(a^{1/2} \cdot a)^2$
36. $(b^{2/3} \cdot b^{1/6})^6$
37. $(x^{-1/2} \cdot x^{3/4})^{-2}$
38. $(a^{1/2} \cdot a^{-2})^3$
39. $(y^{-1/2} \cdot y^{2/3})^{2/3}$
40. $(b^{-2/3} \cdot b^{1/4})^{-4/3}$
41. $(x^8y^2)^{1/2}$
42. $(a^3b^9)^{2/3}$

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43. $(x^4y^2z^6)^{3/2}$

44. $(a^8b^4c^4)^{3/4}$

45. $(x^{-3}y^6)^{-1/3}$

46. $(a^2b^{-6})^{-1/2}$

47. $(x^{-2}y^{1/3})^{-3/4}$

48. $(a^{-2/3}b^{2/3})^{3/2}$

49. $\left(\dfrac{x^{1/2}}{y^2}\right)^4$

50. $\left(\dfrac{b^{-3/4}}{a^{-1/2}}\right)^8$

51. $\dfrac{x^{1/4}\cdot x^{-1/2}}{x^{2/3}}$

52. $\dfrac{b^{1/2}\cdot b^{-3/4}}{b^{1/4}}$

53. $\left(\dfrac{y^{2/3}\cdot y^{-5/6}}{y^{1/9}}\right)^9$

54. $\left(\dfrac{a^{1/3}\cdot a^{-2/3}}{a^{1/2}}\right)^4$

55. $\left(\dfrac{b^2\cdot b^{-3/4}}{b^{-1/2}}\right)^{-1/2}$

56. $\dfrac{(x^{-5/6}\cdot x^3)^{-2/3}}{x^{4/3}}$

57. $(a^{2/3}b^2)^6(a^3b^3)^{1/3}$

58. $(x^3y^{-1/2})^{-2}(x^{-3}y^2)^{1/6}$

59. $(16m^{-2}n^4)^{-1/2}(mn^{1/2})$

60. $(27m^3n^{-6})^{1/3}(m^{-1/3}n^{5/6})^6$

61. $\left(\dfrac{x^{1/2}y^{-3/4}}{y^{2/3}}\right)^{-6}$

62. $\left(\dfrac{x^{1/2}y^{-5/4}}{y^{-3/4}}\right)^{-4}$

63. $\left(\dfrac{2^{-6}b^{-3}}{a^{-1/2}}\right)^{-2/3}$

64. $\left(\dfrac{49c^{5/3}}{a^{-1/4}b^{5/6}}\right)^{-3/2}$

65. $y^{3/2}(y^{1/2}-y^{-1/2})$

66. $y^{3/5}(y^{2/5}+y^{-3/5})$

67. $a^{-1/4}(a^{5/4}-a^{9/4})$

68. $x^{4/3}(x^{2/3}+x^{-1/3})$

69. $x^n\cdot x^{3n}$

70. $a^{2n}\cdot a^{-5n}$

71. $x^n\cdot x^{n/2}$

72. $a^{n/2}\cdot a^{-n/3}$

73. $\dfrac{y^{n/2}}{y^{-n}}$

74. $\dfrac{b^{m/3}}{b^m}$

75. $(x^{2n})^n$

76. $(x^{5n})^{2n}$

77. $(x^{n/4}y^{n/8})^8$

78. $(x^{n/2}y^{n/3})^6$

79. $(x^{n/5}y^{n/10})^{20}$

80. $(x^{n/2}y^{n/5})^{10}$

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Objective B

Rewrite the exponential expression as a radical expression.

81. $3^{1/4}$ **82.** $5^{1/2}$ **83.** $a^{3/2}$ **84.** $b^{4/3}$

85. $(2t)^{5/2}$ **86.** $(3x)^{2/3}$ **87.** $-2x^{2/3}$ **88.** $-3a^{2/5}$

89. $(a^2b)^{2/3}$ **90.** $(x^2y^3)^{3/4}$ **91.** $(a^2b^4)^{3/5}$ **92.** $(a^3b^7)^{3/2}$

93. $(4x - 3)^{3/4}$ **94.** $(3x - 2)^{1/3}$ **95.** $x^{-2/3}$ **96.** $b^{-3/4}$

Rewrite the radical expression as an exponential expression.

97. $\sqrt{14}$ **98.** $\sqrt{7}$ **99.** $\sqrt[3]{x}$ **100.** $\sqrt[4]{x}$

101. $\sqrt[3]{x^4}$ **102.** $\sqrt[4]{a^3}$ **103.** $\sqrt[5]{b^3}$ **104.** $\sqrt[4]{b^5}$

105. $\sqrt[3]{2x^2}$ **106.** $\sqrt[5]{4y^7}$ **107.** $-\sqrt{3x^5}$ **108.** $-\sqrt[4]{4x^5}$

109. $3x\sqrt[3]{y^2}$ **110.** $2y\sqrt{x^3}$ **111.** $\sqrt{a^2 - 2}$ **112.** $\sqrt{3 - y^2}$

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Objective C

Simplify.

113. $\sqrt{x^{16}}$ **114.** $\sqrt{y^{14}}$ **115.** $-\sqrt{x^8}$ **116.** $-\sqrt{a^6}$

117. $\sqrt[3]{x^3y^9}$ **118.** $\sqrt[3]{a^6b^{12}}$ **119.** $-\sqrt[3]{x^{15}y^3}$ **120.** $-\sqrt[3]{a^9b^9}$

121. $\sqrt{16a^4b^{12}}$ **122.** $\sqrt{25x^8y^2}$ **123.** $\sqrt{-16x^4y^2}$ **124.** $\sqrt{-9a^4b^8}$

125. $\sqrt[3]{27x^9}$ **126.** $\sqrt[3]{8a^{21}b^6}$ **127.** $\sqrt[3]{-64x^9y^{12}}$ **128.** $\sqrt[3]{-27a^3b^{15}}$

129. $-\sqrt[4]{x^8y^{12}}$ **130.** $-\sqrt[4]{a^{16}b^4}$ **131.** $\sqrt[5]{x^{20}y^{10}}$ **132.** $\sqrt[5]{a^5b^{25}}$

133. $\sqrt[4]{81x^4y^{20}}$ **134.** $\sqrt[4]{16a^8b^{20}}$ **135.** $\sqrt[5]{32a^5b^{10}}$ **136.** $\sqrt[5]{-32x^{15}y^{20}}$

APPLYING THE CONCEPTS

137. Determine whether the following statements are true or false. If the statement is false, correct the right-hand side of the equation.

a. $\sqrt{(-2)^2} = -2$ **b.** $\sqrt[3]{(-3)^3} = -3$ **c.** $\sqrt[n]{a} = a^{1/n}$

d. $\sqrt[n]{a^n + b^n} = a + b$ **e.** $(a^{1/2} + b^{1/2})^2 = a + b$ **f.** $\sqrt[m]{a^n} = a^{mn}$

138. Simplify.

a. $\sqrt[3]{\sqrt{x^6}}$ **b.** $\sqrt[4]{\sqrt{a^8}}$ **c.** $\sqrt{\sqrt{81y^8}}$

d. $\sqrt{\sqrt[n]{a^{4n}}}$ **e.** $\sqrt[n]{\sqrt{b^{6n}}}$ **f.** $\sqrt{\sqrt[3]{x^{12}y^{24}}}$

139. If x is any real number, is $\sqrt{x^2} = x$ always true? Show why or why not.

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9.2 Operations on Radical Expressions

To simplify radical expressions

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Point of Interest

The Latin expression for irrational numbers was *numerus surdus*, which literally means "inaudible number." A prominent 16th-century mathematician wrote of irrational numbers, "Just as an infinite number is not a number, so an irrational number is not a true number, but lies hidden in some sort of cloud of infinity." In 1872, Richard Dedekind wrote a paper that established the first logical treatment of irrational numbers.

If a number is not a perfect power, its root can only be approximated; examples include $\sqrt{5}$ and $\sqrt[3]{3}$. These numbers are **irrational numbers**. Their decimal representations never terminate or repeat.

$$\sqrt{5} = 2.2360679\ldots \qquad \sqrt[3]{3} = 1.4422495\ldots$$

A radical expression is not in simplest form when the radicand contains a factor greater than 1 that is a perfect power. The Product Property of Radicals is used to simplify radical expressions whose radicands are not perfect powers.

The Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

➡ Simplify: $\sqrt{48}$

$\sqrt{48} = \sqrt{16 \cdot 3}$ • Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{16}\sqrt{3}$ • Use the Product Property of Radicals to write the expression as a product.

$= 4\sqrt{3}$ • Simplify $\sqrt{16}$.

Note that 48 must be written as the product of a perfect square and *a factor that does not contain a perfect square.* Therefore, it would not be correct to rewrite $\sqrt{48}$ as $\sqrt{4 \cdot 12}$ and simplify the expression as shown at the right. Although 4 is a perfect-square factor of 48, 12 contains a perfect square ($12 = 4 \cdot 3$) and $\sqrt{12}$ can be simplified. Remember to find the largest perfect power that is a factor of the radicand.

$$\sqrt{48} = \sqrt{4 \cdot 12}$$
$$= \sqrt{4}\sqrt{12}$$
$$= 2\sqrt{12}$$

Not in simplest form

➡ Simplify: $\sqrt{18x^2y^3}$

$\sqrt{18x^2y^3} = \sqrt{9x^2y^2 \cdot 2y}$ • Write the radicand as the product of a perfect square and factors that do not contain a perfect square.

$= \sqrt{9x^2y^2}\sqrt{2y}$ • Use the Product Property of Radicals to write the expression as a product.

$= 3xy\sqrt{2y}$ • Simplify.

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➡ Simplify: $\sqrt[3]{x^7}$

$\sqrt[3]{x^7} = \sqrt[3]{x^6 \cdot x}$ • Write the radicand as the product of a perfect cube and a factor that does not contain a perfect cube.

$= \sqrt[3]{x^6}\sqrt[3]{x}$ • Use the Product Property of Radicals to write the expression as a product.

$= x^2\sqrt[3]{x}$ • Simplify.

➡ Simplify: $\sqrt[4]{32x^7}$

$\sqrt[4]{32x^7} = \sqrt[4]{2^5x^7}$ • Write the prime factorization of the coefficient of the radicand in exponential form.

$= \sqrt[4]{2^4x^4(2x^3)}$ • Write the radicand as the product of a perfect fourth power and factors that do not contain a perfect fourth power.

$= \sqrt[4]{2^4x^4}\sqrt[4]{2x^3}$ • Use the Product Property of Radicals to write the expression as a product.

$= 2x\sqrt[4]{2x^3}$ • Simplify.

Example 1 Simplify: $\sqrt[4]{x^9}$

Solution $\sqrt[4]{x^9} = \sqrt[4]{x^8 \cdot x} = \sqrt[4]{x^8}\sqrt[4]{x}$
$= x^2\sqrt[4]{x}$

You Try It 1 Simplify: $\sqrt[5]{x^7}$

Your solution

Example 2 Simplify: $\sqrt[3]{-27a^5b^{12}}$

Solution $\sqrt[3]{-27a^5b^{12}} = \sqrt[3]{(-3)^3a^5b^{12}}$
$= \sqrt[3]{(-3)^3a^3b^{12}(a^2)}$
$= \sqrt[3]{(-3)^3a^3b^{12}}\sqrt[3]{a^2}$
$= -3ab^4\sqrt[3]{a^2}$

You Try It 2 Simplify: $\sqrt[3]{-64x^8y^{18}}$

Your solution

Solutions on p. S28

Objective B To add or subtract radical expressions

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The Distributive Property is used to simplify the sum or difference of radical expressions that have the same radicand and the same index. For example,

$$3\sqrt{5} + 8\sqrt{5} = (3 + 8)\sqrt{5} = 11\sqrt{5}$$

$$2\sqrt[3]{3x} - 9\sqrt[3]{3x} = (2 - 9)\sqrt[3]{3x} = -7\sqrt[3]{3x}$$

TAKE NOTE
Adding and subtracting radicals is similar to combining like terms.

Radical expressions that are in simplest form and have unlike radicands or different indices cannot be simplified by the Distributive Property. The expressions below cannot be simplified by the Distributive Property.

$$3\sqrt[4]{2} - 6\sqrt[4]{3} \qquad 2\sqrt[4]{4x} + 3\sqrt[3]{4x}$$

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➡ Simplify: $3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2}$

$$\begin{aligned} &3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2} \\ &= 3\sqrt{16x^2}\,\sqrt{2} - 2x\sqrt{2} + \sqrt{64x^2}\,\sqrt{2} \\ &= 3 \cdot 4x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2} \\ &= 12x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2} \\ &= 18x\sqrt{2} \end{aligned}$$

- First simplify each term. Then combine like terms by using the Distributive Property.

Example 3

Simplify: $5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9}$

Solution

$$\begin{aligned} &5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9} \\ &= 5b\sqrt[4]{16a^4b^4}\,\sqrt[4]{2a^3b} - 2a\sqrt[4]{81b^8}\,\sqrt[4]{2a^3b} \\ &= 5b \cdot 2ab\sqrt[4]{2a^3b} - 2a \cdot 3b^2\sqrt[4]{2a^3b} \\ &= 10ab^2\sqrt[4]{2a^3b} - 6ab^2\sqrt[4]{2a^3b} \\ &= 4ab^2\sqrt[4]{2a^3b} \end{aligned}$$

You Try It 3

Simplify: $3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4}$

Your solution

Solution on p. S28

Objective C — To multiply radical expressions

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The Product Property of Radicals is used to multiply radical expressions with the same index.

$$\sqrt{3x} \cdot \sqrt{5y} = \sqrt{3x \cdot 5y} = \sqrt{15xy}$$

➡ Simplify: $\sqrt[3]{2a^5b}\,\sqrt[3]{16a^2b^2}$

$$\begin{aligned} \sqrt[3]{2a^5b}\,\sqrt[3]{16a^2b^2} &= \sqrt[3]{32a^7b^3} \\ &= \sqrt[3]{8a^6b^3}\,\sqrt[3]{4a} \\ &= 2a^2b\sqrt[3]{4a} \end{aligned}$$

- Use the Product Property of Radicals to multiply the radicands.
- Simplify.

➡ Simplify: $\sqrt{2x}(\sqrt{8x} - \sqrt{3})$

$$\begin{aligned} \sqrt{2x}(\sqrt{8x} - \sqrt{3}) &= \sqrt{2x}(\sqrt{8x}) - \sqrt{2x}(\sqrt{3}) \\ &= \sqrt{16x^2} - \sqrt{6x} \\ &= 4x - \sqrt{6x} \end{aligned}$$

- Use the Distributive Property.
- Simplify.

➡ Simplify: $(2\sqrt{5} - 3)(3\sqrt{5} + 4)$

$$\begin{aligned} (2\sqrt{5} - 3)(3\sqrt{5} + 4) &= 6(\sqrt{5})^2 + 8\sqrt{5} - 9\sqrt{5} - 12 \\ &= 30 + 8\sqrt{5} - 9\sqrt{5} - 12 \\ &= 18 - \sqrt{5} \end{aligned}$$

- Use the FOIL method to multiply the numbers.
- Combine like terms.

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➡ Simplify: $(4\sqrt{a} - \sqrt{b})(2\sqrt{a} + 5\sqrt{b})$

$$(4\sqrt{a} - \sqrt{b})(2\sqrt{a} + 5\sqrt{b})$$
$$= 8(\sqrt{a})^2 + 20\sqrt{ab} - 2\sqrt{ab} - 5(\sqrt{b})^2$$
$$= 8a + 18\sqrt{ab} - 5b$$

• Use the FOIL method.

TAKE NOTE

The concept of conjugate is used in a number of different instances. Make sure you understand this idea.

The conjugate of $\sqrt{3} - 4$ is $\sqrt{3} + 4$.

The conjugate of $\sqrt{3} + 4$ is $\sqrt{3} - 4$.

The conjugate of $\sqrt{5a} + \sqrt{b}$ is $\sqrt{5a} - \sqrt{b}$.

The expressions $a + b$ and $a - b$ are **conjugates** of each other. Recall that $(a + b)(a - b) = a^2 - b^2$. This identity is used to simplify conjugate radical expressions.

➡ Simplify: $(\sqrt{11} - 3)(\sqrt{11} + 3)$

$$(\sqrt{11} - 3)(\sqrt{11} + 3) = (\sqrt{11})^2 - 3^2 = 11 - 9$$
$$= 2$$

• The radical expressions are conjugates.

Example 4

Simplify: $\sqrt{3x}(\sqrt{27x^2} - \sqrt{3x})$

Solution

$$\sqrt{3x}(\sqrt{27x^2} - \sqrt{3x}) = \sqrt{81x^3} - \sqrt{9x^2}$$
$$= \sqrt{81x^2}\sqrt{x} - \sqrt{9x^2}$$
$$= 9x\sqrt{x} - 3x$$

You Try It 4

Simplify: $\sqrt{5b}(\sqrt{3b} - \sqrt{10})$

Your solution

Example 5

Simplify: $(2\sqrt[3]{x} - 3)(3\sqrt[3]{x} - 4)$

Solution

$$(2\sqrt[3]{x} - 3)(3\sqrt[3]{x} - 4)$$
$$= 6\sqrt[3]{x^2} - 8\sqrt[3]{x} - 9\sqrt[3]{x} + 12$$
$$= 6\sqrt[3]{x^2} - 17\sqrt[3]{x} + 12$$

You Try It 5

Simplify: $(2\sqrt[3]{2x} - 3)(\sqrt[3]{2x} - 5)$

Your solution

Example 6

Simplify: $(2\sqrt{x} - \sqrt{2y})(2\sqrt{x} + \sqrt{2y})$

Solution

$$(2\sqrt{x} - \sqrt{2y})(2\sqrt{x} + \sqrt{2y})$$
$$= (2\sqrt{x})^2 - (\sqrt{2y})^2$$
$$= 4x - 2y$$

You Try It 6

Simplify: $(\sqrt{a} - 3\sqrt{y})(\sqrt{a} + 3\sqrt{y})$

Your solution

Solutions on p. S28

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Objective D

To divide radical expressions



The Quotient Property of Radicals is used to divide radical expressions with the same index.

The Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Simplify: $\sqrt[3]{\dfrac{81x^5}{y^6}}$

$$\sqrt[3]{\frac{81x^5}{y^6}} = \frac{\sqrt[3]{81x^5}}{\sqrt[3]{y^6}}$$

- Use the Quotient Property of Radicals.

$$= \frac{\sqrt[3]{27x^3}\sqrt[3]{3x^2}}{\sqrt[3]{y^6}} = \frac{3x\sqrt[3]{3x^2}}{y^2}$$

- Simplify each radical expression.

Simplify: $\dfrac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}}$

$$\frac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}} = \sqrt{\frac{5a^4b^7c^2}{ab^3c}}$$

- Use the Quotient Property of Radicals.

$$= \sqrt{5a^3b^4c}$$

$$= \sqrt{a^2b^4}\sqrt{5ac} = ab^2\sqrt{5ac}$$

- Simplify the radicand.

A radical expression is in simplest form when no radical remains in the denominator of the radical expression. The procedure used to remove a radical from the denominator is called **rationalizing the denominator.**

Simplify: $\dfrac{5}{\sqrt{2}}$

$$\frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot 1 = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

- Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$, which equals 1.

$$= \frac{5\sqrt{2}}{2}$$

- $\sqrt{2} \cdot \sqrt{2} = (\sqrt{2})^2 = 2$

Simplify: $\dfrac{3}{\sqrt[3]{4x}}$

$$\frac{3}{\sqrt[3]{4x}} = \frac{3}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}$$

- Because $\sqrt[3]{4x} \cdot \sqrt[3]{2x^2} = \sqrt[3]{8x^3}$, a perfect cube, multiply the expression by $\frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}$, which equals 1.

$$= \frac{3\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}} = \frac{3\sqrt[3]{2x^2}}{2x}$$

- Simplify.

TAKE NOTE

Multiplying by $\frac{\sqrt[3]{4x}}{\sqrt[3]{4x}}$ will not rationalize the denominator of $\frac{3x}{\sqrt[3]{4x}}$.

$$\frac{3}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{4x}}{\sqrt[3]{4x}} = \frac{3\sqrt[3]{4x}}{\sqrt[3]{16x^2}}$$

Because $\sqrt[3]{16x^2}$ is not a perfect cube, the denominator still contains a radical expression.

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To simplify a fraction that has a square-root expression with two terms in the denominator, multiply the numerator and denominator by the conjugate of the denominator. Then simplify.

Simplify: $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

TAKE NOTE
Here is an example of using a conjugate to simplify a radical expression.

$$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\cdot\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$
$$=\frac{(\sqrt{x})^2-\sqrt{xy}-\sqrt{xy}+(\sqrt{y})^2}{(\sqrt{x})^2-(\sqrt{y})^2}=\frac{x-2\sqrt{xy}+y}{x-y}$$

Example 7

Simplify: $\dfrac{5}{\sqrt{5x}}$

Solution

$$\frac{5}{\sqrt{5x}}=\frac{5}{\sqrt{5x}}\cdot\frac{\sqrt{5x}}{\sqrt{5x}}=\frac{5\sqrt{5x}}{(\sqrt{5x})^2}$$
$$=\frac{5\sqrt{5x}}{5x}=\frac{\sqrt{5x}}{x}$$

You Try It 7

Simplify: $\dfrac{y}{\sqrt{3y}}$

Your solution

Example 8

Simplify: $\dfrac{3x}{\sqrt[4]{2x}}$

Solution

$$\frac{3x}{\sqrt[4]{2x}}=\frac{3x}{\sqrt[4]{2x}}\cdot\frac{\sqrt[4]{8x^3}}{\sqrt[4]{8x^3}}$$
$$=\frac{3x\sqrt[4]{8x^3}}{\sqrt[4]{16x^4}}=\frac{3x\sqrt[4]{8x^3}}{2x}$$
$$=\frac{3\sqrt[4]{8x^3}}{2}$$

You Try It 8

Simplify: $\dfrac{3x}{\sqrt[3]{3x^2}}$

Your solution

Example 9

Simplify: $\dfrac{3}{5-2\sqrt{3}}$

Solution

$$\frac{3}{5-2\sqrt{3}}=\frac{3}{5-2\sqrt{3}}\cdot\frac{5+2\sqrt{3}}{5+2\sqrt{3}}=\frac{15+6\sqrt{3}}{5^2-(2\sqrt{3})^2}$$
$$=\frac{15+6\sqrt{3}}{25-12}=\frac{15+6\sqrt{3}}{13}$$

You Try It 9

Simplify: $\dfrac{3+\sqrt{6}}{2-\sqrt{6}}$

Your solution

Solutions on p. S28

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9.2 Exercises

Objective A

Simplify.

1. $\sqrt{x^4y^3z^5}$
2. $\sqrt{x^3y^6z^9}$
3. $\sqrt{8a^3b^8}$
4. $\sqrt{24a^9b^6}$
5. $\sqrt{45x^2y^3z^5}$
6. $\sqrt{60xy^7z^{12}}$
7. $\sqrt{-9x^3}$
8. $\sqrt{-x^2y^5}$
9. $\sqrt[3]{a^{16}b^8}$
10. $\sqrt[3]{a^5b^8}$
11. $\sqrt[3]{-125x^2y^4}$
12. $\sqrt[3]{-216x^5y^9}$
13. $\sqrt[3]{a^4b^5c^6}$
14. $\sqrt[3]{a^8b^{11}c^{15}}$
15. $\sqrt[4]{16x^9y^5}$
16. $\sqrt[4]{64x^8y^{10}}$

Objective B

Add or subtract.

17. $2\sqrt{x} - 8\sqrt{x}$
18. $3\sqrt{y} + 12\sqrt{y}$
19. $\sqrt{8} - \sqrt{32}$
20. $\sqrt{27a} - \sqrt{8a}$
21. $\sqrt{18b} + \sqrt{75b}$
22. $2\sqrt{2x^3} + 4x\sqrt{8x}$
23. $3\sqrt{8x^2y^3} - 2x\sqrt{32y^3}$
24. $2\sqrt{32x^2y^3} - xy\sqrt{98y}$
25. $2a\sqrt{27ab^5} + 3b\sqrt{3a^3b}$
26. $\sqrt[3]{128} + \sqrt[3]{250}$
27. $\sqrt[3]{16} - \sqrt[3]{54}$
28. $2\sqrt[3]{3a^4} - 3a\sqrt[3]{81a}$
29. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$
30. $3\sqrt[3]{x^5y^7} - 8xy\sqrt[3]{x^2y^4}$

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31. $3\sqrt[4]{32a^5} - a\sqrt[4]{162a}$

32. $2a\sqrt[4]{16ab^5} + 3b\sqrt[4]{256a^5b}$

33. $2\sqrt{50} - 3\sqrt{125} + \sqrt{98}$

34. $3\sqrt{108} - 2\sqrt{18} - 3\sqrt{48}$

35. $\sqrt{9b^3} - \sqrt{25b^3} + \sqrt{49b^3}$

36. $\sqrt{4x^7y^5} + 9x^2\sqrt{x^3y^5} - 5xy\sqrt{x^5y^3}$

37. $2x\sqrt{8xy^2} - 3y\sqrt{32x^3} + \sqrt{4x^3y^3}$

38. $5a\sqrt{3a^3b} + 2a^2\sqrt{27ab} - 4\sqrt{75a^5b}$

39. $\sqrt[3]{54xy^3} - 5\sqrt[3]{2xy^3} + y\sqrt[3]{128x}$

40. $2\sqrt[3]{24x^3y^4} + 4x\sqrt[3]{81y^4} - 3y\sqrt[3]{24x^3y}$

41. $2a\sqrt[4]{32b^5} - 3b\sqrt[4]{162a^4b} + \sqrt[4]{2a^4b^5}$

42. $6y\sqrt[4]{48x^5} - 2x\sqrt[4]{243xy^4} - 4\sqrt[4]{3x^5y^4}$

Objective C

Multiply.

43. $\sqrt{8}\sqrt{32}$

44. $\sqrt{14}\sqrt{35}$

45. $\sqrt[3]{4}\sqrt[3]{8}$

46. $\sqrt[3]{6}\sqrt[3]{36}$

47. $\sqrt{x^2y^5}\sqrt{xy}$

48. $\sqrt{a^3b}\sqrt{ab^4}$

49. $\sqrt{2x^2y}\sqrt{32xy}$

50. $\sqrt{5x^3y}\sqrt{10x^3y^4}$

51. $\sqrt[3]{x^2y}\sqrt[3]{16x^4y^2}$

52. $\sqrt[3]{4a^2b^3}\sqrt[3]{8ab^5}$

53. $\sqrt[4]{12ab^3}\sqrt[4]{4a^5b^2}$

54. $\sqrt[4]{36a^2b^4}\sqrt[4]{12a^5b^3}$

55. $\sqrt{3}(\sqrt{27} - \sqrt{3})$

56. $\sqrt{10}(\sqrt{10} - \sqrt{5})$

57. $\sqrt{x}(\sqrt{x} - \sqrt{2})$

58. $\sqrt{y}(\sqrt{y} - \sqrt{5})$

59. $\sqrt{2x}(\sqrt{8x} - \sqrt{32})$

60. $\sqrt{3a}(\sqrt{27a^2} - \sqrt{a})$

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61. $(\sqrt{x} - 3)^2$

62. $(\sqrt{2x} + 4)^2$

63. $(4\sqrt{5} + 2)^2$

64. $2\sqrt{3x^2} \cdot 3\sqrt{12xy^3} \cdot \sqrt{6x^3y}$

65. $2\sqrt{14xy} \cdot 4\sqrt{7x^2y} \cdot 3\sqrt{8xy^2}$

66. $\sqrt[3]{8ab}\sqrt[3]{4a^2b^3}\sqrt[3]{9ab^4}$

67. $\sqrt[3]{2a^2b}\sqrt[3]{4a^3b^2}\sqrt[3]{8a^5b^6}$

68. $(\sqrt{2} - 3)(\sqrt{2} + 4)$

69. $(\sqrt{5} - 5)(2\sqrt{5} + 2)$

70. $(\sqrt{y} - 2)(\sqrt{y} + 2)$

71. $(\sqrt{x} - y)(\sqrt{x} + y)$

72. $(\sqrt{2x} - 3\sqrt{y})(\sqrt{2x} + 3\sqrt{y})$

73. $(2\sqrt{3x} - \sqrt{y})(2\sqrt{3x} + \sqrt{y})$

Objective D

74. When is a radical expression in simplest form?

75. Explain what it means to rationalize the denominator of a radical expression and how to do so.

Simplify.

76. $\dfrac{\sqrt{32x^2}}{\sqrt{2x}}$

77. $\dfrac{\sqrt{60y^4}}{\sqrt{12y}}$

78. $\dfrac{\sqrt{42a^3b^5}}{\sqrt{14a^2b}}$

79. $\dfrac{\sqrt{65ab^4}}{\sqrt{5ab}}$

80. $\dfrac{1}{\sqrt{5}}$

81. $\dfrac{1}{\sqrt{2}}$

82. $\dfrac{1}{\sqrt{2x}}$

83. $\dfrac{2}{\sqrt{3y}}$

84. $\dfrac{5}{\sqrt{5x}}$

85. $\dfrac{9}{\sqrt{3a}}$

86. $\sqrt{\dfrac{x}{5}}$

87. $\sqrt{\dfrac{y}{2}}$

88. $\dfrac{3}{\sqrt[3]{2}}$

89. $\dfrac{5}{\sqrt[3]{9}}$

90. $\dfrac{3}{\sqrt[3]{4x^2}}$

91. $\dfrac{5}{\sqrt[3]{3y}}$

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92. $\dfrac{\sqrt{40x^3y^2}}{\sqrt{80x^2y^3}}$ **93.** $\dfrac{\sqrt{15a^2b^5}}{\sqrt{30a^5b^3}}$ **94.** $\dfrac{\sqrt{24a^2b}}{\sqrt{18ab^4}}$ **95.** $\dfrac{\sqrt{12x^3y}}{\sqrt{20x^4y}}$

96. $\dfrac{5}{\sqrt{3}-2}$ **97.** $\dfrac{-2}{1-\sqrt{2}}$ **98.** $\dfrac{-3}{2-\sqrt{3}}$ **99.** $\dfrac{-4}{3-\sqrt{2}}$

100. $\dfrac{2}{\sqrt{5}+2}$ **101.** $\dfrac{5}{2-\sqrt{7}}$ **102.** $\dfrac{3}{\sqrt{y}-2}$ **103.** $\dfrac{-7}{\sqrt{x}-3}$

104. $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ **105.** $\dfrac{\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}}$ **106.** $\dfrac{2+3\sqrt{7}}{5-2\sqrt{7}}$

107. $\dfrac{2+3\sqrt{5}}{1-\sqrt{5}}$ **108.** $\dfrac{2\sqrt{3}-1}{3\sqrt{3}+2}$ **109.** $\dfrac{2\sqrt{a}-\sqrt{b}}{4\sqrt{a}+3\sqrt{b}}$

110. $\dfrac{2\sqrt{x}-4}{\sqrt{x}+2}$ **111.** $\dfrac{3\sqrt{y}-y}{\sqrt{y}+2y}$ **112.** $\dfrac{3\sqrt{x}-4\sqrt{y}}{3\sqrt{x}-2\sqrt{y}}$

APPLYING THE CONCEPTS

113. Determine whether the following statements are true or false. If the statement is false, correct the right side of the equation.

a. $\sqrt[2]{3}\cdot\sqrt[3]{4}=\sqrt[5]{12}$ **b.** $\sqrt{3}\cdot\sqrt{3}=3$ **c.** $\sqrt[3]{x}\cdot\sqrt[3]{x}=x$

d. $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$ **e.** $\sqrt[2]{2}\cdot\sqrt[3]{3}=\sqrt[5]{2+3}$ **f.** $8\sqrt[5]{a}-2\sqrt[5]{a}=6\sqrt[5]{a}$

114. Multiply: $(\sqrt[3]{a}+\sqrt[3]{b})(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})$

115. Rewrite $\dfrac{\sqrt[4]{(a+b)^3}}{\sqrt{a+b}}$ as an expression with a single radical.

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9.3 Complex Numbers

To simplify a complex number

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The radical expression $\sqrt{-4}$ is not a real number, because there is no real number whose square is -4. However, the solution of an algebraic equation is sometimes the square root of a negative number.

For example, the equation $x^2 + 1 = 0$ does not have a real number solution, because there is no real number whose square is a negative number.

$$x^2 + 1 = 0$$
$$x^2 = -1$$

Around the 17th century, a new number, called an **imaginary number**, was defined so that a negative number would have a square root. The letter i was chosen to represent the number whose square is -1.

$$i^2 = -1$$

An imaginary number is defined in terms of i.

Point of Interest

The first written occurrence of an imaginary number was in a book published in 1545 by Girolamo Cardan, where he wrote (in our modern notation) $5 + \sqrt{-15}$. He went on to say that the number "is as refined as it is useless." It was not until the 20th century that applications of complex numbers were found.

Definition of $\sqrt{-a}$

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$.

$$\sqrt{-a} = i\sqrt{a}$$

Here are some examples.

$$\sqrt{-16} = i\sqrt{16} = 4i$$
$$\sqrt{-12} = i\sqrt{12} = 2i\sqrt{3}$$
$$\sqrt{-21} = i\sqrt{21}$$
$$\sqrt{-1} = i\sqrt{1} = i$$

It is customary to write i in front of a radical to avoid confusing $\sqrt{a}\,i$ with $\sqrt{ai}$.

The real numbers and imaginary numbers make up the complex numbers.

Complex Number

A **complex number** is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The number a is the **real part** of $a + bi$, and b is the **imaginary part**.

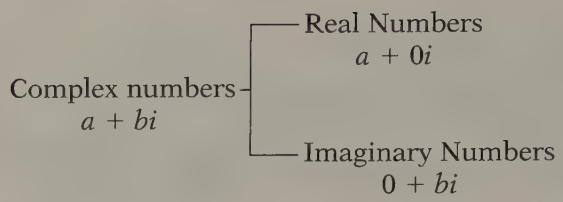
TAKE NOTE

The *imaginary part* of a complex number is a real number. As another example, the imaginary part of $6 - 8i$ is -8.

Examples of complex numbers are shown at the right.

Real Part	*Imaginary Part*
a	bi
3	$2i$
8	$-10i$

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A *real number* is a complex number in which $b = 0$.

An *imaginary number* is a complex number in which $a = 0$.

Simplify: $\sqrt{20} - \sqrt{-50}$

$$\sqrt{20} - \sqrt{-50} = \sqrt{20} - i\sqrt{50}$$
$$= \sqrt{4 \cdot 5} - i\sqrt{25 \cdot 2}$$
$$= 2\sqrt{5} - 5i\sqrt{2}$$

- Write the complex number in the form $a + bi$.
- Use the Product Property of Radicals to simplify each radical.

Example 1
Simplify: $\sqrt{-80}$

Solution
$\sqrt{-80} = i\sqrt{80} = i\sqrt{16 \cdot 5} = 4i\sqrt{5}$

You Try It 1
Simplify: $\sqrt{-45}$

Your solution

Example 2
Simplify: $\sqrt{25} + \sqrt{-40}$

Solution
$$\sqrt{25} + \sqrt{-40} = \sqrt{25} + i\sqrt{40}$$
$$= \sqrt{25} + i\sqrt{4 \cdot 10}$$
$$= 5 + 2i\sqrt{10}$$

You Try It 2
Simplify: $\sqrt{98} - \sqrt{-60}$

Your solution

Solutions on p. S28

Objective E To add or subtract complex numbers

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Addition and Subtraction of Complex Numbers

To add two complex numbers, add the real parts and add the imaginary parts. To subtract two complex numbers, subtract the real parts and subtract the imaginary parts.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Subtract: $(3 - 7i) - (4 - 2i)$

$$(3 - 7i) - (4 - 2i) = (3 - 4) + [-7 - (-2)]i$$
$$= -1 - 5i$$

- Subtract the real parts and subtract the imaginary parts of the complex numbers.

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Add: $(3 + \sqrt{-12}) + (7 - \sqrt{-27})$

$$(3 + \sqrt{-12}) + (7 - \sqrt{-27})$$
$$= (3 + i\sqrt{12}) + (7 - i\sqrt{27})$$ • Write each complex number in the form $a + bi$.
$$= (3 + i\sqrt{4 \cdot 3}) + (7 - i\sqrt{9 \cdot 3})$$ • Use the Product Property of Radicals to simplify each radical.
$$= (3 + 2i\sqrt{3}) + (7 - 3i\sqrt{3})$$
$$= 10 - i\sqrt{3}$$ • Add the complex numbers.

Example 3
Add: $(3 + 2i) + (6 - 5i)$

Solution
$(3 + 2i) + (6 - 5i) = 9 - 3i$

You Try It 3
Subtract: $(-4 + 2i) - (6 - 8i)$

Your solution

Example 4
Subtract: $(9 - \sqrt{-8}) - (5 + \sqrt{-32})$

Solution
$$(9 - \sqrt{-8}) - (5 + \sqrt{-32})$$
$$= (9 - i\sqrt{8}) - (5 + i\sqrt{32})$$
$$= (9 - i\sqrt{4 \cdot 2}) - (5 + i\sqrt{16 \cdot 2})$$
$$= (9 - 2i\sqrt{2}) - (5 + 4i\sqrt{2})$$
$$= 4 - 6i\sqrt{2}$$

You Try It 4
Subtract: $(16 - \sqrt{-45}) - (3 + \sqrt{-20})$

Your solution

Example 5
Add: $(6 + 4i) + (-6 - 4i)$

Solution
$(6 + 4i) + (-6 - 4i) = 0 + 0i = 0$

This illustrates that the additive inverse of $a + bi$ is $-a - bi$.

You Try It 5
Add: $(3 - 2i) + (-3 + 2i)$

Your solution

Solutions on p. S28

Objective B To multiply complex numbers

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When multiplying complex numbers, we often find that i^2 is a part of the product. Recall that $i^2 = -1$.

Multiply: $2i \cdot 3i$

$$2i \cdot 3i = 6i^2$$ • Multiply the imaginary numbers.
$$= 6(-1)$$ • Replace i^2 by -1.
$$= -6$$ • Simplify.

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TAKE NOTE
This example illustrates an important point. When working with an expression that has a square root of a negative number, always rewrite the number as the product of a real number and i before continuing.

➡ Multiply: $\sqrt{-6} \cdot \sqrt{-24}$

$$\sqrt{-6} \cdot \sqrt{-24} = i\sqrt{6} \cdot i\sqrt{24}$$
- Write each radical as the product of a real number and i.

$$= i^2\sqrt{144}$$
- Multiply the imaginary numbers.

$$= -\sqrt{144}$$
- Replace i^2 by -1.

$$= -12$$
- Simplify the radical expression.

Note from the last example that it would have been incorrect to multiply the radicands of the two radical expressions. To illustrate,

$$\sqrt{-6} \cdot \sqrt{-24} = \sqrt{(-6)(-24)} = \sqrt{144} = 12, \textit{ not } -12$$

➡ Multiply: $4i(3 - 2i)$

$$4i(3 - 2i) = 12i - 8i^2$$
- Use the Distributive Property to remove parentheses.

$$= 12i - 8(-1)$$
- Replace i^2 by -1.

$$= 8 + 12i$$
- Write the answer in the form $a + bi$.

The product of two complex numbers is defined as follows.

The Product of Two Complex Numbers

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

One way to remember this rule is to use the FOIL method.

➡ Multiply: $(2 + 4i)(3 - 5i)$

$$(2 + 4i)(3 - 5i) = 6 - 10i + 12i - 20i^2$$
- Use the FOIL method to find the product.

$$= 6 + 2i - 20i^2$$

$$= 6 + 2i - 20(-1)$$
- Replace i^2 by -1.

$$= 26 + 2i$$
- Write the answer in the form $a + bi$.

The conjugate of $a + bi$ is $a - bi$.
The product of conjugates, $(a + bi)(a - bi)$, is the real number $a^2 + b^2$.

$$(a + bi)(a - bi) = a^2 - b^2i^2$$
$$= a^2 - b^2(-1)$$
$$= a^2 + b^2$$

➡ Multiply: $(2 + 3i)(2 - 3i)$

$$(2 + 3i)(2 - 3i) = 2^2 + 3^2$$
- The product of conjugates is $a^2 + b^2$.

$$= 4 + 9$$

$$= 13$$

Note that the product of a complex number and its conjugate is a real number.

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Example 6
Multiply: $(2i)(-5i)$

Solution
$(2i)(-5i) = -10i^2 = (-10)(-1) = 10$

You Try It 6
Multiply: $(-3i)(-10i)$

Your solution

Example 7
Multiply: $\sqrt{-10} \cdot \sqrt{-5}$

Solution
$\sqrt{-10} \cdot \sqrt{-5} = i\sqrt{10} \cdot i\sqrt{5}$
$= i^2\sqrt{50} = -\sqrt{25 \cdot 2} = -5\sqrt{2}$

You Try It 7
Multiply: $-\sqrt{-8} \cdot \sqrt{-5}$

Your solution

Example 8
Multiply: $3i(2 - 4i)$

Solution:
$3i(2 - 4i) = 6i - 12i^2 = 6i - 12(-1)$
$= 12 + 6i$

You Try It 8
Multiply: $-6i(3 + 4i)$

Your solution

Example 9
Multiply: $\sqrt{-8}(\sqrt{6} - \sqrt{-2})$

Solution
$\sqrt{-8}(\sqrt{6} - \sqrt{-2}) = i\sqrt{8}(\sqrt{6} - i\sqrt{2})$
$= i\sqrt{48} - i^2\sqrt{16}$
$= i\sqrt{16 \cdot 3} - (-1)\sqrt{16}$
$= 4i\sqrt{3} + 4 = 4 + 4i\sqrt{3}$

You Try It 9
Multiply: $\sqrt{-3}(\sqrt{27} - \sqrt{-6})$

Your solution

Example 10
Multiply: $(3 - 4i)(2 + 5i)$

Solution
$(3 - 4i)(2 + 5i) = 6 + 15i - 8i - 20i^2$
$= 6 + 7i - 20i^2$
$= 6 + 7i - 20(-1) = 26 + 7i$

You Try It 10
Multiply: $(4 - 3i)(2 - i)$

Your solution

Example 11
Multiply: $(4 + 5i)(4 - 5i)$

Solution
$(4 + 5i)(4 - 5i) = 4^2 + 5^2 = 16 + 25 = 41$

You Try It 11
Multiply: $(3 + 6i)(3 - 6i)$

Your solution

Example 12
Multiply: $\left(\frac{9}{10} + \frac{3}{10}i\right)\left(1 - \frac{1}{3}i\right)$

Solution
$\left(\frac{9}{10} + \frac{3}{10}i\right)\left(1 - \frac{1}{3}i\right) = \frac{9}{10} - \frac{3}{10}i + \frac{3}{10}i - \frac{1}{10}i^2$
$= \frac{9}{10} - \frac{1}{10}i^2 = \frac{9}{10} - \frac{1}{10}(-1)$
$= \frac{9}{10} + \frac{1}{10} = 1$

You Try It 12
Multiply: $(3 - i)\left(\frac{3}{10} + \frac{1}{10}i\right)$

Your solution

Solutions on pp. S28–S29

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Objective D To divide complex numbers



A rational expression containing one or more complex numbers is in simplest form when no imaginary number remains in the denominator.

Simplify: $\frac{2-3i}{2i}$

$$\frac{2-3i}{2i} = \frac{2-3i}{2i}\cdot\frac{i}{i}$$ • Multiply the expression by $\frac{i}{i}$.

$$= \frac{2i-3i^2}{2i^2}$$

$$= \frac{2i-3(-1)}{2(-1)}$$ • Replace i^2 by -1.

$$= \frac{3+2i}{-2}$$ • Simplify.

$$= -\frac{3}{2} - i$$ • Write the answer in the form $a + bi$.

Simplify: $\frac{3+2i}{1+i}$

$$\frac{3+2i}{1+i} = \frac{3+2i}{1+i}\cdot\frac{1-i}{1-i}$$ • Multiply the numerator and denominator by the conjugate of $1 + i$.

$$= \frac{3-3i+2i-2i^2}{1^2+1^2}$$ • In $1 + i$, $a = 1$ and $b = 1$. $a^2 + b^2 = 1^2 + 1^2$.

$$= \frac{3-i-2(-1)}{2}$$ • Replace i^2 by -1 and simplify.

$$= \frac{5-i}{2} = \frac{5}{2} - \frac{1}{2}i$$ • Write the answer in the form $a + bi$.

Example 13

Simplify: $\frac{5+4i}{3i}$

Solution

$$\frac{5+4i}{3i} = \frac{5+4i}{3i}\cdot\frac{i}{i} = \frac{5i+4i^2}{3i^2}$$

$$= \frac{5i+4(-1)}{3(-1)} = \frac{-4+5i}{-3} = \frac{4}{3} - \frac{5}{3}i$$

You Try It 13

Simplify: $\frac{2-3i}{4i}$

Your solution

Example 14

Simplify: $\frac{5-3i}{4+2i}$

Solution

$$\frac{5-3i}{4+2i} = \frac{5-3i}{4+2i}\cdot\frac{4-2i}{4-2i}$$

$$= \frac{20-10i-12i+6i^2}{4^2+2^2}$$

$$= \frac{20-22i+6(-1)}{20}$$

$$= \frac{14-22i}{20} = \frac{14}{20} - \frac{22}{20}i = \frac{7}{10} - \frac{11}{10}i$$

You Try It 14

Simplify: $\frac{2+5i}{3-2i}$

Your solution

Solutions on p. S29

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9.3 Exercises

Objective A

1. What is an imaginary number? What is a complex number?

2. Are all real numbers also complex numbers? Are all complex numbers also real numbers?

Simplify.

3. $\sqrt{-4}$

4. $\sqrt{-64}$

5. $\sqrt{-98}$

6. $\sqrt{-72}$

7. $\sqrt{-27}$

8. $\sqrt{-75}$

9. $\sqrt{16} + \sqrt{-4}$

10. $\sqrt{25} + \sqrt{-9}$

11. $\sqrt{12} - \sqrt{-18}$

12. $\sqrt{60} - \sqrt{-48}$

13. $\sqrt{160} - \sqrt{-147}$

14. $\sqrt{96} - \sqrt{-125}$

Objective B

Add or subtract.

15. $(2 + 4i) + (6 - 5i)$

16. $(6 - 9i) + (4 + 2i)$

17. $(-2 - 4i) - (6 - 8i)$

18. $(3 - 5i) + (8 - 2i)$

19. $(8 - \sqrt{-4}) - (2 + \sqrt{-16})$

20. $(5 - \sqrt{-25}) - (11 - \sqrt{-36})$

21. $(12 - \sqrt{-50}) + (7 - \sqrt{-8})$

22. $(5 - \sqrt{-12}) - (9 + \sqrt{-108})$

23. $(\sqrt{8} + \sqrt{-18}) + (\sqrt{32} - \sqrt{-72})$

24. $(\sqrt{40} - \sqrt{-98}) - (\sqrt{90} + \sqrt{-32})$

Objective C

Multiply.

25. $(7i)(-9i)$

26. $(-6i)(-4i)$

27. $\sqrt{-2}\sqrt{-8}$

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28. $\sqrt{-5}\sqrt{-45}$ **29.** $\sqrt{-3}\sqrt{-6}$ **30.** $\sqrt{-5}\sqrt{-10}$

31. $2i(6 + 2i)$ **32.** $-3i(4 - 5i)$ **33.** $\sqrt{-2}(\sqrt{8} + \sqrt{-2})$

34. $\sqrt{-3}(\sqrt{12} - \sqrt{-6})$ **35.** $(5 - 2i)(3 + i)$ **36.** $(2 - 4i)(2 - i)$

37. $(6 + 5i)(3 + 2i)$ **38.** $(4 - 7i)(2 + 3i)$ **39.** $(1 - i)\left(\frac{1}{2} + \frac{1}{2}i\right)$

40. $\left(\frac{4}{5} - \frac{2}{5}i\right)\left(1 + \frac{1}{2}i\right)$ **41.** $\left(\frac{6}{5} + \frac{3}{5}i\right)\left(\frac{2}{3} - \frac{1}{3}i\right)$ **42.** $(2 - i)\left(\frac{2}{5} + \frac{1}{5}i\right)$

Simplify.

43. $\frac{3}{i}$ **44.** $\frac{4}{5i}$ **45.** $\frac{2 - 3i}{-4i}$ **46.** $\frac{16 + 5i}{-3i}$

47. $\frac{4}{5 + i}$ **48.** $\frac{6}{5 + 2i}$ **49.** $\frac{2}{2 - i}$ **50.** $\frac{5}{4 - i}$

51. $\frac{1 - 3i}{3 + i}$ **52.** $\frac{2 + 12i}{5 + i}$ **53.** $\frac{\sqrt{-10}}{\sqrt{8} - \sqrt{-2}}$ **54.** $\frac{\sqrt{-2}}{\sqrt{12} - \sqrt{-8}}$

55. $\frac{2 - 3i}{3 + i}$ **56.** $\frac{3 + 5i}{1 - i}$ **57.** $\frac{5 + 3i}{3 - i}$ **58.** $\frac{3 - 2i}{2i + 3}$

APPLYING THE CONCEPTS

59. **a.** Is $3i$ a solution of $2x^2 + 18 = 0$?
b. Is $3 + i$ a solution of $x^2 - 6x + 10 = 0$?

60. Evaluate i^n for $n = 0, 1, 2, 3, 4, 5, 6$, and 7. Make a conjecture about the value of i^n for any natural number. Using your conjecture, evaluate i^{76}.

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9.4 Equations Containing Radical Expressions

Objective A

To solve a radical equation

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An equation that contains a variable expression in a radicand is a **radical equation**.

$\sqrt[3]{2x-5}+x=7$
$\sqrt{x+1}-\sqrt{x}=4$
} Radical Equations

The following property is used to solve a radical equation.

The Property of Raising Each Side of an Equation to a Power

If two numbers are equal, then the same powers of the numbers are equal.

If $a = b$, then $a^n = b^n$.

➡ Solve: $\sqrt{x-2}-6=0$

$$\sqrt{x-2}-6=0$$
$$\sqrt{x-2}=6$$ • Isolate the radical by adding 6 to each side of the equation.
$$(\sqrt{x-2})^2=6^2$$ • Square each side of the equation.
$$x-2=36$$ • Simplify and solve for x.
$$x=38$$

Check:

$$\sqrt{x-2}-6=0$$

$\sqrt{38-2}-6$	0
$\sqrt{36}-6$	0
$6-6$	0
$0=0$	

38 checks as a solution. The solution is 38.

➡ Solve: $\sqrt[3]{x+2}=-3$

$$\sqrt[3]{x+2}=-3$$
$$(\sqrt[3]{x+2})^3=(-3)^3$$ • Cube each side of the equation.
$$x+2=-27$$ • Solve the resulting equation.
$$x=-29$$

Check:

$$\sqrt[3]{x+2}=-3$$

$\sqrt[3]{-29+2}$	-3
$\sqrt[3]{-27}$	-3
$-3=-3$	

-29 checks as a solution. The solution is -29.

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Raising each side of an equation to an even power may result in an equation that has a solution that is not a solution of the original equation. This is called an **extraneous solution**. Here is an example:

Solve: $\sqrt{2x-1}+\sqrt{x}=2$

$$\sqrt{2x-1}+\sqrt{x}=2$$

$$\sqrt{2x-1}=2-\sqrt{x}$$ • Solve for one of the radical expressions.

$$(\sqrt{2x-1})^2=(2-\sqrt{x})^2$$ • Square each side. Recall that $(a-b)^2=a^2-2ab+b^2$.

$$2x-1=4-4\sqrt{x}+x$$

$$x-5=-4\sqrt{x}$$

$$(x-5)^2=(-4\sqrt{x})^2$$ • Square each side.

$$x^2-10x+25=16x$$

$$x^2-26x+25=0$$ • Solve the quadratic equation by factoring.

$$(x-25)(x-1)=0$$

$$x=25 \quad \text{or} \quad x=1$$

Check:

$\sqrt{2x-1}+\sqrt{x}=2$	
$\sqrt{2(25)-1}+\sqrt{25}$	2
$7+5$	2
$12\neq 2$	

$\sqrt{2x-1}+\sqrt{x}=2$	
$\sqrt{2(1)-1}+\sqrt{1}$	2
$1+1$	2
$2=2$	

25 does not check as a solution. 1 checks as a solution. The solution is 1.

TAKE NOTE

Note that

$(2-\sqrt{x})^2=(2-\sqrt{x})(2-\sqrt{x})=4-4\sqrt{x}+x$

TAKE NOTE

You must always check the proposed solutions to radical equations. The proposed solutions of the equation on the right were 1 and 25. However, 25 did not check as a solution. Here 25 is an extraneous solution.

Example 1

Solve: $\sqrt[3]{3x-1}=-4$

Solution

$$\sqrt[3]{3x-1}=-4$$

$$(\sqrt[3]{3x-1})^3=(-4)^3$$

$$3x-1=-64$$

$$3x=-63$$

$$x=-21$$

Check:

$\sqrt[3]{3x-1}=-4$	
$\sqrt[3]{3(-21)-1}$	-4
$\sqrt[3]{-64}$	-4
$-4=-4$	

The solution is -21.

You Try It 1

Solve: $\sqrt[4]{x-8}=3$

Your solution

Example 2

Solve: $\sqrt{x-1}+\sqrt{x+4}=5$

Solution

$$\sqrt{x-1}+\sqrt{x+4}=5$$

$$\sqrt{x+4}=5-\sqrt{x-1}$$

$$(\sqrt{x+4})^2=(5-\sqrt{x-1})^2$$

$$x+4=25-10\sqrt{x-1}+x-1$$

$$2=\sqrt{x-1}$$

$$2^2=(\sqrt{x-1})^2$$

$$4=x-1$$

$$5=x$$

5 checks as a solution. The solution is 5.

You Try It 2

Solve: $\sqrt{x}-\sqrt{x+5}=1$

Your solution

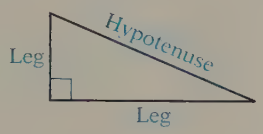
Solutions on p. S29

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Objective B To solve application problems

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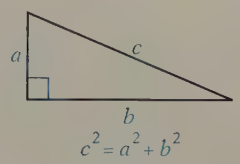
A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse**. The other two sides are called **legs**.



Point of Interest

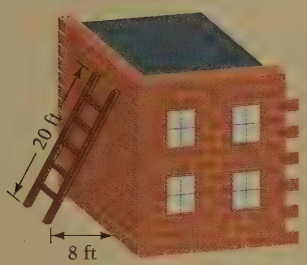
The first known proof of this theorem occurs in a Chinese text, *Arithmetic Classic*, which was first written around 600 B.C.

Pythagoras, a Greek mathematician, discovered that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. This is called the **Pythagorean Theorem**.



Example 3

A ladder 20 ft long is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 8 ft from the building? Round to the nearest tenth.



Strategy

To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 20^2 &= 8^2 + b^2 \\ 400 &= 64 + b^2 \\ 336 &= b^2 \\ (336)^{1/2} &= (b^2)^{1/2} \\ \sqrt{336} &= b \\ 18.3 &\approx b \end{aligned}$$

The distance is 18.3 ft.

You Try It 3

Find the diagonal of a rectangle that is 6 cm long and 3 cm wide. Round to the nearest tenth.

Your strategy

Your solution

Solution on p. S29

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Example 4

An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 96 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

Strategy

To find the distance the object has fallen, replace v in the equation with the given value and solve for d.

Solution

$$v = \sqrt{64d}$$
$$96 = \sqrt{64d}$$
$$(96)^2 = (\sqrt{64d})^2$$
$$9216 = 64d$$
$$144 = d$$

The object has fallen 144 ft.

You Try It 4

How far would a submarine periscope have to be above the water for the lookout to locate a ship 5.5 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

Your strategy

Your solution

Example 5

Find the length of a pendulum that makes one swing in 1.5 s. The equation for the time of one swing is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

Strategy

To find the length of the pendulum, replace T in the equation with the given value and solve for L.

Solution

$$T = 2\pi\sqrt{\frac{L}{32}}$$
$$1.5 = 2\pi\sqrt{\frac{L}{32}}$$
$$\frac{1.5}{2\pi} = \sqrt{\frac{L}{32}} \quad \text{• Divide each side by } 2\pi.$$
$$\left(\frac{1.5}{2\pi}\right)^2 = \frac{L}{32} \quad \text{• Square each side.}$$
$$32\left(\frac{1.5}{2\pi}\right)^2 = L \quad \text{• Multiply each side by 32.}$$
$$1.82 \approx L \quad \text{• Use the } \pi \text{ key on your calculator.}$$

The length of the pendulum is 1.82 ft.

You Try It 5

Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 22 ft/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

Your strategy

Your solution

Solutions on p. S29

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9.4 Exercises

Objective A

Solve.

1. $\sqrt{x} = 5$
2. $\sqrt{y} = 2$
3. $\sqrt[3]{a} = 3$
4. $\sqrt[3]{y} = 5$
5. $\sqrt{3x} = 12$
6. $\sqrt{5x} = 10$
7. $\sqrt[3]{4x} = -2$
8. $\sqrt[3]{6x} = -3$
9. $\sqrt{2x} = -4$
10. $\sqrt{5x} = -5$
11. $\sqrt{3x - 2} = 5$
12. $\sqrt{5x - 4} = 9$
13. $\sqrt{3 - 2x} = 7$
14. $\sqrt{9 - 4x} = 4$
15. $7 = \sqrt{1 - 3x}$
16. $6 = \sqrt{8 - 7x}$
17. $\sqrt[3]{4x - 1} = 2$
18. $\sqrt[3]{5x + 2} = 3$
19. $\sqrt[3]{1 - 2x} = -3$
20. $\sqrt[3]{3 - 2x} = -2$
21. $\sqrt[3]{9x + 1} = 4$
22. $\sqrt{3x + 9} - 12 = 0$
23. $\sqrt{4x - 3} - 5 = 0$
24. $\sqrt{x - 2} = 4$
25. $\sqrt[3]{x - 3} + 5 = 0$
26. $\sqrt[3]{x - 2} = 3$
27. $\sqrt[3]{2x - 6} = 4$
28. $\sqrt{x^2 - 8x} = 3$
29. $\sqrt{x^2 + 7x + 11} = 1$
30. $\sqrt[4]{4x + 1} = 2$
31. $\sqrt[4]{2x - 9} = 3$
32. $\sqrt{2x - 3} - 2 = 1$
33. $\sqrt{3x - 5} - 5 = 3$

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34. $\sqrt[3]{2x-3}+5=2$

35. $\sqrt[3]{x-4}+7=5$

36. $\sqrt{5x-16}+1=4$

37. $\sqrt{3x-5}-2=3$

38. $\sqrt{2x-1}-8=-5$

39. $\sqrt{7x+2}-10=-7$

40. $\sqrt[3]{4x-3}-2=3$

41. $\sqrt[3]{1-3x}+5=3$

42. $1-\sqrt{4x+3}=-5$

43. $7-\sqrt{3x+1}=-1$

44. $\sqrt{x+1}=2-\sqrt{x}$

45. $\sqrt{2x+4}=3-\sqrt{2x}$

46. $\sqrt{x^2+3x-2}-x=1$

47. $\sqrt{x^2-4x-1}+3=x$

48. $\sqrt{x^2-3x-1}=3$

49. $\sqrt{x^2-2x+1}=3$

50. $\sqrt{2x+5}-\sqrt{3x-2}=1$

51. $\sqrt{4x+1}-\sqrt{2x+4}=1$

52. $\sqrt{5x-1}-\sqrt{3x-2}=1$

53. $\sqrt{5x+4}-\sqrt{3x+1}=1$

54. $\sqrt[3]{x^2+2}-3=0$

55. $\sqrt[3]{x^2+4}-2=0$

56. $\sqrt[4]{x^2+2x+8}-2=0$

57. $\sqrt[4]{x^2+x-1}-1=0$

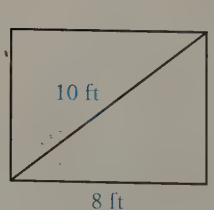
58. $4\sqrt{x+1}-x=1$

59. $3\sqrt{x-2}+2=x$

60. $x+3\sqrt{x-2}=12$

Objective 6 *Application Problems*

61. Find the width of a rectangle that has a diagonal of 10 ft and a length of 8 ft.

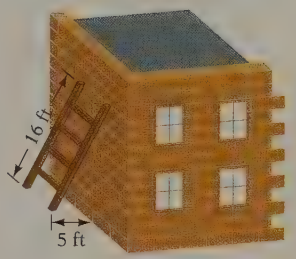


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62. Find the length of a rectangle that has a diagonal of 15 m and a width of 9 m.

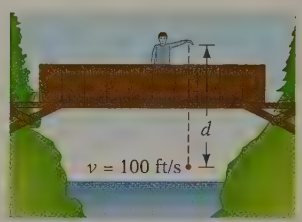
63. A 26-foot ladder is leaning against a building. How far is the bottom of the ladder from the building when the ladder reaches a height of 24 ft on the building?

64. A 16-foot ladder is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 5 ft from the building? Round to the nearest tenth.



65. An object is dropped from an airplane. Find the distance the object has fallen when its speed reaches 400 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance.

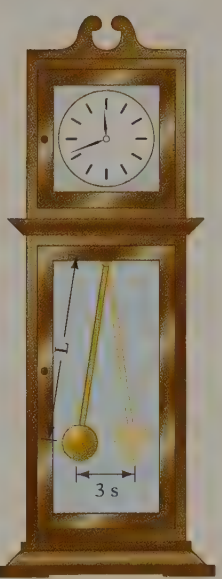
66. An object is dropped from a bridge. Find the distance the object has fallen when its speed reaches 100 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance.



67. How far would a submarine periscope have to be above the water for the lookout to locate a ship 3.6 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water.

68. How far would a submarine periscope have to be above the water for the lookout to locate a ship 4.2 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water.

69. Find the length of a pendulum on a clock that makes one swing in 2.4 s. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.



70. Find the length of a pendulum that makes one swing in 3 s. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

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71. Find the distance required for a car to reach a velocity of 60 m/s when the acceleration is 10 m/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity, a is the acceleration, and s is the distance.

72. The time it takes for an object to fall a certain distance is given by the equation $t = \sqrt{\frac{2d}{g}}$, where t is the time in seconds, d is the distance in feet, and g is the acceleration due to gravity. The acceleration due to gravity on Earth is 32 feet per second. If an object is dropped from the top of a tall building, how far will it fall in 6 s?

73. High definition television (HDTV) gives consumers a wider viewing area, more like a film in a theater. A regular television with a 27-inch diagonal measurement has a screen 16.2 in. tall. An HDTV screen with the same 16.2-inch height would have a diagonal measuring 33 in. How many inches wider is the HDTV screen? Round to the nearest hundredth.



74. At what height above Earth's surface would a satellite be in orbit if it is traveling at a speed of 7500 m/s? Use the equation $v = \sqrt{\frac{4 \times 10^{14}}{h + 6.4 \times 10^6}}$, where v is the speed of the satellite in meters per second and h is the height above Earth's surface in meters. Round to the nearest thousand.

APPLYING THE CONCEPTS

75. Solve the following equations. Describe the solution by using the following terms: integer, rational number, irrational number, real number, and imaginary number. Note that more than one term may be used to describe the answer.

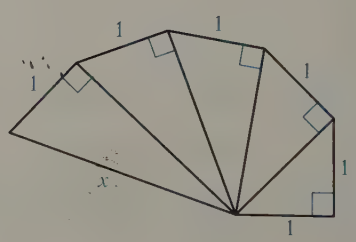
a. $x^2 + 3 = 7$ **b.** $x^2 + 1 = 0$ **c.** $x^{3/4} = 8$

76. Solve: $\sqrt{3x - 2} = \sqrt{2x - 3} + \sqrt{x - 1}$

77. Solve $a^2 + b^2 = c^2$ for a.

78. Solve $V = \frac{4}{3}\pi r^3$ for r.

79. Find the length of the side labeled x in the diagram at the right.



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Focus on Problem Solving

Another Look at Polya's Four-Step Process

Polya's four general steps to follow when attempting to solve a problem are to understand the problem, devise a plan, carry out the plan, and review the solution. (See the Focus on Problem Solving in the chapter entitled "Factoring.") In the process of devising a plan (Step 2), it may be appropriate to write a mathematical expression or an equation. We will illustrate this with the following problem.

Number the letters of the alphabet in sequence from 1 to 26. (See the list at the left.) Find a word for which the product of the numerical values of the letters of the word equals 1,000,000. We will agree that a "word" is any sequence of letters that contains at least one vowel; it need not be in the dictionary.

A = 1
B = 2
C = 3
D = 4
E = 5
F = 6
G = 7
H = 8
I = 9
J = 10
K = 11
L = 12
M = 13
N = 14
O = 15
P = 16
Q = 17
R = 18
S = 19
T = 20
U = 21
V = 22
W = 23
X = 24
Y = 25
Z = 26

1. Understand the Problem

Consider REZB. The product of the values of the letters is $18 \cdot 5 \cdot 26 \cdot 2 = 4680$. This "word" is a sequence of letters with at least one vowel. However, the product of the numerical values of the letters is not 1,000,000. Thus this word does not solve our problem.

2. Devise a Plan

Actually, we should have known that the product of the values of the letters in REZB could not equal 1,000,000. The letter R has a factor of 9, and the letter Z has a factor of 13. Neither of these two numbers is a factor of 1,000,000. Consequently, R and Z cannot be letters in the word we are trying to find. This observation leads to an important observation: Each of the letters that make up our word must be a factor of 1,000,000. To find these letters, consider the prime factorization of 1,000,000.

$$1{,}000{,}000 = 2^6 \cdot 5^6$$

Looking at the prime factorization, we note that only letters that contain 2 or 5 as factors are possible candidates. These letters are B, D, E, H, J, P, T, and Y. One additional point: Because 1 times any number is the number, the letter A can be part of any word we construct.

Our task is now to construct a word from these letters such that the product is 1,000,000. From the prime factorization above, we must have 2 as a factor six times and 5 as a factor six times.

3. Carry Out the Plan

We must construct a word with the characteristics described in our plan. Here is a possibility:

THEBEYE

4. Review Your Solution

You should multiply the values of all the letters and verify that the product is 1,000,000. To ensure that you have an understanding of the problem, try to find other "words" that satisfy the conditions of the problem.

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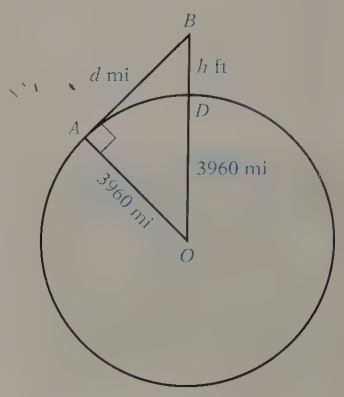
Projects and Group Activities

Distance to the Horizon

In Section 9.4, we used the formula $d = \sqrt{1.5h}$ to calculate the approximate distance d (in miles) that a person using a periscope h feet above the water could see. That formula is derived by using the Pythagorean Theorem.

Consider the diagram (not to scale) at the right, which shows Earth as a sphere and the periscope extending h feet above the surface. From geometry, because AB is tangent to the circle and OA is a radius, triangle AOB is a right triangle. Therefore,

$$(OA)^2 + (AB)^2 = (OB)^2$$



Substituting into this formula, we have

$$3960^2 + d^2 = \left(3960 + \frac{h}{5280}\right)^2$$

• Because h is in feet, $\frac{h}{5280}$ is in miles.

$$3960^2 + d^2 = 3960^2 + \frac{2 \cdot 3960}{5280}h + \left(\frac{h}{5280}\right)^2$$

$$d^2 = \frac{3}{2}h + \left(\frac{h}{5280}\right)^2$$

$$d = \sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2}$$

At this point, an assumption is made that $\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$, where we have written $\frac{3}{2}$ as 1.5. Thus $d \approx \sqrt{1.5h}$ is used to approximate the distance that can be seen using a periscope h feet above the water.

1. Write a paragraph that justifies the assumption that

$$\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$$

(*Suggestion:* Evaluate each expression for various values of h. Because h is the height of a periscope above the water, it is unlikely that $h > 25$ ft.)

2. The distance d is the distance from the top of the periscope to A. The distance along the surface of the water is given by arc AD. This distance, L, can be approximated by the equation

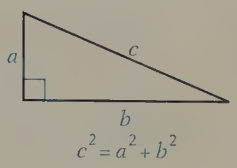
$$L \approx \sqrt{1.5h} + 0.306186\left(\sqrt{\frac{h}{5280}}\right)^3$$

Using this formula, calculate L when $h = 10$.

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Pythagorean Triples

The Pythagorean Theorem states that if a and b are the legs of a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.



For instance, the triangle with legs 3 and 4 and hypotenuse 5 is a right triangle because $3^2 + 4^2 = 5^2$. The numbers 3, 4, and 5 are called a **Pythagorean triple** because they are natural numbers that satisfy the equation of the Pythagorean Theorem.

1. Determine whether the numbers are a Pythagorean triple.
a. 5, 7, and 9
b. 8, 15, and 17
c. 11, 60, and 61
d. 28, 45, and 53

Mathematicians have investigated Pythagorean triples and have found formulas that will generate these triples. One such formula is

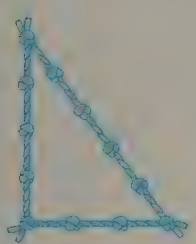
$$a = m^2 - n^2 \qquad b = 2mn \qquad c = m^2 + n^2, \text{ where } m > n$$

For instance, let $m = 2$ and $n = 1$. Then $a = 2^2 - 1^2 = 3$, $b = 2(2)(1) = 4$, and $c = 2^2 + 1^2 = 5$. This is the Pythagorean triple given above.

2. Find the Pythagorean triple produced by each of the following.
a. $m = 3$ and $n = 1$
b. $m = 5$ and $n = 2$
c. $m = 4$ and $n = 2$
d. $m = 6$ and $n = 1$

3. Find values of m and n that yield the Pythagorean triple 11, 60, 61.

4. Verify that $a^2 + b^2 = c^2$ when $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$.



5. The early Greek builders used a rope with 12 equally spaced knots to make right-angle corners for buildings. Explain how they used the rope.

6. Find three odd integers, a, b, c, such that $a^2 + b^2 = c^2$.

Chapter Summary

Key Words

The *nth root of a* is $a^{1/n}$. The expression $\sqrt[n]{a}$ is another symbol for the nth root of a. [pp. 497, 499]

In the expression $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called a *radical sign*, n is the *index*, and a is the *radicand*. [p. 499]

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The symbol $\sqrt{\ }$ is used to indicate the positive or *principal square root* of a number. [p. 501]

The expressions $a + b$ and $a - b$ are called *conjugates* of each other. The product of conjugates of the form $(a + b)(a - b)$ is $a^2 - b^2$. [p. 510]

The procedure used to remove a radical from the denominator of a radical expression is called *rationalizing the denominator.* [p. 511]

A *complex number* is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. For the complex number $a + bi$, a is the *real part* of the complex number, and b is the *imaginary part* of the complex number. [p. 517]

A *radical equation* is an equation that contains a variable expression in a radicand. [p. 525]

Essential Rules

Rule for Rational Exponents [p. 497]

If m and n are positive integers and $a^{1/n}$ is a real number, then $a^{m/n} = (a^{1/n})^m$.

Rule for Writing Exponential Expressions as Radical Expressions [p. 499]

If $a^{1/n}$ is a real number, then $a^{m/n} = \sqrt[n]{a^m}$.

The Product Property of Radicals [p. 507]

If a and b are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$.

The Quotient Property of Radicals [p. 511]

If a and b are positive real numbers, then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

Definition of $\sqrt{-a}$ [p. 517]

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$: $\sqrt{-a} = i\sqrt{a}$.

Addition of Complex Numbers [p. 518]

If $a + bi$ and $c + di$ are complex numbers, then
$(a + bi) + (c + di) = (a + c) + (b + d)i$.

Subtraction of Complex Numbers [p. 518]

If $a + bi$ and $c + di$ are complex numbers, then
$(a + bi) - (c + di) = (a - c) + (b - d)i$.

The Property of Raising Each Side of an Equation to a Power [p. 525]

If a and b are real numbers and $a = b$, then $a^n = b^n$.

The Pythagorean Theorem [p. 527]

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.
$c^2 = a^2 + b^2$

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Chapter Review

1. Simplify: $(16x^{-4}y^{12})^{1/4}(100x^6y^{-2})^{1/2}$

2. Solve: $\sqrt[4]{3x-5} = 2$

3. Multiply: $(6 - 5i)(4 + 3i)$

4. Rewrite $7y\sqrt[3]{x^2}$ as an exponential expression.

5. Multiply: $(\sqrt{3} + 8)(\sqrt{3} - 2)$

6. Solve: $\sqrt{4x + 9} + 10 = 11$

7. Simplify: $\dfrac{x^{-3/2}}{x^{7/2}}$

8. Simplify: $\dfrac{8}{\sqrt{3y}}$

9. Simplify: $\sqrt[3]{-8a^6b^{12}}$

10. Subtract: $\sqrt{50a^4b^3} - ab\sqrt{18a^2b}$

11. Simplify: $\dfrac{x+2}{\sqrt{x}+\sqrt{2}}$

12. Simplify: $\dfrac{5+2i}{3i}$

13. Simplify: $\sqrt{18a^3b^6}$

14. Subtract: $(\sqrt{50} + \sqrt{-72}) - (\sqrt{162} - \sqrt{-8})$

15. Subtract: $3x\sqrt[3]{54x^8y^{10}} - 2x^2y\sqrt[3]{16x^5y^7}$

16. Multiply: $\sqrt[3]{16x^4y}\,\sqrt[3]{4xy^5}$

17. Multiply: $i(3 - 7i)$

18. Rewrite $3x^{3/4}$ as a radical expression.

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19. Simplify: $\sqrt[5]{-64a^8b^{12}}$

20. Simplify: $\frac{5 + 9i}{1 - i}$

21. Multiply: $\sqrt{-12}\sqrt{-6}$

22. Solve: $\sqrt{x - 5} + \sqrt{x + 6} = 11$

23. Simplify: $\sqrt[4]{81a^8b^{12}}$

24. Simplify: $\sqrt{-50}$

25. Subtract: $(-8 + 3i) - (4 - 7i)$

26. Simplify: $(5 - \sqrt{6})^2$

27. Simplify: $4x\sqrt{12x^2y} + \sqrt{3x^4y} - x^2\sqrt{27y}$

28. The velocity of the wind determines the amount of power generated by a windmill. A typical equation for this relationship is $v = 4.05\sqrt[3]{P}$, where v is the velocity in miles per hour and P is the power in watts. Find the amount of power generated by a 20-mph wind. Round to the nearest whole number.

29. Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 16 ft/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

30. A 12-foot ladder is leaning against a building. How far from the building is the bottom of the ladder when the top of the ladder touches the building 10 ft above the ground? Round to the nearest hundredth.

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Chapter Test

1. Write $\frac{1}{2}\sqrt[4]{x^3}$ as an exponential expression.

2. Subtract: $\sqrt[3]{54x^7y^3} - x\sqrt[3]{128x^4y^3} - x^2\sqrt[3]{2xy^3}$

3. Write $3y^{2/5}$ as a radical expression.

4. Multiply: $(2 + 5i)(4 - 2i)$

5. Simplify: $(2\sqrt{x} + \sqrt{y})^2$

6. Simplify: $\frac{r^{2/3}r^{-1}}{r^{-1/2}}$

7. Solve: $\sqrt{x + 12} - \sqrt{x} = 2$

8. Simplify: $\sqrt[3]{8x^3y^6}$

9. Multiply: $\sqrt{3x}(\sqrt{x} - \sqrt{25x})$

10. Subtract: $(5 - 2i) - (8 - 4i)$

11. Simplify: $\sqrt{32x^4y^7}$

12. Multiply: $(2\sqrt{3} + 4)(3\sqrt{3} - 1)$

13. Add: $(2 + i) + (2 - i)$

14. Simplify: $\frac{4 - 2\sqrt{5}}{2 - \sqrt{5}}$

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15. Add: $\sqrt{18a^3} + a\sqrt{50a}$

16. Multiply: $(\sqrt{a} - 3\sqrt{b})(2\sqrt{a} + 5\sqrt{b})$

17. Simplify: $\dfrac{(2x^{1/3}y^{-2/3})^6}{(x^{-4}y^8)^{1/4}}$

18. Simplify: $\dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{y}}$

19. Simplify: $\dfrac{2 + 3i}{1 - 2i}$

20. Solve: $\sqrt[3]{2x - 2} + 4 = 2$

21. Simplify: $\left(\dfrac{4a^4}{b^2}\right)^{-3/2}$

22. Simplify: $\sqrt[3]{27a^4b^3c^7}$

23. Simplify: $\dfrac{\sqrt{32x^5y}}{\sqrt{2xy^3}}$

24. Multiply: $(\sqrt{-8})(\sqrt{-2})$

25. An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 192 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

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Cumulative Review

1. Simplify: $2^3 \cdot 3 - 4(3 - 4 \cdot 5)$

2. Evaluate $4a^2b - a^3$ when $a = -2$ and $b = 3$.

3. Simplify: $-3(4x - 1) - 2(1 - x)$

4. Solve: $5 - \frac{2}{3}x = 4$

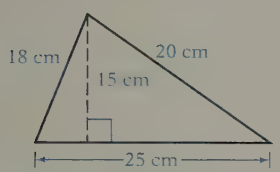
5. Solve: $2[4 - 2(3 - 2x)] = 4(1 - x)$

6. Solve: $6x - 3(2x + 2) > 3 - 3(x + 2)$

7. Solve: $2 + |4 - 3x| = 5$

8. Solve: $|2x + 3| \leq 9$

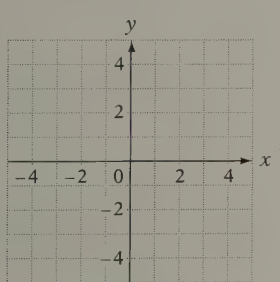
9. Find the area of the triangle shown in the figure below.



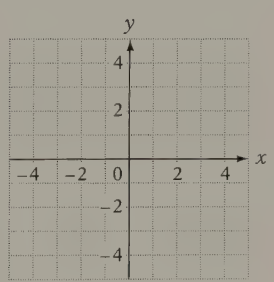
10. Find the value of the determinant:

$$\begin{vmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 3 & 1 & -2 \end{vmatrix}$$

11. Graph $3x - 2y = -6$. State the slope and y-intercept.



12. Graph the solution set of $3x + 2y \leq 4$.



13. Find the equation of the line that passes through the points (2, 3) and (−1, 2).

14. Solve by using Cramer's Rule:

$$2x - y = 4$$
$$-2x + 3y = 5$$

15. Simplify: $(2^{-1}x^2y^{-6})(2^{-1}y^{-4})^{-2}$

16. Factor: $81x^2 - y^2$

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17. Factor: $x^5 + 2x^3 - 3x$

18. Solve $P = \dfrac{R - C}{n}$ for C.

19. Simplify: $\left(\dfrac{x^{-2/3}y^{1/2}}{y^{-1/3}}\right)^6$

20. Subtract: $\sqrt{40x^3} - x\sqrt{90x}$

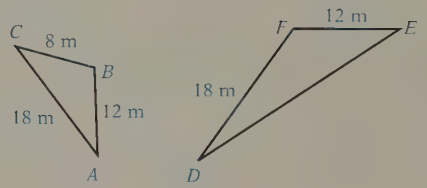
21. Multiply: $(\sqrt{3} - 2)(\sqrt{3} - 5)$

22. Simplify: $\dfrac{4}{\sqrt{6} - \sqrt{2}}$

23. Simplify: $\dfrac{2i}{3 - i}$

24. Solve: $\sqrt[3]{3x - 4} + 5 = 1$

25. The two triangles are similar triangles. Find the length of side DE.

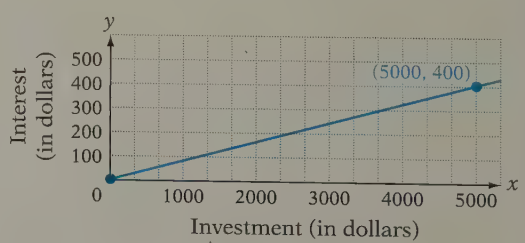


26. An investment of \$2500 is made at an annual simple interest rate of 7.2%. How much additional money must be invested at an annual simple interest rate of 8.4% so that the total interest earned is \$516?

27. A sales executive traveled 25 mi by car and then an additional 625 mi by plane. The rate of the plane was five times the rate of the car. The total time of the trip was 3 h. Find the rate of the plane.

28. How long does it take light to travel to Earth from the moon when the moon is 232,500 mi from Earth? Light travels 1.86×10^5 mi/s.

29. The graph shows the amount invested and the annual interest earned on the investment. Find the slope of the line between the two points shown on the graph. Then write a sentence that states the meaning of the slope.



30. How far would a submarine periscope have to be above the water for the lookout to locate a ship 7 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest tenth of a foot.

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Chapter 10 Quadratic Equations



A rainstorm like the one shown here directly affects highway driving conditions. How quickly a car is able to stop once the brakes are applied is determined by many factors, including how wet the road is, the type of pavement, and the speed of the car. Some factors, such as the tread on a car's tires, can be controlled, and good tread can minimize the adverse effects of braking on dangerously wet roads. The braking distance of a car can be modeled by quadratic equations such as the ones in **Exercises 5 and 9 on page 573.**

Objectives

Section 10.1

A To solve a quadratic equation by factoring
B To write a quadratic equation given its solutions
C To solve a quadratic equation by taking square roots

Section 10.2

A To solve a quadratic equation by completing the square

Section 10.3

A To solve a quadratic equation by using the quadratic formula

Section 10.4

A To solve an equation that is quadratic in form
B To solve a radical equation that is reducible to a quadratic equation
C To solve a fractional equation that is reducible to a quadratic equation

Section 10.5

A To solve application problems

Section 10.6

A To solve a nonlinear inequality

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Prep Test

1. Simplify: $\sqrt{18}$

2. Simplify: $\sqrt{-9}$

3. Simplify: $\dfrac{3x-2}{x-1}-1$

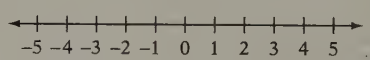
4. Evaluate b^2-4ac when $a=2$, $b=-4$, and $c=1$.

5. Is $4x^2+28x+49$ a perfect-square trinomial?

6. Factor: $4x^2-4x+1$

7. Factor: $9x^2-4$

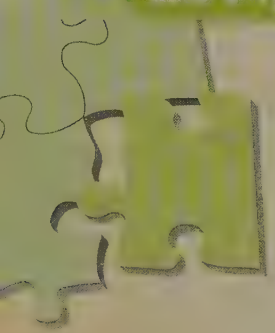
8. Graph: $\{x|x<-1\}\cap\{x|x<4\}$



9. Solve: $x(x-1)=x+15$

10. Solve: $\dfrac{4}{x-3}=\dfrac{16}{x}$

Go Figure



The numeral $0.AAA\ldots$ is a repeating decimal. Given that $A\neq B$ and $\sqrt{0.AAA\ldots}=\sqrt{B}$, find the value of A.

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10.1 Solving Quadratic Equations by Factoring or by Taking Square Roots

Objective A: To solve a quadratic equation by factoring



Recall that a **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$.

Quadratic equations

$3x^2 - x + 2 = 0$, $a = 3$, $b = -1$, $c = 2$

$-x^2 + 4 = 0$, $a = -1$, $b = 0$, $c = 4$

$6x^2 - 5x = 0$, $a = 6$, $b = -5$, $c = 0$

A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. Because the degree of the polynomial $ax^2 + bx + c$ is 2, a quadratic equation is also called a **second-degree equation**.

As we discussed earlier, quadratic equations sometimes can be solved by using the Principle of Zero Products. This method is reviewed here.

The Principle of Zero Products

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors must be zero.

TAKE NOTE

Recall that the steps involved in solving a quadratic equation by factoring are

1. Write the equation in standard form.
2. Factor.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

➡ Solve by factoring: $3x^2 = 2 - 5x$

$$3x^2 = 2 - 5x$$
$$3x^2 + 5x - 2 = 0$$
$$(3x - 1)(x + 2) = 0$$
$$3x - 1 = 0 \qquad x + 2 = 0$$
$$3x = 1 \qquad x = -2$$
$$x = \frac{1}{3}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products to write two equations.
- Solve each equation.

$\frac{1}{3}$ and -2 check as solutions. The solutions are $\frac{1}{3}$ and -2.

TAKE NOTE

When a quadratic equation has two solutions that are the same number, the solution is called a **double root** of the equation. 3 is a double root of $x^2 - 6x = -9$.

➡ Solve by factoring: $x^2 - 6x = -9$

$$x^2 - 6x = -9$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)(x - 3) = 0$$
$$x - 3 = 0 \qquad x - 3 = 0$$
$$x = 3 \qquad x = 3$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products.
- Solve each equation.

3 checks as a solution. The solution is 3.

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Example 1
Solve by factoring: $2x(x - 3) = x + 4$

Solution

$$2x(x - 3) = x + 4$$
$$2x^2 - 6x = x + 4$$
$$2x^2 - 7x - 4 = 0$$
$$(2x + 1)(x - 4) = 0$$
$$2x + 1 = 0 \qquad x - 4 = 0$$
$$2x = -1 \qquad x = 4$$
$$x = -\frac{1}{2}$$

The solutions are $-\frac{1}{2}$ and 4.

You Try It 1
Solve by factoring: $2x^2 = 7x - 3$

Your solution

Example 2
Solve for x by factoring: $x^2 - 4ax - 5a^2 = 0$

Solution
This is a literal equation. Solve for x in terms of a.

$$x^2 - 4ax - 5a^2 = 0$$
$$(x + a)(x - 5a) = 0$$
$$x + a = 0 \qquad x - 5a = 0$$
$$x = -a \qquad x = 5a$$

The solutions are $-a$ and $5a$.

You Try It 2
Solve for x by factoring: $x^2 - 3ax - 4a^2 = 0$

Your solution

Solutions on p. S30

Objective C

To write a quadratic equation given its solutions

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As shown below, the solutions of the equation $(x - r_1)(x - r_2) = 0$ are r_1 and r_2.

$$(x - r_1)(x - r_2) = 0$$
$$x - r_1 = 0 \qquad x - r_2 = 0$$
$$x = r_1 \qquad x = r_2$$

Check:

$$\frac{(x - r_1)(x - r_2) = 0}{(r_1 - r_1)(r_1 - r_2) \mid 0}$$
$$0 \cdot (r_1 - r_2) \mid 0$$
$$0 = 0$$

$$\frac{(x - r_1)(x - r_2) = 0}{(r_2 - r_1)(r_2 - r_2) \mid 0}$$
$$(r_2 - r_1) \cdot 0 \mid 0$$
$$0 = 0$$

Using the equation $(x - r_1)(x - r_2) = 0$ and the fact that r_1 and r_2 are solutions of this equation, it is possible to write a quadratic equation given its solutions.

Write a quadratic equation that has solutions 4 and -5.

$$(x - r_1)(x - r_2) = 0$$
$$(x - 4)[x - (-5)] = 0 \quad \text{• Replace } r_1 \text{ by 4 and } r_2 \text{ by } -5.$$
$$(x - 4)(x + 5) = 0 \quad \text{• Simplify.}$$
$$x^2 + x - 20 = 0 \quad \text{• Multiply.}$$

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Write a quadratic equation with integer coefficients and solutions $\frac{2}{3}$ and $\frac{1}{2}$.

$$(x - r_1)(x - r_2) = 0$$
- Replace r_1 by $\frac{2}{3}$ and r_2 by $\frac{1}{2}$.

$$\left(x - \frac{2}{3}\right)\left(x - \frac{1}{2}\right) = 0$$

$$x^2 - \frac{7}{6}x + \frac{1}{3} = 0$$
- Multiply.

$$6\left(x^2 - \frac{7}{6}x + \frac{1}{3}\right) = 6 \cdot 0$$
- Multiply each side of the equation by the LCM of the denominators.

$$6x^2 - 7x + 2 = 0$$

Example 3

Write a quadratic equation with integer coefficients and solutions $\frac{1}{2}$ and -4.

Solution

$$(x - r_1)(x - r_2) = 0$$

$$\left(x - \frac{1}{2}\right)[x - (-4)] = 0$$

$$\left(x - \frac{1}{2}\right)(x + 4) = 0$$

$$x^2 + \frac{7}{2}x - 2 = 0$$

$$2\left(x^2 + \frac{7}{2}x - 2\right) = 2 \cdot 0$$

$$2x^2 + 7x - 4 = 0$$

You Try It 3

Write a quadratic equation with integer coefficients and solutions 3 and $-\frac{1}{2}$.

Your solution

Solution on p. S30

Objective C — To solve a quadratic equation by taking square roots



The solution of the quadratic equation $x^2 = 16$ is shown at the right.

$$x^2 = 16$$
$$x^2 - 16 = 0$$
$$(x - 4)(x + 4) = 0$$
$$x - 4 = 0 \qquad x + 4 = 0$$
$$x = 4 \qquad x = -4$$

The solutions can also be found by taking the square root of each side of the equation and writing the positive and negative square roots of the number. The notation $x = \pm 4$ means $x = 4$ or $x = -4$.

$$x^2 = 16$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = \pm 4$$

The solutions are 4 and -4.

Solve by taking square roots: $3x^2 = 54$

$$3x^2 = 54$$
- Solve for x^2.

$$x^2 = 18$$
- Take the square root of each side of the equation.

$$\sqrt{x^2} = \sqrt{18}$$
- Simplify.

$$x = \pm\sqrt{18} = \pm 3\sqrt{2}$$
- $3\sqrt{2}$ and $-3\sqrt{2}$ check as solutions.

The solutions are $3\sqrt{2}$ and $-3\sqrt{2}$.

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Solving a quadratic equation by taking the square root of each side of the equation can lead to solutions that are complex numbers.

Solve by taking square roots: $2x^2 + 18 = 0$

$$2x^2 + 18 = 0$$
$$2x^2 = -18$$ • Solve for x^2.
$$x^2 = -9$$
$$\sqrt{x^2} = \sqrt{-9}$$ • Take the square root of each side of the equation.
$$x = \pm\sqrt{-9}$$ • Simplify.
$$x = \pm 3i$$

Check:

$2x^2 + 18 = 0$	
$2(3i)^2 + 18$	0
$2(-9) + 18$	0
$-18 + 18$	0
$0 = 0$	

$2x^2 + 18 = 0$	
$2(-3i)^2 + 18$	0
$2(-9) + 18$	0
$-18 + 18$	0
$0 = 0$	

The solutions are $3i$ and $-3i$.

An equation containing the square of a binomial can be solved by taking square roots.

Solve by taking square roots: $(x + 2)^2 - 24 = 0$

$$(x + 2)^2 - 24 = 0$$
$$(x + 2)^2 = 24$$ • Solve for $(x + 2)^2$.
$$\sqrt{(x + 2)^2} = \sqrt{24}$$ • Take the square root of each side of the equation. Then simplify.
$$x + 2 = \pm\sqrt{24}$$
$$x + 2 = \pm 2\sqrt{6}$$
$$x + 2 = 2\sqrt{6} \qquad x + 2 = -2\sqrt{6}$$ • Solve for x.
$$x = -2 + 2\sqrt{6} \qquad x = -2 - 2\sqrt{6}$$

The solutions are $-2 + 2\sqrt{6}$ and $-2 - 2\sqrt{6}$.

Example 4

Solve by taking square roots:
$3(x - 2)^2 + 12 = 0$

Solution

$$3(x - 2)^2 + 12 = 0$$
$$3(x - 2)^2 = -12$$
$$(x - 2)^2 = -4$$
$$\sqrt{(x - 2)^2} = \sqrt{-4}$$
$$x - 2 = \pm\sqrt{-4}$$
$$x - 2 = \pm 2i$$
$$x - 2 = 2i \qquad x - 2 = -2i$$
$$x = 2 + 2i \qquad x = 2 - 2i$$

The solutions are $2 + 2i$ and $2 - 2i$.

You Try It 4

Solve by taking square roots:
$2(x + 1)^2 - 24 = 0$

Your solution

Solution on p. S30

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10.1 Exercises

Objective A

1. Explain why the restriction $a \neq 0$ is necessary in the definition of a quadratic equation.

2. What does the Principle of Zero Products state? How is it used to solve a quadratic equation?

Write the quadratic equation in standard form with the coefficient of x^2 positive. Give the values of a, b, and c.

3. $2x^2 - 4x = 5$ **4.** $x^2 = 3x + 1$ **5.** $5x = 4x^2 + 6$ **6.** $3x^2 = 7$

Solve by factoring.

7. $x^2 - 4x = 0$ **8.** $y^2 + 6y = 0$ **9.** $t^2 - 25 = 0$

10. $p^2 - 81 = 0$ **11.** $s^2 - s - 6 = 0$ **12.** $v^2 + 4v - 5 = 0$

13. $y^2 - 6y + 9 = 0$ **14.** $x^2 + 10x + 25 = 0$ **15.** $9z^2 - 18z = 0$

16. $4y^2 + 20y = 0$ **17.** $r^2 - 3r = 10$ **18.** $p^2 + 5p = 6$

19. $v^2 + 10 = 7v$ **20.** $t^2 - 16 = 15t$ **21.** $2x^2 - 9x - 18 = 0$

22. $3y^2 - 4y - 4 = 0$ **23.** $4z^2 - 9z + 2 = 0$ **24.** $2s^2 - 9s + 9 = 0$

25. $3w^2 + 11w = 4$ **26.** $2r^2 + r = 6$ **27.** $6x^2 = 23x + 18$

28. $6x^2 = 7x - 2$ **29.** $4 - 15u - 4u^2 = 0$ **30.** $3 - 2y - 8y^2 = 0$

31. $x + 18 = x(x - 6)$ **32.** $t + 24 = t(t + 6)$ **33.** $4s(s + 3) = s - 6$

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34. $3v(v - 2) = 11v + 6$

35. $u^2 - 2u + 4 = (2u - 3)(u + 2)$

36. $(3v - 2)(2v + 1) = 3v^2 - 11v - 10$

37. $(3x - 4)(x + 4) = x^2 - 3x - 28$

Solve for x by factoring.

38. $x^2 + 14ax + 48a^2 = 0$

39. $x^2 - 9bx + 14b^2 = 0$

40. $x^2 + 9xy - 36y^2 = 0$

41. $x^2 - 6cx - 7c^2 = 0$

42. $x^2 - ax - 20a^2 = 0$

43. $2x^2 + 3bx + b^2 = 0$

44. $3x^2 - 4cx + c^2 = 0$

45. $3x^2 - 14ax + 8a^2 = 0$

46. $3x^2 - 11xy + 6y^2 = 0$

47. $3x^2 - 8ax - 3a^2 = 0$

48. $3x^2 - 4bx - 4b^2 = 0$

49. $4x^2 + 8xy + 3y^2 = 0$

50. $6x^2 - 11cx + 3c^2 = 0$

51. $6x^2 + 11ax + 4a^2 = 0$

52. $12x^2 - 5xy - 2y^2 = 0$

Objective B

Write a quadratic equation that has integer coefficients and has as solutions the given pair of numbers.

53. 2 and 5

54. 3 and 1

55. -2 and -4

56. -1 and -3

57. 6 and -1

58. -2 and 5

59. 3 and -3

60. 5 and -5

61. 4 and 4

62. 2 and 2

63. 0 and 5

64. 0 and -2

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65. 0 and 3

66. 0 and -1

67. 3 and $\frac{1}{2}$

68. 2 and $\frac{2}{3}$

69. $-\frac{3}{4}$ and 2

70. $-\frac{1}{2}$ and 5

71. $-\frac{5}{3}$ and -2

72. $-\frac{3}{2}$ and -1

73. $-\frac{2}{3}$ and $\frac{2}{3}$

74. $-\frac{1}{2}$ and $\frac{1}{2}$

75. $\frac{1}{2}$ and $\frac{1}{3}$

76. $\frac{3}{4}$ and $\frac{2}{3}$

77. $\frac{6}{5}$ and $-\frac{1}{2}$

78. $\frac{3}{4}$ and $-\frac{3}{2}$

79. $-\frac{1}{4}$ and $-\frac{1}{2}$

80. $-\frac{5}{6}$ and $-\frac{2}{3}$

81. $\frac{3}{5}$ and $-\frac{1}{10}$

82. $\frac{7}{2}$ and $-\frac{1}{4}$

Solve by taking square roots.

83. $y^2 = 49$

84. $x^2 = 64$

85. $z^2 = -4$

86. $v^2 = -16$

87. $s^2 - 4 = 0$

88. $r^2 - 36 = 0$

89. $4x^2 - 81 = 0$

90. $9x^2 - 16 = 0$

91. $y^2 + 49 = 0$

92. $z^2 + 16 = 0$

93. $v^2 - 48 = 0$

94. $s^2 - 32 = 0$

95. $r^2 - 75 = 0$

96. $u^2 - 54 = 0$

97. $z^2 + 18 = 0$

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98. $t^2 + 27 = 0$

99. $(x - 1)^2 = 36$

100. $(x + 2)^2 = 25$

101. $3(y + 3)^2 = 27$

102. $4(s - 2)^2 = 36$

103. $5(z + 2)^2 = 125$

104. $2(y - 3)^2 = 18$

105. $\left(v - \frac{1}{2}\right)^2 = \frac{1}{4}$

106. $\left(r + \frac{2}{3}\right)^2 = \frac{1}{9}$

107. $(x + 5)^2 - 6 = 0$

108. $(t - 1)^2 - 15 = 0$

109. $(v - 3)^2 + 45 = 0$

110. $(x + 5)^2 + 32 = 0$

111. $\left(u - \frac{2}{3}\right)^2 - 18 = 0$

112. $\left(z - \frac{1}{2}\right)^2 - 20 = 0$

APPLYING THE CONCEPTS

Write a quadratic equation that has as solutions the given pair of numbers.

113. $\sqrt{2}$ and $-\sqrt{2}$

114. $2i$ and $-2i$

115. $3\sqrt{2}$ and $-3\sqrt{2}$

116. $2i\sqrt{3}$ and $-2i\sqrt{3}$

Solve for x.

117. $4a^2x^2 = 36b^2, a > 0, b > 0$

118. $(x + a)^2 - 4 = 0$

119. $(2x - 1)^2 = (2x + 3)^2$

120. Show that the solutions of the equation $ax^2 + bx = 0, a > 0, b > 0$, are 0 and $-\frac{b}{a}$.

121. Show that the solutions of the equation $ax^2 + c = 0, a > 0, c > 0$, are $\frac{\sqrt{ca}}{a}i$ and $-\frac{\sqrt{ca}}{a}i$.

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10.2 Solving Quadratic Equations by Completing the Square

Objective A

To solve a quadratic equation by completing the square

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Recall that a perfect-square trinomial is the square of a binomial.

Perfect-Square Trinomial		*Square of a Binomial*
$x^2 + 8x + 16$	=	$(x + 4)^2$
$x^2 - 10x + 25$	=	$(x - 5)^2$
$x^2 + 2ax + a^2$	=	$(x + a)^2$

For each perfect-square trinomial, the square of $\frac{1}{2}$ of the coefficient of x equals the constant term.

$$\left(\frac{1}{2}\text{ coefficient of } x\right)^2 = \text{constant term}$$

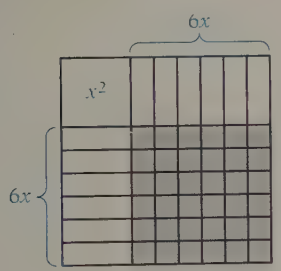
$x^2 + 8x + 16$, $\left(\frac{1}{2} \cdot 8\right)^2 = 16$

$x^2 - 10x + 25$, $\left[\frac{1}{2}(-10)\right]^2 = 25$

$x^2 + 2ax + a^2$, $\left(\frac{1}{2} \cdot 2a\right)^2 = a^2$

Point of Interest

Early attempts to solve quadratic equations were primarily geometric. The Persian mathematician al-Khowarizmi (c. A.D. 800) essentially completed a square of $x^2 + 12x$ as follows.



This relationship can be used to write the constant term for a perfect-square trinomial. Adding to a binomial the constant term that makes it a perfect-square trinomial is called **completing the square**.

➡ Complete the square on $x^2 + 12x$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left[\frac{1}{2}(12)\right]^2 = (6)^2 = 36$ • Find the constant term.

$x^2 + 12x + 36$ • Complete the square on $x^2 + 12x$ by adding the constant term.

$x^2 + 12x + 36 = (x + 6)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

➡ Complete the square on $z^2 - 3z$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left[\frac{1}{2} \cdot (-3)\right]^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$ • Find the constant term.

$z^2 - 3z + \frac{9}{4}$ • Complete the square on $z^2 - 3z$ by adding the constant term.

$z^2 - 3z + \frac{9}{4} = \left(z - \frac{3}{2}\right)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

Any quadratic equation can be solved by completing the square. Add to each side of the equation the term that completes the square. Rewrite the equation in the form $(x + a)^2 = b$. Then take the square root of each side of the equation.

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➡ Solve by completing the square: $x^2 - 6x - 15 = 0$

$$x^2 - 6x - 15 = 0$$

$$x^2 - 6x = 15$$ • Add 15 to each side of the equation.

$$x^2 - 6x + 9 = 15 + 9$$ • Complete the square. Add $\left[\frac{1}{2}(-6)\right]^2 = (-3)^2 = 9$ to each side of the equation.

$$(x - 3)^2 = 24$$ • Factor the perfect-square trinomial.

$$\sqrt{(x - 3)^2} = \sqrt{24}$$ • Take the square root of each side of the equation.

$$x - 3 = \pm\sqrt{24}$$ • Solve for x.

$$x - 3 = \pm 2\sqrt{6}$$

$$x - 3 = 2\sqrt{6} \qquad x - 3 = -2\sqrt{6}$$

$$x = 3 + 2\sqrt{6} \qquad x = 3 - 2\sqrt{6}$$

Check:

$$\begin{array}{r|l} x^2 - 6x - 15 = 0 & \\ \hline (3 + 2\sqrt{6})^2 - 6(3 + 2\sqrt{6}) - 15 & 0 \\ 9 + 12\sqrt{6} + 24 - 18 - 12\sqrt{6} - 15 & 0 \\ 0 = 0 & \end{array}$$

$$\begin{array}{r|l} x^2 - 6x - 15 = 0 & \\ \hline (3 - 2\sqrt{6})^2 - 6(3 - 2\sqrt{6}) - 15 & 0 \\ 9 - 12\sqrt{6} + 24 - 18 + 12\sqrt{6} - 15 & 0 \\ 0 = 0 & \end{array}$$

The solutions are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$.

TAKE NOTE
The exact solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. 7.899 and -1.899 are approximate solutions of the equation.

In the example above, the solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. These are the exact solutions. However, in some situations it may be preferable to have decimal approximations of the solutions to a quadratic equation. Approximate solutions can be found by using a calculator and then rounding to the desired degree of accuracy.

$$3 + 2\sqrt{6} \approx 7.899 \quad \text{and} \quad 3 - 2\sqrt{6} \approx -1.899$$

To the nearest thousandth, the solutions of the equation $x^2 - 6x - 15 = 0$ are 7.899 and -1.899.

➡ Solve $2x^2 - x - 2 = 0$ by completing the square. Find the exact solutions and approximate the solutions to the nearest thousandth.

In order for us to complete the square on an expression, the coefficient of the squared term must be 1. After adding the opposite of the constant term to each side of the equation, multiply each side of the equation by $\frac{1}{2}$.

$$2x^2 - x - 2 = 0$$

$$2x^2 - x = 2$$ • Add 2 to each side of the equation.

$$\frac{1}{2}(2x^2 - x) = \frac{1}{2} \cdot 2$$ • Multiply each side of the equation by $\frac{1}{2}$.

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TAKE NOTE

This example illustrates all the steps required to solve a quadratic equation by completing the square.

1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x.
7. Check the solutions.

$$x^2 - \frac{1}{2}x = 1$$

- The coefficient of x^2 is now 1.

$$x^2 - \frac{1}{2}x + \frac{1}{16} = 1 + \frac{1}{16}$$

- Complete the square. Add $\left[\frac{1}{2}\left(-\frac{1}{2}\right)\right]^2 = \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$ to each side of the equation.

$$\left(x - \frac{1}{4}\right)^2 = \frac{17}{16}$$

- Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{17}{16}}$$

- Take the square root of each side of the equation.

$$x - \frac{1}{4} = \pm\frac{\sqrt{17}}{4}$$

- Solve for x.

$$x - \frac{1}{4} = \frac{\sqrt{17}}{4} \qquad x - \frac{1}{4} = -\frac{\sqrt{17}}{4}$$

$$x = \frac{1}{4} + \frac{\sqrt{17}}{4} \qquad x = \frac{1}{4} - \frac{\sqrt{17}}{4}$$

The exact solutions are $\frac{1 + \sqrt{17}}{4}$ and $\frac{1 - \sqrt{17}}{4}$.

$$\frac{1 + \sqrt{17}}{4} \approx 1.281 \qquad \frac{1 - \sqrt{17}}{4} \approx -0.781$$

To the nearest thousandth, the solutions are 1.281 and −0.781.

Point of Interest

Mathematicians have studied quadratic equations for centuries. Many of the initial equations were a result of trying to solve a geometry problem. One of the most famous, which dates from around 500 B.C., is "squaring the circle." The question was "Is it possible to construct a square whose area is that of a given circle?" For these early mathematicians, to *construct* meant to draw with only a straightedge and a compass. It was approximately 2300 years later that mathematicians were able to prove that such a construction is impossible.

➨ Solve by completing the square: $x^2 - 3x + 5 = 0$

$$x^2 - 3x + 5 = 0$$

$$x^2 - 3x = -5$$

- Add the opposite of the constant term to each side of the equation.

$$x^2 - 3x + \frac{9}{4} = -5 + \frac{9}{4}$$

- Add to each side of the equation the term that completes the square on $x^2 - 3x$.

$$\left(x - \frac{3}{2}\right)^2 = -\frac{11}{4}$$

- Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{-\frac{11}{4}}$$

- Take the square root of each side of the equation.

$$x - \frac{3}{2} = \pm\frac{i\sqrt{11}}{2}$$

- Simplify.

$$x - \frac{3}{2} = \frac{i\sqrt{11}}{2} \qquad x - \frac{3}{2} = -\frac{i\sqrt{11}}{2}$$

- Solve for x.

$$x = \frac{3}{2} + \frac{i\sqrt{11}}{2} \qquad x = \frac{3}{2} - \frac{i\sqrt{11}}{2}$$

$\frac{3}{2} + \frac{\sqrt{11}}{2}i$ and $\frac{3}{2} - \frac{\sqrt{11}}{2}i$ check as solutions.

The solutions are $\frac{3}{2} + \frac{\sqrt{11}}{2}i$ and $\frac{3}{2} - \frac{\sqrt{11}}{2}i$.

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Example 1

Solve by completing the square:
$4x^2 - 8x + 1 = 0$

Solution

$4x^2 - 8x + 1 = 0$

$4x^2 - 8x = -1$

$\frac{1}{4}(4x^2 - 8x) = \frac{1}{4}(-1)$ • The coefficient of x^2 must be 1.

$x^2 - 2x = -\frac{1}{4}$

$x^2 - 2x + 1 = -\frac{1}{4} + 1$ • Complete the square.

$(x - 1)^2 = \frac{3}{4}$

$\sqrt{(x - 1)^2} = \sqrt{\frac{3}{4}}$

$x - 1 = \pm\frac{\sqrt{3}}{2}$

$x - 1 = \frac{\sqrt{3}}{2}$ $\qquad$ $x - 1 = -\frac{\sqrt{3}}{2}$

$x = 1 + \frac{\sqrt{3}}{2}$ $\qquad$ $x = 1 - \frac{\sqrt{3}}{2}$

$= \frac{2 + \sqrt{3}}{2}$ $\qquad$ $= \frac{2 - \sqrt{3}}{2}$

The solutions are $\frac{2 + \sqrt{3}}{2}$ and $\frac{2 - \sqrt{3}}{2}$.

You Try It 1

Solve by completing the square:
$4x^2 - 4x - 1 = 0$

Your solution

Example 2

Solve by completing the square:
$x^2 + 4x + 5 = 0$

Solution

$x^2 + 4x + 5 = 0$

$x^2 + 4x = -5$

$x^2 + 4x + 4 = -5 + 4$ • Complete the square.

$(x + 2)^2 = -1$

$\sqrt{(x + 2)^2} = \sqrt{-1}$

$x + 2 = \pm i$

$x + 2 = i$ $\qquad$ $x + 2 = -i$

$x = -2 + i$ $\qquad$ $x = -2 - i$

The solutions are $-2 + i$ and $-2 - i$.

You Try It 2

Solve by completing the square:
$2x^2 + x - 5 = 0$

Your solution

Solutions on p. S30

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10.2 Exercises

Objective A

1. What is the meaning of the phrase *complete the square*?

2. Write out the steps for solving a quadratic equation by completing the square.

Solve by completing the square.

3. $x^2 - 4x - 5 = 0$

4. $y^2 + 6y + 5 = 0$

5. $v^2 + 8v - 9 = 0$

6. $w^2 - 2w - 24 = 0$

7. $z^2 - 6z + 9 = 0$

8. $u^2 + 10u + 25 = 0$

9. $r^2 + 4r - 7 = 0$

10. $s^2 + 6s - 1 = 0$

11. $x^2 - 6x + 7 = 0$

12. $y^2 + 8y + 13 = 0$

13. $z^2 - 2z + 2 = 0$

14. $t^2 - 4t + 8 = 0$

15. $s^2 - 5s - 24 = 0$

16. $v^2 + 7v - 44 = 0$

17. $x^2 + 5x - 36 = 0$

18. $y^2 - 9y + 20 = 0$

19. $p^2 - 3p + 1 = 0$

20. $r^2 - 5r - 2 = 0$

21. $t^2 - t - 1 = 0$

22. $u^2 - u - 7 = 0$

23. $y^2 - 6y = 4$

24. $w^2 + 4w = 2$

25. $x^2 = 8x - 15$

26. $z^2 = 4z - 3$

27. $v^2 = 4v - 13$

28. $x^2 = 2x - 17$

29. $p^2 + 6p = -13$

30. $x^2 + 4x = -20$

31. $y^2 - 2y = 17$

32. $x^2 + 10x = 7$

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33. $z^2 = z + 4$

34. $r^2 = 3r - 1$

35. $x^2 + 13 = 2x$

36. $6v^2 - 7v = 3$

37. $4x^2 - 4x + 5 = 0$

38. $4t^2 - 4t + 17 = 0$

39. $9x^2 - 6x + 2 = 0$

40. $9y^2 - 12y + 13 = 0$

41. $2s^2 = 4s + 5$

42. $3u^2 = 6u + 1$

43. $2r^2 = 3 - r$

44. $2x^2 = 12 - 5x$

45. $y - 2 = (y - 3)(y + 2)$

46. $8s - 11 = (s - 4)(s - 2)$

47. $6t - 2 = (2t - 3)(t - 1)$

48. $2z + 9 = (2z + 3)(z + 2)$

49. $(x - 4)(x + 1) = x - 3$

50. $(y - 3)^2 = 2y + 10$

Solve by completing the square. Approximate the solutions to the nearest thousandth.

51. $z^2 + 2z = 4$

52. $t^2 - 4t = 7$

53. $2x^2 = 4x - 1$

54. $3y^2 = 5y - 1$

55. $4z^2 + 2z = 1$

56. $4w^2 - 8w = 3$

APPLYING THE CONCEPTS

Solve for x by completing the square.

57. $x^2 - ax - 2a^2 = 0$

58. $x^2 + 3ax - 4a^2 = 0$

59. $x^2 + 3ax - 10a^2 = 0$

60. When a baseball is hit, the height h, in feet, of the ball above the ground t seconds after it is hit can be approximated by the equation $h = -16t^2 + 70t + 4$. The distance s, in feet, that the ball is from home plate t seconds after it is hit can be approximated by the equation $s = 44.5t$.

a. Determine when the ball will hit the ground. Round to the nearest hundredth. (*Hint*: The ball hits the ground when $h = 0$.)

b. Determine whether the ball will clear a 6-foot fence 325 ft from home plate.

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10.3 Solving Quadratic Equations by Using the Quadratic Formula

Objective A

To solve a quadratic equation by using the quadratic formula

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A general formula known as the **quadratic formula** can be derived by applying the method of completing the square to the standard form of a quadratic equation. This formula can be used to solve any quadratic equation. The equation $ax^2 + bx + c = 0$ is solved by completing the square as follows.

$$ax^2 + bx + c = 0$$

Add the opposite of the constant term to each side of the equation.

$$ax^2 + bx + c + (-c) = 0 + (-c)$$

$$ax^2 + bx = -c$$

Multiply each side of the equation by the reciprocal of a, the coefficient of x^2.

$$\frac{1}{a}(ax^2 + bx) = \frac{1}{a}(-c)$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square by adding $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$ to each side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \left(\frac{c}{a}\cdot\frac{4a}{4a}\right)$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Factor the perfect-square trinomial on the left side of the equation.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take the square root of each side of the equation.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solve for x.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Point of Interest

Although mathematicians have studied quadratic equations since around 500 B.C., it was not until the 18th century that the formula was written as it is today. Of further note, the word *quadratic* has the same Latin root as does the word *square*.

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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The quadratic formula is frequently written as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

➡ Solve by using the quadratic formula: $2x^2 + 5x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}$$

$$x = \frac{-5 + 1}{4} = \frac{-4}{4} = -1 \qquad x = \frac{-5 - 1}{4} = \frac{-6}{4} = -\frac{3}{2}$$

The solutions are -1 and $-\frac{3}{2}$.

- The equation $2x^2 + 5x + 3 = 0$ is in standard form. $a = 2$, $b = 5$, $c = 3$ Replace a, b, and c in the quadratic formula with these values.

TAKE NOTE

The solutions of this quadratic equation are rational numbers. When this happens, the equation could have been solved by factoring and using the Principle of Zero Products. This may be easier than applying the quadratic formula.

➡ Solve $3x^2 = 4x + 6$ by using the quadratic formula. Find the exact solutions and approximate the solutions to the nearest thousandth.

$$3x^2 = 4x + 6$$

$$3x^2 - 4x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{16 - (-72)}}{6}$$

$$= \frac{4 \pm \sqrt{88}}{6} = \frac{4 \pm 2\sqrt{22}}{6}$$

$$= \frac{2(2 \pm \sqrt{22})}{2 \cdot 3} = \frac{2 \pm \sqrt{22}}{3}$$

- Write the equation in standard form. Subtract $4x$ and 6 from each side of the equation. $a = 3$, $b = -4$, $c = -6$ Replace a, b, and c in the quadratic formula with these values.

Check:

$3x^2 = 4x + 6$

$$3\left(\frac{2 + \sqrt{22}}{3}\right)^2 \quad \Big| \quad 4\left(\frac{2 + \sqrt{22}}{3}\right) + 6$$

$$3\left(\frac{4 + 4\sqrt{22} + 22}{9}\right) \quad \Big| \quad \frac{8}{3} + \frac{4\sqrt{22}}{3} + \frac{18}{3}$$

$$3\left(\frac{26 + 4\sqrt{22}}{9}\right) \quad \Big| \quad \frac{26}{3} + \frac{4\sqrt{22}}{3}$$

$$\frac{26 + 4\sqrt{22}}{3} = \frac{26 + 4\sqrt{22}}{3}$$

$3x^2 = 4x + 6$

$$3\left(\frac{2 - \sqrt{22}}{3}\right)^2 \quad \Big| \quad 4\left(\frac{2 - \sqrt{22}}{3}\right) + 6$$

$$3\left(\frac{4 - 4\sqrt{22} + 22}{9}\right) \quad \Big| \quad \frac{8}{3} - \frac{4\sqrt{22}}{3} + \frac{18}{3}$$

$$3\left(\frac{26 - 4\sqrt{22}}{9}\right) \quad \Big| \quad \frac{26}{3} - \frac{4\sqrt{22}}{3}$$

$$\frac{26 - 4\sqrt{22}}{3} = \frac{26 - 4\sqrt{22}}{3}$$

The solutions are $\dfrac{2 + \sqrt{22}}{3}$ and $\dfrac{2 - \sqrt{22}}{3}$.

$$\frac{2 + \sqrt{22}}{3} \approx 2.230 \qquad \frac{2 - \sqrt{22}}{3} \approx -0.897$$

To the nearest thousandth, the solutions are 2.230 and -0.897.

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Solve by using the quadratic formula: $4x^2 = 8x - 13$

$$4x^2 = 8x - 13$$

$$4x^2 - 8x + 13 = 0$$ • Write the equation in standard form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ • Use the quadratic formula.

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 13}}{2 \cdot 4}$$ • $a = 4, b = -8, c = 13$

$$= \frac{8 \pm \sqrt{64 - 208}}{8} = \frac{8 \pm \sqrt{-144}}{8}$$

$$= \frac{8 \pm 12i}{8} = \frac{2 + 3i}{2}$$

The solutions are $1 + \frac{3}{2}i$ and $1 - \frac{3}{2}i$.

Of the three preceding examples, the first two had real number solutions; the last one had complex number solutions.

In the quadratic formula, the quantity $b^2 - 4ac$ is called the **discriminant**. When a, b, and c are real numbers, the discriminant determines whether a quadratic equation will have a double root, two real number solutions that are not equal, or two complex number solutions.

THE EFFECT OF THE DISCRIMINANT ON THE SOLUTIONS OF A QUADRATIC EQUATION

1. If $b^2 - 4ac = 0$, the equation has one real number solution, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

Use the discriminant to determine whether $x^2 - 4x - 5 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

$b^2 - 4ac$

$(-4)^2 - 4(1)(-5) = 16 + 20 = 36$ • Evaluate the discriminant. $a = 1, b = -4, c = -5$

$36 > 0$

Because $b^2 - 4ac > 0$, the equation has two real number solutions.

Example 1 Solve by using the quadratic formula: $2x^2 - x + 5 = 0$

Solution

$2x^2 - x + 5 = 0$

$a = 2, b = -1, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2 \cdot 2}$$

$$= \frac{1 \pm \sqrt{1 - 40}}{4} = \frac{1 \pm \sqrt{-39}}{4}$$

$$= \frac{1 \pm i\sqrt{39}}{4}$$

The solutions are $\frac{1}{4} + \frac{\sqrt{39}}{4}i$ and $\frac{1}{4} - \frac{\sqrt{39}}{4}i$.

You Try It 1 Solve by using the quadratic formula: $x^2 - 2x + 10 = 0$

Your solution

Solution on p. S31

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Example 2

Solve by using the quadratic formula:
$2x^2 = (x - 2)(x - 3)$

Solution

$2x^2 = (x - 2)(x - 3)$
$2x^2 = x^2 - 5x + 6$

$x^2 + 5x - 6 = 0$
$a = 1, b = 5, c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2 \cdot 1}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2}$$

$$= \frac{-5 \pm 7}{2}$$

$$x = \frac{-5 + 7}{2} \qquad x = \frac{-5 - 7}{2}$$

$$= \frac{2}{2} = 1 \qquad = \frac{-12}{2} = -6$$

The solutions are 1 and -6.

You Try It 2

Solve by using the quadratic formula:
$4x^2 = 4x - 1$

Your solution

Example 3

Use the discriminant to determine whether $4x^2 - 2x + 5 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

Solution

$a = 4, b = -2, c = 5$

$$b^2 - 4ac = (-2)^2 - 4(4)(5)$$
$$= 4 - 80$$
$$= -76$$

$-76 < 0$

Because the discriminant is less than zero, the equation has two complex number solutions.

You Try It 3

Use the discriminant to determine whether $3x^2 - x - 1 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

Your solution

Solutions on p. S31

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10.3 Exercises

Objective A

1. Write the quadratic formula. What does each variable in the formula represent?

2. Write the expression that appears under the radical symbol in the quadratic formula. What is this quantity called? What can it be used to determine?

Solve by using the quadratic formula.

3. $x^2 - 3x - 10 = 0$

4. $z^2 - 4z - 8 = 0$

5. $y^2 + 5y - 36 = 0$

6. $z^2 - 3z - 40 = 0$

7. $w^2 = 8w + 72$

8. $t^2 = 2t + 35$

9. $v^2 = 24 - 5v$

10. $x^2 = 18 - 7x$

11. $2y^2 + 5y - 1 = 0$

12. $4p^2 - 7p + 1 = 0$

13. $8s^2 = 10s + 3$

14. $12t^2 = 5t + 2$

15. $x^2 = 14x - 4$

16. $v^2 = 12v - 24$

17. $2z^2 - 2z - 1 = 0$

18. $6w^2 = 9w - 1$

19. $z^2 + 2z + 2 = 0$

20. $p^2 - 4p + 5 = 0$

21. $y^2 - 2y + 5 = 0$

22. $x^2 + 6x + 13 = 0$

23. $s^2 - 4s + 13 = 0$

24. $t^2 - 6t + 10 = 0$

25. $2w^2 - 2w - 5 = 0$

26. $v^2 + 8v + 3 = 0$

27. $2x^2 + 6x + 5 = 0$

28. $2y^2 + 2y + 13 = 0$

29. $4t^2 - 6t + 9 = 0$

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Solve by using the quadratic formula. Approximate the solutions to the nearest thousandth.

30. $x^2 - 6x - 6 = 0$

31. $p^2 - 8p + 3 = 0$

32. $r^2 - 2r = 4$

33. $w^2 + 4w = 1$

34. $3t^2 = 7t + 1$

35. $2y^2 = y + 5$

Use the discriminant to determine whether the quadratic equation has one real number solution, two real number solutions, or two complex number solutions.

36. $2z^2 - z + 5 = 0$

37. $3y^2 + y + 1 = 0$

38. $9x^2 - 12x + 4 = 0$

39. $4x^2 + 20x + 25 = 0$

40. $2v^2 - 3v - 1 = 0$

41. $3w^2 + 3w - 2 = 0$

APPLYING THE CONCEPTS

42. Name the three methods of solving a quadratic equation that have been discussed. What are the advantages and disadvantages of each?

For what values of p does the quadratic equation have two real number solutions that are not equal? Write the answer in set-builder notation.

43. $x^2 - 6x + p = 0$

44. $x^2 + 10x + p = 0$

For what values of p does the quadratic equation have two complex number solutions? Write the answer in set-builder notation.

45. $x^2 - 2x + p = 0$

46. $x^2 + 4x + p = 0$

47. Find all values of x that satisfy the equation $x^2 + ix + 2 = 0$.

48. Show that the equation $x^2 + bx - 1 = 0$ always has real number solutions regardless of the value of b.

49. Can the quadratic formula be used to solve *any* quadratic equation? If so, explain why. If not, give an example of a quadratic equation that cannot be solved by using the quadratic formula.

50. One of the steps in the derivation of the quadratic formula is $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ or $\pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$. Carefully explain the occurrence of the $\pm$ sign. (*Hint:* Recall that $\sqrt{x^2} = |x|$.)

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10.4 Solving Equations That Are Reducible to Quadratic Equations

Objective A

To solve an equation that is quadratic in form

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Certain equations that are not quadratic can be expressed in quadratic form by making suitable substitutions. An equation is **quadratic in form** if it can be written as $au^2 + bu + c = 0$.

The equation $x^4 - 4x^2 - 5 = 0$ is quadratic in form.

$$\begin{aligned} x^4 - 4x^2 - 5 &= 0 \\ (x^2)^2 - 4(x^2) - 5 &= 0 \\ u^2 - 4u - 5 &= 0 \end{aligned}$$

• Let $x^2 = u$.

The equation $y - y^{1/2} - 6 = 0$ is quadratic in form.

$$\begin{aligned} y - y^{1/2} - 6 &= 0 \\ (y^{1/2})^2 - (y^{1/2}) - 6 &= 0 \\ u^2 - u - 6 &= 0 \end{aligned}$$

• Let $y^{1/2} = u$.

Here is the key to recognizing equations that are quadratic in form: When the equation is written in standard form, the exponent on one variable term is $\frac{1}{2}$ the exponent on the other variable term.

➡ Solve: $z + 7z^{1/2} - 18 = 0$

$$z + 7z^{1/2} - 18 = 0$$

• The equation is quadratic in form.

$$(z^{1/2})^2 + 7(z^{1/2}) - 18 = 0$$

$$u^2 + 7u - 18 = 0$$

• Let $z^{1/2} = u$.

$$(u - 2)(u + 9) = 0$$

• Solve by factoring.

$$\begin{array}{ll} u - 2 = 0 & u + 9 = 0 \\ u = 2 & u = -9 \end{array}$$

• Replace u by $z^{1/2}$.

$$\begin{array}{ll} z^{1/2} = 2 & z^{1/2} = -9 \\ \sqrt{z} = 2 & \sqrt{z} = -9 \end{array}$$

• Solve for z.

$$\begin{array}{ll} (\sqrt{z})^2 = 2^2 & (\sqrt{z})^2 = (-9)^2 \\ z = 4 & z = 81 \end{array}$$

Check:

$$\begin{array}{r|l} z + 7z^{1/2} - 18 = 0 & \\ \hline 4 + 7(4)^{1/2} - 18 & 0 \\ 4 + 7 \cdot 2 - 18 & \\ 4 + 14 - 18 & \\ 0 = 0 & \end{array}$$

$$\begin{array}{r|l} z + 7z^{1/2} - 18 = 0 & \\ \hline 81 + 7(81)^{1/2} - 18 & 0 \\ 81 + 7 \cdot 9 - 18 & \\ 81 + 63 - 18 & \\ 126 \neq 0 & \end{array}$$

4 checks as a solution, but 81 does not check as a solution.
The solution is 4.

TAKE NOTE
When each side of an equation is squared, the resulting equation may have a solution that is not a solution of the original equation.

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Example 1 Solve: $x^4 + x^2 - 12 = 0$

Solution

$$x^4 + x^2 - 12 = 0$$
$$(x^2)^2 + (x^2) - 12 = 0$$
$$u^2 + u - 12 = 0$$
$$(u - 3)(u + 4) = 0$$

$u - 3 = 0$ $\quad$ $u + 4 = 0$
$u = 3$ $\quad$ $u = -4$

Replace u by x^2.

$x^2 = 3$ $\quad$ $x^2 = -4$
$\sqrt{x^2} = \sqrt{3}$ $\quad$ $\sqrt{x^2} = \sqrt{-4}$
$x = \pm\sqrt{3}$ $\quad$ $x = \pm 2i$

The solutions are $\sqrt{3}$, $-\sqrt{3}$, $2i$, and $-2i$.

You Try It 1 Solve: $x - 5x^{1/2} + 6 = 0$

Your solution

Solution on p. S31

To solve a radical equation that is reducible to a quadratic equation

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Certain equations that contain radicals can be expressed as quadratic equations.

Solve: $\sqrt{x + 2} + 4 = x$

$$\sqrt{x + 2} + 4 = x$$
$$\sqrt{x + 2} = x - 4$$ • Solve for the radical expression.
$$(\sqrt{x + 2})^2 = (x - 4)^2$$ • Square each side of the equation.
$$x + 2 = x^2 - 8x + 16$$ • Simplify.
$$0 = x^2 - 9x + 14$$ • Write the equation in standard form.
$$0 = (x - 7)(x - 2)$$ • Solve for x.

$x - 7 = 0$ $\quad$ $x - 2 = 0$
$x = 7$ $\quad$ $x = 2$

TAKE NOTE
You should always check your solutions by substituting the proposed solutions back into the *original* equation.

Check: $\sqrt{x + 2} + 4 = x$

$\sqrt{7 + 2} + 4$	7
$\sqrt{9} + 4$	
$3 + 4$	
$7 = 7$	

$\sqrt{x + 2} + 4 \stackrel{?}{=} x$

$\sqrt{2 + 2} + 4$	2
$\sqrt{4} + 4$	
$2 + 4$	
$6 \neq 2$	

7 checks as a solution, but 2 does not check as a solution.
The solution is 7.

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Example 2

Solve: $\sqrt{7y - 3} + 3 = 2y$

Solution

$$\sqrt{7y - 3} + 3 = 2y$$
$$\sqrt{7y - 3} = 2y - 3$$
$$(\sqrt{7y - 3})^2 = (2y - 3)^2$$
$$7y - 3 = 4y^2 - 12y + 9$$
$$0 = 4y^2 - 19y + 12$$
$$0 = (4y - 3)(y - 4)$$

$4y - 3 = 0$ $\qquad$ $y - 4 = 0$

$4y = 3$ $\qquad$ $y = 4$

$y = \frac{3}{4}$

4 checks as a solution.

$\frac{3}{4}$ does not check as a solution.

The solution is 4.

You Try It 2

Solve: $\sqrt{2x + 1} + x = 7$

Your solution

Example 3

Solve: $\sqrt{2y + 1} - \sqrt{y} = 1$

Solution

$$\sqrt{2y + 1} - \sqrt{y} = 1$$

Solve for one of the radical expressions.

$$\sqrt{2y + 1} = \sqrt{y} + 1$$
$$(\sqrt{2y + 1})^2 = (\sqrt{y} + 1)^2$$
$$2y + 1 = y + 2\sqrt{y} + 1$$
$$y = 2\sqrt{y}$$

Square each side of the equation.

$$y^2 = (2\sqrt{y})^2$$
$$y^2 = 4y$$
$$y^2 - 4y = 0$$
$$y(y - 4) = 0$$

$y = 0$ $\qquad$ $y - 4 = 0$

$y = 4$

0 and 4 check as solutions.

The solutions are 0 and 4.

You Try It 3

Solve: $\sqrt{2x - 1} + \sqrt{x} = 2$

Your solution

Solutions on p. S31

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Objective C

To solve a fractional equation that is reducible to a quadratic equation



After each side of a fractional equation has been multiplied by the LCM of the denominators, the resulting equation may be a quadratic equation.

Solve: $\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$

$$\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$$

$$2r(r+1)\left(\frac{1}{r} + \frac{1}{r+1}\right) = 2r(r+1) \cdot \frac{3}{2}$$

- Multiply each side of the equation by the LCM of the denominators.

$$2(r+1) + 2r = r(r+1) \cdot 3$$
$$2r + 2 + 2r = 3r(r+1)$$
$$4r + 2 = 3r^2 + 3r$$
$$0 = 3r^2 - r - 2$$

- Write the equation in standard form.
- Solve for r by factoring.

$$0 = (3r+2)(r-1)$$

$3r + 2 = 0$ $\qquad$ $r - 1 = 0$

$3r = -2$ $\qquad$ $r = 1$

$r = -\frac{2}{3}$

$-\frac{2}{3}$ and 1 check as solutions. The solutions are $-\frac{2}{3}$ and 1.

Example 4

Solve: $\frac{9}{x-3} = 2x + 1$

Solution

$$\frac{9}{x-3} = 2x + 1$$

$$(x-3)\frac{9}{x-3} = (x-3)(2x+1)$$

$$9 = 2x^2 - 5x - 3$$
$$0 = 2x^2 - 5x - 12$$
$$0 = (2x+3)(x-4)$$

$2x + 3 = 0$ $\qquad$ $x - 4 = 0$

$2x = -3$ $\qquad$ $x = 4$

$x = -\frac{3}{2}$

$-\frac{3}{2}$ and 4 check as solutions.

The solutions are $-\frac{3}{2}$ and 4.

You Try It 4

Solve: $3y + \frac{25}{3y-2} = -8$

Your solution

Solution on p. S31

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10.4 Exercises

Objective A

Solve.

1. $x^4 - 13x^2 + 36 = 0$
2. $y^4 - 5y^2 + 4 = 0$
3. $z^4 - 6z^2 + 8 = 0$
4. $t^4 - 12t^2 + 27 = 0$
5. $p - 3p^{1/2} + 2 = 0$
6. $v - 7v^{1/2} + 12 = 0$
7. $x - x^{1/2} - 12 = 0$
8. $w - 2w^{1/2} - 15 = 0$
9. $z^4 + 3z^2 - 4 = 0$
10. $y^4 + 5y^2 - 36 = 0$
11. $x^4 + 12x^2 - 64 = 0$
12. $x^4 - 81 = 0$
13. $p + 2p^{1/2} - 24 = 0$
14. $v + 3v^{1/2} - 4 = 0$
15. $y^{2/3} - 9y^{1/3} + 8 = 0$
16. $z^{2/3} - z^{1/3} - 6 = 0$
17. $9w^4 - 13w^2 + 4 = 0$
18. $4y^4 - 7y^2 - 36 = 0$

Objective B

Solve.

19. $\sqrt{x + 1} + x = 5$
20. $\sqrt{x - 4} + x = 6$
21. $x = \sqrt{x} + 6$
22. $\sqrt{2y - 1} = y - 2$
23. $\sqrt{3w + 3} = w + 1$
24. $\sqrt{2s + 1} = s - 1$
25. $\sqrt{4y + 1} - y = 1$
26. $\sqrt{3s + 4} + 2s = 12$
27. $\sqrt{10x + 5} - 2x = 1$

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28. $\sqrt{t+8} = 2t+1$

29. $\sqrt{p+11} = 1-p$

30. $x-7 = \sqrt{x-5}$

31. $\sqrt{x-1} - \sqrt{x} = -1$

32. $\sqrt{y}+1 = \sqrt{y+5}$

33. $\sqrt{2x-1} = 1 - \sqrt{x-1}$

34. $\sqrt{x+6} + \sqrt{x+2} = 2$

35. $\sqrt{t+3} + \sqrt{2t+7} = 1$

36. $\sqrt{5-2x} = \sqrt{2-x}+1$

Solve.

37. $x = \dfrac{10}{x-9}$

38. $z = \dfrac{5}{z-4}$

39. $\dfrac{t}{t+1} = \dfrac{-2}{t-1}$

40. $\dfrac{2v}{v-1} = \dfrac{5}{v+2}$

41. $\dfrac{y-1}{y+2} + y = 1$

42. $\dfrac{2p-1}{p-2} + p = 8$

43. $\dfrac{3r+2}{r+2} - 2r = 1$

44. $\dfrac{2v+3}{v+4} + 3v = 4$

45. $\dfrac{2}{2x+1} + \dfrac{1}{x} = 3$

46. $\dfrac{3}{s} - \dfrac{2}{2s-1} = 1$

47. $\dfrac{16}{z-2} + \dfrac{16}{z+2} = 6$

48. $\dfrac{2}{y+1} + \dfrac{1}{y-1} = 1$

49. $\dfrac{t}{t-2} + \dfrac{2}{t-1} = 4$

50. $\dfrac{4t+1}{t+4} + \dfrac{3t-1}{t+1} = 2$

51. $\dfrac{5}{2p-1} + \dfrac{4}{p+1} = 2$

APPLYING THE CONCEPTS

52. Solve: $(\sqrt{x}-2)^2 - 5\sqrt{x} + 14 = 0$. (*Hint:* Let $u = \sqrt{x} - 2$.)

53. Solve: $(\sqrt{x}+3)^2 - 4\sqrt{x} - 17 = 0$. (*Hint:* Let $u = \sqrt{x} + 3$.)

54. The fourth power of a number is twenty-five less than ten times the square of the number. Find the number.

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10.5 Applications of Quadratic Equations

Objective 1

To solve application problems

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The application problems in this section are similar to problems solved earlier in the text. Each of the strategies for the problems in this section will result in a quadratic equation.

➡ A small pipe takes 16 min longer to empty a tank than does a larger pipe. Working together, the pipes can empty the tank in 6 min. How long would it take the smaller pipe working alone to empty the tank?

Strategy for Solving an Application Problem

1. Determine the type of problem. Is it a uniform motion problem, a geometry problem, an integer problem, or a work problem?

The problem is a work problem.

2. Choose a variable to represent the unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in a table.

The unknown time of the larger pipe: t
The unknown time of the smaller pipe: $t + 16$

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Larger pipe	$\frac{1}{t}$	·	6	=	$\frac{6}{t}$
Smaller pipe	$\frac{1}{t+16}$	·	6	=	$\frac{6}{t+16}$

3. Determine how the quantities are related.

$$\frac{6}{t} + \frac{6}{t+16} = 1$$

• The sum of the parts of the task completed must equal 1.

$$t(t+16)\left(\frac{6}{t} + \frac{6}{t+16}\right) = t(t+16) \cdot 1$$
$$(t+16)6 + 6t = t^2 + 16t$$
$$6t + 96 + 6t = t^2 + 16t$$
$$0 = t^2 + 4t - 96$$
$$0 = (t+12)(t-8)$$
$$t + 12 = 0 \qquad t - 8 = 0$$
$$t = -12 \qquad t = 8$$

Because time cannot be negative, the solution $t = -12$ is not possible.

The time for the smaller pipe is $t + 16$.

$t + 16 = 8 + 16 = 24$ • Replace t by 8 and evaluate.

The smaller pipe requires 24 min to empty the tank.

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Example 1

In 8 h, two campers rowed 15 mi down a river and then rowed back to their campsite. The rate of the river's current was 1 mph. Find the rate at which the campers rowed.

Strategy

- This is a uniform motion problem.
- Unknown rowing rate of the campers: r

	Distance	*Rate*	*Time*
Down river	15	$r + 1$	$\frac{15}{r+1}$
Up river	15	$r - 1$	$\frac{15}{r-1}$

- The total time of the trip was 8 h.

Solution

$$\frac{15}{r+1} + \frac{15}{r-1} = 8$$

$$(r+1)(r-1)\left(\frac{15}{r+1} + \frac{15}{r-1}\right) = (r+1)(r-1)8$$

$$(r-1)15 + (r+1)15 = (r^2 - 1)8$$

$$15r - 15 + 15r + 15 = 8r^2 - 8$$

$$30r = 8r^2 - 8$$

$$0 = 8r^2 - 30r - 8$$

$$0 = 2(4r^2 - 15r - 4)$$

$$0 = 2(4r + 1)(r - 4)$$

$4r + 1 = 0$ $\qquad$ $r - 4 = 0$

$4r = -1$ $\qquad$ $r = 4$

$r = -\frac{1}{4}$

The solution $r = -\frac{1}{4}$ is not possible, because the rate cannot be a negative number.

The rowing rate was 4 mph.

You Try It 1

The length of a rectangle is 3 m more than the width. The area is 54 m^2. Find the length of the rectangle.

Your strategy

Your solution

Solution on p. S32

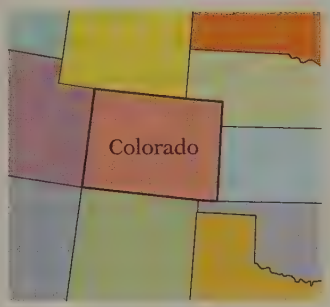
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10.5 Exercises

Objective A Application Problems

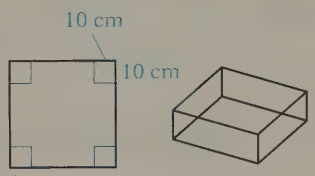
1. The base of a triangle is 1 cm less than five times the height of the triangle. The area of the triangle is 21 cm^2. Find the height and the length of the base of the triangle.

2. The length of a rectangle is 2 ft less than three times the width of the rectangle. The area of the rectangle is 65 ft^2. Find the length and width of the rectangle.



3. The state of Colorado is almost perfectly rectangular, with its north border 111 mi longer than its west border. If the state encompasses 104,000 mi^2, estimate the dimensions of Colorado. Round to the nearest mile.

4. A square piece of cardboard is formed into a box by cutting 10-centimeter squares from each of the four corners and then folding up the sides, as shown in the figure. If the volume, V, of the box is to be 49,000 cm^3, what size square piece of cardboard is needed? Recall that $V = LWH$.



5. A car with good tire tread can stop in less distance than a car with poor tread. The formula for the stopping distance d, in feet, of a car with good tread on dry cement is approximated by $d = 0.04v^2 + 0.5v$, where v is the speed of the car. If the driver must be able to stop within 60 ft, what is the maximum safe speed, to the nearest mile per hour, of the car?



6. A model rocket is launched with an initial velocity of 200 ft/s. The height h, in feet, of the rocket t seconds after the launch is given by $h(t) = -16t^2 + 200t$. How many seconds after the launch will the rocket be 300 ft above the ground? Round to the nearest hundredth of a second.

7. The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to return to Earth if it has an initial velocity of 200 ft/s.

8. The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to reach a height of 64 ft if it has an initial velocity of 128 ft/s. Round to the nearest hundredth of a second.

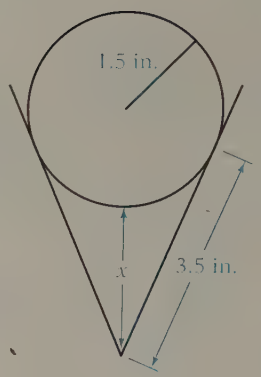
9. In Germany, there is no speed limit on some portions of the autobahn (highway). Other portions have a speed limit of 180 km/h (approximately 112 mph). The distance d, in meters, required to stop a car traveling at v kilometers per hour is $d(v) = 0.019v^2 + 0.69v$. Approximate, to the nearest tenth, the maximum speed a driver can be traveling and still be able to stop within 150 m.



German Autobahn System

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10. A perfectly spherical scoop of mint chocolate chip ice cream is placed in a cone, as shown in the figure. How far is the bottom of the scoop of ice cream from the bottom of the cone? Round to the nearest tenth. (*Hint:* A line segment from the center of the ice cream to the point at which the ice cream touches the cone is perpendicular to the edge of the cone.)



11. A small pipe can fill a tank in 6 min more time than it takes a larger pipe to fill the same tank. Working together, both pipes can fill the tank in 4 min. How long would it take each pipe working alone to fill the tank?

12. A cruise ship made a trip of 100 mi in 8 h. The ship traveled the first 40 mi at a constant rate before increasing its speed by 5 mph. Then it traveled another 60 mi at the increased speed. Find the rate of the cruise ship for the first 40 mi.

13. The Concorde's speed in calm air is 1320 mph. Flying with the wind, the Concorde can fly from New York to London, a distance of approximately 4000 mi, in 0.5 h less than the time required to make the return trip. Find the rate of the wind to the nearest mile per hour.



14. A car travels 120 mi. A second car, traveling 10 mph faster than the first car, makes the same trip in 1 h less time. Find the speed of each car.

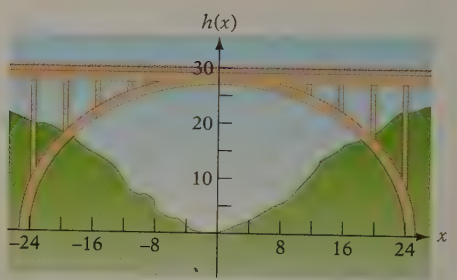
15. For a portion of the Green River in Utah, the rate of the river's current is 4 mph. A tour guide can row 5 mi down this river and back in 3 h. Find the rowing rate of the guide in calm water.

16. The height h, in feet, of an arch is given by the equation $h(x) = -\frac{3}{64}x^2 + 27$, where $|x|$ is the distance in feet from the center of the arch.

a. What is the maximum height of the arch?

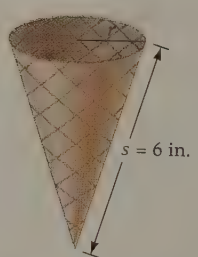
b. What is the height of the arch 8 ft to the right of the center?

c. How far from the center is the arch 8 ft tall?

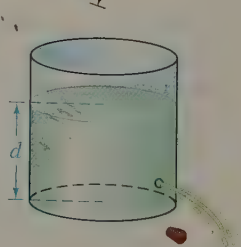


APPLYING THE CONCEPTS

17. The surface area of the ice cream cone shown at the right is given by $A = \pi r^2 + \pi rs$, where r is the radius of the circular top of the cone and s is the slant height of the cone. If the surface area of the cone is 11.25π in^2 and its slant height is 6 in., find the radius of the cone.



18. Using Torricelli's Principle, it can be shown that the depth, d, of a liquid in a bottle with a hole of area 0.5 cm^2 in its side can be approximated by $d = 0.0034t^2 - 0.52518t + 20$, where t is the time since a stopper was removed from the hole. When will the depth be 10 cm? Round to the nearest tenth of a second.



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10.6 Quadratic Inequalities and Rational Inequalities

Objective A

To solve a nonlinear inequality

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A **quadratic inequality** is one that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where $a \neq 0$. The symbols $\leq$ and $\geq$ can also be used. The solution set of a quadratic inequality can be found by solving a compound inequality.

To solve $x^2 - 3x - 10 > 0$, first factor the trinomial.

$$x^2 - 3x - 10 > 0$$
$$(x + 2)(x - 5) > 0$$

There are two cases for which the product of the factors will be positive: (1) both factors are positive, or (2) both factors are negative.

(1) $x + 2 > 0$ and $x - 5 > 0$
(2) $x + 2 < 0$ and $x - 5 < 0$

Solve each pair of compound inequalities.

(1) $x + 2 > 0$ and $x - 5 > 0$
$x > -2$ $x > 5$
$\{x|x > -2\} \cap \{x|x > 5\} = \{x|x > 5\}$

(2) $x + 2 < 0$ and $x - 5 < 0$
$x < -2$ $x < 5$
$\{x|x < -2\} \cap \{x|x < 5\} = \{x|x < -2\}$

Because the two cases for which the product will be positive are connected by *or*, the solution set is the union of the solution sets of the individual inequalities.

$$\{x|x > 5\} \cup \{x|x < -2\} = \{x|x > 5 \text{ or } x < -2\}$$

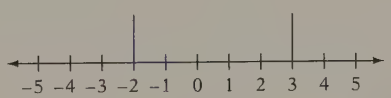
Although the solution set of any quadratic inequality can be found by using the method outlined above, a graphical method is often easier to use.

Solve and graph the solution set of $x^2 - x - 6 < 0$.

$$x^2 - x - 6 < 0$$
$$(x - 3)(x + 2) < 0$$

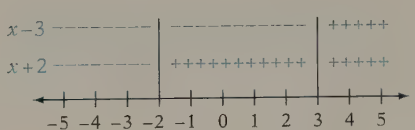
Factor the trinomial.

On a number line, draw lines indicating the numbers that make each factor equal to zero.



$x - 3 = 0$ $x + 2 = 0$
$x = 3$ $x = -2$

For each factor, place plus signs above the number line for those regions where the factor is positive and minus signs where the factor is negative.



Because $x^2 - x - 6 < 0$, the solution set will be the regions where one factor is positive and the other factor is negative.

Write the solution set.

$$\{x|-2 < x < 3\}$$

The graph of the solution set of $x^2 - x - 6 < 0$ is shown at the right.



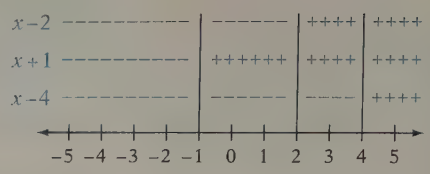
TAKE NOTE

For each factor, choose a number in each region. For example: when $x = -4$, $x - 3$ is negative; when $x = 1$, $x - 3$ is negative; and when $x = 4$, $x - 3$ is positive. When $x = -4$, $x + 2$ is negative; when $x = 1$, $x + 2$ is positive; and when $x = 4$, $x + 2$ is positive.

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➡ Solve and graph the solution set of $(x - 2)(x + 1)(x - 4) > 0$.

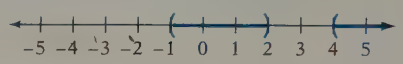
On a number line, identify for each factor the regions where the factor is positive and those where the factor is negative.



There are two regions where the product of the three factors is positive.

Write the solution set. $\{x \mid -1 < x < 2 \text{ or } x > 4\}$

The graph of the solution set of $(x - 2)(x + 1)(x - 4) > 0$ is shown at the right.



➡ Solve: $\dfrac{2x - 5}{x - 4} \le 1$

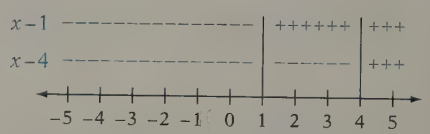
Rewrite the inequality so that 0 appears on the right side of the inequality.

$$\frac{2x - 5}{x - 4} \le 1$$
$$\frac{2x - 5}{x - 4} - 1 \le 0$$

Simplify.

$$\frac{2x - 5}{x - 4} - \frac{x - 4}{x - 4} \le 0$$
$$\frac{x - 1}{x - 4} \le 0$$

On a number line, identify for each factor of the numerator and each factor of the denominator the regions where the factor is positive and those where the factor is negative.



The region where the quotient of the two factors is negative is between 1 and 4.

Write the solution set. $\{x \mid 1 \le x < 4\}$

Note that 1 is part of the solution set but 4 is not because the denominator of the rational expression is zero when $x = 4$.

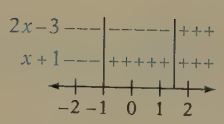
Example 1

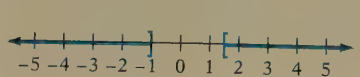
Solve and graph the solution set of $2x^2 - x - 3 \ge 0$.

Solution

$$2x^2 - x - 3 \ge 0$$
$$(2x - 3)(x + 1) \ge 0$$

$\left\{x \mid x \le -1 \text{ or } x \ge \frac{3}{2}\right\}$

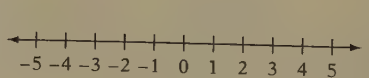




You Try It 1

Solve and graph the solution set of $2x^2 - x - 10 \le 0$.

Your solution



Solution on p. S32

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10.6 Exercises

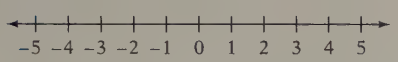
Objective A

1. If $(x - 3)(x - 5) > 0$, what must be true of the values of $x - 3$ and $x - 5$?

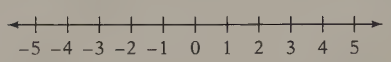
2. For the inequality $\frac{x - 2}{x - 3} \leq 1$, which of the values 1, 2, and 3 is not a possible element of the solution set? Why?

Solve and graph the solution set.

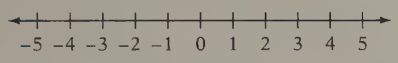
3. $(x - 4)(x + 2) > 0$



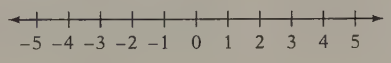
4. $(x + 1)(x - 3) > 0$



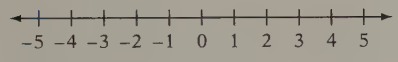
5. $x^2 - 3x + 2 \geq 0$



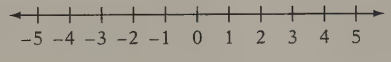
6. $x^2 + 5x + 6 > 0$



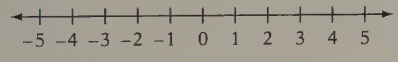
7. $x^2 - x - 12 < 0$



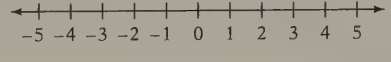
8. $x^2 + x - 20 < 0$



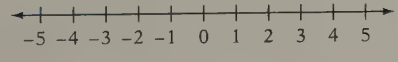
9. $(x - 1)(x + 2)(x - 3) < 0$



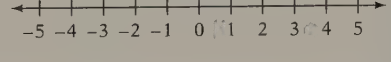
10. $(x + 4)(x - 2)(x + 1) > 0$



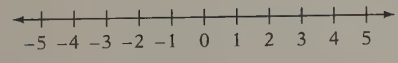
11. $(x + 4)(x - 2)(x - 1) \geq 0$



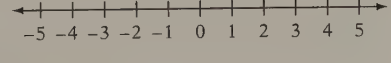
12. $(x - 1)(x + 5)(x - 2) \leq 0$



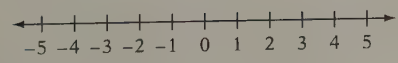
13. $\frac{x - 4}{x + 2} > 0$



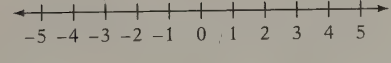
14. $\frac{x + 2}{x - 3} > 0$



15. $\frac{x - 3}{x + 1} \leq 0$

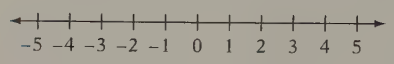


16. $\frac{x - 1}{x} > 0$

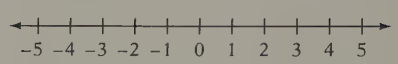


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17. $\dfrac{(x-1)(x+2)}{x-3} \le 0$



18. $\dfrac{(x+3)(x-1)}{x-2} \ge 0$



Solve.

19. $x^2 - 16 > 0$

20. $x^2 - 4 \ge 0$

21. $x^2 - 9x \le 36$

22. $x^2 + 4x > 21$

23. $4x^2 - 8x + 3 < 0$

24. $2x^2 + 11x + 12 \ge 0$

25. $\dfrac{3}{x-1} < 2$

26. $\dfrac{x}{(x-1)(x+2)} \ge 0$

27. $\dfrac{x-2}{(x+1)(x-1)} \le 0$

28. $\dfrac{1}{x} < 2$

29. $\dfrac{x}{2x-1} \ge 1$

30. $\dfrac{x}{2x-3} \le 1$

31. $\dfrac{x}{2-x} \le -3$

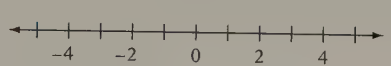
32. $\dfrac{3}{x-2} > \dfrac{2}{x+2}$

33. $\dfrac{3}{x-5} > \dfrac{1}{x+1}$

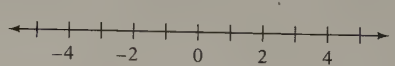
APPLYING THE CONCEPTS

Graph the solution set.

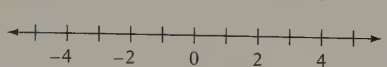
34. $(x+2)(x-3)(x+1)(x+4) > 0$



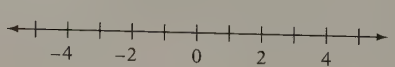
35. $(x-1)(x+3)(x-2)(x-4) \ge 0$



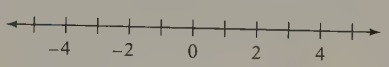
36. $(x^2 + 2x - 8)(x^2 - 2x - 3) < 0$



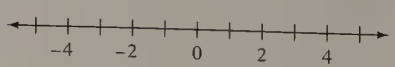
37. $(x^2 + 2x - 3)(x^2 + 3x + 2) \ge 0$



38. $(x^2 + 1)(x^2 - 3x + 2) > 0$



39. $\dfrac{x^2(3-x)(2x+1)}{(x+4)(x+2)} \ge 0$



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Focus on Problem Solving

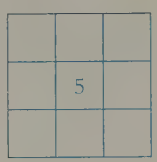
Use a Variety of Problem-Solving Techniques

We have examined several problem-solving strategies throughout the text. See if you can apply those techniques to the following problems.

1. Eight coins look exactly alike, but one is lighter than the others. Explain how the different coin can be found in two weighings on a balance scale.

2. For the sequence of numbers 1, 1, 2, 3, 5, 8, 13, . . . , identify a possible pattern and then use that pattern to determine the next number in the sequence.

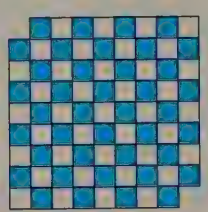
3. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the squares at the right so that the sum of any row, column, or diagonal is 15. (*Suggestion:* Note that 1, 5, 9; 2, 5, 8; 3, 5, 7; and 4, 5, 6 all add to 15. Because 5 is part of each sum, this suggests that 5 be placed in the center of the squares.)



4. A restaurant charges $10.00 for a pizza that has a diameter of 9 in. Determine the selling price of a pizza with a diameter of 18 in. so that the selling price per square inch is the same as for the 9-inch pizza.

5. You have a balance scale and weights of 1 g, 4 g, 8 g, and 16 g. Using only these weights, can you weigh something that weighs 7 g? 9 g? 12 g? 19 g?

6. Can the checkerboard at the right be covered with dominos (which look like ▭) so that every square on the board is covered by a domino? Why or why not? (*Note:* The dominos cannot overlap.)



Projects and Group Activities

Completing the Square

Essentially all of the investigations into mathematics before the Renaissance were geometric. The solutions of quadratic equations were calculated from a construction of a certain area. Proofs of theorems, even theorems about numbers, were based entirely on geometry. In this project, we will examine the geometric solution of a quadratic equation.

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➡ Solve: $x^2 + 6x = 7$

Begin with a line of unknown length, x, and one of length 6, the coefficient of x. Using these lines, construct a rectangle as shown.

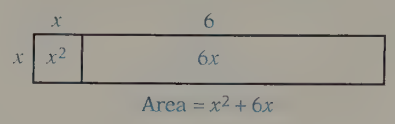


Figure 1

Now draw another area that has exactly the same area as Figure 1 by cutting off one-half of the rectangle of area $6x$ and placing it on the bottom of the square labeled x^2. See Figure 2.

The unshaded area in Figure 2 has exactly the same area as Figure 1. However, when the shaded area is added to Figure 2 to make a square, the total area is 9 square units larger than that of Figure 1. In equation form,

(Area of Figure 1) + 9 = area of Figure 2

or

$$x^2 + 6x + 9 = (x + 3)^2$$

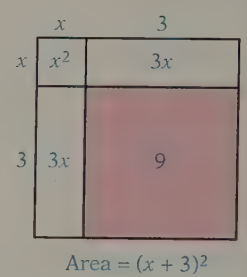


Figure 2

From the original equation, $x^2 + 6x = 7$. Thus,

$$\begin{aligned} x^2 + 6x + 9 &= (x + 3)^2 \\ 7 + 9 &= (x + 3)^2 \quad \bullet\ x^2 + 6x = 7 \\ 16 &= (x + 3)^2 \\ 4 &= x + 3 \quad \bullet\ \text{See note below.} \\ 1 &= x \end{aligned}$$

Note: Although early mathematicians knew that a quadratic equation may have two solutions, both solutions were allowed only if they were positive. After all, a geometric construction could not have a negative length. Therefore, the solution of this equation was 1; the solution -7 would have been dismissed as *fictitious*, the actual word that was frequently used through the 15th century for negative-number solutions of an equation.

Try to solve the quadratic equation $x^2 + 4x = 12$ by geometrically completing the square.

Using a Graphing Calculator to Solve a Quadratic Equation

Recall that an x-intercept of the graph of an equation is a point at which the graph crosses the x-axis. For the graph in Figure 3, the x-intercepts are $(-2, 0)$ and $(3, 0)$.

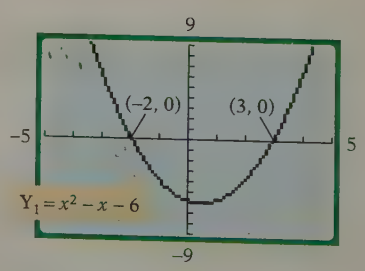


Figure 3

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Recall also that to find the x-intercept of a graph, set $y = 0$ and then solve for x. For the equation in Figure 3, if we set $y = 0$, the resulting equation is $0 = x^2 - x - 6$, which is a quadratic equation. Solving this equation by factoring, we have

$$0 = x^2 - x - 6$$
$$0 = (x + 2)(x - 3)$$

$$x + 2 = 0 \qquad x - 3 = 0$$
$$x = -2 \qquad x = 3$$

Thus the solutions of the equation are the x-coordinates of the x-intercepts of the graph.

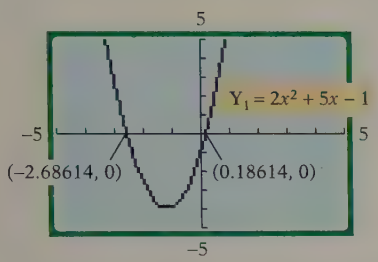


Figure 4

This connection between the solutions of an equation and the x-intercepts of its graph enables us to find approximations of the real number solutions of an equation graphically. For example, to use a TI-83 to approximate the solutions of $2x^2 + 5x - 1 = 0$ graphically, use the following keystrokes to graph $y = 2x^2 + 5x - 1$ and find the decimal approximations of the x-coordinates of the x-intercepts. Use the window shown in the graph in Figure 4.

Y= CLEAR 2 X,T,θ,n x^2 + 5 X,T,θ,n − 1
2nd CALC 2

Use the arrow keys to move to the left of the leftmost x-intercept. Press ENTER. Use the arrow keys to move to a point just to the right of the leftmost x-intercept. Press ENTER twice. The x-coordinate at the bottom of the screen is the approximation of one solution of the equation. To find the other solution, use the same procedure, but move the cursor first to the left and then to the right of the rightmost x-intercept.

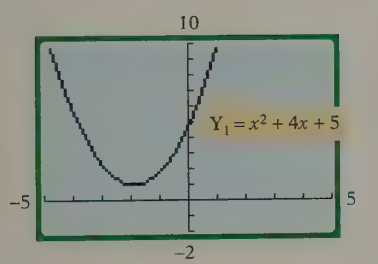


Figure 5

Attempting to find the solutions of an equation graphically will not necessarily find all the solutions. Because the x-coordinates of the x-intercepts of a graph are *real* numbers, only real number solutions can be found. For instance, consider the equation $x^2 + 4x + 5 = 0$. The graph of $y = x^2 + 4x + 5$ is shown in Figure 5. Note that the graph has no x-intercepts and, consequently, no real number solutions. However, $x^2 + 4x + 5 = 0$ does have complex number solutions that can be obtained by using the quadratic formula. They are $-2 + i$ and $-2 - i$.

Chapter Summary

Key Words

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$. A quadratic equation is also called a *second-degree equation.* [p. 545]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. [p. 545]

When a quadratic equation has two solutions that are the same number, the solution is called a *double root* of the equation. [p. 545]

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Methods of Solving a Quadratic Equation:

1. **Factoring**
   This method is used only when the polynomial $ax^2 + bx + c$ is factorable.
2. **Taking square roots**
   This method is used only when the value of b in $ax^2 + bx + c$ is 0.
   It is also used when an equation can be written in the form

   Square of a binomial = constant
3. **Completing the square**
   This method can be used to solve any quadratic equation.
4. **Quadratic formula**
   This method can be used to solve any quadratic equation.
   [pp. 545, 547–548, 553–555, 559–561]

Adding to a binomial the constant term that makes it a perfect-square trinomial is called *completing the square.* [p. 553]

For an equation of the form $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the *discriminant.* [p. 561]

An equation is *quadratic in form* if it can be written as $au^2 + bu + c = 0$. [p. 565]

A *quadratic inequality* is one that can be written in the form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$, where $a \neq 0$. The symbols $\leq$ or $\geq$ can also be used. [p. 575]

Essential Rules

The Principle of Zero Products [p. 545] If $ab = 0$, then $a = 0$ or $b = 0$.

To Solve a Quadratic Equation by Factoring [p. 545]
1. Write the equation in standard form.
2. Factor.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

To Write a Quadratic Equation Given Its Solutions [p. 547]
Use the equation $(x - r_1)(x - r_2) = 0$. Replace r_1 with one solution and r_2 with the other solution. Then multiply the two factors.

To Complete the Square [p. 553]
Add to a binomial of the form $x^2 + bx$ the square of $\frac{1}{2}$ of the coefficient of x.

To Solve a Quadratic Equation by Completing the Square [p. 555]
1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x.
7. Check the solutions.

The Quadratic Formula [p. 559]
The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The Effect of the Discriminant on the Solutions of a Quadratic Equation [p. 561]
1. If $b^2 - 4ac = 0$, the equation has one real number solution, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

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Chapter Review

1. Solve by factoring: $2x^2 - 3x = 0$

2. Solve by factoring: $6x^2 + 9cx = 6c^2$

3. Solve by taking square roots:
$x^2 = 48$

4. Solve by taking square roots:
$\left(x + \frac{1}{2}\right)^2 + 4 = 0$

5. Solve by completing the square:
$x^2 + 4x + 3 = 0$

6. Solve by completing the square:
$7x^2 - 14x + 3 = 0$

7. Solve by using the quadratic formula:
$12x^2 - 25x + 12 = 0$

8. Solve by using the quadratic formula:
$x^2 - x + 8 = 0$

9. Write a quadratic equation that has integer coefficients and has solutions 0 and -3.

10. Write a quadratic equation that has integer coefficients and has solutions $\frac{3}{4}$ and $-\frac{2}{3}$.

11. Solve by completing the square:
$x^2 - 2x + 8 = 0$

12. Solve by completing the square:
$(x - 2)(x + 3) = x - 10$

13. Solve by using the quadratic formula:
$3x(x - 3) = 2x - 4$

14. Use the discriminant to determine whether $3x^2 - 5x + 1 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

15. Solve: $(x + 3)(2x - 5) < 0$

16. Solve: $(x - 2)(x + 4)(2x + 3) \leq 0$

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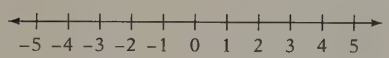
17. Solve: $x^{2/3} + x^{1/3} - 12 = 0$

18. Solve: $2(x - 1) + 3\sqrt{x - 1} - 2 = 0$

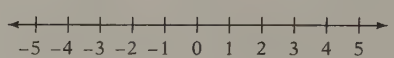
19. Solve: $3x = \frac{9}{x - 2}$

20. Solve: $\frac{3x + 7}{x + 2} + x = 3$

21. Solve and graph the solution set:
$\frac{x - 2}{2x - 3} \geq 0$



22. Solve and graph the solution set:
$\frac{(2x - 1)(x + 3)}{x - 4} \leq 0$



23. Solve: $x = \sqrt{x} + 2$

24. Solve: $2x = \sqrt{5x + 24} + 3$

25. Solve: $\frac{x - 2}{2x + 3} - \frac{x - 4}{x} = 2$

26. Solve: $1 - \frac{x + 4}{2 - x} = \frac{x - 3}{x + 2}$

27. The length of a rectangle is two more centimeters than twice the width. The area of the rectangle is 60 cm^2. Find the length and width of the rectangle.

28. The sum of the squares of three consecutive even integers is fifty-six. Find the three integers.

29. An older computer requires 12 min longer to print the payroll than does a newer computer. Together the computers can print the payroll in 8 min. Find the time for the new computer working alone to print the payroll.

30. A car travels 200 mi. A second car, making the same trip, travels 10 mph faster than the first car and makes the trip in 1 h less time. Find the speed of each car.

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Chapter Test

1. Solve by factoring: $3x^2 + 10x = 8$

2. Solve by factoring: $6x^2 - 5x - 6 = 0$

3. Write a quadratic equation that has integer coefficients and has solutions 3 and -3.

4. Write a quadratic equation that has integer coefficients and has solutions $\frac{1}{2}$ and -4.

5. Solve by taking square roots: $3(x - 2)^2 - 24 = 0$

6. Solve by completing the square: $x^2 - 6x - 2 = 0$

7. Solve by completing the square: $3x^2 - 6x = 2$

8. Solve by using the quadratic formula: $2x^2 - 2x = 1$

9. Solve by using the quadratic formula: $x^2 + 4x + 12 = 0$

10. Use the discriminant to determine whether $3x^2 - 4x = 1$ has one real number solution, two real number solutions, or two complex number solutions.

11. Use the discriminant to determine whether $x^2 - 6x = -15$ has one real number solution, two real number solutions, or two complex number solutions.

12. Solve: $2x + 7x^{1/2} - 4 = 0$

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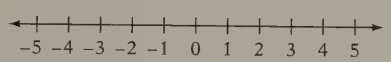
13. Solve: $x^4 - 4x^2 + 3 = 0$

14. Solve: $\sqrt{2x + 1} + 5 = 2x$

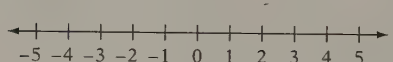
15. Solve: $\sqrt{x - 2} = \sqrt{x} - 2$

16. Solve: $\frac{2x}{x - 3} + \frac{5}{x - 1} = 1$

17. Solve and graph the solution set of $(x - 2)(x + 4)(x - 4) < 0$.



18. Solve and graph the solution set of $\frac{2x - 3}{x + 4} \leq 0$.



19. The base of a triangle is 3 ft more than three times the height. The area of the triangle is 30 ft^2. Find the base and height of the triangle.

20. The rate of a river's current is 2 mph. A canoe was rowed 6 mi down the river and back in 4 h. Find the rowing rate in calm water.

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Cumulative Review

1. Evaluate $2a^2 - b^2 \div c^2$ when $a = 3$, $b = -4$, and $c = -2$.

2. Solve: $|3x - 2| < 8$

3. Find the volume of a cylinder with a height of 6 m and a radius of 3 m. Give the exact measure.

4. Given $f(x) = \dfrac{2x - 3}{x^2 - 1}$, find $f(-2)$.

5. Find the slope of the line containing the points $(3, -4)$ and $(-1, 2)$.

6. Find the x- and y-intercepts of the graph of $6x - 5y = 15$.

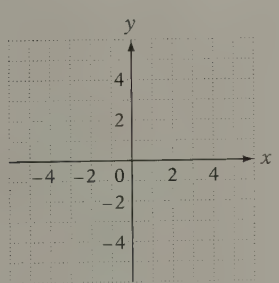
7. Find the equation of the line that contains the point $(1, 2)$ and is parallel to the line $x - y = 1$.

8. Solve the system of equations.

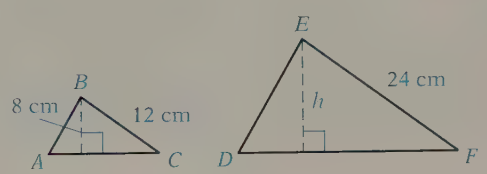
$$\begin{aligned} x + y + z &= 2 \\ -x + 2y - 3z &= -9 \\ x - 2y - 2z &= -1 \end{aligned}$$

9. Graph the solution set:

$$\begin{aligned} x + y &\le 3 \\ 2x - y &< 4 \end{aligned}$$



10. Triangles ABC and DEF are similar. Find the height of triangle DEF.



11. Divide: $(3x^3 - 13x^2 + 10) \div (3x - 4)$

12. Factor: $-3x^3y + 6x^2y^2 - 9xy^3$

13. Factor: $6x^2 - 7x - 20$

14. Multiply: $\dfrac{x^2 + 2x + 1}{8x^2 + 8x} \cdot \dfrac{4x^3 - 4x^2}{x^2 - 1}$

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15. Solve: $\frac{x}{x+2} - \frac{4x}{x+3} = 1$

16. Solve $S = \frac{n}{2}(a + b)$ for b.

17. Multiply: $a^{-1/2}(a^{1/2} - a^{3/2})$

18. Multiply: $-2i(7 - 4i)$

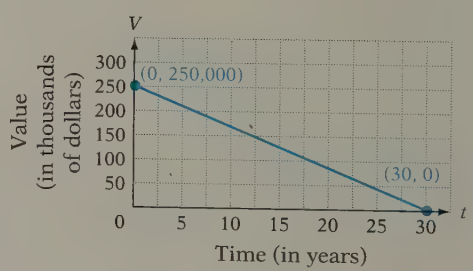
19. Solve: $\sqrt{3x + 1} - 1 = x$

20. Solve: $x^4 - 6x^2 + 8 = 0$

21. A piston rod for an automobile is $9\frac{3}{8}$ in. with a tolerance of $\frac{1}{64}$ in. Find the lower and upper limits of the length of the piston rod.

22. The base of a triangle is $(x + 8)$ ft. The height is $(2x - 4)$ ft. Find the area of the triangle in terms of the variable x.

23. The graph shows the relationship between the value of a building and the time, in years, since depreciation began. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



24. How high on a building will a 17-foot ladder reach when the bottom of the ladder is 8 ft from the building?

25. Use the discriminant to determine whether $2x^2 + 4x + 3 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

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Chapter 11 Functions and Relations



The Statuary Hall in the Capitol Building in Washington, D.C., pictured above, is a whispering gallery. Two people standing at what are called the foci of the elliptical ceiling can whisper and yet hear each other even though they are a considerable distance apart. The whisper from one person is reflected to the person standing at the other focus. The **Project on page 640** shows how to calculate the foci of an ellipse.

Objectives

Section 11.1

A To graph a linear function
B To solve application problems

Section 11.2

A To graph a quadratic function
B To find the x-intercepts of a parabola
C To find the minimum or maximum of a quadratic function
D To solve application problems

Section 11.3

A To graph functions

Section 11.4

A To perform operations on functions
B To find the composition of two functions

Section 11.5

A To determine whether a function is one-to-one
B To find the inverse of a function

Section 11.6

A To graph a parabola
B To find the equation of a circle and to graph a circle
C To graph an ellipse with center at the origin
D To graph a hyperbola with center at the origin

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Evaluate $-\frac{b}{2a}$ for $b = -4$ and $a = 2$.

2. Given $y = -x^2 + 2x + 1$, find the value of y when $x = -2$.

3. Given $f(x) = x^2 - 3x + 2$, find $f(-4)$.

4. Evaluate $p(r) = r^2 - 5$ when $r = 2 + h$.

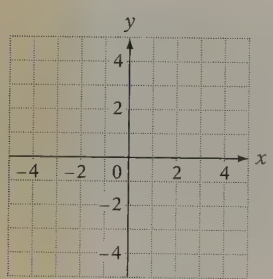
5. Solve: $0 = 3x^2 - 7x - 6$

6. Solve: $0 = x^2 - 4x + 1$

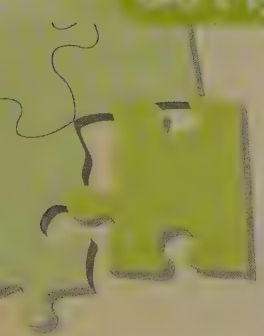
7. Solve $x = 2y + 4$ for y.

8. Find the domain and range of the relation $\{(-2, 4), (3, 5), (4, 6), (6, 5)\}$. Is the relation a function?

9. Graph: $x = -2$



Go Figure



Each time the two hands of a certain standard 12-hour clock form a 180° angle, a bell chimes once. From noon today until noon tomorrow, how many chimes will be heard?

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11.1 Linear Functions

Objective A

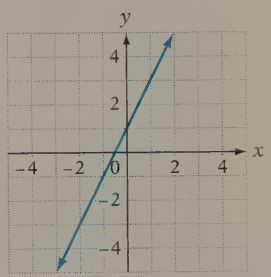
To graph a linear function

VIDEO & DVD CD TUTOR WEB SSM

The graph of a function is the graph of the ordered pairs (x, y) that belong to the function. Because y and $f(x)$ are interchangeable, the ordered pairs of a function can be written as (x, y) or $(x, f(x))$. A function that can be written in the form $y = mx + b$ or $f(x) = mx + b$ is called a **linear function**. The graph of a linear function has certain characteristics. It is a straight line with slope m and y-intercept $(0, b)$.

Graph: $f(x) = 2x + 1$.

This is a linear function. You can think of the function as the equation $y = 2x + 1$. The y-intercept is $(0, 1)$. The slope is 2.



Beginning at the y-intercept, move right 1 and up 2. The point $(1, 3)$ is another point on the graph. Draw a straight line through the points $(0, 1)$ and $(1, 3)$.

When a function is given by an equation, the domain of the function is all real numbers for which the function evaluates to a real number. For instance,

- The domain of $f(x) = 2x + 1$ is all real numbers because the value of $2x + 1$ is a real number for any value of x.
- The domain of $g(x) = \frac{1}{x - 2}$ is all real numbers except 2; when $x = 2$, $g(2) = \frac{1}{2 - 2} = \frac{1}{0}$, which is not a real number.

Example 1

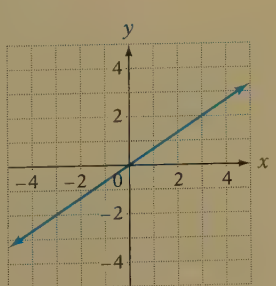
Graph: $f(x) = \frac{2}{3}x$

Solution

$b = 0$

y-intercept: $(0, 0)$

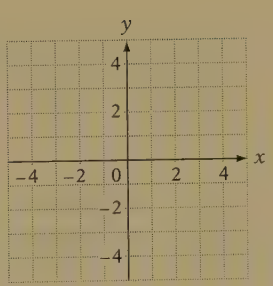
$m = \frac{2}{3}$



You Try It 1

Graph: $f(x) = \frac{3}{5}x - 4$

Your solution



Solution on p. S32

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Objective B To solve application problems



Linear functions can be used to model a variety of applications in science and business. For each application, data are collected and the independent and dependent variables are selected. Then a linear function is determined that models the data.

Example 2

Suppose a manufacturer has determined that at a price of \$115, consumers will purchase 1 million portable CD players and that at a price of \$90, consumers will purchase 1.25 million portable CD players. Describe this situation with a linear function. Use this function to predict how many portable CD players consumers will purchase if the price is \$80.

Strategy

- Select the independent and dependent variables. Because you are trying to determine the number of CD players, that quantity is the *dependent* variable, y. The price of CD players is the *independent* variable, x. From the given data, two ordered pairs are (115, 1) and (90, 1.25). (The ordinates are in millions of units.) Use these ordered pairs to determine the linear function.
- Evaluate the function for $x = 80$ to predict how many CD players consumers will purchase if the price is \$80.

Solution

Let $(x_1, y_1) = (115, 1)$ and $(x_2, y_2) = (90, 1.25)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.25 - 1}{90 - 115} = -\frac{0.25}{25} = -0.01$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -0.01(x - 115)$$
$$y - 1 = -0.01x + 1.15$$
$$y = -0.01x + 2.15$$

The linear function is $f(x) = -0.01x + 2.15$.

$$f(80) = -0.01(80) + 2.15 = 1.35$$

Consumers will purchase 1.35 million CD players at a price of \$80.

You Try It 2

Gabriel Daniel Fahrenheit invented the mercury thermometer in 1717. In terms of readings on this thermometer, water freezes at 32°F and boils at 212°F. In 1742 Anders Celsius invented the Celsius temperature scale. On this scale, water freezes at 0°C and boils at 100°C. Determine a linear function that can be used to predict the Celsius temperature when the Fahrenheit temperature is known.

Your strategy

Your solution

Solution on p. S32

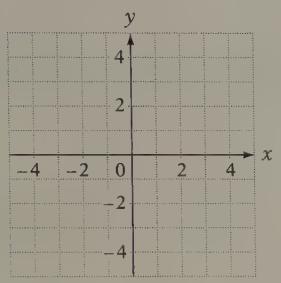
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11.1 Exercises

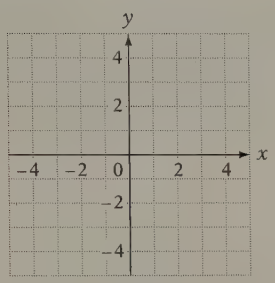
Objective A

Graph.

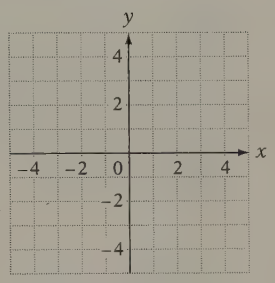
1. $f(x) = 3x - 4$



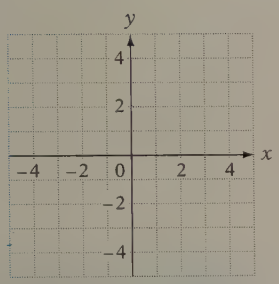
2. $f(x) = -2x + 3$



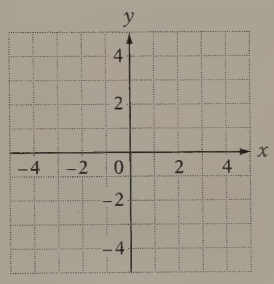
3. $f(x) = -\frac{2}{3}x$



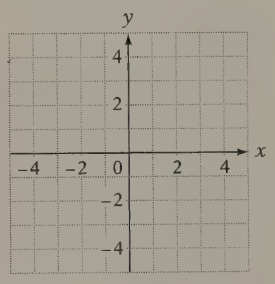
4. $f(x) = \frac{3}{2}x$



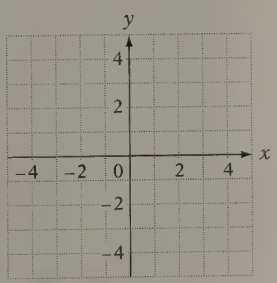
5. $f(x) = \frac{2}{3}x - 4$



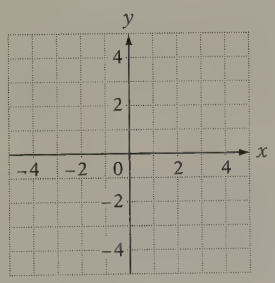
6. $f(x) = \frac{3}{4}x + 2$



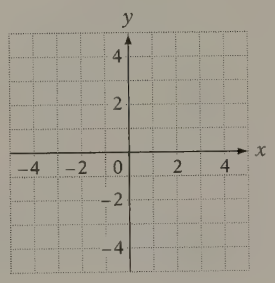
7. $f(x) = -\frac{1}{3}x + 2$



8. $f(x) = -\frac{3}{2}x - 3$



9. $f(x) = \frac{3}{5}x - 1$



Objective B *Application Problems*

10. The sale price of an item is a function, s, of the original price, p, where $s(p) = 0.80p$. If an item's original price is \$200, what is the sale price of the item?

11. The markup on an item is a function, m, of its cost, c, where $m(c) = 0.25c$. If the cost of an item is \$150, what is the markup on the item?

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12. A manufacturer of graphing calculators has determined that 10,000 calculators per week will be sold at a price of \$95. At a price of \$90, it is estimated that 12,000 calculators would be sold. Determine a linear function that will predict the number of calculators that would be sold at a given price. Use this method to predict the number of calculators per week that would be sold at a price of \$75.

13. The operator of a hotel estimates that 500 rooms per night will be rented if the room rate per night is \$75. For each \$10 increase in the price of a room, 6 fewer rooms will be rented. Determine a linear function that will predict the number of rooms that will be rented for a given price per room. Use this model to predict the number of rooms that will be rented if the room rate is \$100.

14. A general building contractor estimates that the cost to build a new home is \$30,000 plus \$85 for each square foot of floor space in the house. Determine a linear function that will give the cost of building a house that contains a given number of square feet. Use this model to determine the cost to build a house that contains 1800 ft^2.

APPLYING THE CONCEPTS

15. A child's height is a function of the child's age. The graph of this function is not linear, as children go through growth spurts as they develop. However, for the graph to be reasonable, the function must be an increasing function (that is, as the age increases, the height increases) because children do not get shorter as they grow older. Match each function described below with a reasonable graph of the function.

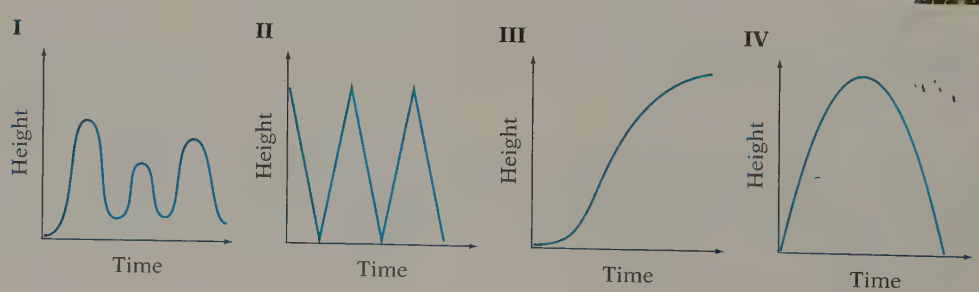
a. The height of a plane above the ground during take-off depends on how long it has been since the plane left the gate.

b. The height of a football above the ground is related to the number of seconds that have passed since it was punted.

c. A basketball player is dribbling a basketball. The basketball's distance from the floor is related to the number of seconds that have passed since the player began dribbling the ball.

d. Two children are seated together on a roller coaster. The height of the children above the ground depends on how long they have been on the ride.





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11.2 Quadratic Functions

Objective A

To graph a quadratic function

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Recall that a linear function is one that can be expressed by the equation $f(x) = mx + b$. The graph of a linear function has certain characteristics. It is a straight line with slope m and y-intercept $(0, b)$. A **quadratic function** is one that can be expressed by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. The graph of this function, called a **parabola**, also has certain characteristics. The graph of a quadratic function can be drawn by finding ordered pairs that belong to the function.

➡ Graph $f(x) = x^2 - 2x - 3$.

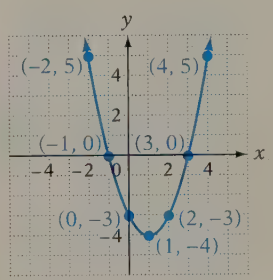
By evaluating the function for various values of x, find enough ordered pairs to determine the shape of the graph.

TAKE NOTE

Sometimes the value of the independent variable is called the **input** because it is *put in* place of the independent variable. The result of evaluating the function is called the **output**.

An **input/output table** shows the results of evaluating a function for various values of the independent variable. An input/output table for $f(x) = x^2 - 2x - 3$ is shown at the right.

x	$f(x) = x^2 - 2x - 3$	$f(x)$	(x, y)
-2	$f(-2) = (-2)^2 - 2(-2) - 3$	5	$(-2, 5)$
-1	$f(-1) = (-1)^2 - 2(-1) - 3$	0	$(-1, 0)$
0	$f(0) = (0)^2 - 2(0) - 3$	-3	$(0, -3)$
1	$f(1) = (1)^2 - 2(1) - 3$	-4	$(1, -4)$
2	$f(2) = (2)^2 - 2(2) - 3$	-3	$(2, -3)$
3	$f(3) = (3)^2 - 2(3) - 3$	0	$(3, 0)$
4	$f(4) = (4)^2 - 2(4) - 3$	5	$(4, 5)$



Because the value of $f(x) = x^2 - 2x - 3$ is a real number for all values of x, the domain of f is $\{x | x \in \text{real numbers}\}$. From the graph, it appears that no value of y is less than -4. Thus the range is $\{y | y \geq -4\}$. The range can also be determined algebraically, as shown below, by completing the square.

TAKE NOTE

In completing the square, 1 is both added and subtracted. Because $1 - 1 = 0$, the expression $x^2 - 2x - 3$ is not changed. Note that

$(x - 1)^2 - 4$
$= (x^2 - 2x + 1) - 4$
$= x^2 - 2x - 3$

which is the original expression.

$$\begin{aligned} f(x) &= x^2 - 2x - 3 \\ &= (x^2 - 2x) - 3 \\ &= (x^2 - 2x + 1) - 1 - 3 \\ &= (x - 1)^2 - 4 \end{aligned}$$

- Group the variable terms.
- Complete the square of $x^2 - 2x$. Add and subtract $\left[\frac{1}{2}(-2)\right]^2 = 1$.
- Factor and combine like terms.

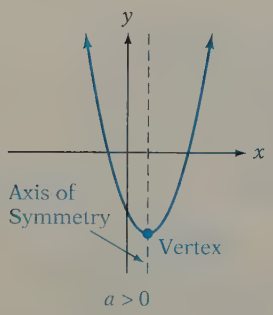
Because the square of a positive number is always positive, we have

$$\begin{aligned} (x - 1)^2 &\geq 0 \\ (x - 1)^2 - 4 &\geq -4 \\ f(x) &\geq -4 \\ y &\geq -4 \end{aligned}$$

- Subtract 4 from each side of the inequality.
- $f(x) = x^2 - 2x - 3 = (x - 1)^2 - 4$

From the last inequality, the range is $\{y | y \geq -4\}$.

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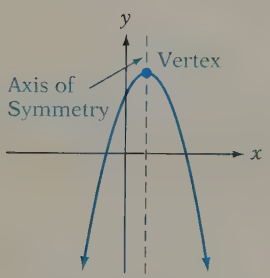


$a > 0$

In general, the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, resembles a "cup" shape as shown at the left. The parabola opens up when $a > 0$ (a is positive) and opens down when $a < 0$ (a is negative). When the parabola opens up, the **vertex** of the parabola is the point with the smallest y-coordinate. When the parabola opens down, the **vertex** is the point with the largest y-coordinate.

The **axis of symmetry** is a line that passes through the vertex of the parabola and is parallel to the y-axis. To understand the axis of symmetry, think of folding the graph along that line. The two portions of the graph will match up.

The following formulas can be used to find the axis of symmetry and the vertex of a parabola.



$a < 0$

The Axis of Symmetry and Vertex of a Parabola

Let $f(x) = ax^2 + bx + c$ be the equation of a parabola.

The equation of the axis of symmetry is $x = -\frac{b}{2a}$.

The coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

➡ Find the axis of symmetry and the vertex of the parabola whose equation is $g(x) = -2x^2 + 3x + 1$. Then graph the equation.

From the equation $g(x) = -2x^2 + 3x + 1$, $a = -2$, $b = 3$, and $c = 1$.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2(-2)} = \frac{3}{4}$

The axis of symmetry is a vertical line passing through the point $\left(\frac{3}{4}, 0\right)$.

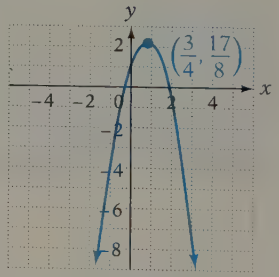
The x-coordinate of the vertex is $\frac{3}{4}$.

Find the y-coordinate of the vertex by replacing x with $\frac{3}{4}$ and evaluating.

$$y = -2x^2 + 3x + 1$$
$$= -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 1 = \frac{17}{8}$$

The vertex is $\left(\frac{3}{4}, \frac{17}{8}\right)$.

Because a is negative ($a = -2$), the graph opens down. Find a few ordered pairs that belong to the function, and then sketch the graph.



$g(x) = -2x^2 + 3x + 1$

TAKE NOTE
Once the coordinates of the vertex are found, the range of a quadratic function can be determined.

Once the y-coordinate of the vertex is known, the range of the function can be determined. Here, the graph of g opens down, so the y-coordinate of the vertex is the largest value of y. Therefore, the range of g is $\left\{y \mid y \leq \frac{17}{8}\right\}$. The value of $-2x^2 + 3x + 1$ is a real number for all values of x; the domain is $\{x \mid x \in \text{real numbers}\}$.

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Example 1

Find the vertex and axis of symmetry of the parabola whose equation is $y = -x^2 + 4x + 1$. Then graph the equation.

Solution

x-coordinate of vertex:

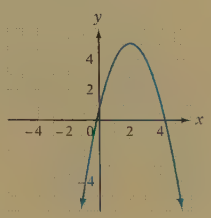
$$-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

y-coordinate of vertex:

$$y = -x^2 + 4x + 1$$
$$= -(2)^2 + 4(2) + 1$$
$$= 5$$

vertex: $(2, 5)$

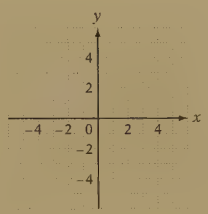
axis of symmetry: $x = 2$



You Try It 1

Find the vertex and axis of symmetry of the parabola whose equation is $y = 4x^2 + 4x + 1$. Then graph the equation.

Your solution



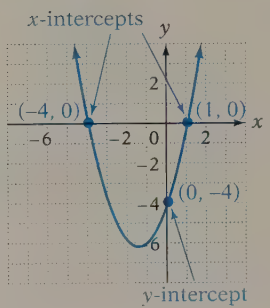
Solution on p. S32

Objective B To find the x-intercepts of a parabola

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Recall that a point at which a graph crosses the x- or y-axis is called an *intercept* of the graph. The x-intercepts of the graph of an equation occur when $y = 0$; the y-intercepts occur when $x = 0$.

The graph of $y = x^2 + 3x - 4$ is shown at the right. The points whose coordinates are $(-4, 0)$ and $(1, 0)$ are x-intercepts of the graph.



Find the x-intercepts of the parabola whose equation is $y = 4x^2 - 4x + 1$.

To find the x-intercepts, let $y = 0$ and then solve for x.

$$y = 4x^2 - 4x + 1$$
$$0 = 4x^2 - 4x + 1$$

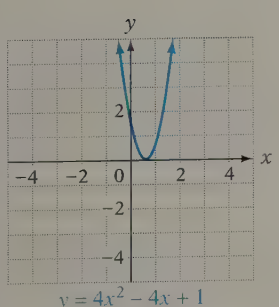
- Let $y = 0$.

$$0 = (2x - 1)(2x - 1)$$

- Solve for x by factoring and using the Principle of Zero Products.

$$2x - 1 = 0 \qquad 2x - 1 = 0$$
$$2x = 1 \qquad 2x = 1$$
$$x = \frac{1}{2} \qquad x = \frac{1}{2}$$

The x-intercept is $\left(\frac{1}{2}, 0\right)$.



$y = 4x^2 - 4x + 1$

In this last example, the parabola has only one x-intercept. In this case, the parabola is said to be **tangent** to the x-axis at $x = \frac{1}{2}$.

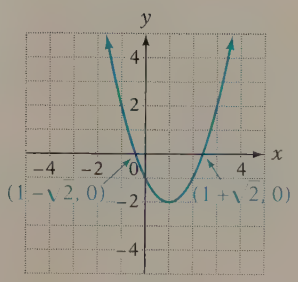
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➡ Find the x-intercepts of $y = x^2 - 2x - 1$.

To find the x-intercepts, let $y = 0$ and solve for x.

$y = x^2 - 2x - 1$

$0 = x^2 - 2x - 1$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

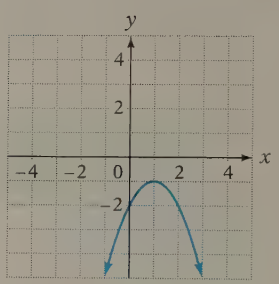
$$= \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

- Because $x^2 - 2x - 1$ does not easily factor, use the quadratic formula to solve for x.
- $a = 1, b = -2, c = -1$

The x-intercepts are $(1 - \sqrt{2}, 0)$ and $(1 + \sqrt{2}, 0)$.

The graph of a parabola may not have x-intercepts. The graph of $y = -x^2 + 2x - 2$ is shown at the right. Note that the graph does not pass through the x-axis and thus there are no x-intercepts. This means there are no real number solutions of $-x^2 + 2x - 2 = 0$.



Using the quadratic formula, we find that the solutions of the equation $-x^2 + 2x - 2 = 0$ are the complex numbers $1 - i$ and $1 + i$.

Recall that the discriminant of the quadratic formula is the expression $b^2 - 4ac$ and that this expression can be used to determine whether $ax^2 + bx + c = 0$ has zero, one, or two real number solutions. Because there is a connection between the solutions of $ax^2 + bx + c = 0$ and the x-intercepts of the graph of $y = ax^2 + bx + c$, the discriminant can be used to determine the number of x-intercepts of a parabola.

The Effect of the Discriminant on the Number of x-Intercepts of a Parabola

1. If $b^2 - 4ac = 0$, the parabola has one x-intercept.
2. If $b^2 - 4ac > 0$, the parabola has two x-intercepts.
3. If $b^2 - 4ac < 0$, the parabola has no x-intercepts.

➡ Use the discriminant to determine the number of x-intercepts of the parabola whose equation is $y = 2x^2 - x + 2$.

$b^2 - 4ac$

$(-1)^2 - 4(2)(2) = 1 - 16 = -15$

$-15 < 0$

- Evaluate the discriminant. $a = 2, b = -1, c = 2$

The discriminant is less than zero, so the parabola has no x-intercepts.

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Example 2

Find the x-intercepts of $y = 2x^2 - 5x + 2$.

Solution

$y = 2x^2 - 5x + 2$
$0 = 2x^2 - 5x + 2$
$0 = (2x - 1)(x - 2)$

$2x - 1 = 0 \qquad x - 2 = 0$
$2x = 1 \qquad x = 2$
$x = \frac{1}{2}$

The x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $(2, 0)$.

You Try It 2

Find the x-intercepts of $y = x^2 + 3x + 4$.

Your solution

Example 3

Use the discriminant to determine the number of x-intercepts of $y = x^2 - 6x + 9$.

Solution

$a = 1, b = -6, c = 9$
$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

Because the discriminant is equal to zero, the parabola has one x-intercept.

You Try It 3

Use the discriminant to determine the number of x-intercepts of $y = x^2 - x - 6$.

Your solution

Solutions on p. S33

Objective C

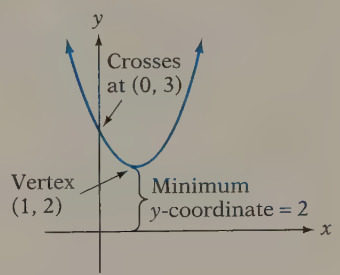
To find the minimum or maximum of a quadratic function

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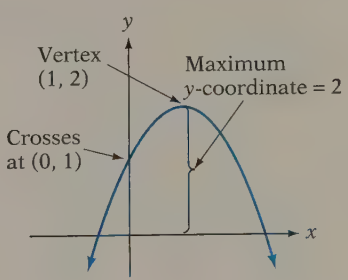
Point of Interest

Calculus is a branch of mathematics that demonstrates, among other things, how to find the maximum or minimum of functions other than quadratic functions. These are very important problems in applied mathematics. For instance, an automotive engineer wants to design a car whose shape will *minimize* the effects of air flow. The same engineer tries to *maximize* the efficiency of a car's engine. Similarly, an economist may try to determine what business practices will *minimize* cost and *maximize* profit.

The graph of $f(x) = x^2 - 2x + 3$ is shown at the right. Because a is positive, the parabola opens up. The vertex of the parabola is the lowest point on the parabola. It is the point that has the minimum y-coordinate. Therefore, the value of the function at this point is a **minimum**.



The graph of $f(x) = -x^2 + 2x + 1$ is shown at the right. Because a is negative, the parabola opens down. The vertex of the parabola is the highest point on the parabola. It is the point that has the maximum y-coordinate. Therefore, the value of the function at this point is a **maximum**.



To find the minimum or maximum value of a quadratic function, first find the x-coordinate of the vertex. Then evaluate the function at that value.

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Example 4

Find the maximum or minimum value of $f(x) = 2x^2 - 3x + 1$.

Solution

$$x = -\frac{b}{2a} = -\frac{-3}{2(2)} = \frac{3}{4}$$

$$f(x) = 2x^2 - 3x + 1$$

$$f\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$$

$$= \frac{9}{8} - \frac{9}{4} + 1 = -\frac{1}{8}$$

Because a is positive, the graph opens up. The function has a minimum value.

The minimum value of the function is $-\frac{1}{8}$.

You Try It 4

Find the maximum or minimum value of $f(x) = -3x^2 + 4x - 1$.

Your solution

Solution on p. S33

To solve application problems

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Example 5

A mining company has determined that the cost C, in dollars per ton, of mining a mineral is given by the equation $C(x) = 0.2x^2 - 2x + 12$, where x is the number of tons of the mineral that are mined. Find the number of tons of the mineral that should be mined to minimize the cost. What is the minimum cost?

Strategy

- To find the number of tons that will minimize the cost, find the x-coordinate of the vertex.
- To find the minimum cost, evaluate the function at the x-coordinate of the vertex.

Solution

$$x = -\frac{b}{2a} = -\frac{-2}{2(0.2)} = 5$$

To minimize cost, 5 tons should be mined.

$$C(x) = 0.2x^2 - 2x + 12$$
$$C(5) = 0.2(5)^2 - 2(5) + 12 = 5 - 10 + 12 = 7$$

The minimum cost per ton is \$7.

You Try It 5

The height s, in feet, of a ball thrown straight up is given by the equation $s(t) = -16t^2 + 64t$, where t is the time in seconds. Find the time it takes the ball to reach its maximum height. What is the maximum height?

Your strategy

Your solution

Solution on p. S33

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11.2 Exercises

Objective A

1. Describe (a) the vertex and (b) the axis of symmetry of a parabola.

2. The axis of symmetry of a parabola is the line $x = -5$. What is the x-coordinate of the vertex of the parabola?

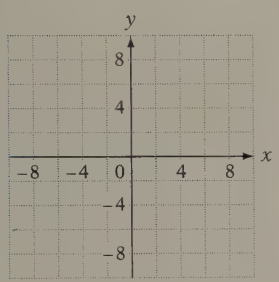
3. The axis of symmetry of a parabola is the line $x = 8$. What is the x-coordinate of the vertex of the parabola?

4. The vertex of a parabola is $(7, -9)$. What is the equation of the axis of symmetry of the parabola?

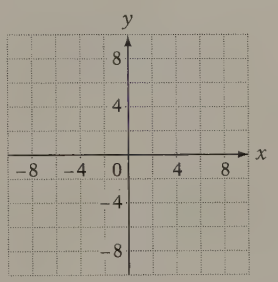
5. The vertex of a parabola is $(-4, 10)$. What is the equation of the axis of symmetry of the parabola?

Find the vertex and axis of symmetry of the parabola. Then sketch its graph.

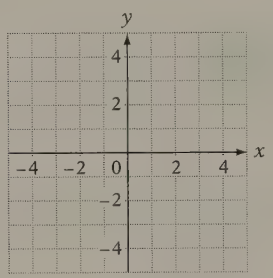
6. $y = x^2 - 2x - 4$



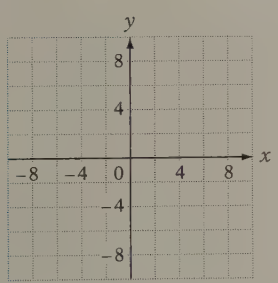
7. $y = x^2 + 4x - 4$



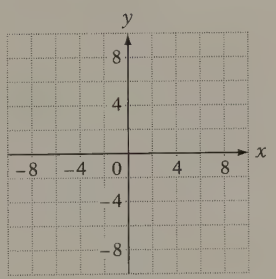
8. $y = -x^2 + 2x - 3$



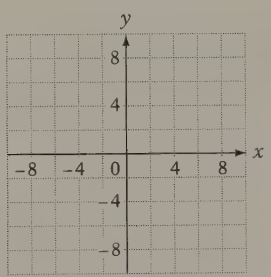
9. $y = -x^2 + 4x - 5$



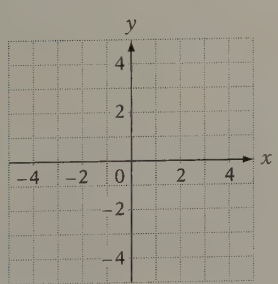
10. $f(x) = x^2 + 6x + 5$



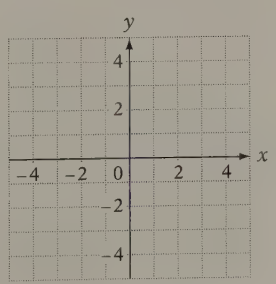
11. $f(x) = x^2 - x - 6$



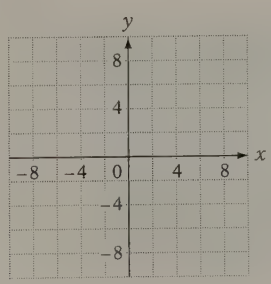
12. $G(x) = x^2 - x - 2$



13. $F(x) = x^2 - 3x + 2$

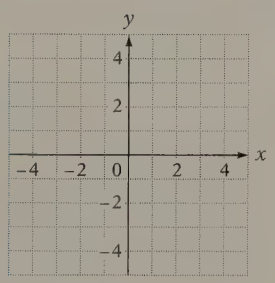


14. $y = -2x^2 + 6x$

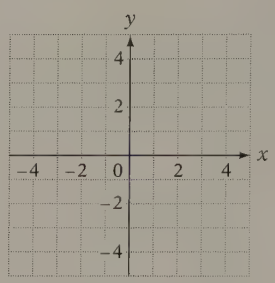


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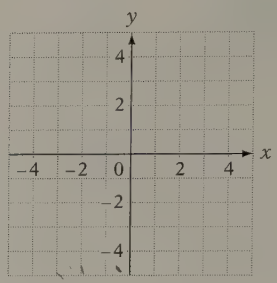
15. $y = -x^2 + 2x - 1$



16. $h(x) = \frac{1}{2}x^2 - x + 1$



17. $P(x) = -\frac{1}{2}x^2 + 2x - 3$



State the domain and the range of the function.

18. $f(x) = 2x^2 - 4x - 5$

19. $f(x) = 2x^2 + 8x + 3$

20. $f(x) = -x^2 + 6x - 9$

21. $f(x) = -x^2 - 4x - 5$

Objective B

22. What are the x-intercepts of a parabola?

Find the x-intercepts of the graph of the parabola.

23. $y = x^2 - 4$

24. $y = x^2 - 9$

25. $y = 2x^2 - 4x$

26. $y = 3x^2 + 6x$

27. $y = x^2 - x - 2$

28. $y = x^2 - 2x - 8$

29. $y = 2x^2 - x - 1$

30. $y = 2x^2 - 5x - 3$

31. $y = x^2 + 2x - 1$

32. $y = x^2 + 4x - 3$

33. $y = x^2 + 6x + 10$

34. $y = -x^2 - 4x - 5$

Use the discriminant to determine the number of x-intercepts of the graph.

35. $y = 2x^2 + x + 1$

36. $y = 2x^2 + 2x - 1$

37. $y = -x^2 - x + 3$

38. $y = -2x^2 + x + 1$

39. $y = x^2 - 8x + 16$

40. $y = x^2 - 10x + 25$

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41. $y = -3x^2 - x - 2$ **42.** $y = -2x^2 + x - 1$ **43.** $y = 4x^2 - x - 2$

Objective C

44. When does a quadratic function have a minimum value and when does it have a maximum value?

45. Does the function have a minimum or a maximum value?
a. $f(x) = -x^2 + 6x - 1$ **b.** $f(x) = 2x^2 - 4$ **c.** $f(x) = -5x^2 + x$

46. Does the function have a minimum or a maximum value?
a. $f(x) = 3x^2 - 2x + 4$ **b.** $f(x) = -x^2 + 9$ **c.** $f(x) = 6x^2 - 3x$

Find the minimum or maximum value of the quadratic function.

47. $f(x) = x^2 - 2x + 3$ **48.** $f(x) = x^2 + 3x - 4$ **49.** $f(x) = -2x^2 + 4x - 3$

50. $f(x) = -2x^2 - 3x + 4$ **51.** $f(x) = 2x^2 + 4x$ **52.** $f(x) = -2x^2 - 3x$

53. $f(x) = -2x^2 + 4x - 5$ **54.** $f(x) = -3x^2 + x - 6$ **55.** $f(x) = 2x^2 + 3x - 8$

56. Which of the following parabolas has the greatest minimum value?
a. $y = x^2 - 2x - 3$ **b.** $y = x^2 - 10x + 20$ **c.** $y = 3x^2 - 6$

57. Which of the following parabolas has the greatest maximum value?
a. $y = -2x^2 + 2x - 1$ **b.** $y = -x^2 + 8x - 2$ **c.** $y = -4x^2 + 3$

Objective D *Application Problems*

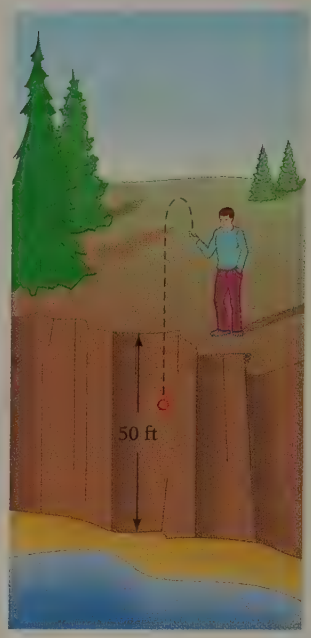
58. A tour operator believes that the profit, P, from selling x tickets is given by $P(x) = 40x - 0.25x^2$. Using this model, what is the maximum profit the tour operator can expect?



59. The suspension cable that supports a small footbridge hangs in the shape of a parabola. The height in feet of the cable above the bridge is given by the function $h(x) = 0.25x^2 - 0.8x + 25$, where x is the distance from one end of the bridge. What is the minimum height of the cable above the bridge?

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60. The height s, in feet, of a rock thrown upward at an initial speed of 64 ft/s from a cliff 50 ft above an ocean beach is given by the function $s(t) = -16t^2 + 64t + 50$, where t is the time in seconds. Find the maximum height above the beach that the rock will attain.



61. An event in the Summer Olympics is 10-meter springboard diving. In this event, the height s, in meters, of a diver above the water t seconds after jumping is given by $s(t) = -4.9t^2 + 7.8t + 10$. What is the maximum height that the diver will be above the water? Round to the nearest tenth.

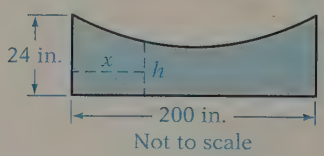
62. A pool is treated with a chemical to reduce the amount of algae in the pool. The amount of algae in the pool t days after the treatment can be approximated by the function $A(t) = 40t^2 - 400t + 500$. How many days after treatment will the pool have the least amount of algae?

63. Some football fields are built in a parabolic mound shape so that water will drain off the field. A model for the shape of the field is given by $h(x) = -0.00023475x^2 + 0.0375x$, where h is the height of the field in feet at a distance of x feet from the sideline. What is the maximum height? Round to the nearest tenth.

64. An equation that models the thickness h, in inches, of the mirror at the Palomar Mountain Observatory is given by

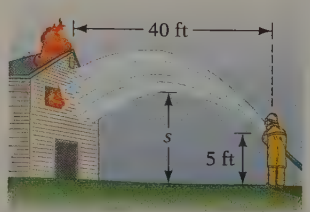
$$h(x) = 0.000379x^2 - 0.0758x + 24$$

where x is measured in inches from the edge of the mirror. Find the minimum thickness of the mirror.



65. The fuel efficiency of an average car is given by the equation $E(v) = -0.018v^2 + 1.476v + 3.4$, where E is the fuel efficiency in miles per gallon and v is the speed of the car in miles per hour. What speed will yield the maximum fuel efficiency? What is the maximum fuel efficiency?

66. The height s, in feet, of water squirting from a fire hose nozzle is given by the equation $s(x) = -\frac{1}{30}x^2 + 2x + 5$, where x is the horizontal distance, in feet, from the nozzle. How high on a building 40 ft from the fire hose will the water land?



APPLYING THE CONCEPTS

67. One root of the quadratic equation $2x^2 - 5x + k = 0$ is 4. What is the other root?

68. What is the value of k if the vertex of the parabola $y = x^2 - 8x + k$ is a point on the x-axis?

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11.3 Graphs of Functions

Objective A

To graph functions

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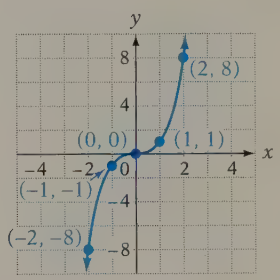
The graphs of the polynomial functions $f(x) = mx + b$ (a straight line) and $f(x) = ax^2 + bx + c$, $a \neq 0$ (a parabola) have been discussed. The graphs of other functions can be drawn by finding ordered pairs that belong to the function, plotting the points that correspond to the ordered pairs, and then drawing a curve through the points.

TAKE NOTE
The units along the x-axis are different from those along the y-axis. If the units along the x-axis were the same as those along the y-axis, the graph would appear narrower than the one shown.

➡ Graph: $F(x) = x^3$

Select several values of x and evaluate the function.

x	$F(x) = x^3$	$F(x)$	(x, y)
-2	$(-2)^3$	-8	$(-2, -8)$
-1	$(-1)^3$	-1	$(-1, -1)$
0	0^3	0	$(0, 0)$
1	1^3	1	$(1, 1)$
2	2^3	8	$(2, 8)$

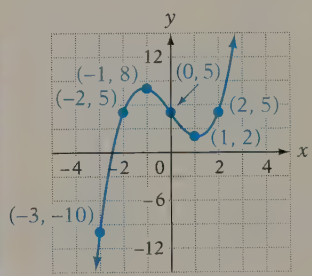


Plot the ordered pairs and draw a graph through the points.

➡ Graph: $g(x) = x^3 - 4x + 5$

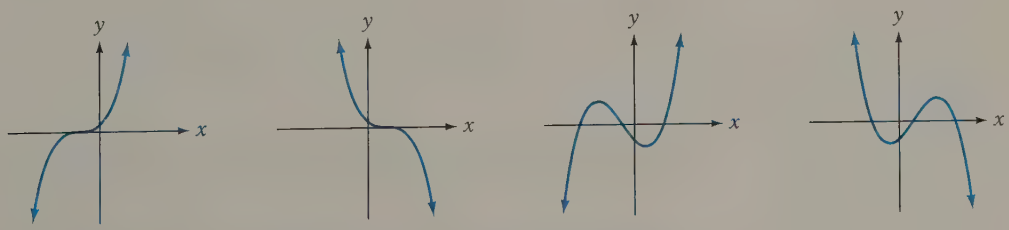
Select several values of x and evaluate the function.

x	$g(x) = x^3 - 4x + 5$	$g(x)$	(x, y)
-3	$(-3)^3 - 4(-3) + 5$	-10	$(-3, -10)$
-2	$(-2)^3 - 4(-2) + 5$	5	$(-2, 5)$
-1	$(-1)^3 - 4(-1) + 5$	8	$(-1, 8)$
0	$(0)^3 - 4(0) + 5$	5	$(0, 5)$
1	$(1)^3 - 4(1) + 5$	2	$(1, 2)$
2	$(2)^3 - 4(2) + 5$	5	$(2, 5)$



Plot the ordered pairs and draw a graph through the points.

Note from the graphs of the two cubic functions above that the shapes of the graphs can be different. The following graphs of typical cubic polynomial functions show their general shapes.



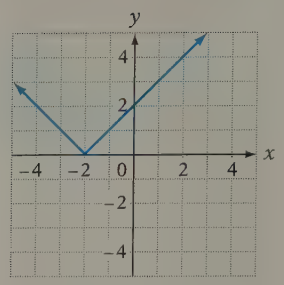
As the degree of a polynomial increases, the graph of the polynomial function can change significantly. In these cases, it may be necessary to plot many points before an accurate graph can be drawn. Only polynomials of degree 3 are considered here.

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➡ Graph: $f(x) = |x + 2|$

This is an absolute value function.

x	$f(x) = \lvert x + 2 \rvert$	$f(x)$	(x, y)
-3	$\lvert -3 + 2 \rvert$	1	$(-3, 1)$
-2	$\lvert -2 + 2 \rvert$	0	$(-2, 0)$
-1	$\lvert -1 + 2 \rvert$	1	$(-1, 1)$
0	$\lvert 0 + 2 \rvert$	2	$(0, 2)$
1	$\lvert 1 + 2 \rvert$	3	$(1, 3)$
2	$\lvert 2 + 2 \rvert$	4	$(2, 4)$



In general, the graph of the absolute value of a linear polynomial is V-shaped.

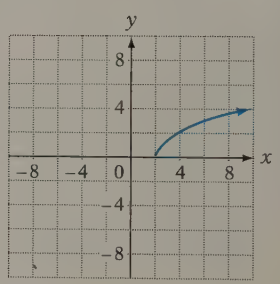
➡ Graph: $R(x) = \sqrt{2x - 4}$

This is a radical function. Because the square root of a negative number is not a real number, the domain of this function requires that $2x - 4 \geq 0$. Solve this inequality for x.

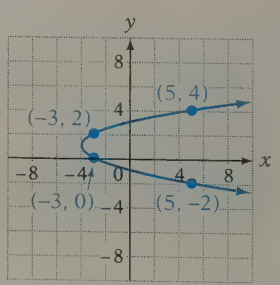
$$\begin{aligned} 2x - 4 &\geq 0 \\ 2x &\geq 4 \\ x &\geq 2 \end{aligned}$$

The domain is $\{x | x \geq 2\}$. This means that only values of x that are greater than or equal to 2 can be chosen as values at which to evaluate the function. In this case, some of the y-coordinates must be approximated.

x	$R(x) = \sqrt{2x - 4}$	$R(x)$	(x, y)
2	$\sqrt{2(2) - 4}$	0	$(2, 0)$
3	$\sqrt{2(3) - 4}$	1.41	$(3, 1.41)$
4	$\sqrt{2(4) - 4}$	2	$(4, 2)$
5	$\sqrt{2(5) - 4}$	2.45	$(5, 2.45)$
6	$\sqrt{2(6) - 4}$	2.83	$(6, 2.83)$



Recall that a function is a special type of relation, one for which no two ordered pairs have the same first coordinate. Graphically, this means that the graph of a function cannot pass through two points that have the same x-coordinate and different y-coordinates. For instance, the graph at the right is not the graph of a function because there are ordered pairs with the same x-coordinate and different y-coordinates.



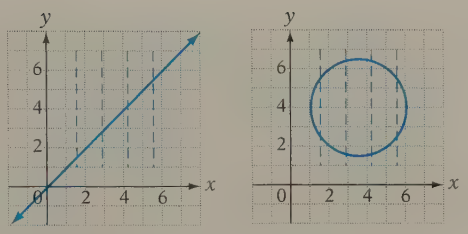
This last graph illustrates a general statement that can be made about whether a graph defines a function. It is called the *vertical-line test*.

Vertical-Line Test

A graph defines a function if any vertical line intersects the graph at no more than one point.

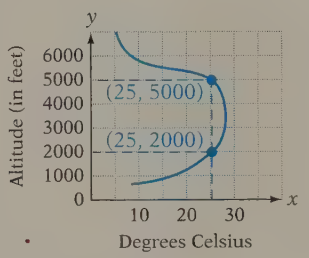
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For example, the graph of a nonvertical straight line is the graph of a function. Any vertical line intersects the graph no more than once. The graph of a circle, however, is not the graph of a function. There are vertical lines that intersect the graph at more than one point.



There are practical situations in which a graph is not the graph of a function. Below is an example of such an application.

One of the causes of smog is an inversion layer of air where temperatures at higher altitudes are warmer than those at lower altitudes. The graph at the right shows the altitudes at which various temperatures were recorded. As shown by the dashed lines in the graph, there are two altitudes at which the temperature was 25°C. This means that there are two ordered pairs (shown in the graph) with the same first coordinate but different second coordinates. The graph does not define a function.

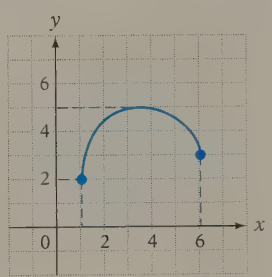


When a graph does define a function, the domain and range can be estimated from the graph.

TAKE NOTE

To determine the domain, think of collapsing the graph onto the x-axis and determining the interval on the x-axis that the graph covers. To determine the range, think of collapsing the graph onto the y-axis and determining the interval on the y-axis that the graph covers.

➡ Determine the domain and range of the function given by the graph at the right.

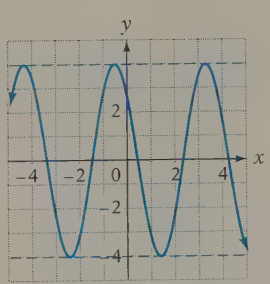


The solid dots on the graph indicate its beginning and ending points.

The domain is the set of x-coordinates.
Domain: $\{x \mid 1 \le x \le 6\}$

The range is the set of y-coordinates.
Range: $\{y \mid 2 \le y \le 5\}$

➡ Determine the domain and range of the function given by the graph at the right.



The arrows on the graph indicate that the graph continues in the same manner.

The domain is the set of x-coordinates.
Domain: $\{x \mid x \in \text{real numbers}\}$

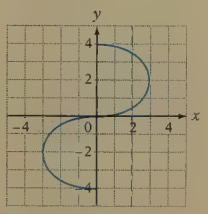
The range is the set of y-coordinates.
Range: $\{y \mid -4 \le y \le 4\}$

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Example 1

Use the vertical-line test to determine whether the graph shown is the graph of a function.

Solution

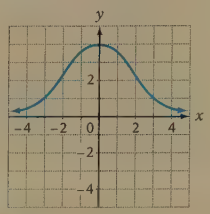


A vertical line intersects the graph more than once. The graph is not the graph of a function.

You Try It 1

Use the vertical-line test to determine whether the graph shown is the graph of a function.

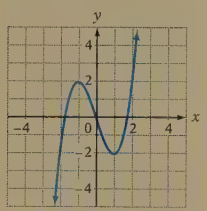
Your solution



Example 2

Graph $f(x) = x^3 - 3x$. State the domain and range of the function.

Solution

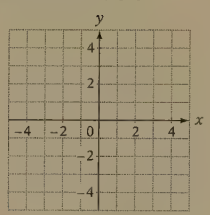


domain: $\{x | x \in \text{real numbers}\}$
range: $\{y | y \in \text{real numbers}\}$

You Try It 2

Graph $f(x) = -\frac{1}{2}x^3 + 2x$. State the domain and range of the function.

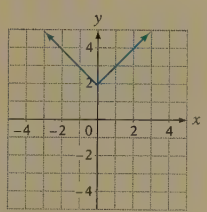
Your solution



Example 3

Graph $f(x) = |x| + 2$. State the domain and range of the function.

Solution

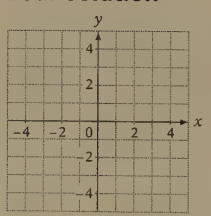


domain: $\{x | x \in \text{real numbers}\}$
range: $\{y | y \geq 2\}$

You Try It 3

Graph $f(x) = |x + 2|$. State the domain and range of the function.

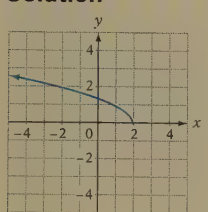
Your solution



Example 4

Graph $f(x) = \sqrt{2 - x}$. State the domain and range of the function.

Solution

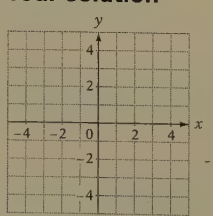


domain: $\{x | x \leq 2\}$
range: $\{y | y \geq 0\}$

You Try It 4

Graph $f(x) = -\sqrt{x - 1}$. State the domain and range of the function.

Your solution



Solutions on p. S33

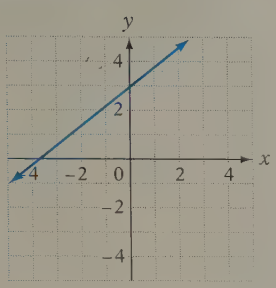
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11.3 Exercises

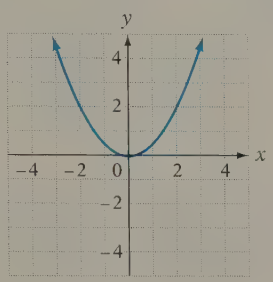
Objective A

Use the vertical-line test to determine whether the graph is the graph of a function.

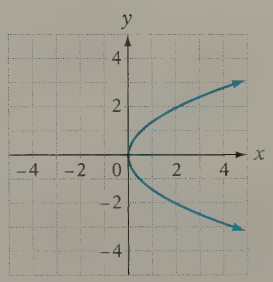
1.



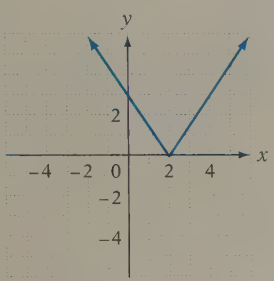
2.



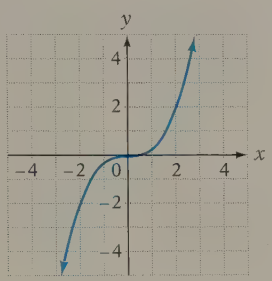
3.



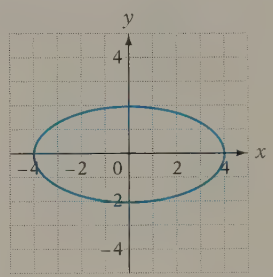
4.



5.

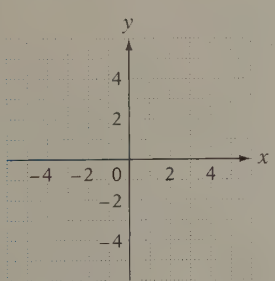


6.

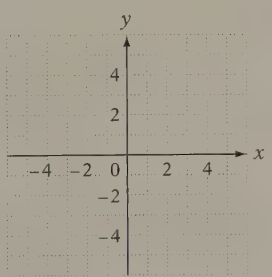


Graph the function and state its domain and range.

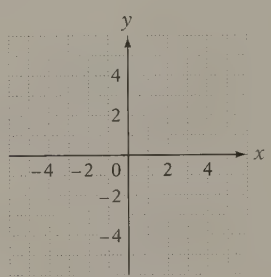
7. $f(x) = 3|2 - x|$



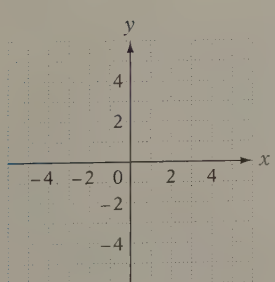
8. $f(x) = x^3 - 1$



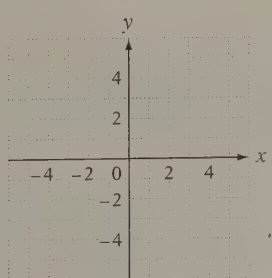
9. $f(x) = 1 - x^3$



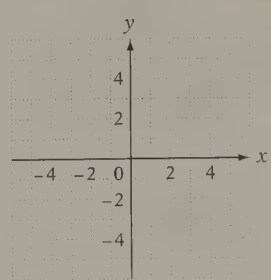
10. $f(x) = \sqrt{1 + x}$



11. $f(x) = \sqrt{4 - x}$

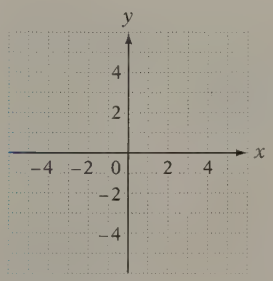


12. $f(x) = |2x - 1|$

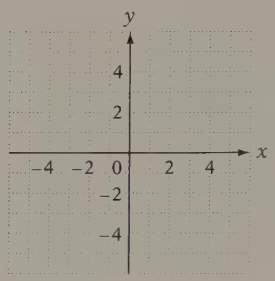


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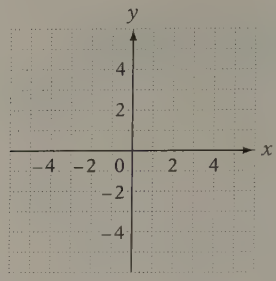
13. $f(x) = x^3 + 4x^2 + 4x$



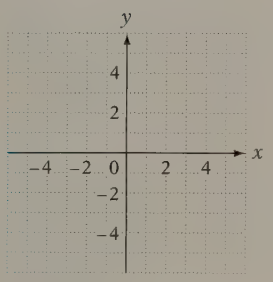
14. $f(x) = x^3 - x^2 - x + 1$



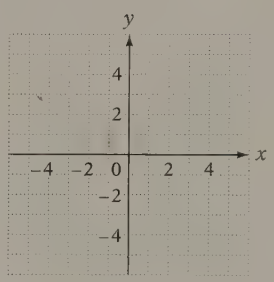
15. $f(x) = -\sqrt{x + 2}$



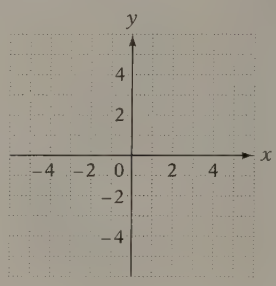
16. $f(x) = -\sqrt{x - 3}$



17. $f(x) = |2x + 2|$



18. $f(x) = 2|x + 1|$



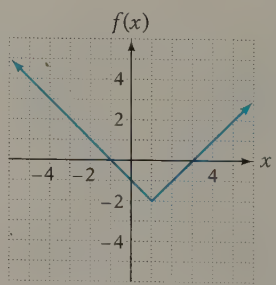
APPLYING THE CONCEPTS

19. If $f(x) = \sqrt{x - 2}$ and $f(a) = 4$, find a.

20. If $f(x) = \sqrt{x + 5}$ and $f(a) = 3$, find a.

21. $f(a, b)$ = the sum of a and b
$g(a, b)$ = the product of a and b
Find $f(2, 5) + g(2, 5)$.

22. The graph of the function f is shown at the right. For this function, which of the following are true?
a. $f(4) = 1$ **b.** $f(0) = 3$ **c.** $f(-3) = 2$



23. Let $f(x)$ be the digit in the xth decimal place of the repeating decimal $0.\overline{387}$. For example, $f(3) = 7$ because 7 is the digit in the third decimal place. Find $f(14)$.

24. Given $f(x) = (x + 1)(x - 1)$, for what values of x is $f(x)$ negative? Write your answer in set-builder notation.

25. Given $f(x) = -|x + 3|$, for what value of x is $f(x)$ greatest?

26. Given $f(x) = |2x - 2|$, for what value of x is $f(x)$ smallest?

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11.4 Algebra of Functions

Objective A

To perform operations on functions



The operations of addition, subtraction, multiplication, and division of functions are defined as follows.

Operations on Functions

If f and g are functions and x is an element of the domain of each function, then

$(f + g)(x) = f(x) + g(x)$ $\qquad$ $(f \cdot g)(x) = f(x) \cdot g(x)$

$(f - g)(x) = f(x) - g(x)$ $\qquad$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

➡ Given $f(x) = x^2 + 1$ and $g(x) = 3x - 2$, find $(f + g)(3)$ and $(f \cdot g)(-1)$.

$$\begin{aligned}(f + g)(3) &= f(3) + g(3)\\ &= [(3)^2 + 1] + [3(3) - 2]\\ &= 10 + 7 = 17\end{aligned}$$

$$\begin{aligned}(f \cdot g)(-1) &= f(-1) \cdot g(-1)\\ &= [(-1)^2 + 1] \cdot [3(-1) - 2]\\ &= 2 \cdot (-5) = -10\end{aligned}$$

Consider the functions f and g from the last example. Let $S(x)$ be the sum of the two functions. Then

$$\begin{aligned}S(x) &= (f + g)(x) = f(x) + g(x)\\ &= (x^2 + 1) + (3x - 2)\\ S(x) &= x^2 + 3x - 1\end{aligned}$$

- The definition of addition of functions
- $f(x) = x^2 + 1$, $g(x) = 3x - 2$

Now evaluate $S(3)$.

$$\begin{aligned}S(3) &= (3)^2 + 3(3) - 1\\ &= 9 + 9 - 1\\ &= 17 = (f + g)(3)\end{aligned}$$

Note that $S(3) = 17$ and $(f + g)(3) = 17$. This shows that adding $f(x) + g(x)$ and then evaluating is the same as evaluating $f(x)$ and $g(x)$ and then adding. The same is true for the other operations on functions. For instance, let $P(x)$ be the product of the functions f and g. Then

$$\begin{aligned}P(x) &= (f \cdot g)(x) = f(x) \cdot g(x)\\ &= (x^2 + 1)(3x - 2)\\ &= 3x^3 - 2x^2 + 3x - 2\\ P(-1) &= 3(-1)^3 - 2(-1)^2 + 3(-1) - 2\\ &= -3 - 2 - 3 - 2\\ &= -10\end{aligned}$$

- Note that $P(-1) = -10$ and $(f \cdot g)(-1) = -10$.

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Given $f(x) = 2x^2 - 5x + 3$ and $g(x) = x^2 - 1$, find $\left(\frac{f}{g}\right)(1)$.

$$\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)}$$
$$= \frac{2(1)^2 - 5(1) + 3}{(1)^2 - 1} = \frac{0}{0}$$ • Not a real number

Because $\frac{0}{0}$ is not defined, the expression $\left(\frac{f}{g}\right)(1)$ cannot be evaluated.

Example 1

Given $f(x) = x^2 - x + 1$ and $g(x) = x^3 - 4$, find $(f - g)(3)$.

Solution

$$(f - g)(3) = f(3) - g(3)$$
$$= (3^2 - 3 + 1) - (3^3 - 4)$$
$$= 7 - 23$$
$$= -16$$

$(f - g)(3) = -16$

You Try It 1

Given $f(x) = x^2 + 2x$ and $g(x) = 5x - 2$, find $(f + g)(-2)$.

Your solution

Example 2

Given $f(x) = x^2 + 2$ and $g(x) = 2x + 3$, find $(f \cdot g)(-2)$.

Solution

$$(f \cdot g)(-2) = f(-2) \cdot g(-2)$$
$$= [(-2)^2 + 2] \cdot [2(-2) + 3]$$
$$= 6 \cdot (-1)$$
$$= -6$$

$(f \cdot g)(-2) = -6$

You Try It 2

Given $f(x) = 4 - x^2$ and $g(x) = 3x - 4$, find $(f \cdot g)(3)$.

Your solution

Example 3

Given $f(x) = x^2 + 4x + 4$ and $g(x) = x^3 - 2$, find $\left(\frac{f}{g}\right)(3)$.

Solution

$$\left(\frac{f}{g}\right)(3) = \frac{f(3)}{g(3)}$$
$$= \frac{3^2 + 4(3) + 4}{3^3 - 2}$$
$$= \frac{25}{25}$$
$$= 1$$

$\left(\frac{f}{g}\right)(3) = 1$

You Try It 3

Given $f(x) = x^2 - 4$ and $g(x) = x^2 + 2x + 1$, find $\left(\frac{f}{g}\right)(4)$.

Your solution

Solutions on pp. S33–S34

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Objective B To find the composition of two functions

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A function can be evaluated at the value of another function. Consider

$$f(x) = 2x + 7 \quad \text{and} \quad g(x) = x^2 + 1$$

The expression $f[g(-2)]$ means to evaluate the function f at $g(-2)$.

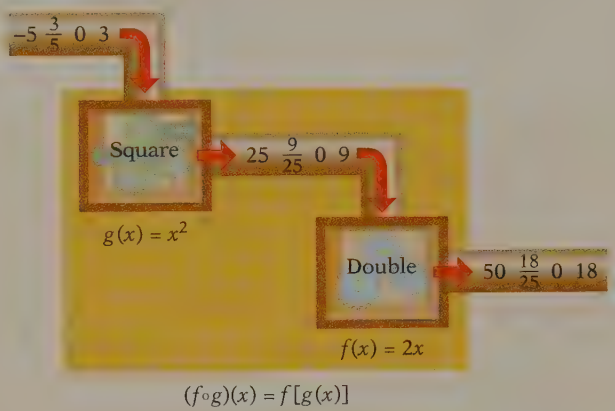
$$g(-2) = (-2)^2 + 1 = 4 + 1 = 5$$
$$f[g(-2)] = f(5) = 2(5) + 7 = 10 + 7 = 17$$

Definition of the Composition of Two Functions

Let f and g be two functions such that $g(x)$ is in the domain of f for all x in the domain of g. Then the **composition** of the two functions, denoted by $f \circ g$, is the function whose value at x is given by $(f \circ g)(x) = f[g(x)]$.

The function defined by $f[g(x)]$ is called the **composite** of f and g.

The function machine at the right illustrates the composition of two functions, $g(x) = x^2$ and $f(x) = 2x$. Note that a composite function combines two functions. First one function pairs an input with an output. Then that output is used as the input for a second function, which in turn produces a final output.



The requirement in the definition of the composition of two functions that $g(x)$ be in the domain of f for all x in the domain of g is important. For instance, let

$$f(x) = \frac{1}{x - 1} \quad \text{and} \quad g(x) = 3x - 5$$

When $x = 2$,

$$g(2) = 3(2) - 5 = 1$$
$$f[g(2)] = f(1) = \frac{1}{1 - 1} = \frac{1}{0}$$

• This is not a real number.

In this case, $g(2)$ is not in the domain of f. Thus the composition is not defined at 2.

➨ Given $f(x) = x^3 - x + 1$ and $g(x) = 2x^2 - 10$, evaluate $(g \circ f)(2)$.

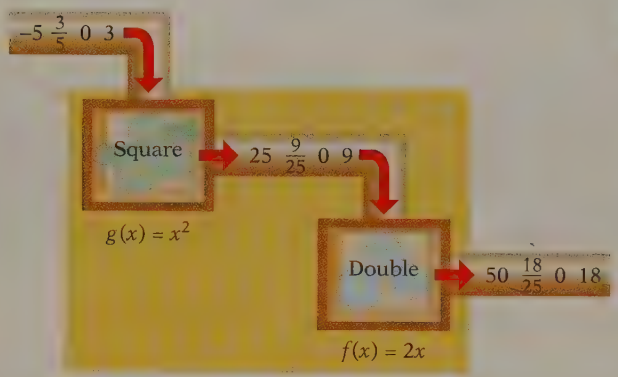
$$f(2) = (2)^3 - (2) + 1 = 7$$
$$(g \circ f)(2) = g[f(2)]$$
$$= g(7)$$
$$= 2(7)^2 - 10 = 88$$
$$(g \circ f)(2) = 88$$

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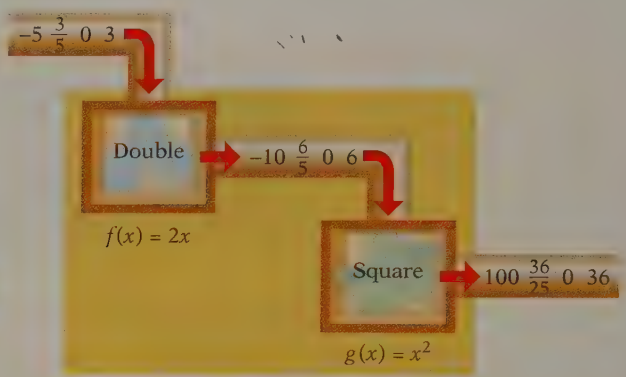
➡ Given $f(x) = 3x - 2$ and $g(x) = x^2 - 2x$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f[g(x)] = 3(x^2 - 2x) - 2$$
$$= 3x^2 - 6x - 2$$

When we are evaluating compositions of functions, the order in which the functions are applied is important. In the two diagrams below, the order of the *square function*, $g(x) = x^2$, and the *double function*, $f(x) = 2x$, is interchanged. Note that the final outputs are different. Therefore, $(g \circ f)(x) \neq (f \circ g)(x)$.



$(f \circ g)(x) = f[g(x)]$



$(g \circ f)(x) = g[f(x)]$

Example 4

Given $f(x) = x^2 - x$ and $g(x) = 3x - 2$, find $f[g(3)]$.

Solution

$$g(x) = 3x - 2$$
$$g(3) = 3(3) - 2 = 9 - 2 = 7$$
$$f(x) = x^2 - x$$
$$f[g(3)] = f(7) = 7^2 - 7 = 42$$

You Try It 4

Given $f(x) = 1 - 2x$ and $g(x) = x^2$, find $f[g(-1)]$.

Your solution

Example 5

Given $s(t) = t^2 + 3t - 1$ and $v(t) = 2t + 1$, determine $s[v(t)]$.

Solution

$$s(t) = t^2 + 3t - 1$$
$$s[v(t)] = (2t + 1)^2 + 3(2t + 1) - 1$$
$$= 4t^2 + 4t + 1 + 6t + 3 - 1$$
$$= 4t^2 + 10t + 3$$

You Try It 5

Given $L(s) = s + 1$ and $M(s) = s^3 + 1$, determine $M[L(s)]$.

Your solution

Solutions on p. S34

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11.4 Exercises

Objective A

For $f(x) = 2x^2 - 3$ and $g(x) = -2x + 4$, find:

1. $(f - g)(2)$
2. $(f - g)(3)$
3. $(f + g)(0)$
4. $(f + g)(1)$
5. $(f \cdot g)(2)$
6. $(f \cdot g)(-1)$
7. $\left(\frac{f}{g}\right)(4)$
8. $\left(\frac{f}{g}\right)(-1)$
9. $\left(\frac{g}{f}\right)(-3)$

For $f(x) = 2x^2 + 3x - 1$ and $g(x) = 2x - 4$, find:

10. $(f + g)(-3)$
11. $(f + g)(1)$
12. $(f - g)(-2)$
13. $(f - g)(4)$
14. $(f \cdot g)(-2)$
15. $(f \cdot g)(1)$
16. $\left(\frac{f}{g}\right)(2)$
17. $\left(\frac{f}{g}\right)(-3)$
18. $(f \cdot g)\left(\frac{1}{2}\right)$

For $f(x) = x^2 + 3x - 5$ and $g(x) = x^3 - 2x + 3$, find:

19. $(f - g)(2)$
20. $(f \cdot g)(-3)$
21. $\left(\frac{f}{g}\right)(-2)$

Objective B

Given $f(x) = 2x - 3$ and $g(x) = 4x - 1$, evaluate the composite function.

22. $f[g(0)]$
23. $g[f(0)]$
24. $f[g(2)]$
25. $g[f(-2)]$
26. $f[g(x)]$
27. $g[f(x)]$

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Given $h(x) = 2x + 4$ and $f(x) = \frac{1}{2}x + 2$, evaluate the composite function.

28. $(h \circ f)(0)$ **29.** $(f \circ h)(0)$ **30.** $(h \circ f)(2)$

31. $(f \circ h)(-1)$ **32.** $(h \circ f)(x)$ **33.** $(f \circ h)(x)$

Given $f(x) = x^2 + x + 1$ and $h(x) = 3x + 2$, evaluate the composite function.

34. $(f \circ h)(0)$ **35.** $(h \circ f)(0)$ **36.** $(f \circ h)(-1)$

37. $(h \circ f)(-2)$ **38.** $(f \circ h)(x)$ **39.** $(h \circ f)(x)$

Given $f(x) = x - 2$ and $g(x) = x^3$, evaluate the composite function.

40. $(f \circ g)(2)$ **41.** $(f \circ g)(-1)$ **42.** $(g \circ f)(2)$

43. $(g \circ f)(-1)$ **44.** $(f \circ g)(x)$ **45.** $(g \circ f)(x)$

APPLYING THE CONCEPTS

For the function $g(x) = x^2 - 1$, find:

46. $g(2 + h)$ **47.** $g(3 + h) - g(3)$ **48.** $g(-1 + h) - g(-1)$

49. $\dfrac{g(1 + h) - g(1)}{h}$ **50.** $\dfrac{g(-2 + h) - g(-2)}{h}$ **51.** $\dfrac{g(a + h) - g(a)}{h}$

Given $f(x) = 2x$, $g(x) = 3x - 1$, and $h(x) = x - 2$, find:

52. $f(g[h(2)])$ **53.** $g(h[f(1)])$ **54.** $h(g[f(-1)])$

55. $f(h[g(0)])$ **56.** $f(g[h(x)])$ **57.** $g(f[h(x)])$

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11.5 One-to-One and Inverse Functions

Objective A

To determine whether a function is one-to-one

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Recall that a function is a set of ordered pairs in which no two ordered pairs that have the same first coordinate have different second coordinates. This means that given any x, there is only one y that can be paired with that x. A **one-to-one function** satisfies the additional condition that given any y, there is only one x that can be paired with the given y. One-to-one functions are commonly written as 1–1.

TAKE NOTE

Recall that a function is a set of ordered pairs in which no two ordered pairs have the same first coordinate. A 1–1 function is a function in which no two ordered pairs have the same second coordinate.

One-to-One Function

A function f is a 1–1 function if, for any a and b in the domain of f, $f(a) = f(b)$ implies that $a = b$.

This definition states that if the y-coordinates of an ordered pair are equal, $f(a) = f(b)$, then the x-coordinates must be equal, $a = b$.

The function defined by $f(x) = 2x + 1$ is a 1–1 function. To show this, determine $f(a)$ and $f(b)$. Then form the equation $f(a) = f(b)$.

$f(a) = 2a + 1 \qquad f(b) = 2b + 1$

$$\begin{aligned} f(a) &= f(b) \\ 2a + 1 &= 2b + 1 \\ 2a &= 2b \qquad \text{• Subtract 1 from each side of the equation.} \\ a &= b \qquad \text{• Divide each side of the equation by 2.} \end{aligned}$$

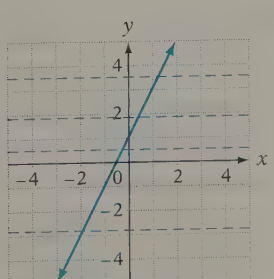
Because $f(a) = f(b)$ implies that $a = b$, the function is a 1–1 function.

Consider the function defined by $g(x) = x^2 - x$. Evaluate the function at -2 and 3.

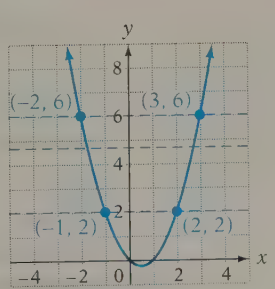
$$g(-2) = (-2)^2 - (-2) = 6 \qquad g(3) = 3^2 - 3 = 6$$

From this evaluation, $g(-2) = 6$ and $g(3) = 6$, but $-2 \neq 3$. Thus g is not a 1–1 function.

The graphs of $f(x) = 2x + 1$ and $g(x) = x^2 - x$ are shown below. Note that a horizontal line intersects the graph of f at no more than one point. However, a horizontal line intersects the graph of g at more than one point.



$f(x) = 2x + 1$



$g(x) = x^2 - x$

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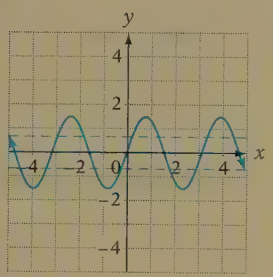
Looking at the graph of f on the previous page, note that for each y-coordinate there is only one x-coordinate. Thus f is a 1–1 function. From the graph of g, however, there are *two* x-coordinates for a given y-coordinate. For instance, $(-2, 6)$ and $(3, 6)$ are the coordinates of two points on the graph for which the y-coordinates are the same and the x-coordinates are different. Therefore, g is not a 1–1 function.

Horizontal-Line Test

The graph of a function represents the graph of a 1–1 function if any horizontal line intersects the graph at no more than one point.

Example 1

Determine whether the graph is the graph of a 1–1 function.

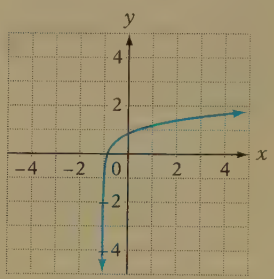


Solution

Because a horizontal line intersects the graph more than once, the graph is not the graph of a 1–1 function.

You Try It 1

Determine whether the graph is the graph of a 1–1 function.



Your solution

Solution on p. S34

Objective B To find the inverse of a function









The **inverse of a function** is the set of ordered pairs formed by reversing the coordinates of each ordered pair of the function.

For example, the set of ordered pairs of the function defined by $f(x) = 2x$ with domain $\{-2, -1, 0, 1, 2\}$ is $\{(-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4)\}$. The set of ordered pairs of the inverse function is $\{(-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2)\}$.

From the ordered pairs of f, we have

$$\text{domain} = \{-2, -1, 0, 1, 2\} \quad \text{and} \quad \text{range} = \{-4, -2, 0, 2, 4\}$$

From the ordered pairs of the inverse function, we have

$$\text{domain} = \{-4, -2, 0, 2, 4\} \quad \text{and} \quad \text{range} = \{-2, -1, 0, 1, 2\}$$

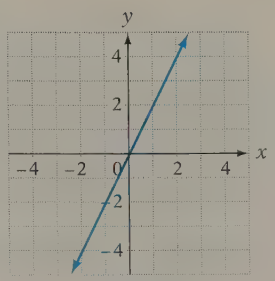
Note that the domain of the inverse function is the range of the original function, and the range of the inverse function is the domain of the original function.

Now consider the function defined by $g(x) = x^2$ with domain $\{-2, -1, 0, 1, 2\}$. The set of ordered pairs of this function is $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$. Reversing the ordered pairs gives $\{(4, -2), (1, -1), (0, 0), (1, 1), (4, 2)\}$. These ordered pairs do not satisfy the condition of a function, because there are ordered pairs with the same first coordinate and different second coordinates. This example illustrates that not all functions have an inverse function.

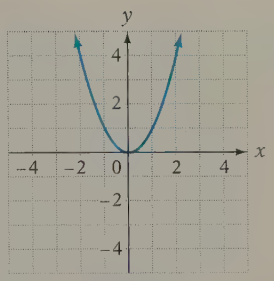
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The graphs of $f(x) = 2x$ and $g(x) = x^2$ with the set of real numbers as the domain are shown at the right.

By the horizontal-line test, f is a 1–1 function but g is not.



$f(x) = 2x$



$g(x) = x^2$

Condition for an Inverse Function

A function f has an inverse function if and only if f is a 1–1 function.

TAKE NOTE

It is important to remember that f^{-1} is the symbol for the *inverse* function and does not denote reciprocal.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

The symbol f^{-1} is used to denote the inverse of the function f. The symbol $f^{-1}(x)$ is read "f inverse of x."

$f^{-1}(x)$ is *not* the reciprocal of $f(x)$, but rather is the notation for the inverse of a 1–1 function.

To find the inverse of a function, interchange x and y. Then solve for y.

➡ Find the inverse of the function defined by $f(x) = 3x + 6$.

$$f(x) = 3x + 6$$

Replace $f(x)$ by y.

$$y = 3x + 6$$

Interchange x and y.

$$x = 3y + 6$$

Solve for y.

$$x - 6 = 3y$$

$$\frac{1}{3}x - 2 = y$$

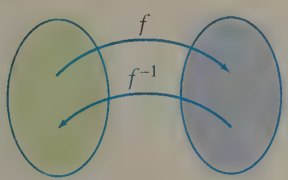
Replace y by $f^{-1}(x)$.

$$f^{-1}(x) = \frac{1}{3}x - 2$$

The inverse of the function $f(x) = 3x + 6$ is given by $f^{-1}(x) = \frac{1}{3}x - 2$.

TAKE NOTE

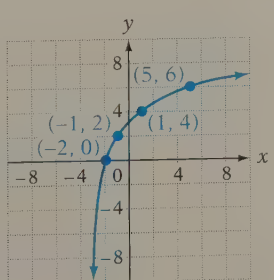
If the ordered pairs of f are given by (x, y), then the ordered pairs of f^{-1} are given by (y, x). That is, x and y are interchanged. This is the reason for Step 2 at the right.

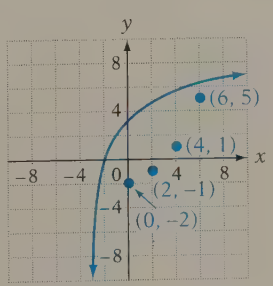


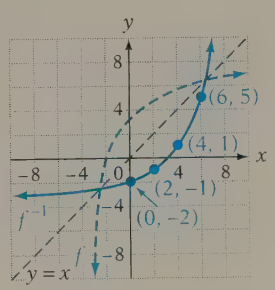
domain of f
range of f^{-1}

range of f
domain of f^{-1}

The fact that the ordered pairs of the inverse of a function are the reverse of those of the original function has a graphical interpretation. In the graph in the middle, the points with the coordinates reversed from the first graph are plotted. The inverse function is graphed by drawing a smooth curve through those points, as shown in the figure on the right.







Note the dashed graph of $y = x$ that is shown in the figure on the right. If two functions are inverses of each other, their graphs are mirror images with respect to the graph of the line $y = x$.

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The composition of a function and its inverse have a special property.

Property of the Composition of Inverse Functions

$$f^{-1}[f(x)] = x \quad \text{and} \quad f[f^{-1}(x)] = x$$

This property can be used to determine whether two functions are inverses of each other.

Are $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$ inverses of each other?

To determine whether the functions are inverses, use the Property of the Composition of Inverse Functions.

$$f[g(x)] = 2\left(\frac{1}{2}x + 2\right) - 4 \qquad g[f(x)] = \frac{1}{2}(2x - 4) + 2$$
$$= x + 4 - 4 \qquad = x - 2 + 2$$
$$= x \qquad = x$$

Because $f[g(x)] = x$ and $g[f(x)] = x$, the functions are inverses of each other.

Example 2

Find the inverse of the function defined by $f(x) = 2x - 3$.

Solution

$f(x) = 2x - 3$
$y = 2x - 3$ • Replace $f(x)$ by y.
$x = 2y - 3$ • Interchange x and y.
$x + 3 = 2y$ • Solve for y.
$\frac{1}{2}x + \frac{3}{2} = y$
$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$ • Replace y by $f^{-1}(x)$.

The inverse of the function is given by $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$.

You Try It 2

Find the inverse of the function defined by $f(x) = \frac{1}{2}x + 4$.

Your solution

Example 3

Are $f(x) = 3x - 6$ and $g(x) = \frac{1}{3}x + 2$ inverses of each other?

Solution

$$f[g(x)] = 3\left(\frac{1}{3}x + 2\right) - 6 = x + 6 - 6 = x$$

$$g[f(x)] = \frac{1}{3}(3x - 6) + 2 = x - 2 + 2 = x$$

Yes, the functions are inverses of each other.

You Try It 3

Are $f(x) = 2x - 6$ and $g(x) = \frac{1}{2}x - 3$ inverses of each other?

Your solution

Solutions on p. S34

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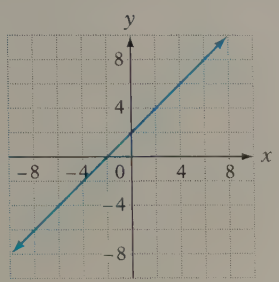
11.5 Exercises

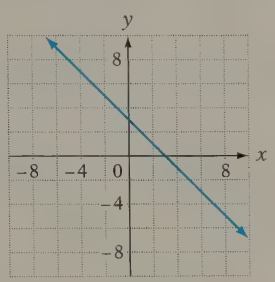
Objective A

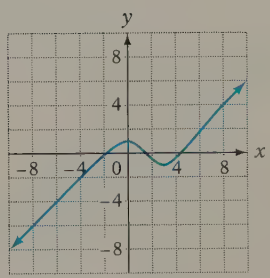
1. What is a 1–1 function?

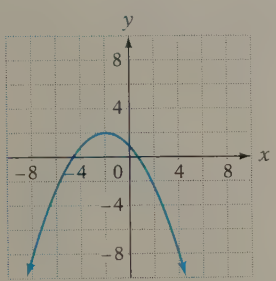
2. What is the horizontal-line test?

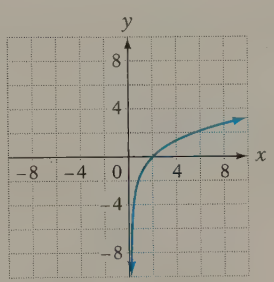
Determine whether the graph represents the graph of a 1–1 function.

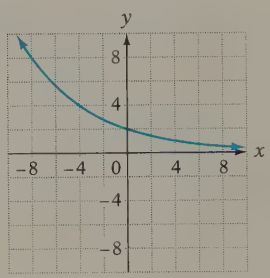
3. 

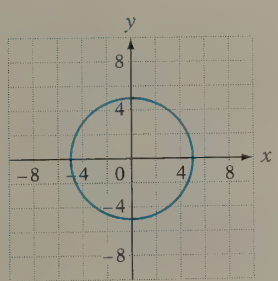
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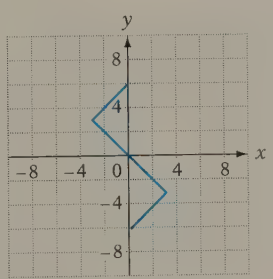
5. 

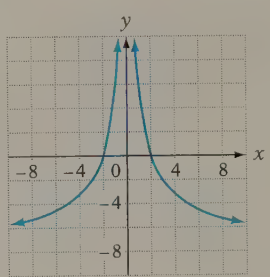
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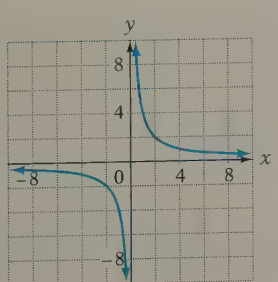
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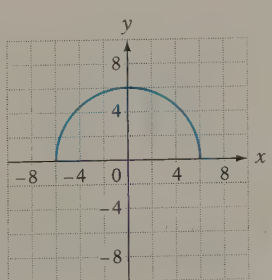
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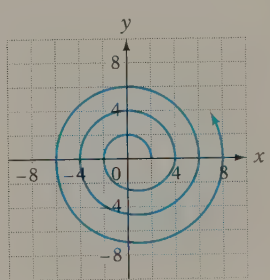
9. 

10. 

11. 

12. 

13. 

14. 

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Objective B

15. What is the inverse of a function?

16. Why is it that not all functions have an inverse function?

Find the inverse of the function. If the function does not have an inverse function, write "no inverse."

17. $\{(1, 0), (2, 3), (3, 8), (4, 15)\}$

18. $\{(1, 0), (2, 1), (-1, 0), (-2, 0)\}$

19. $\{(3, 5), (-3, -5), (2, 5), (-2, -5)\}$

20. $\{(-5, -5), (-3, -1), (-1, 3), (1, 7)\}$

21. $\{(0, -2), (-1, 5), (3, 3), (-4, 6)\}$

22. $\{(-2, -2), (0, 0), (2, 2), (4, 4)\}$

23. $\{(-2, -3), (-1, 3), (0, 3), (1, 3)\}$

24. $\{(2, 0), (1, 0), (3, 0), (4, 0)\}$

Find $f^{-1}(x)$.

25. $f(x) = 4x - 8$

26. $f(x) = 3x + 6$

27. $f(x) = 2x + 4$

28. $f(x) = x - 5$

29. $f(x) = \frac{1}{2}x - 1$

30. $f(x) = \frac{1}{3}x + 2$

31. $f(x) = -2x + 2$

32. $f(x) = -3x - 9$

33. $f(x) = \frac{2}{3}x + 4$

34. $f(x) = \frac{3}{4}x - 4$

35. $f(x) = -\frac{1}{3}x + 1$

36. $f(x) = -\frac{1}{2}x + 2$

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37. $f(x) = 2x - 5$

38. $f(x) = 3x + 4$

39. $f(x) = 5x - 2$

40. $f(x) = 4x - 2$

41. $f(x) = 6x - 3$

42. $f(x) = -8x + 4$

Given $f(x) = 3x - 5$, find:

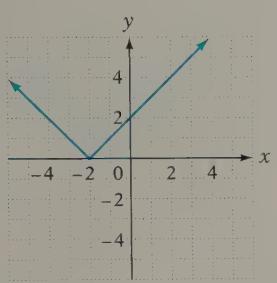
43. $f^{-1}(0)$

44. $f^{-1}(2)$

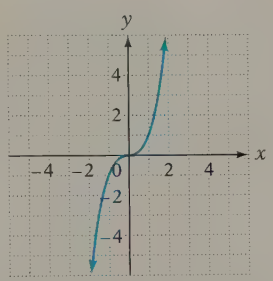
45. $f^{-1}(4)$

State whether the graph is the graph of a function. If it is the graph of a function, does it have an inverse?

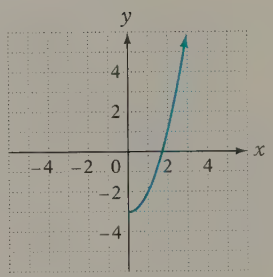
46.



47.



48.



Use the Property of the Composition of Inverse Functions to determine whether the functions are inverses of each other.

49. $f(x) = 4x; g(x) = \frac{x}{4}$

50. $g(x) = x + 5; h(x) = x - 5$

51. $f(x) = 3x; h(x) = \frac{1}{3x}$

52. $h(x) = x + 2; g(x) = 2 - x$

53. $g(x) = 3x + 2; f(x) = \frac{1}{3}x - \frac{2}{3}$

54. $h(x) = 4x - 1; f(x) = \frac{1}{4}x + \frac{1}{4}$

55. $f(x) = \frac{1}{2}x - \frac{3}{2}; g(x) = 2x + 3$

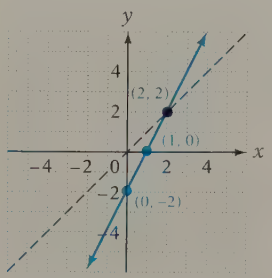
56. $g(x) = -\frac{1}{2}x - \frac{1}{2}; h(x) = -2x + 1$

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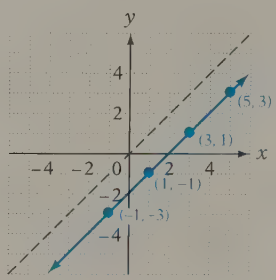
APPLYING THE CONCEPTS

Given the graph of the 1–1 function, draw the graph of the inverse of the function by using the technique shown in Objective B of this section.

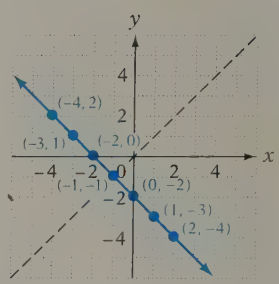
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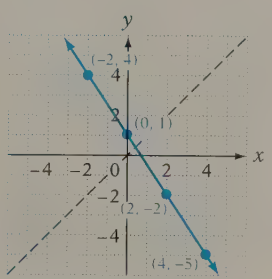
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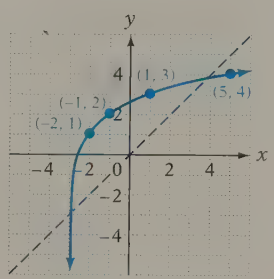
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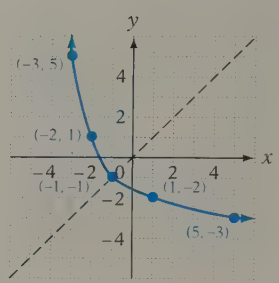
60.



61.



62.



Each of the tables below defines a function. Is the inverse of the function a function? Explain your answer.

63. Grading Scale Table

Score	*Grade*
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

64. First-Class Postage

Weight	*Cost*
$0 < w < 1$	\$ .34
$1 < w \leq 2$	\$ .57
$2 < w \leq 3$	\$ .80
$3 < w \leq 4$	\$1.03

If f is a 1–1 function and $f(0) = 1$, $f(3) = -1$, and $f(5) = -3$, find:

65. $f^{-1}(-3)$ **66.** $f^{-1}(-1)$ **67.** $f^{-1}(1)$

If f is a 1–1 function and $f(-3) = 3$, $f(-4) = 7$, and $f(0) = 8$, find:

68. $f^{-1}(3)$ **69.** $f^{-1}(7)$ **70.** $f^{-1}(8)$

71. Is the inverse of a constant function a function? Explain your answer.

72. The graphs of all functions given by $f(x) = mx + b$, $m \neq 0$, are straight lines. Are all of these functions 1–1 functions? If so, explain why. If not, give an example of a linear function that is not 1–1.

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11.6 Conic Sections

Objective A

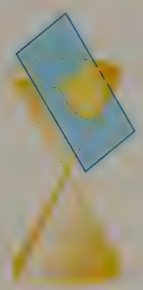
To graph a parabola

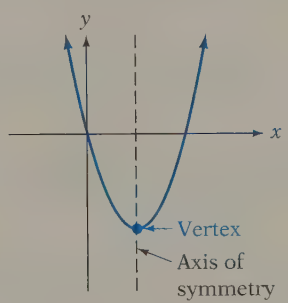
VIDEO & DVD CD TUTOR WEB SSM

Point of Interest

Hypatia (c. 340–415) is considered the first prominent woman mathematician. She lectured in mathematics and philosophy at the Museum in Alexandria, the most distinguished place of learning in the world. One of the topics on which Hypatia lectured was conic sections. One historian has claimed that with the death (actually the murder) of Hypatia, "the long and glorious history of Greek mathematics was at an end."

The **conic sections** are curves that can be constructed from the intersection of a plane and a right circular cone. The parabola, which was introduced earlier, is one of these curves. Here we will review some of that previous discussion and look at equations of parabolas that were not discussed before.





Every parabola has an axis of symmetry and a vertex that is on the axis of symmetry. To understand the axis of symmetry, think of folding the paper along that axis. The two halves of the curve will match up.

The graph of the equation $y = ax^2 + bx + c$, $a \neq 0$, is a parabola with the axis of symmetry parallel to the y-axis. The parabola opens up when $a > 0$ and opens down when $a < 0$. When the parabola opens up, the vertex is the lowest point on the parabola. When the parabola opens down, the vertex is the highest point on the parabola.

The coordinates of the vertex can be found by completing the square.

Find the vertex of the parabola whose equation is $y = x^2 - 4x + 5$.

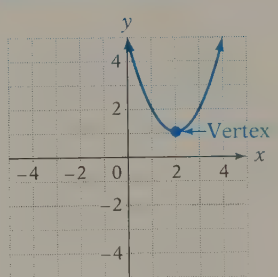
$y = x^2 - 4x + 5$
$y = (x^2 - 4x) + 5$
$y = (x^2 - 4x + 4) - 4 + 5$

- Group the terms involving x.
- Complete the square on $x^2 - 4x$. Note that 4 is added and subtracted. Because $4 - 4 = 0$, the equation is not changed.

$y = (x - 2)^2 + 1$

- Factor the trinomial and combine like terms.

The coefficient of x^2 is positive, so the parabola opens up. The vertex is the lowest point on the parabola, or the point that has the least y-coordinate.



Because $(x - 2)^2 \geq 0$ for all x, the least y-coordinate occurs when $(x - 2)^2 = 0$, which occurs when $x = 2$. This means the x-coordinate of the vertex is 2.

To find the y-coordinate of the vertex, replace x in $y = (x - 2)^2 + 1$ by 2 and solve for y.

$y = (x - 2)^2 + 1$
$= (2 - 2)^2 + 1 = 1$

The vertex is (2, 1).

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Point of Interest

The suspension cables for some bridges, such as the Golden Gate bridge, hang in the shape of a parabola. Parabolic shapes are also used for mirrors in telescopes and in certain antenna designs.

By following the procedure of the last example and completing the square on the equation $y = ax^2 + bx + c$, we find that the **x-coordinate of the vertex** is $-\frac{b}{2a}$. The y-coordinate of the vertex can then be determined by substituting this value of x into $y = ax^2 + bx + c$ and solving for y.

Because the axis of symmetry is parallel to the y-axis and passes through the vertex, the equation of the **axis of symmetry** is $x = -\frac{b}{2a}$.

➡ Find the vertex and axis of symmetry of the parabola whose equation is $y = -3x^2 + 6x + 1$. Then sketch its graph.

x-coordinate: $-\frac{b}{2a} = -\frac{6}{2(-3)} = 1$

• Find the x-coordinate of the vertex and the axis of symmetry, $a = -3$, $b = 6$.

The x-coordinate of the vertex is 1.
The axis of symmetry is the line $x = 1$.

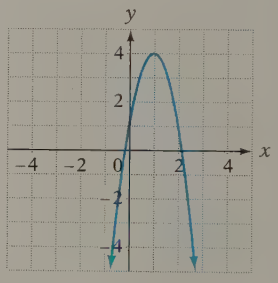
To find the y-coordinate of the vertex, replace x by 1 and solve for y.

$y = -3x^2 + 6x + 1$
$= -3(1)^2 + 6(1) + 1 = 4$

The vertex is (1, 4).

Because a is negative, the parabola opens down.

Find a few ordered pairs and use symmetry to sketch the graph.



➡ Find the vertex and axis of symmetry of the parabola whose equation is $y = x^2 - 2$. Then sketch its graph.

x-coordinate: $-\frac{b}{2a} = -\frac{0}{2(1)} = 0$

• Find the x-coordinate of the vertex and the axis of symmetry. $a = 1$, $b = 0$

The x-coordinate of the vertex is 0.
The axis of symmetry is the line $x = 0$.

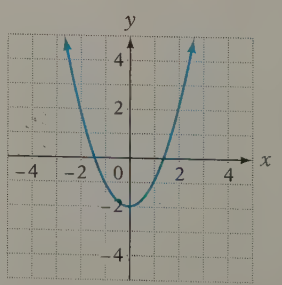
To find the y-coordinate of the vertex, replace x by 0 and solve for y.

$y = x^2 - 2$
$= 0^2 - 2 = -2$

The vertex is (0, −2).

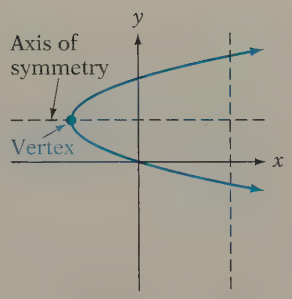
Because a is positive, the parabola opens up.

Find a few ordered pairs and use symmetry to sketch the graph.



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The graph of an equation of the form $x = ay^2 + by + c$, $a \neq 0$, is also a parabola. In this case, the parabola opens to the right when a is positive and opens to the left when a is negative.



For a parabola of this form, the **y-coordinate of the vertex** is $-\frac{b}{2a}$. The **axis of symmetry** is the line $y = -\frac{b}{2a}$.

Using the vertical line test, the graph of a parabola of this form is not the graph of a function. The graph of $x = ay^2 + by + c$ is a relation.

Find the vertex and axis of symmetry of the parabola whose equation is $x = 2y^2 - 8y + 5$. Then sketch its graph.

y-coordinate: $-\frac{b}{2a} = -\frac{-8}{2(2)} = 2$

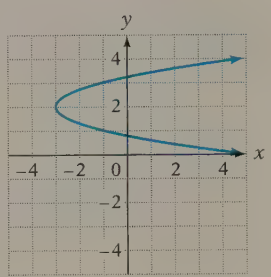
• **Find the y-coordinate of the vertex and the axis of symmetry. $a = 2$, $b = -8$**

The y-coordinate of the vertex is 2.
The axis of symmetry is the line $y = 2$.

To find the x-coordinate of the vertex, replace y by 2 and solve for x.

$$x = 2y^2 - 8y + 5$$
$$= 2(2)^2 - 8(2) + 5 = -3$$

The vertex is $(-3, 2)$.



Since a is positive, the parabola opens to the right.

Find a few ordered pairs and use symmetry to sketch the graph.

Find the vertex and axis of symmetry of the parabola whose equation is $x = -2y^2 - 4y - 3$. Then sketch its graph.

y-coordinate: $-\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$

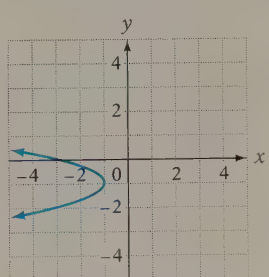
• **Find the y-coordinate of the vertex and the axis of symmetry. $a = -2$, $b = -4$**

The y-coordinate of the vertex is -1.
The axis of symmetry is the line $y = -1$.

To find the x-coordinate of the vertex, replace y by -1 and solve for x.

$$x = -2y^2 - 4y - 3$$
$$= -2(-1)^2 - 4(-1) - 3 = -1$$

The vertex is $(-1, -1)$.



Because a is negative, the parabola opens to the left.

Find a few ordered pairs and use symmetry to sketch the graph.

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Example 1

Find the vertex and axis of symmetry of the parabola whose equation is $y = x^2 - 4x + 3$. Then sketch its graph.

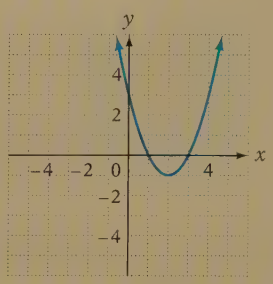
Solution

$$-\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

axis of symmetry: $x = 2$

$$y = 2^2 - 4(2) + 3$$
$$= -1$$

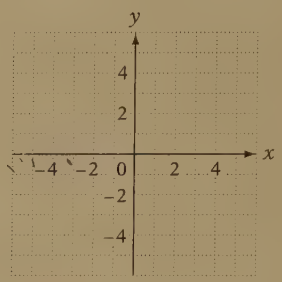
vertex: $(2, -1)$



You Try It 1

Find the vertex and axis of symmetry of the parabola whose equation is $y = x^2 + 2x + 1$. Then sketch its graph.

Your solution



Example 2

Find the vertex and axis of symmetry of the parabola whose equation is $x = 2y^2 - 4y + 1$. Then sketch its graph.

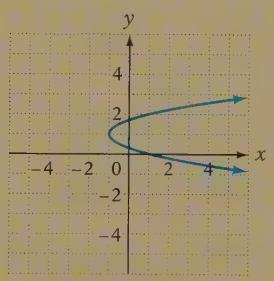
Solution

$$-\frac{b}{2a} = -\frac{-4}{2(2)} = 1$$

axis of symmetry: $y = 1$

$$x = 2(1)^2 - 4(1) + 1$$
$$= -1$$

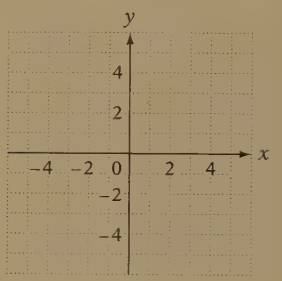
vertex: $(-1, 1)$



You Try It 2

Find the vertex and axis of symmetry of the parabola whose equation is $x = -y^2 - 2y + 2$. Then sketch its graph.

Your solution



Example 3

Find the vertex and axis of symmetry of the parabola whose equation is $y = x^2 + 1$. Then sketch its graph.

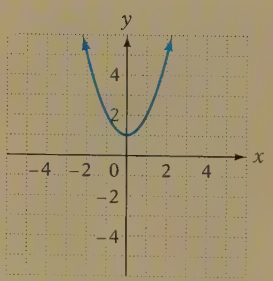
Solution

$$-\frac{b}{2a} = -\frac{0}{2(1)} = 0$$

axis of symmetry: $x = 0$

$$y = 0^2 + 1$$
$$= 1$$

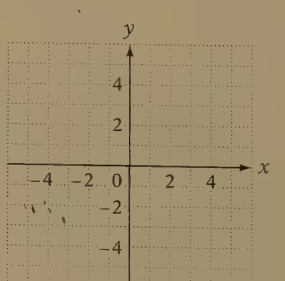
vertex: $(0, 1)$



You Try It 3

Find the vertex and axis of symmetry of the parabola whose equation is $y = x^2 - 2x - 1$. Then sketch its graph.

Your solution



Solutions on p. S34

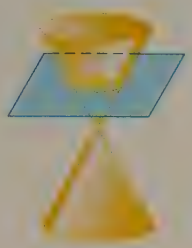
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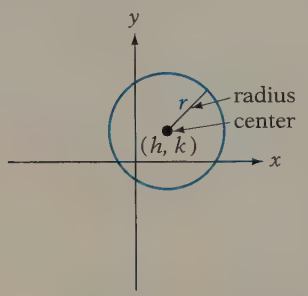
Objective B **To find the equation of a circle and to graph a circle**

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TAKE NOTE
As the angle of the plane that intersects the cone changes, different conic sections are formed. For a parabola, the plane was *parallel to the side* of the cone. For a circle, the plane is *parallel to the base* of the cone.

A **circle** is a conic section formed by the intersection of a cone and a plane parallel to the base of the cone.





A **circle** can be defined as all points (x, y) in the plane that are a fixed distance from a given point (h, k) called the **center**. The fixed distance is the **radius** of the circle.

The Standard Form of the Equation of a Circle

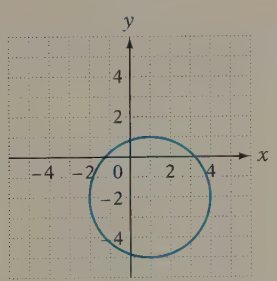
Let r be the radius of a circle and let (h, k) be the coordinates of the center of the circle. Then the equation of the circle is given by

$$(x - h)^2 + (y - k)^2 = r^2$$

➡ Sketch a graph of $(x - 1)^2 + (y + 2)^2 = 9$.

$(x - 1)^2 + [y - (-2)]^2 = 3^2$ • Rewrite the equation in standard form.

center: $(1, -2)$ radius: 3



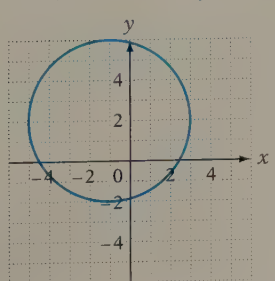
➡ Find the equation of the circle with radius 4 and center $(-1, 2)$. Then sketch its graph.

$(x - h)^2 + (y - k)^2 = r^2$ • Use the standard form of the equation of a circle.

$[x - (-1)]^2 + (y - 2)^2 = 4^2$ • Replace r by 4, h by -1, and k by 2.

$(x + 1)^2 + (y - 2)^2 = 16$

• Sketch the graph by drawing a circle with center $(-1, 2)$ and radius 4.



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Applying the vertical-line test reveals that the graph of a circle is not the graph of a function. The graph of a circle is the graph of a relation.

Example 4

Sketch a graph of $(x + 2)^2 + (y - 1)^2 = 4$.

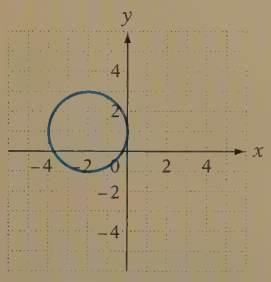
Solution

$(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 1)^2 = 2^2$

center: $(h, k) = (-2, 1)$

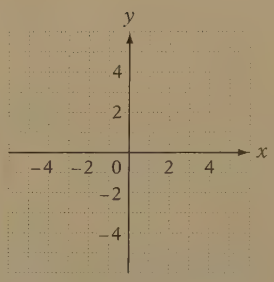
radius: $r = 2$



You Try It 4

Sketch a graph of $(x - 2)^2 + (y + 3)^2 = 9$.

Your solution



Example 5

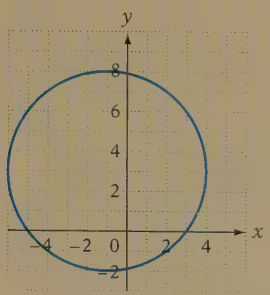
Find the equation of the circle with radius 5 and center $(-1, 3)$. Then sketch its graph.

Solution

$(x - h)^2 + (y - k)^2 = r^2$

$[x - (-1)]^2 + (y - 3)^2 = 5^2$

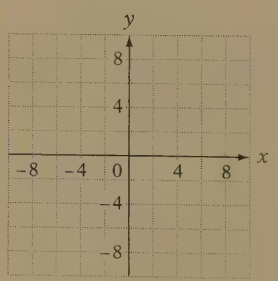
$(x + 1)^2 + (y - 3)^2 = 25$



You Try It 5

Find the equation of the circle with radius 4 and center $(2, -3)$. Then sketch its graph.

Your solution



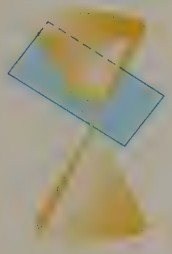
Solutions on p. S34

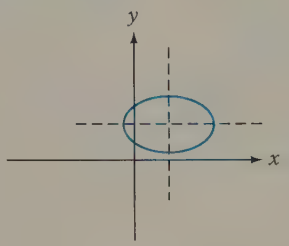
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Objective C To graph an ellipse with center at the origin

VIDEO & DVD CD TUTOR WEB SSM

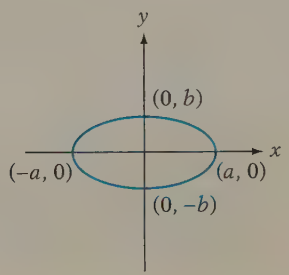
The orbits of the planets around the sun are "oval" shaped. This oval shape can be described as an **ellipse**, which is another of the conic sections.





There are two **axes of symmetry** for an ellipse. The intersection of these two axes is the **center** of the ellipse.

An ellipse with center at the origin is shown at the right. Note that there are two x-intercepts and two y-intercepts.



Point of Interest

The word *ellipse* comes from the Greek word *ellipsis*, which means "deficient." The method by which the early Greeks analyzed the conics caused a certain area in the construction of the ellipse to be less than another area (deficient). The word *ellipsis* in English, which means "omission," has the same Greek root as the word *ellipse*.

The Standard Form of the Equation of an Ellipse with Center at the Origin

The equation of an ellipse with center at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The x-intercepts are $(a, 0)$ and $(-a, 0)$. The y-intercepts are $(0, b)$ and $(0, -b)$.

By finding the x- and y-intercepts of an ellipse and using the fact that the ellipse is "oval" shaped, we can sketch a graph of the ellipse.

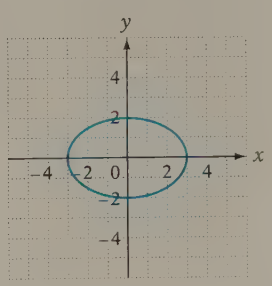
➡ Sketch the graph of the ellipse whose equation is $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Comparing $\frac{x^2}{9} + \frac{y^2}{4} = 1$ with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have $a^2 = 9$ and $b^2 = 4$.

Therefore, $a = 3$ and $b = 2$.

The x-intercepts are $(3, 0)$ and $(-3, 0)$.
The y-intercepts are $(0, 2)$ and $(0, -2)$.

Use the intercepts to sketch a graph of the ellipse.



Using the vertical-line test, we find that the graph of an ellipse is not the graph of a function. The graph of an ellipse is the graph of a relation.

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Sketch a graph of the ellipse whose equation is $\frac{x^2}{16} + \frac{y^2}{16} = 1$.

Point of Interest

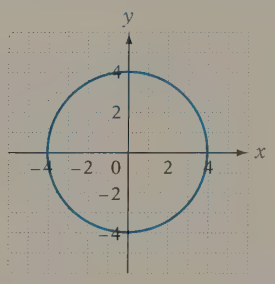
For a circle, $a = b$ and thus $\frac{a}{b} = 1$. Early Greek astronomers thought that each planet had a circular orbit. Today we know that the planets have elliptical orbits. However, in most cases the ellipse is very nearly a circle.

For Earth, $\frac{a}{b} \approx 1.00014$. The most elliptical orbit is Pluto's, for which $\frac{a}{b} \approx 1.0328$.

The x-intercepts are $(4, 0)$ and $(-4, 0)$.

The y-intercepts are $(0, 4)$ and $(0, -4)$.

- $a^2 = 16$, $b^2 = 16$
- **Use the intercepts and symmetry to sketch the graph of the ellipse.**



The graph in this example is the graph of a circle. A circle is a special case of an ellipse. It occurs when $a^2 = b^2$ in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

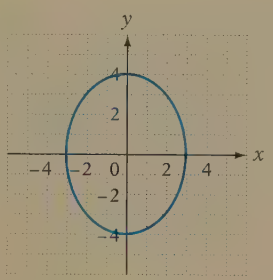
Example 6

Sketch a graph of the ellipse whose equation is $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

Solution

x-intercepts:
$(3, 0)$ and $(-3, 0)$

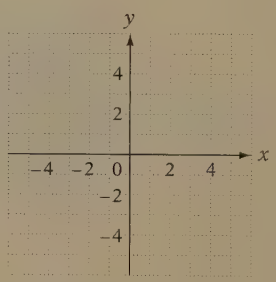
y-intercepts:
$(0, 4)$ and $(0, -4)$



You Try It 6

Sketch a graph of the ellipse whose equation is $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

Your solution



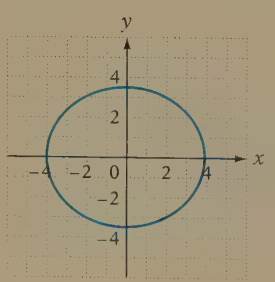
Example 7

Sketch a graph of the ellipse whose equation is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

Solution

x-intercepts:
$(4, 0)$ and $(-4, 0)$

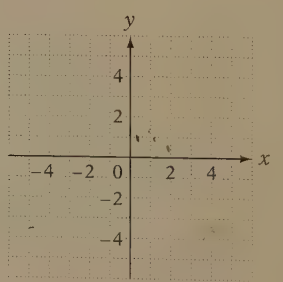
y-intercepts:
$(0, 2\sqrt{3})$
and $(0, -2\sqrt{3})$
$(2\sqrt{3} \approx 3.5)$



You Try It 7

Sketch a graph of the ellipse whose equation is $\frac{x^2}{18} + \frac{y^2}{9} = 1$.

Your solution



Solutions on pp. S34–S35

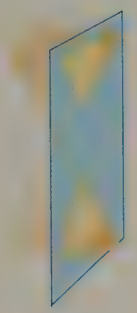
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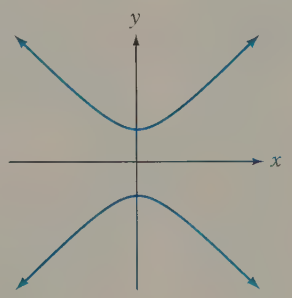
Objective D

To graph a hyperbola with center at the origin

VIDEO & DVD CD TUTOR WEB SSM

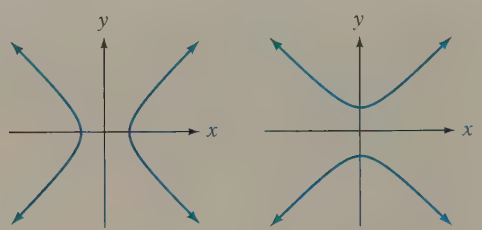
A **hyperbola** is a conic section that is formed by the intersection of a cone and a plane perpendicular to the base of the cone.





The hyperbola has two **vertices** and an **axis of symmetry** that passes through the vertices. The **center** of a hyperbola is the point halfway between the two vertices.

The graphs at the right show two possible graphs of a hyperbola with center at the origin.



In the first graph, an axis of symmetry is the x-axis and the vertices are x-intercepts.

In the second graph, an axis of symmetry is the y-axis and the vertices are y-intercepts.

Note that in either case, the graph of a hyperbola is not the graph of a function. The graph of a hyperbola is the graph of a relation.

Point of Interest

The word *hyperbola* comes from the Greek word *yperboli*, which means "exceeding." The method by which the early Greeks analyzed the conics caused a certain area in the construction of the hyperbola to be greater than (to exceed) another area. The word *hyperbole* in English, meaning "exaggeration," has the same Greek root as the word *hyperbola*.

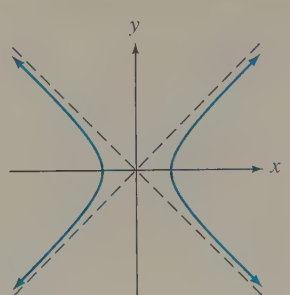
The word *asymptote* comes from the Greek word *asymptotos*, which means "not capable of meeting."

The Standard Form of the Equation of a Hyperbola with Center at the Origin

The equation of a hyperbola for which an axis of symmetry is the x-axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The vertices are $(a, 0)$ and $(-a, 0)$.

The equation of a hyperbola for which an axis of symmetry is the y-axis is $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$. The vertices are $(0, b)$ and $(0, -b)$.

To sketch a hyperbola, it is helpful to draw two lines that are "approached" by the hyperbola. These two lines are called **asymptotes**. As a point on the hyperbola gets farther from the origin, the hyperbola "gets closer to" the asymptotes.

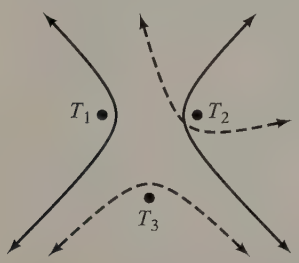


Because the asymptotes are straight lines, their equations are linear equations. The equations of the asymptotes for a hyperbola with center at the origin are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

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Point of Interest

Hyperbolas are used in LORAN (LOng RAnge Navigation) as a method by which a ship's navigator can determine the position of the ship, as shown in the figure below. They are also used as mirrors in some telescopes to focus incoming light.



➡ Sketch a graph of the hyperbola whose equation is $\frac{y^2}{9} - \frac{x^2}{4} = 1$.

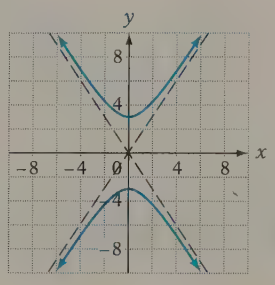
An axis of symmetry is the y-axis.

$b^2 = 9, a^2 = 4$

The vertices are $(0, 3)$ and $(0, -3)$.

The asymptotes are $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$.

- The vertices are $(0, b)$ and $(0, -b)$.
- The asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.
- Sketch the asymptotes. Use symmetry and the fact that the hyperbola will approach the asymptotes to sketch its graph.



Example 8

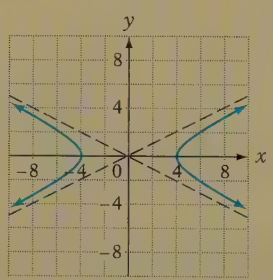
Sketch a graph of the hyperbola whose equation is $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

Solution

axis of symmetry: x-axis

vertices: $(4, 0)$ and $(-4, 0)$

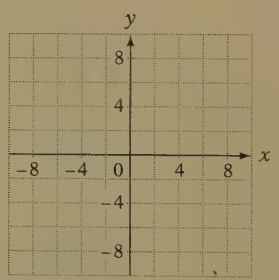
asymptotes: $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$



You Try It 8

Sketch a graph of the hyperbola whose equation is $\frac{x^2}{9} - \frac{y^2}{25} = 1$.

Your solution



Example 9

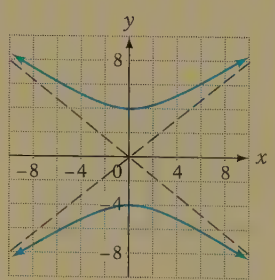
Sketch a graph of the hyperbola whose equation is $\frac{y^2}{16} - \frac{x^2}{25} = 1$.

Solution

axis of symmetry: y-axis

vertices: $(0, 4)$ and $(0, -4)$

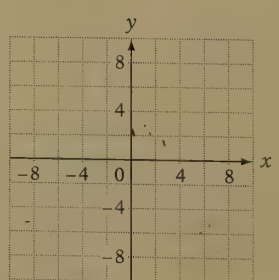
asymptotes: $y = \frac{4}{5}x$ and $y = -\frac{4}{5}x$



You Try It 9

Sketch a graph of the hyperbola whose equation is $\frac{y^2}{9} - \frac{x^2}{9} = 1$.

Your solution



Solutions on p. S35

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11.6 Exercises

Objective A

State (a) whether the axis of symmetry is a vertical or a horizontal line and (b) in what direction the parabola opens.

1. $y = 3x^2 - 4x + 7$

2. $y = -x^2 + 5x - 2$

3. $x = y^2 + 2y - 8$

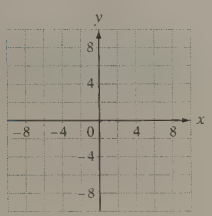
4. $x = -3y^2 - y + 9$

5. $x = -\frac{1}{2}y^2 - 4y - 7$

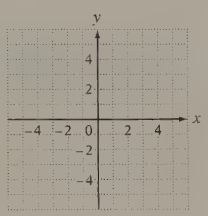
6. $y = \frac{1}{4}x^2 + 6x - 1$

Find the vertex and axis of symmetry of the parabola given by the equation. Then sketch its graph.

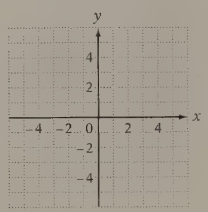
7. $x = y^2 - 3y - 4$



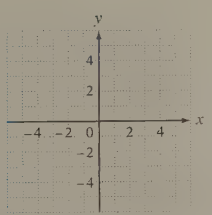
8. $y = x^2 - 2$



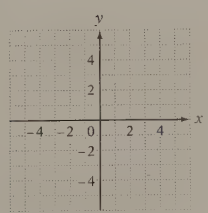
9. $y = x^2 + 2$



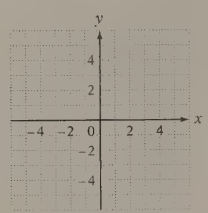
10. $x = -\frac{1}{2}y^2 + 4$



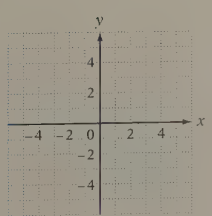
11. $x = -\frac{1}{4}y^2 - 1$



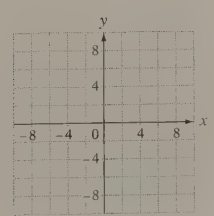
12. $x = \frac{1}{2}y^2 - y + 1$



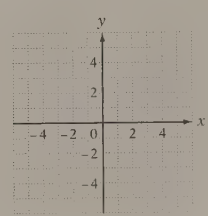
13. $x = -\frac{1}{2}y^2 + 2y - 3$



14. $y = -\frac{1}{2}x^2 + 2x + 6$



15. $y = \frac{1}{2}x^2 + x - 3$

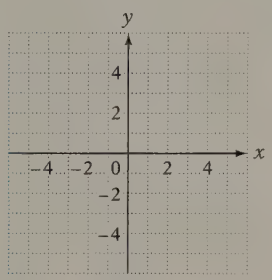


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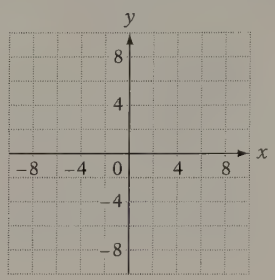
Objective B

Sketch a graph of the circle given by the equation.

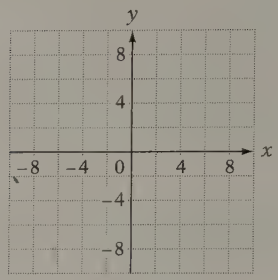
16. $(x - 2)^2 + (y + 2)^2 = 9$



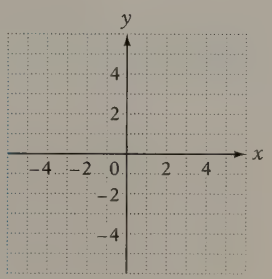
17. $(x + 2)^2 + (y - 3)^2 = 16$



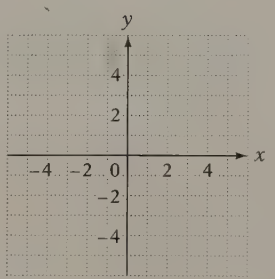
18. $(x + 3)^2 + (y - 1)^2 = 25$



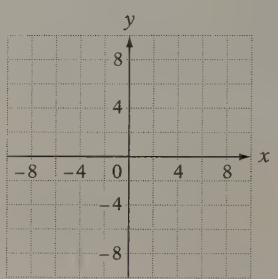
19. $(x - 2)^2 + (y + 3)^2 = 4$



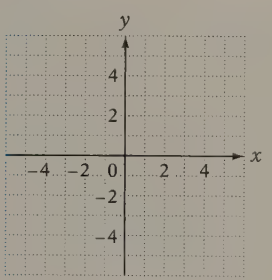
20. $(x + 2)^2 + (y + 2)^2 = 4$



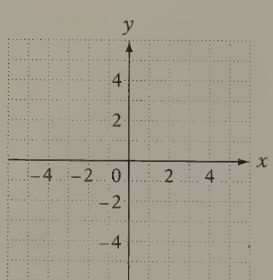
21. $(x - 1)^2 + (y - 2)^2 = 25$



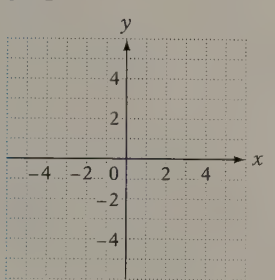
22. Find the equation of the circle with radius 2 and center $(2, -1)$. Then sketch its graph.



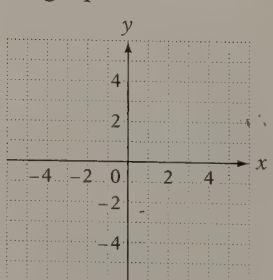
23. Find the equation of the circle with radius 3 and center $(-1, -2)$. Then sketch its graph.



24. Find the equation of the circle with radius $\sqrt{5}$ and center $(-1, 1)$. Then sketch its graph.



25. Find the equation of the circle with radius $\sqrt{5}$ and center $(-2, 1)$. Then sketch its graph.

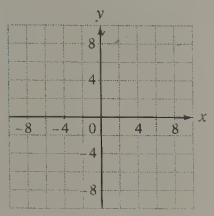


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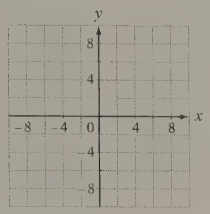
Objective E

Sketch a graph of the ellipse given by the equation.

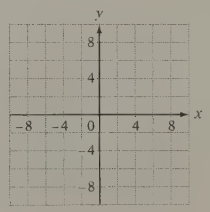
26. $\frac{x^2}{4} + \frac{y^2}{9} = 1$



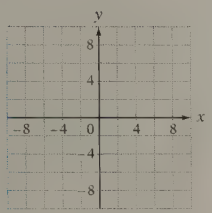
27. $\frac{x^2}{25} + \frac{y^2}{16} = 1$



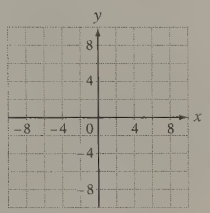
28. $\frac{x^2}{25} + \frac{y^2}{9} = 1$



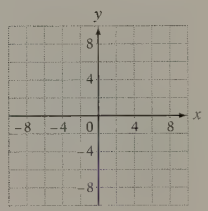
29. $\frac{x^2}{16} + \frac{y^2}{9} = 1$



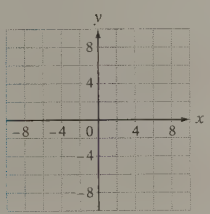
30. $\frac{x^2}{36} + \frac{y^2}{16} = 1$



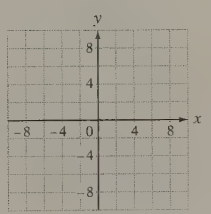
31. $\frac{x^2}{49} + \frac{y^2}{64} = 1$



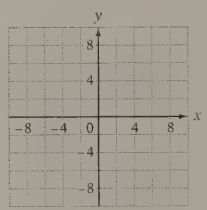
32. $\frac{x^2}{16} + \frac{y^2}{49} = 1$



33. $\frac{x^2}{25} + \frac{y^2}{36} = 1$



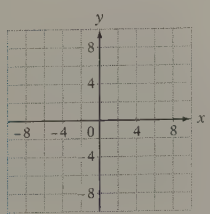
34. $\frac{x^2}{4} + \frac{y^2}{25} = 1$



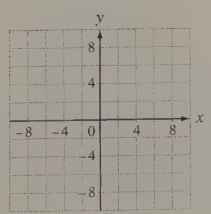
Objective F

Sketch a graph of the hyperbola given by the equation.

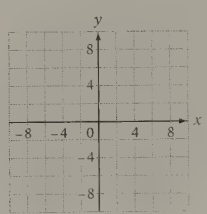
35. $\frac{x^2}{9} - \frac{y^2}{16} = 1$



36. $\frac{x^2}{25} - \frac{y^2}{4} = 1$

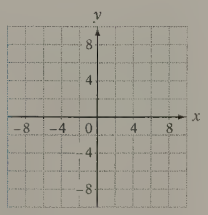


37. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

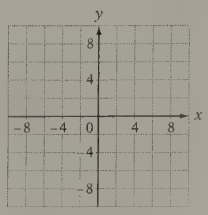


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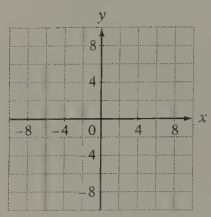
38. $\frac{y^2}{16} - \frac{x^2}{25} = 1$



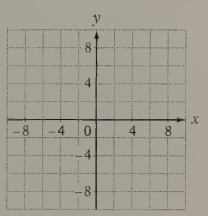
39. $\frac{x^2}{16} - \frac{y^2}{4} = 1$



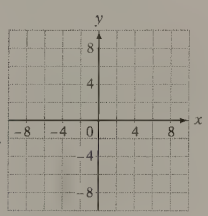
40. $\frac{x^2}{9} - \frac{y^2}{49} = 1$



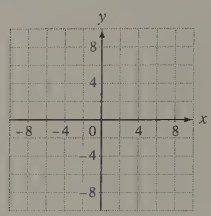
41. $\frac{y^2}{25} - \frac{x^2}{9} = 1$



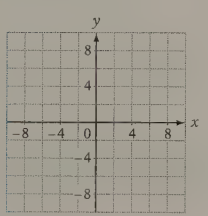
42. $\frac{y^2}{4} - \frac{x^2}{16} = 1$



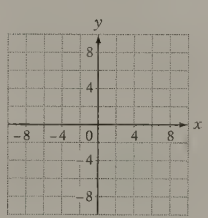
43. $\frac{x^2}{4} - \frac{y^2}{25} = 1$



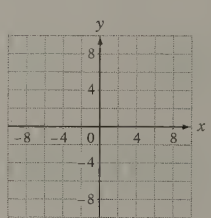
44. $\frac{x^2}{36} - \frac{y^2}{9} = 1$



45. $\frac{y^2}{9} - \frac{x^2}{36} = 1$



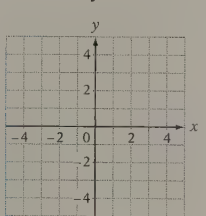
46. $\frac{y^2}{25} - \frac{x^2}{4} = 1$



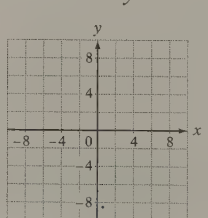
APPLYING THE CONCEPTS

Write the equation in standard form. Identify the graph, and then graph the equation.

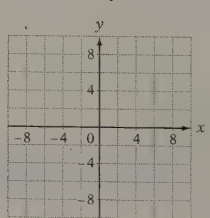
47. $4x^2 + 9y^2 = 36$



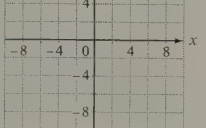
48. $16x^2 + 25y^2 = 400$



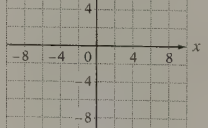
49. $9x^2 - 25y^2 = 225$



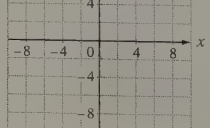
50. $25y^2 - 4x^2 = -100$



51. $9y^2 - 16x^2 = 144$



52. $4y^2 - x^2 = 36$



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Focus on Problem Solving

Algebraic Manipulation and Graphing Techniques

Problem solving is often easier when we have both algebraic manipulation and graphing techniques at our disposal. Solving quadratic equations and graphing quadratic equations in two variables are used here to solve problems involving profit.

A company's revenue, R, is the total amount of money the company earned by selling its products. The cost, C, is the total amount of money the company spent to manufacture and sell its products. A company's profit, P, is the difference between the revenue and cost: $P = R - C$. A company's revenue and cost may be represented by equations.

A company manufactures and sells woodstoves. The total monthly cost, in dollars, to produce n woodstoves is $C = 30n + 2000$. Write a variable expression for the company's monthly profit if the revenue, in dollars, obtained from selling all n woodstoves is $R = 150n - 0.4n^2$.

$$P = R - C$$
$$P = 150n - 0.4n^2 - (30n + 2000)$$
$$P = -0.4n^2 + 120n - 2000$$

- Replace R by $150n - 0.4n^2$ and C by $30n + 2000$. Then simplify.

How many woodstoves must the company manufacture and sell in order to make a profit of \$6000 a month?

$$P = -0.4n^2 + 120n - 2000$$
$$6000 = -0.4n^2 + 120n - 2000$$
- Substitute 6000 for P.

$$0 = -0.4n^2 + 120n - 8000$$
- Write the equation in standard form.

$$0 = n^2 - 300n + 20{,}000$$
- Divide each side of the equation by -0.4.

$$0 = (n - 100)(n - 200)$$
- Factor.

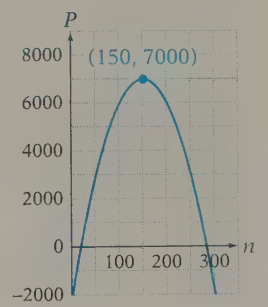
$$n - 100 = 0 \qquad n - 200 = 0$$
- Solve for n.

$$n = 100 \qquad n = 200$$

The company will make a monthly profit of \$6000 if either 100 or 200 woodstoves are manufactured and sold.

The graph of $P = -0.4n^2 + 120n - 2000$ is shown at the right. Note that when $P = 6000$, the values of n are 100 and 200.

Also note that the coordinates of the highest point on the graph are (150, 7000). This means that the company makes a *maximum* profit of \$7000 per month when 150 woodstoves are manufactured and sold.



1. The total cost, in dollars, for a company to produce and sell n guitars per month is $C = 240n + 1200$. The company's revenue, in dollars, from selling all n guitars is $R = 400n - 2n^2$.

a. How many guitars must the company produce and sell each month in order to make a monthly profit of \$1200?

b. Graph the profit equation. What is the maximum monthly profit the company can make?

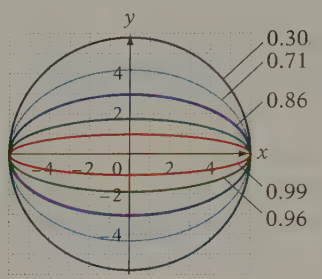
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Projects and Group Activities

The Eccentricity and Foci of an Ellipse

The graph of an ellipse can be long and thin, or it can have a shape that is very close to a circle. The **eccentricity,** e, of an ellipse is a measure of its "roundness."

The shapes of ellipses with various eccentricities are shown below.

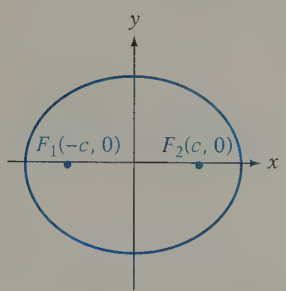


Planet	Eccentricity
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.049
Saturn	0.051
Uranus	0.046
Neptune	0.005
Pluto	0.250

1. Based on the eccentricities of the ellipses shown above, complete the sentence. "As the eccentricity of an ellipse gets closer to 1, the ellipses get *flatter/rounder.*"

2. The planets travel around the sun in elliptical orbits. The eccentricities of the orbits of the planets are shown in the table at the left. Which planet has the most nearly circular orbit?

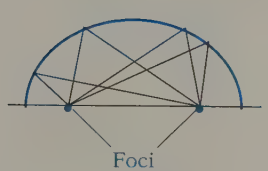
For an ellipse given by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$, a formula for eccentricity is $e = \frac{\sqrt{a^2 - b^2}}{a}$. Use this formula to find the eccentricity of the ellipses in Exercise 3 and 4. If necessary, round to the nearest hundredth.



3. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

4. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Ellipses have a reflective property that has been used in the design of some buildings. The **foci** of an ellipse are two points on the longer axis, called the **major axis,** of the ellipse.



If light or sound emanates from one focus, it is reflected to the other focus. This phenomenon results in what are called "whispering galleries." The rotunda of the Capitol Building in Washington, D.C., is a whispering gallery. A whisper spoken by a person at one focus can be heard clearly by a person at the other focus.

The foci are c units from the center of an ellipse, where $c = \sqrt{a^2 - b^2}$ for an ellipse whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$.

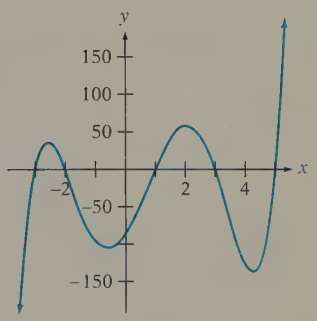
5. Find the foci for the ellipse whose equation is $\frac{x^2}{100} + \frac{y^2}{64} = 1$.

6. Find the foci for the ellipse whose equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

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x-intercepts and Solutions of Equations

The x-coordinate of an x-intercept of the graph of $y = f(x)$ is a real number solution of the equation $f(x) = 0$. For instance, the graph of $P(x) = x^5 - 4x^4 - 16x^3 + 46x^2 + 63x - 90$ is shown at the right. The x-intercepts of the graph are $(-3, 0)$, $(-2, 0)$, $(1, 0)$, $(3, 0)$, and $(5, 0)$, and the roots of the equation $x^5 - 4x^4 - 16x^3 + 46x^2 + 63x - 90 = 0$ are -3, -2, 1, 3, and 5.



1. Graph $P(x) = x^5 + x^4 - 7x^3 - x^2 + 6x$ and determine the x-intercepts. Use Xmin $= -4$, Xmax $= 4$, Xscl $= 1$, Ymin $= -10$, Ymax $= 40$, and Yscl $= 10$.
2. What are the real number solutions of $x^5 + x^4 - 7x^3 - x^2 + 6x = 0$?
3. Graph $P(x) = x^5 - 2x^4 + 5x^3 - 10x^2 + 4x - 8$ and determine the x-intercepts. Use Xmin $= -4$, Xmax $= 4$, Xscl $= 1$, Ymin $= -100$, Ymax $= 100$, and Yscl $= 50$.
4. What are the real number solutions of $x^5 - 2x^4 + 5x^3 - 10x^2 + 4x - 8 = 0$?
5. On the basis of the graph shown above and your answers to Exercises 2 and 4, make a conjecture about the number of real number solutions a fifth-degree equation can have.
6. Make up a few more fifth-degree equations and produce their graphs. Do the numbers of real number solutions of these equations bear out your conjecture?

Chapter Summary

Key Words

A function that can be written in the form $y = mx + b$ or $f(x) = mx + b$ is a *linear function*. The graph of a linear function is a straight line with slope m and y-intercept $(0, b)$. [p. 591]

A *quadratic function* is one that can be expressed by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. The graph of this function is a *parabola*. [p. 595]

The *inverse of a function* is the set of ordered pairs formed by reversing the coordinates of each ordered pair of the function. [p. 618]

The graph of a *conic section* can be represented by the intersection of a plane and a cone. The four conic sections are the *parabola*, *ellipse*, *hyperbola*, and *circle*. [pp. 625, 629, 631, 633]

A *circle* is the set of all points (x, y) in the plane that are a fixed distance from a given point (h, k) called the *center*. The fixed distance is the *radius* of the circle. [p. 629]

The *asymptotes* of a hyperbola are the two straight lines that are "approached" by the hyperbola. As the graph of the hyperbola gets farther from the origin, the hyperbola "gets closer to" the asymptotes. [p. 633]

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Essential Rules

Vertical-Line Test A graph defines a function if any vertical line intersects the graph at no more than one point. [p. 606]

Operations on Functions If f and g are functions and x is an element of the domain of each function, then

$(f + g)(x) = f(x) + g(x)$ $\qquad$ $(f - g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$ $\qquad$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ [p. 611]

Composition of Two Functions The composition of two functions, $f \circ g$, is the function whose value at x is given by $(f \circ g)(x) = f[g(x)]$. [p. 613]

One-to-One Function A function f is a 1–1 function if, for any a and b in the domain of f, $f(a) = f(b)$ implies that $a = b$. [p. 617]

Horizontal-Line Test The graph of a function represents the graph of a 1–1 function if any horizontal line intersects the graph at no more than one point. [p. 618]

Condition for an Inverse Function A function f has an inverse function if and only if f is a 1–1 function. [p. 619]

Property of the Composition of Inverse Functions $f^{-1}[f(x)] = x$ and $f[f^{-1}(x)] = x$ [p. 620]

Equation of a Parabola

$y = ax^2 + bx + c$ [p. 596]

When $a > 0$, the parabola opens up.
When $a < 0$, the parabola opens down.

The x-coordinate of the vertex is $-\frac{b}{2a}$.

The axis of symmetry is the line $x = -\frac{b}{2a}$.

$x = ay^2 + by + c$ [p. 627]

When $a > 0$, the parabola opens to the right.
When $a < 0$, the parabola opens to the left.

The y-coordinate of the vertex is $-\frac{b}{2a}$.

The axis of symmetry is the line $y = -\frac{b}{2a}$.

Equation of a Circle [p. 629]

$(x - h)^2 + (y - k)^2 = r^2$

The center is (h, k) and the radius is r.

Equation of an Ellipse [p. 631]

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The x-intercepts are $(a, 0)$ and $(-a, 0)$.
The y-intercepts are $(0, b)$ and $(0, -b)$.

Equation of a Hyperbola [p. 633]

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

An axis of symmetry is the x-axis.
The vertices are $(a, 0)$ and $(-a, 0)$.

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

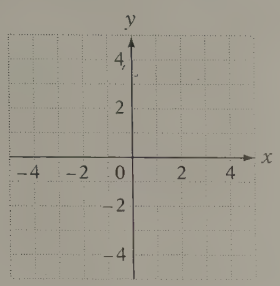
An axis of symmetry is the y-axis.
The vertices are $(0, b)$ and $(0, -b)$.

The equations of the asymptotes are $y = \pm\frac{b}{a}x$.

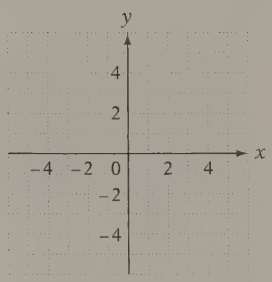
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Chapter Review

1. Graph: $f(x) = \frac{1}{4}x + 3$



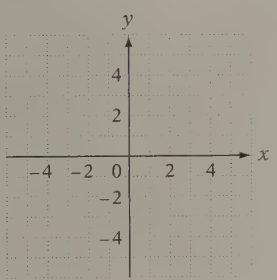
2. Sketch a graph of $f(x) = x^2 - 2x + 3$.



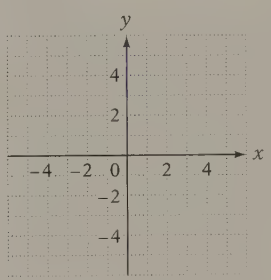
3. Find the x-intercepts of $y = 2x^2 - 3x + 4$.

4. Find the maximum value of the function $f(x) = -x^2 + 8x - 7$.

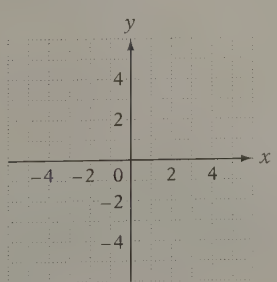
5. Graph $f(x) = -\sqrt{3 - x}$. State the domain and range.



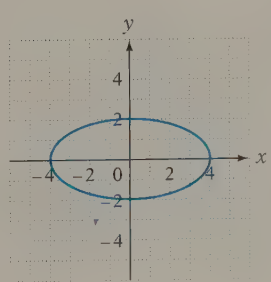
6. Graph $f(x) = \left|\frac{1}{2}x\right| - 2$. State the domain and range.



7. Graph $f(x) = x^3 - 3x + 2$. State the domain and range.



8. Determine whether the graph is the graph of a 1–1 function.



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9. Given $f(x) = x^2 + 2x - 3$ and $g(x) = x^3 - 1$, find $(f - g)(2)$.

10. Given $f(x) = 4x - 5$ and $g(x) = x^2 + 3x + 4$, find $\left(\frac{f}{g}\right)(-2)$.

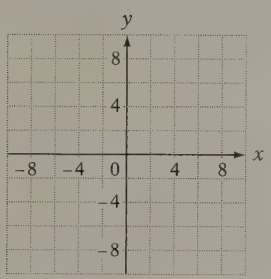
11. Given $f(x) = 4x + 2$ and $g(x) = \frac{x}{x + 1}$, find $f[g(3)]$.

12. Given $f(x) = 2x^2 - 7$ and $g(x) = x - 1$, find $f[g(x)]$.

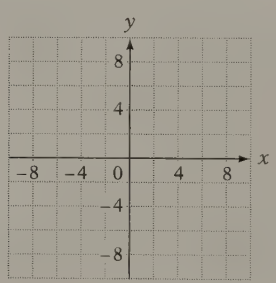
13. Find the inverse of the function $\{(2, 6), (3, 5), (4, 4), (5, 3)\}$.

14. Find the inverse of the function $f(x) = \frac{1}{4}x - 4$.

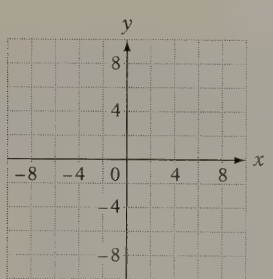
15. Sketch a graph of $x = y^2 - y - 2$.



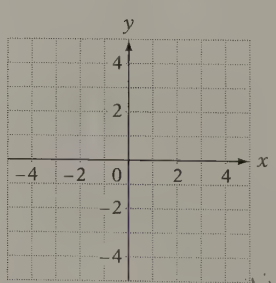
16. Sketch a graph of $(x - 2)^2 + (y + 1)^2 = 9$.



17. Sketch a graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$.



18. Sketch a graph of $\frac{x^2}{16} + \frac{y^2}{4} = 1$.



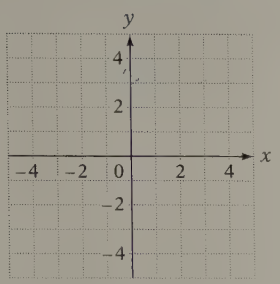
19. Find the equation of the circle with radius 4 and center $(-3, -3)$.

20. The perimeter of a rectangle is 200 cm. What dimensions would give the rectangle a maximum area? What is the maximum area?

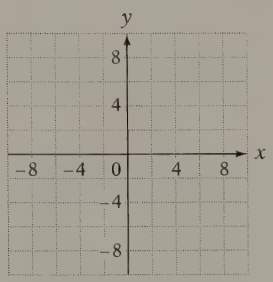
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Chapter Test

1. Graph: $f(x) = 2x - 4$



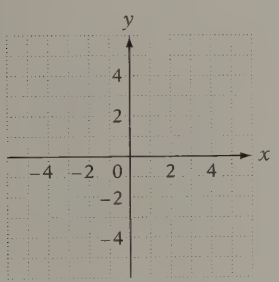
2. Sketch a graph of $f(x) = -2x^2 + x - 2$.



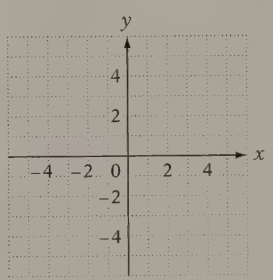
3. Find the x-intercepts of $y = 3x^2 + 9x$.

4. Find the maximum value of $f(x) = -2x^2 + 4x + 1$.

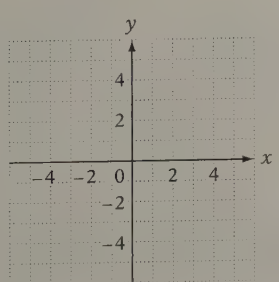
5. Graph $f(x) = \sqrt{x + 4}$. State the domain and range.



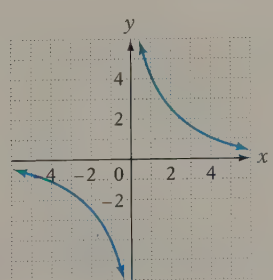
6. Graph $f(x) = |x| - 3$. State the domain and range.



7. Graph $f(x) = 3x^3 - 2$. State the domain and range.



8. Is the graph below the graph of a 1–1 function?



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For problems 9 to 12, use $f(x) = x^2 + 2x - 3$ and $g(x) = x^2 - 2$.

9. Evaluate: $(f + g)(2)$

10. Evaluate: $(f - g)(-4)$

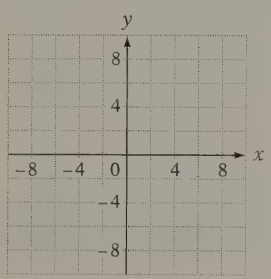
11. Evaluate: $(f \cdot g)(-4)$

12. Evaluate: $\left(\frac{f}{g}\right)(3)$

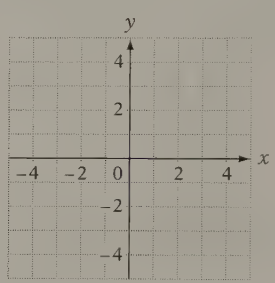
13. Given $f(x) = 2x^2 + x - 5$ and $g(x) = 3x - 1$, find $g[f(x)]$.

14. Find the inverse of $f(x) = -6x + 4$.

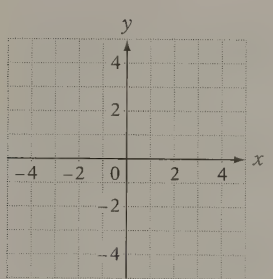
15. Sketch a graph of $x = 2y^2 - 6y + 5$.



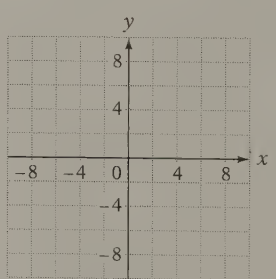
16. Sketch a graph of $(x + 3)^2 + (y + 1)^2 = 1$.



17. Sketch a graph of $\frac{x^2}{1} + \frac{y^2}{9} = 1$.



18. Sketch of graph of $\frac{x^2}{25} - \frac{y^2}{1} = 1$.



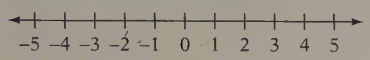
19. Are the functions given by $f(x) = -\frac{1}{4}x + \frac{5}{4}$ and $g(x) = -4x + 5$ inverses of each other?

20. The perimeter of a rectangle is 28 ft. What dimensions would give the rectangle a maximum area?

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Cumulative Review

1. Graph the solution set of $\{x|x < 4\} \cap \{x|x > 2\}$.



2. Solve: $\frac{5x-2}{3} - \frac{1-x}{5} = \frac{x+4}{10}$

3. Evaluate the function $f(x) = -x^2 + 3x - 2$ at $x = -3$.

4. Find the equation of the line that contains the point $(2, -3)$ and has slope $-\frac{3}{2}$.

5. Find the equation of the line that contains the point $(4, -2)$ and is perpendicular to the line $y = -x + 5$.

6. Simplify: $\frac{ax - bx}{ax + ay - bx - by}$

7. Subtract: $\frac{x-4}{3x-2} - \frac{1+x}{3x^2+x-2}$

8. Solve: $\frac{6x}{2x-3} - \frac{1}{2x-3} = 7$

9. Simplify: $\left(\frac{12a^2b^2}{a^{-3}b^{-4}}\right)^{-1}\left(\frac{ab}{4^{-1}a^{-2}b^4}\right)^2$

10. Write $2\sqrt[4]{x^3}$ as an exponential expression.

11. Simplify: $\sqrt{18} - \sqrt{-25}$

12. Solve: $2x^2 + 2x - 3 = 0$

13. Solve: $x - \sqrt{2x-3} = 3$

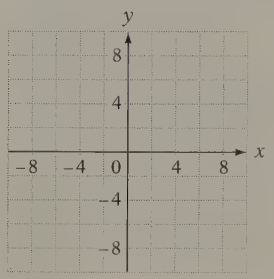
14. Solve: $\frac{3x-2}{x+4} \leq 1$

15. Find the maximum value of the function $f(x) = -2x^2 + 4x - 2$.

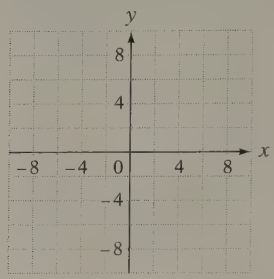
16. Find the inverse of the function $f(x) = 4x + 8$.

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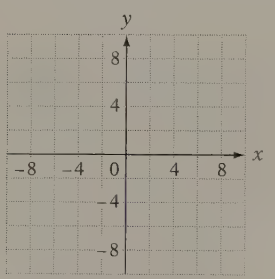
17. Graph the solution set of $5x + 2y > 10$.



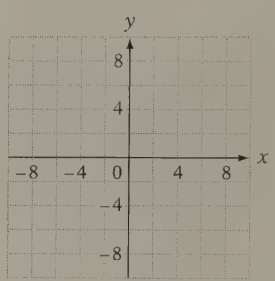
18. Graph: $x = y^2 - 2y + 3$



19. Graph: $\frac{x^2}{25} + \frac{y^2}{4} = 1$



20. Graph: $\frac{y^2}{4} - \frac{x^2}{25} = 1$



21. Tickets for a school play sold for \$4.00 for each adult and \$1.50 for each child. The total receipts for the 192 tickets sold were \$493. Find the number of adult tickets sold.

22. A motorcycle travels 180 mi in the same amount of time that it takes a car to travel 144 mi. The rate of the motorcycle is 12 mph greater than the rate of the car. Find the rate of the motorcycle.

23. The rate of a river's current is 1.5 mph. A rowing crew can row 12 mi down this river and 12 mi back in 6 h. Find the rowing rate of the crew in calm water.

24. The speed (v) of a gear varies inversely as the number of teeth (t). If a gear that has 36 teeth makes 30 revolutions per minute, how many revolutions per minute will a gear that has 60 teeth make?

25. Find the maximum product of two numbers whose sum is 40.

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Chapter 12

Exponential and Logarithmic Functions



This researcher is examining sauropod bones in Jurassic sediments, hoping to determine the age of these long-necked dinosaurs' bones. Archaeologists use the carbon-dating method, which involves an exponential function, to calculate just how old the bones are. Instruments are used to detect the amount of carbon-14 left in an object. Since carbon-14 occurs naturally in living things and gradually decays after death, the measure of the amount of carbon-14 remaining reveals the object's age. The process of using carbon dating is illustrated in the **Example on page 680**.

Objectives

Section 12.1

A To evaluate an exponential function
B To graph an exponential function

Section 12.2

A To write equivalent exponential and logarithmic equations
B To use the Properties of Logarithms
C To use the Change-of-Base Formula

Section 12.3

A To graph a logarithmic function

Section 12.4

A To solve an exponential equation
B To solve a logarithmic equation

Section 12.5

A To solve application problems

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Simplify: 3^{-2}

2. Simplify: $\left(\frac{1}{2}\right)^{-4}$

3. Complete: $\frac{1}{8} = 2^{?}$

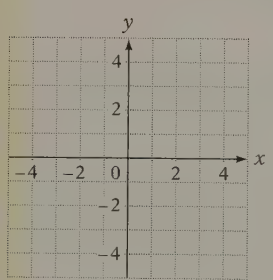
4. Evaluate $f(x) = x^4 + x^3$ for $x = -1$ and $x = 3$.

5. Solve: $3x + 7 = x - 5$

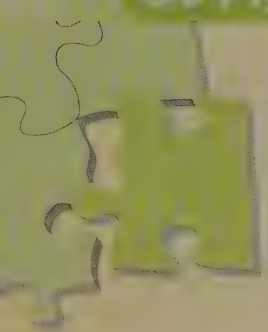
6. Solve: $16 = x^2 - 6x$

7. Evaluate $A(1 + i)^n$ for $A = 5000$, $i = 0.04$, and $n = 6$. Round to the nearest hundredth.

8. Graph: $f(x) = x^2 - 1$



Go Figure



What is the ones digit of $1 + 9 + 9^2 + 9^3 + 9^4 + \ldots + 9^{2000} + 9^{2001}$?

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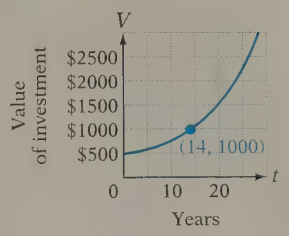
12.1 Exponential Functions

Objective A

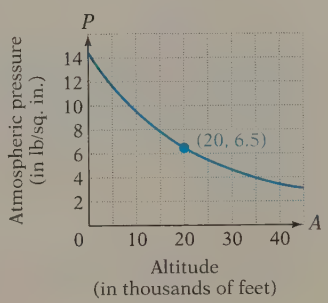
To evaluate an exponential function

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The growth of a $500 savings account that earns 5% annual interest compounded daily is shown in the graph at the right. In 14 years, the savings account contains approximately $1000, twice the initial amount. The growth of this savings account is an example of an exponential function.



The pressure of the atmosphere at a certain height is shown in the graph at the right. This is another example of an exponential function. From the graph, we read that the air pressure is approximately 6.5 lb/in^2 at an altitude of 20,000 ft.



TAKE NOTE

It is important to distinguish between $F(x) = 2^x$ and $P(x) = x^2$. The first is an exponential function; the second is a polynomial function. Exponential functions are characterized by a constant base and a variable exponent. Polynomial functions have a variable base and a constant exponent.

Definition of an Exponential Function

The **exponential function** with base b is defined by

$$f(x) = b^x$$

where $b > 0$, $b \neq 1$, and x is any real number.

In the definition of an exponential function, b, the base, is required to be positive. If the base were a negative number, the value of the function would be a complex number for some values of x. For instance, the value of $f(x) = (-4)^x$ when $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = (-4)^{1/2} = \sqrt{-4} = 2i$. To avoid complex number values of a function, the base of the exponential function is a positive number.

➡ Evaluate $f(x) = 2^x$ at $x = 3$ and $x = -2$.

$f(3) = 2^3 = 8$ • Substitute 3 for x and simplify.

$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ • Substitute -2 for x and simplify.

To evaluate an exponential expression for an irrational number such as $\sqrt{2}$, we obtain an approximation to the value of the function by approximating the irrational number. For instance, the value of $f(x) = 4^x$ when $x = \sqrt{2}$ can be approximated by using an approximation of $\sqrt{2}$.

$$f(\sqrt{2}) = 4^{\sqrt{2}} \approx 4^{1.4142} \approx 7.1029$$

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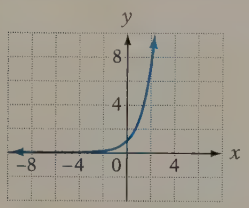
TAKE NOTE
The natural exponential function is an extremely important function. It is used extensively in applied problems in virtually all disciplines, from archaeology to zoology. Leonhard Euler (1707–1783) was the first to use the letter e as the base of the natural exponential function.

Because $f(x) = b^x$ $(b > 0, b \neq 1)$ can be evaluated at both rational and irrational numbers, the domain of f is all real numbers. And because $b^x > 0$ for all values of x, the range of f is the positive real numbers.

A frequently used base in applications of exponential functions is an irrational number designated by e. The number e is approximately 2.71828183. It is an irrational number, so it has a nonterminating, nonrepeating decimal representation.

Natural Exponential Function

The function defined by $f(x) = e^x$ is called the **natural exponential function.**



The e^x key on a calculator can be used to evaluate the natural exponential function. The graph of $y = e^x$ is shown at the left.

Example 1

Evaluate $f(x) = \left(\frac{1}{2}\right)^x$ at $x = 2$ and $x = -3$.

Solution

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$f(2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$f(-3) = \left(\frac{1}{2}\right)^{-3} = 2^3 = 8$$

You Try It 1

Evaluate $f(x) = \left(\frac{2}{3}\right)^x$ at $x = 3$ and $x = -2$.

Your solution

Example 2

Evaluate $f(x) = 2^{3x-1}$ at $x = 1$ and $x = -1$.

Solution

$$f(x) = 2^{3x-1}$$

$$f(1) = 2^{3(1)-1} = 2^2 = 4$$

$$f(-1) = 2^{3(-1)-1} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

You Try It 2

Evaluate $f(x) = 2^{2x+1}$ at $x = 0$ and $x = -2$.

Your solution

Example 3

Evaluate $f(x) = e^{2x}$ at $x = 1$ and $x = -1$. Round to the nearest ten-thousandth.

Solution

$$f(x) = e^{2x}$$

$$f(1) = e^{2\cdot 1} = e^2 \approx 7.3891$$

$$f(-1) = e^{2(-1)} = e^{-2} \approx 0.1353$$

You Try It 3

Evaluate $f(x) = e^{2x-1}$ at $x = 2$ and $x = -2$. Round to the nearest ten-thousandth.

Your solution

Solutions on p. S35

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Objective B To graph an exponential function

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Some properties of an exponential function can be seen in its graph.

➡ Graph $f(x) = 2^x$.

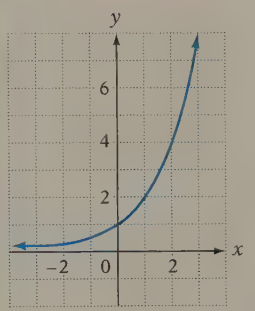
Think of this as the equation $y = 2^x$.

Choose values of x and find the corresponding values of y.

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	$f(x) = y$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



Note that any vertical line would intersect the graph at only one point. Therefore, by the vertical-line test, the graph of $f(x) = 2^x$ is the graph of a function. Also note that any horizontal line would intersect the graph at only one point. Therefore, the graph of $f(x) = 2^x$ is the graph of a one-to-one function.

➡ Graph $f(x) = \left(\frac{1}{2}\right)^x$.

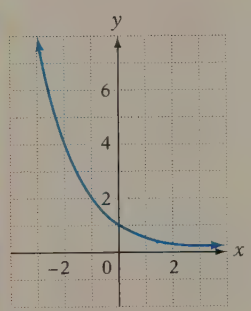
Think of this as the equation $y = \left(\frac{1}{2}\right)^x$.

Choose values of x and find the corresponding values of y.

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	$f(x) = y$
-3	$\left(\frac{1}{2}\right)^{-3} = 8$
-2	$\left(\frac{1}{2}\right)^{-2} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^{0} = 1$
1	$\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$



Applying the vertical-line and horizontal-line tests reveals that the graph of $f(x) = \left(\frac{1}{2}\right)^x$ is also the graph of a one-to-one function.

➡ Graph $f(x) = 2^{-x}$.

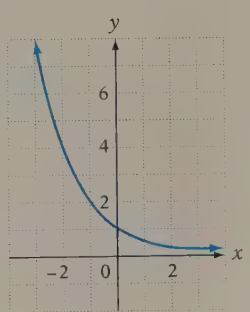
Think of this as the equation $y = 2^{-x}$.

Choose values of x and find the corresponding values of y.

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$



Note that because $2^{-x} = (2^{-1})^x = \left(\frac{1}{2}\right)^x$, the graphs of $f(x) = 2^{-x}$ and $f(x) = \left(\frac{1}{2}\right)^x$ are the same.

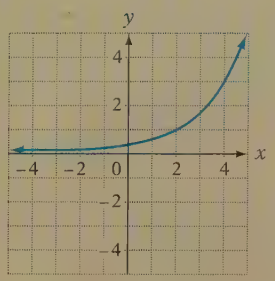
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Example 4

Graph: $f(x) = 3^{\frac{1}{2}x-1}$

Solution

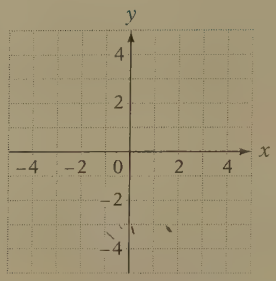
x	y
-2	$\frac{1}{9}$
0	$\frac{1}{3}$
2	1
4	3



You Try It 4

Graph: $f(x) = 2^{-\frac{1}{2}x}$

Your solution

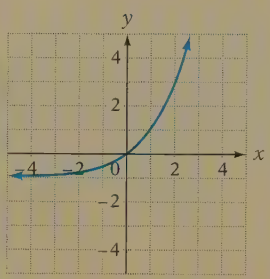


Example 5

Graph: $f(x) = 2^x - 1$

Solution

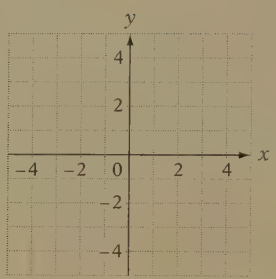
x	y
-2	$-\frac{3}{4}$
-1	$-\frac{1}{2}$
0	0
1	1
2	3
3	7



You Try It 5

Graph: $f(x) = 2^x + 1$

Your solution

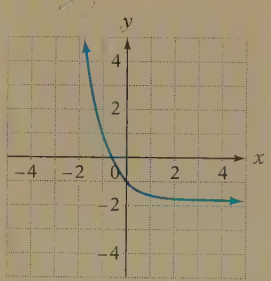


Example 6

Graph: $f(x) = \left(\frac{1}{3}\right)^x - 2$

Solution

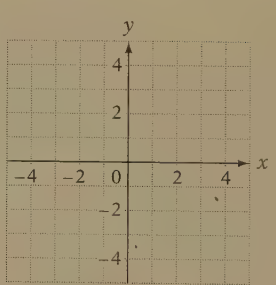
x	y
-2	7
-1	1
0	-1
1	$-\frac{5}{3}$
2	$-\frac{17}{9}$



You Try It 6

Graph: $f(x) = 2^{-x} + 2$

Your solution

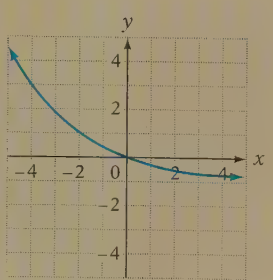


Example 7

Graph: $f(x) = 2^{-\frac{1}{2}x} - 1$

Solution

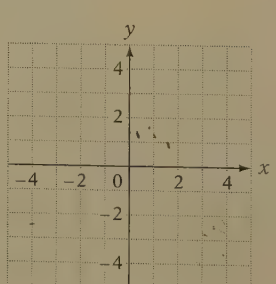
x	y
-6	7
-4	3
-2	1
0	0
2	$-\frac{1}{2}$
4	$-\frac{3}{4}$



You Try It 7

Graph: $f(x) = \left(\frac{1}{2}\right)^{-\frac{1}{2}x} + 2$

Your solution



Solutions on p. S35

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12.1 Exercises

Objective A

1. What is an exponential function?

2. What is the natural exponential function?

3. Which of the following cannot be the base of an exponential function?

a. 7 **b.** $\frac{1}{4}$ **c.** -5 **d.** 0.01

4. Which of the following cannot be the base of an exponential function?

a. 0.9 **b.** 476 **c.** 8 **d.** $-\frac{1}{2}$

5. Given $f(x) = 3^x$, evaluate:

a. $f(2)$ **b.** $f(0)$ **c.** $f(-2)$

6. Given $H(x) = 2^x$, evaluate:

a. $H(-3)$ **b.** $H(0)$ **c.** $H(2)$

7. Given $g(x) = 2^{x+1}$, evaluate:

a. $g(3)$ **b.** $g(1)$ **c.** $g(-3)$

8. Given $F(x) = 3^{x-2}$, evaluate:

a. $F(-4)$ **b.** $F(-1)$ **c.** $F(0)$

9. Given $P(x) = \left(\frac{1}{2}\right)^{2x}$, evaluate:

a. $P(0)$ **b.** $P\left(\frac{3}{2}\right)$ **c.** $P(-2)$

10. Given $R(t) = \left(\frac{1}{3}\right)^{3t}$, evaluate:

a. $R\left(-\frac{1}{3}\right)$ **b.** $R(1)$ **c.** $R(-2)$

11. Given $G(x) = e^{x/2}$, evaluate the following. Round to the nearest ten-thousandth.

a. $G(4)$ **b.** $G(-2)$ **c.** $G\left(\frac{1}{2}\right)$

12. Given $f(x) = e^{2x}$, evaluate the following. Round to the nearest ten-thousandth.

a. $f(-2)$ **b.** $f\left(-\frac{2}{3}\right)$ **c.** $f(2)$

13. Given $H(r) = e^{-r+3}$, evaluate the following. Round to the nearest ten-thousandth.

a. $H(-1)$ **b.** $H(3)$ **c.** $H(5)$

14. Given $P(t) = e^{-\frac{1}{2}t}$, evaluate the following. Round to the nearest ten-thousandth.

a. $P(-3)$ **b.** $P(4)$ **c.** $P\left(\frac{1}{2}\right)$

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15. Given $F(x) = 2^{x^2}$, evaluate:

a. $F(2)$ **b.** $F(-2)$ **c.** $F\left(\frac{3}{4}\right)$

16. Given $Q(x) = 2^{-x^2}$, evaluate:

a. $Q(3)$ **b.** $Q(-1)$ **c.** $Q(-2)$

17. Given $f(x) = e^{-x^2/2}$, evaluate the following. Round to the nearest ten-thousandth.

a. $f(-2)$ **b.** $f(2)$ **c.** $f(-3)$

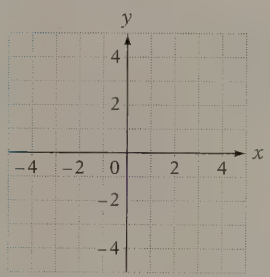
18. Given $f(x) = e^{-2x} + 1$, evaluate the following. Round to the nearest ten-thousandth.

a. $f(-1)$ **b.** $f(3)$ **c.** $f(-2)$

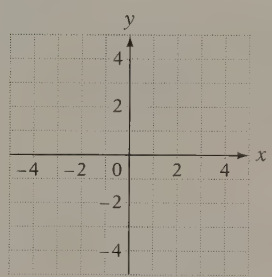
Objective B

Graph.

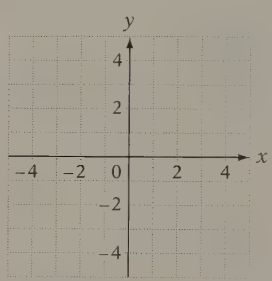
19. $f(x) = 3^x$



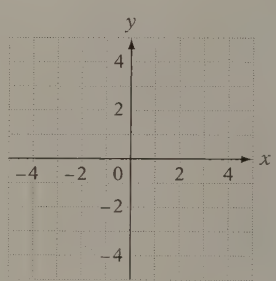
20. $f(x) = 3^{-x}$



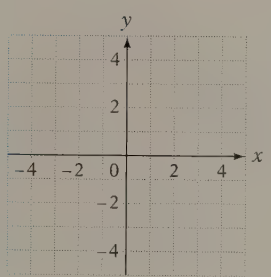
21. $f(x) = 2^{x+1}$



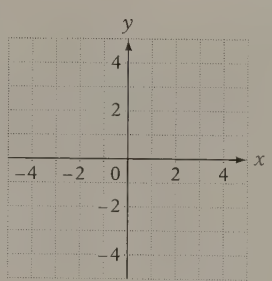
22. $f(x) = 2^{x-1}$



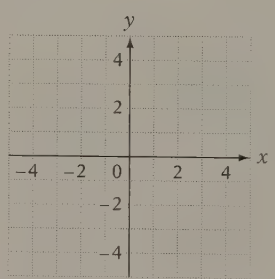
23. $f(x) = \left(\frac{1}{3}\right)^x$



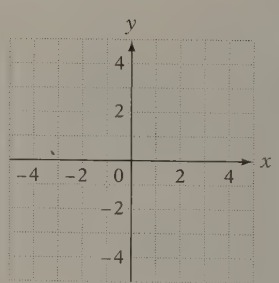
24. $f(x) = \left(\frac{2}{3}\right)^x$



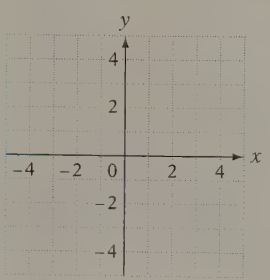
25. $f(x) = 2^{-x} + 1$



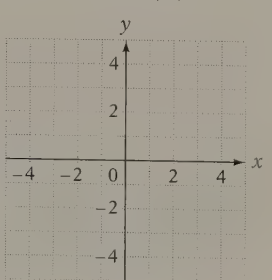
26. $f(x) = 2^x - 3$



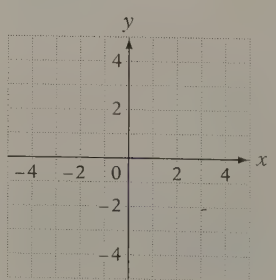
27. $f(x) = \left(\frac{1}{3}\right)^{-x}$



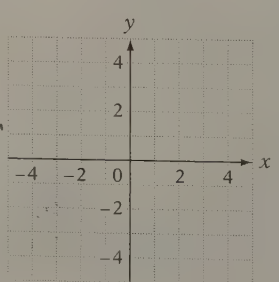
28. $f(x) = \left(\frac{3}{2}\right)^{-x}$



29. $f(x) = \left(\frac{1}{2}\right)^{-x} + 2$



30. $f(x) = \left(\frac{1}{2}\right)^x - 1$



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31. Which of the following functions have the same graph?

a. $f(x) = 3^x$ **b.** $f(x) = \left(\frac{1}{3}\right)^x$ **c.** $f(x) = x^3$ **d.** $f(x) = 3^{-x}$

32. Which of the following functions have the same graph?

a. $f(x) = x^4$ **b.** $f(x) = 4^{-x}$ **c.** $f(x) = 4^x$ **d.** $f(x) = \left(\frac{1}{4}\right)^x$

33. Graph $f(x) = 3^x$ and $f(x) = 3^{-x}$ and find the point of intersection of the two graphs.

34. Graph $f(x) = 2^{x+1}$ and $f(x) = 2^{-x+1}$ and find the point of intersection of the two graphs.

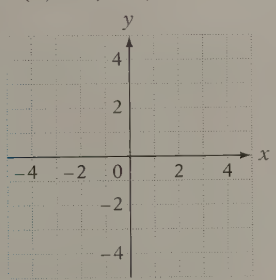
35. Graph $f(x) = \left(\frac{1}{3}\right)^x$. What are the x- and y-intercepts of the graph of the function?

36. Graph $f(x) = \left(\frac{1}{3}\right)^{-x}$. What are the x- and y-intercepts of the graph of the function?

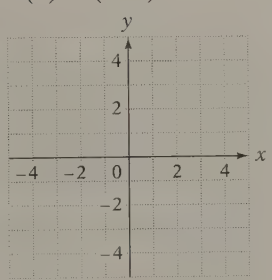
APPLYING THE CONCEPTS

Use a graphing calculator to graph the function.

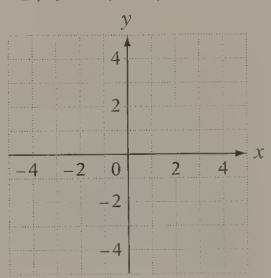
37. $P(x) = (\sqrt{3})^x$



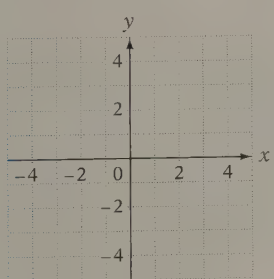
38. $F(x) = (\sqrt{5})^x$



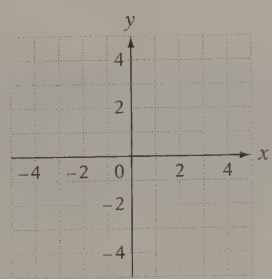
39. $Q(x) = (\sqrt{3})^{-x}$



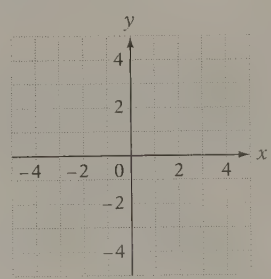
40. $A(x) = (\sqrt{2})^{-x}$



41. $f(x) = \pi^x$



42. $g(x) = \pi^{-x}$



43. Evaluate $\left(1 + \frac{1}{n}\right)^n$ for $n = 100$, 1000, 10,000, and 100,000 and compare the results with the value of e, the base of the natural exponential function. On the basis of your evaluation, complete the following sentence: As n increases, $\left(1 + \frac{1}{n}\right)^n$ becomes closer to _____.

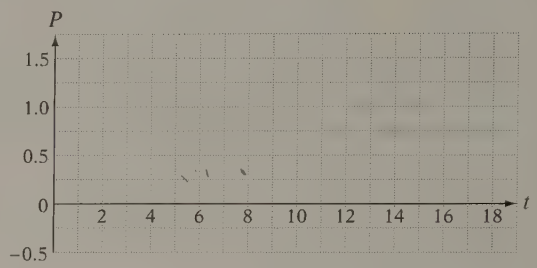
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44. Evaluate $(1 + x)^{1/x}$ for $x = 0.1$, 0.01, 0.001, 0.00001, and 0.0000001 and compare the results with the value of e, the base of the natural exponential function. On the basis of your evaluation, complete the following sentence: As x gets closer to 0, $(1 + x)^{1/x}$ becomes closer to _____.

45. According to population studies, the population of China can be approximated by the equation $P(t) = 1.17(1.012)^t$, where $t = 0$ corresponds to 1993 and $P(t)$ is the population, in billions, of China in t years.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 19, Ymin = −0.5, Ymax = 1.75, and Yscl = 0.25.

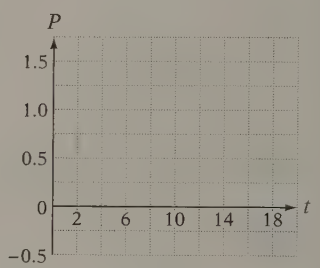
b. The point whose approximate coordinates are (9, 1.303) is on this graph. Write a sentence that explains the meaning of these coordinates.



46. The growth of the population of India can be approximated by the equation $P(t) = 0.883(1.017)^t$, where $t = 0$ corresponds to 1993 and $P(t)$ is the population, in billions, of India in t years.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 20, Ymin = −0.5, Ymax = 1.75, and Yscl = 0.25.

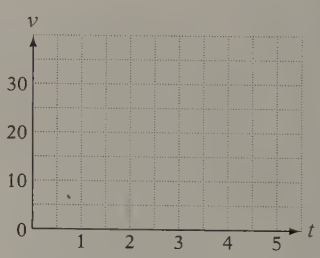
b. The point whose approximate coordinates are (8, 1.01) is on this graph. Write a sentence that explains the meaning of these coordinates.



47. If air resistance is ignored, the speed v, in feet per second, of an object t seconds after it has been dropped is given by $v = 32t$. However, if air resistance is considered, then the speed depends on the mass (and on other things). For a certain mass, the speed t seconds after it has been dropped is given by $v = 32(1 - e^{-t})$.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 5.5, Ymin = 0, Ymax = 40, and Yscl = 5.

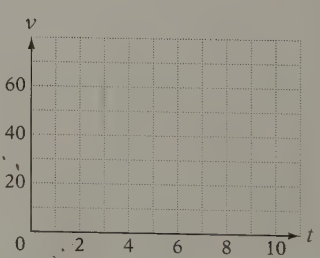
b. The point whose approximate coordinates are (2, 27.7) is on this graph. Write a sentence that explains the meaning of these coordinates.



48. If air resistance is ignored, the speed v, in feet per second, of an object t seconds after it has been dropped is given by $v = 32t$. However, if air resistance is considered, then the speed depends on the mass (and on other things). For a certain mass, the speed t seconds after it has been dropped is given by $v = 64(1 - e^{-t/2})$.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 11, Ymin = 0, Ymax = 80, and Yscl = 10.

b. The point whose approximate coordinates are (4, 55.3) is on this graph. Write a sentence that explains the meaning of these coordinates.



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12.2 Introduction to Logarithms

Objective A

To write equivalent exponential and logarithmic equations

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Because the exponential function is a 1–1 function, it has an inverse function, which is called a **logarithm**. A logarithm is used to answer a question similar to the following: "If $16 = 2^y$, what is the value of y?" Because $16 = 2^4$, the logarithm, base 2, of 16 is 4. This is written as $\log_2 16 = 4$. Note that a logarithm is an exponent that solves a certain equation.

Definition of Logarithm

For $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $x = b^y$.

Read $\log_b x$ as "the logarithm of x, base b" or "log base b of x."

The table at the right shows equivalent statements written in both exponential and logarithmic form.

Exponential Form	*Logarithmic Form*
$2^4 = 16$	$\log_2 16 = 4$
$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$	$\log_{2/3}\left(\frac{4}{9}\right) = 2$
$10^{-1} = 0.1$	$\log_{10}(0.1) = -1$

➡ Write $\log_3 81 = 4$ in exponential form.

$\log_3 81 = 4$ is equivalent to $3^4 = 81$.

➡ Write $10^{-2} = 0.01$ in logarithmic form.

$10^{-2} = 0.01$ is equivalent to $\log_{10}(0.01) = -2$.

The 1–1 property of exponential functions can be used to evaluate some logarithms.

1–1 Property of Exponential Functions

For $b > 0$, $b \neq 1$, if $b^u = b^v$, then $u = v$.

➡ Evaluate $\log_2 8$.

$\log_2 8 = x$ • Write an equation.

$8 = 2^x$ • Write the equation in its equivalent exponential form.

$2^3 = 2^x$ • Write 8 as 2^3.

$3 = x$ • Use the 1–1 Property of Exponential Functions.

$\log_2 8 = 3$

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Solve $\log_4 x = -2$ for x.

$\log_4 x = -2$

$4^{-2} = x$ • Write the equation in its equivalent exponential form.

$\frac{1}{16} = x$ • Simplify.

The solution is $\frac{1}{16}$.

In this example, $\frac{1}{16}$ is called the antilogarithm base 4 of negative 2. In general, if $\log_b M = N$, then M is the antilogarithm base b of N. The antilogarithm of a number can be determined by rewriting the logarithmic expression in exponential form. For instance, the exponential form of $\log_5 x = 3$ is $5^3 = x$; x is the antilogarithm base 5 of 3.

Definition of Antilogarithm

If $\log_b M = N$, the **antilogarithm** base b of N is M. In exponential form, $M = b^N$.

Logarithms base 10 are called **common logarithms**. Usually the base, 10, is omitted when we write the common logarithm of a number. Therefore, $\log_{10} x$ is written $\log x$. To find the common logarithm of most numbers, a calculator is necessary. A calculator was used to find the value of log 384, shown below.

TAKE NOTE
The logarithms of most numbers are irrational numbers. Therefore, the value displayed by a calculator is an approximation.

$$\log 384 \approx 2.5843312$$

(2.5843312: Mantissa; Characteristic)

The decimal part of a common logarithm is called the **mantissa**; the integer part is called the **characteristic**.

When e (the base of the natural exponential function) is used as a base of a logarithm, the logarithm is referred to as the **natural logarithm** and is abbreviated $\ln x$. This is read "el en x." Using a calculator, we learn that

$$\ln 23 \approx 3.135494216$$

The integer and decimal parts of a natural logarithm do not have special names.

Example 1

Evaluate: $\log_3 \left(\frac{1}{9}\right)$

Solution

$\log_3 \left(\frac{1}{9}\right) = x$

$\frac{1}{9} = 3^x$

$3^{-2} = 3^x$

$-2 = x$

$\log_3 \left(\frac{1}{9}\right) = -2$

You Try It 1

Evaluate: $\log_4 64$

Your solution

Solution on p. S36

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Example 2 Solve for x: $\log_5 x = 2$

Solution

$\log_5 x = 2$
$5^2 = x$
$25 = x$

The solution is 25.

You Try It 2 Solve for x: $\log_2 x = -4$

Your solution

Example 3 Solve $\log x = -1.5$ for x. Round to the nearest ten-thousandth.

Solution

$\log x = -1.5$
$10^{-1.5} = x$
$0.0316 \approx x$ • Use a calculator.

You Try It 3 Solve $\ln x = 3$ for x. Round to the nearest ten-thousandth.

Your solution

Solutions on p. S36

Objective 3 To use the Properties of Logarithms

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Because a logarithm is a special kind of exponent, the Properties of Logarithms are similar to the Properties of Exponents.

The property of logarithms that states that the logarithm of the product of two numbers equals the sum of the logarithms of the two numbers is similar to the property of exponents that states that to multiply two exponential expressions with the same base, we add the exponents.

TAKE NOTE

Pay close attention to this theorem. Note, for instance, that this theorem states that

$\log_3 (4 \cdot p) = \log_3 4 + \log_3 p$.

It also states that

$\log_5 9 + \log_5 z = \log_5 (9z)$.

It does *not* state any relationship regarding the expression $\log_b (x + y)$. **This expression cannot be simplified.**

The Logarithm Property of the Product of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b (xy) = \log_b x + \log_b y$.

A proof of this property can be found in the Appendix.

➡ Write $\log_b (6z)$ in expanded form.

$\log_b (6z) = \log_b 6 + \log_b z$ • Use the Logarithm Property of Products.

➡ Write $\log_b 12 + \log_b r$ as a single logarithm.

$\log_b 12 + \log_b r = \log_b (12r)$ • Use the Logarithm Property of Products.

The Logarithm Property of Products can be extended to include the logarithm of the product of more than two factors. For instance,

$$\log_b (xyz) = \log_b x + \log_b y + \log_b z$$
$$\log_b (7rt) = \log_b 7 + \log_b r + \log_b t$$

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A second property of logarithms involves the logarithm of the quotient of two numbers. This property of logarithms is also based on the fact that a logarithm is an exponent and that to divide two exponential expressions with the same base, we subtract the exponents.

The Logarithm Property of the Quotient of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

TAKE NOTE

This theorem is used to rewrite expressions such as $\log_5 \left(\frac{m}{8}\right) = \log_5 m - \log_5 8$. It does *not* state any relationship regarding the expression $\frac{\log_b x}{\log_b y}$. **This expression cannot be simplified.**

A proof of this property can be found in the Appendix.

➡ Write $\log_b \frac{p}{8}$ in expanded form.

$$\log_b \frac{p}{8} = \log_b p - \log_b 8$$

• Use the Logarithm Property of Quotients.

➡ Write $\log_b y - \log_b v$ as a single logarithm.

$$\log_b y - \log_b v = \log_b \frac{y}{v}$$

• Use the Logarithm Property of Quotients.

A third property of logarithms is used to simplify powers of a number.

The Logarithm Property of the Power of a Number

For any positive real numbers x and b, $b \neq 1$, and for any real number r, $\log_b x^r = r \log_b x$.

A proof of this property can be found in the Appendix.

➡ Rewrite $\log_b x^3$ in terms of $\log_b x$.

$$\log_b x^3 = 3 \log_b x$$

• Use the Logarithm Property of Powers.

➡ Rewrite $\frac{2}{3} \log_b x$ with a coefficient of 1.

$$\frac{2}{3} \log_b x = \log_b x^{2/3}$$

• Use the Logarithm Property of Powers.

The following table summarizes the properties of logarithms that we have discussed, along with two other properties.

Summary of the Properties of Logarithms

Let x, y, and b be positive real numbers with $b \neq 1$. Then

Product Property	$\log_b (x \cdot y) = \log_b x + \log_b y$
Quotient Property	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
Power Property	$\log_b x^r = r \log_b x$
Logarithm of One	$\log_b 1 = 0$
1–1 Property	If $\log_b x = \log_b y$, then $x = y$.

Point of Interest

Logarithms were developed independently by Jobst Burgi (1552–1632) and John Napier (1550–1617) as a means of simplifying the calculations of astronomers. The idea was to devise a method by which two numbers could be multiplied by performing additions. Napier is usually given credit for logarithms because he published his results first.

In Napier's original work, the logarithm of 10,000,000 was 0. After this work was published, Napier, in discussions with Henry Briggs (1561–1631), decided that tables of logarithms would be easier to use if the logarithm of 1 were 0. Napier died before new tables could be determined, and Briggs took on the task. His table consisted of logarithms accurate to 30 decimal places, all accomplished without a calculator!

The logarithms Briggs calculated are the common logarithms mentioned earlier.

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➡ Write $\log_b \frac{xy}{z}$ in expanded form.

$$\log_b \frac{xy}{z} = \log_b (xy) - \log_b z \quad \bullet \text{ Use the Logarithm Property of Quotients.}$$
$$= \log_b x + \log_b y - \log_b z \quad \bullet \text{ Use the Logarithm Property of Products.}$$

➡ Write $\log_b \frac{x^2}{y^3}$ in expanded form.

$$\log_b \frac{x^2}{y^3} = \log_b x^2 - \log_b y^3 \quad \bullet \text{ Use the Logarithm Property of Quotients.}$$
$$= 2 \log_b x - 3 \log_b y \quad \bullet \text{ Use the Logarithm Property of Powers.}$$

➡ Write $2 \log_b x + 4 \log_b y$ as a single logarithm with a coefficient of 1.

$$2 \log_b x + 4 \log_b y = \log_b x^2 + \log_b y^4 \quad \bullet \text{ Use the Logarithm Property of Powers.}$$
$$= \log_b x^2 y^4 \quad \bullet \text{ Use the Logarithm Property of Products.}$$

Example 4

Write $\log \sqrt{x^3 y}$ in expanded form.

Solution

$$\log \sqrt{x^3 y} = \log (x^3 y)^{1/2} = \frac{1}{2} \log (x^3 y)$$
$$= \frac{1}{2} (\log x^3 + \log y)$$
$$= \frac{1}{2} (3 \log x + \log y)$$
$$= \frac{3}{2} \log x + \frac{1}{2} \log y$$

You Try It 4

Write $\log_8 \sqrt[3]{xy^2}$ in expanded form.

Your solution

Example 5

Write $\frac{1}{2}(\log_3 x - 3 \log_3 y + \log_3 z)$ as a single logarithm with a coefficient of 1.

Solution

$$\frac{1}{2} (\log_3 x - 3 \log_3 y + \log_3 z)$$
$$= \frac{1}{2} (\log_3 x - \log_3 y^3 + \log_3 z)$$
$$= \frac{1}{2} \left(\log_3 \frac{x}{y^3} + \log_3 z \right)$$
$$= \frac{1}{2} \left(\log_3 \frac{xz}{y^3} \right) = \log_3 \left(\frac{xz}{y^3} \right)^{1/2} = \log_3 \sqrt{\frac{xz}{y^3}}$$

You Try It 5

Write $\frac{1}{3}(\log_4 x - 2 \log_4 y + \log_4 z)$ as a single logarithm with a coefficient of 1.

Your solution

Example 6

Find $\log_4 1$.

Solution

$\log_4 1 = 0$ • Logarithm of One

You Try It 6

Find $\log_9 1$.

Your solution

Solutions on p. S36

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Objective 6 To use the Change-of-Base Formula

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Although only common logarithms and natural logarithms are programmed into calculators, the logarithms for other positive bases can be found.

Evaluate $\log_5 22$.

$$\log_5 22 = x$$ • Write an equation.
$$5^x = 22$$ • Write the equation in its equivalent exponential form.
$$\log 5^x = \log 22$$ • Take the common logarithm of each side of the equation.
$$x \log 5 = \log 22$$ • Use the Power Property of Logarithms.
$$x = \frac{\log 22}{\log 5}$$ • Divide each side by log 5. This is the exact answer.
$$x \approx 1.9206$$ • This is an approximate answer.
$$\log_5 22 \approx 1.9206$$

In the third step above, the natural logarithm, instead of the common logarithm, could have been applied to each side of the equation. The same result would have been obtained.

Using a procedure similar to the one used to evaluate $\log_5 22$, a formula for changing bases can be derived.

Change-of-Base Formula

$$\log_a N = \frac{\log_b N}{\log_b a}$$

Evaluate $\log_2 14$.

$$\log_2 14 = \frac{\log 14}{\log 2}$$ • Use the Change-of-Base Formula with $N = 14$, $a = 2$, $b = 10$.
$$\approx 3.8074$$

For the last example, common logarithms were used. Here is the same example using natural logarithms. Note that the answers are the same.

$$\log_2 14 = \frac{\ln 14}{\ln 2} \approx 3.8074$$

Example 7
Evaluate $\log_2 90.813$ by using common logarithms.

Solution

$$\log_2 90.813 = \frac{\log 90.813}{\log 2} \approx 6.5048$$

You Try It 7
Evaluate $\log_7 6.45$ by using common logarithms.

Your solution

Example 8
Evaluate $\log_8 0.137$ by using natural logarithms.

Solution

$$\log_8 0.137 = \frac{\ln 0.137}{\ln 8} \approx -0.9559$$

You Try It 8
Evaluate $\log_3 0.834$ by using natural logarithms.

Your solution

Solutions on p. S36

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12.2 Exercises

Objective A

1. **a.** What is a common logarithm?
b. How is the common logarithm of $4z$ written?

2. **a.** What is a natural logarithm?
b. How is the natural logarithm of $3x$ written?

Write the exponential equation in logarithmic form.

3. $5^2 = 25$ **4.** $10^3 = 1000$ **5.** $4^{-2} = \frac{1}{16}$ **6.** $3^{-3} = \frac{1}{27}$

7. $10^y = x$ **8.** $e^y = x$ **9.** $a^x = w$ **10.** $b^y = c$

Write the logarithmic equation in exponential form.

11. $\log_3 9 = 2$ **12.** $\log_2 32 = 5$ **13.** $\log 0.01 = -2$ **14.** $\log_5 \frac{1}{5} = -1$

15. $\ln x = y$ **16.** $\log x = y$ **17.** $\log_b u = v$ **18.** $\log_c x = y$

Evaluate.

19. $\log_3 81$ **20.** $\log_7 49$ **21.** $\log_2 128$ **22.** $\log_5 125$

23. $\log 100$ **24.** $\log 0.001$ **25.** $\ln e^3$ **26.** $\ln e^2$

27. $\log_8 1$ **28.** $\log_3 243$ **29.** $\log_5 625$ **30.** $\log_2 64$

Solve for x.

31. $\log_3 x = 2$ **32.** $\log_5 x = 1$ **33.** $\log_4 x = 3$ **34.** $\log_2 x = 6$

35. $\log_7 x = -1$ **36.** $\log_8 x = -2$ **37.** $\log_6 x = 0$ **38.** $\log_4 x = 0$

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Solve for x. Round to the nearest hundredth.

39. $\log x = 2.5$ **40.** $\log x = 3.2$ **41.** $\log x = -1.75$ **42.** $\log x = -2.1$

43. $\ln x = 2$ **44.** $\ln x = 1.4$ **45.** $\ln x = -\frac{1}{2}$ **46.** $\ln x = -1.7$

Objective B

47. What is the Product Property of Logarithms?

48. What is the Quotient Property of Logarithms?

Express as a single logarithm with a coefficient of 1.

49. $\log_3 x^3 + \log_3 y^2$ **50.** $\log_7 x + \log_7 z^2$ **51.** $\ln x^4 - \ln y^2$

52. $\ln x^2 - \ln y$ **53.** $3 \log_7 x$ **54.** $4 \log_8 y$

55. $3 \ln x + 4 \ln y$ **56.** $2 \ln x - 5 \ln y$ **57.** $2(\log_4 x + \log_4 y)$

58. $3(\log_5 r + \log_5 t)$ **59.** $\frac{1}{2}(\log_6 x - \log_6 y)$ **60.** $\frac{1}{3}(\log_8 x - \log_8 y)$

61. $2 \log_3 x - \log_3 y + 2 \log_3 z$ **62.** $4 \log_5 r - 3 \log_5 s + \log_5 t$

63. $\ln x - (2 \ln y + \ln z)$ **64.** $2 \log_b x - 3(\log_b y + \log_b z)$

65. $2(\log_4 s - 2 \log_4 t + \log_4 r)$ **66.** $3(\log_9 x + 2 \log_9 y - 2 \log_9 z)$

67. $3\ln x - 4(\ln y + \ln z)$ **68.** $\ln t - 3(\ln u + \ln v)$

69. $3 \log_2 t - 2(\log_2 r - \log_2 v)$ **70.** $2 \log_{10} x - 3(\log_{10} y - \log_{10} z)$

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71. $\frac{1}{2}(3 \log_4 x - 2 \log_4 y + \log_4 z)$

72. $\frac{1}{3}(4 \log_5 t - 5 \log_5 u - 7 \log_5 v)$

73. $\frac{1}{2}(\ln x - 3 \ln y)$

74. $\frac{1}{3} \ln a + \frac{2}{3} \ln b$

75. $\frac{1}{2}\log_2 x - \frac{2}{3}\log_2 y + \frac{1}{2}\log_2 z$

76. $\frac{2}{3}\log_3 x + \frac{1}{3}\log_3 y - \frac{1}{2}\log_3 z$

Write the logarithm in expanded form.

77. $\log_8 (xz)$

78. $\log_7 (rt)$

79. $\log_3 x^5$

80. $\log_2 y^7$

81. $\log_b \left(\frac{r}{s}\right)$

82. $\log_c \left(\frac{z}{4}\right)$

83. $\log_3 (x^2y^6)$

84. $\log_4 (t^4u^2)$

85. $\log_7 \left(\frac{u^3}{v^4}\right)$

86. $\log \left(\frac{s^5}{t^2}\right)$

87. $\log_2 (rs)^2$

88. $\log_3 (x^2y)^3$

89. $\ln(x^2yz)$

90. $\ln(xy^2z^3)$

91. $\log_5 \left(\frac{xy^2}{z^4}\right)$

92. $\log_b \left(\frac{r^2s}{t^3}\right)$

93. $\log_8 \left(\frac{x^2}{yz^2}\right)$

94. $\log_9 \left(\frac{x}{y^2z^3}\right)$

95. $\log_4 \sqrt{x^3y}$

96. $\log_3 \sqrt{x^5y^3}$

97. $\log_7 \sqrt{\frac{x^3}{y}}$

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98. $\log_b \sqrt[3]{\frac{r^2}{t}}$ **99.** $\log_3 \frac{t}{\sqrt{x}}$ **100.** $\log_4 \frac{x}{\sqrt{y^2 z}}$

Objective C

Evaluate. Round to the nearest ten-thousandth.

101. $\log_{10} 7$ **102.** $\log_{10} 9$ **103.** $\log_{10}\left(\frac{3}{5}\right)$ **104.** $\log_{10}\left(\frac{13}{3}\right)$

105. $\ln 4$ **106.** $\ln 6$ **107.** $\ln\left(\frac{17}{6}\right)$ **108.** $\ln\left(\frac{13}{17}\right)$

109. $\log_8 6$ **110.** $\log_4 8$ **111.** $\log_5 30$ **112.** $\log_6 28$

113. $\log_3 (0.5)$ **114.** $\log_5 (0.6)$ **115.** $\log_7 (1.7)$ **116.** $\log_6 (3.2)$

117. $\log_5 15$ **118.** $\log_3 25$ **119.** $\log_{12} 120$ **120.** $\log_9 90$

121. $\log_4 2.55$ **122.** $\log_8 6.42$ **123.** $\log_5 67$ **124.** $\log_8 35$

APPLYING THE CONCEPTS

125. For each of the following, answer True or False. Assume all variables represent positive numbers.

a. $\log_3 (-9) = -2$

b. $x^y = z$ and $\log_x z = y$ are equivalent equations.

c. $\log (x^{-1}) = \frac{1}{\log x}$

d. $\log \left(\frac{x}{y}\right) = \log x - \log y$

e. $\log (x \cdot y) = \log x \cdot \log y$

f. If $\log x = \log y$, then $x = y$.

126. Complete each statement using the equation $\log_a b = c$.

a. $a^c =$ _____

b. $\text{antilog}_a (\log_a b) =$ _____

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12.3 Graphs of Logarithmic Functions

Objective A

To graph a logarithmic function

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The graph of a logarithmic function can be drawn by using the relationship between the exponential and logarithmic functions.

Point of Interest

Although logarithms were originally developed to assist with computations, logarithmic functions have a much broader use today. These functions occur in geology, acoustics, chemistry, and economics, for example.

Graph: $f(x) = \log_2 x$

Think of this as the equation $y = \log_2 x$.

$$f(x) = \log_2 x$$
$$y = \log_2 x$$

Write the equivalent exponential equation.

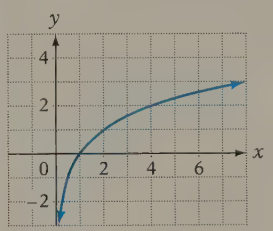
$$x = 2^y$$

Because the equation is solved for x in terms of y, it is easier to choose values of y and find the corresponding values of x. The results can be recorded in a table.

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.



Applying the vertical-line and horizontal-line tests reveals that $f(x) = \log_2 x$ is a 1–1 function.

TAKE NOTE

See the Projects and Group Activities at the end of this chapter for suggestions on graphing logarithmic functions using a graphing calculator.

Graph: $f(x) = \log_2 (x) + 1$

Think of $f(x) = \log_2 (x) + 1$ as the equation $y = \log_2 (x) + 1$.

$$f(x) = \log_2 (x) + 1$$
$$y = \log_2 (x) + 1$$

Solve for $\log_2 (x)$.

$$y - 1 = \log_2 (x)$$

Write the equivalent exponential equation.

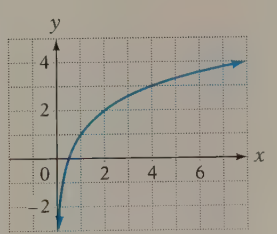
$$2^{y-1} = x$$

Choose values of y and find the corresponding values of x.

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	y
$\frac{1}{4}$	-1
$\frac{1}{2}$	0
1	1
2	2
4	3



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Example 1

Graph: $f(x) = \log_3 x$

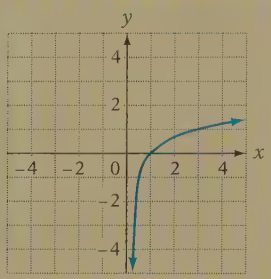
Solution

$f(x) = \log_3 x$

$y = \log_3 x$

$3^y = x$ • Write the equivalent exponential equation.

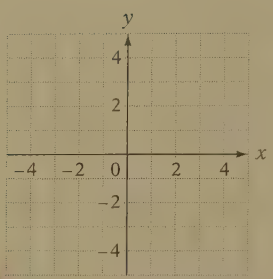
x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1



You Try It 1

Graph: $f(x) = \log_2 (x - 1)$

Your solution



Example 2

Graph: $f(x) = 2 \log_3 x$

Solution

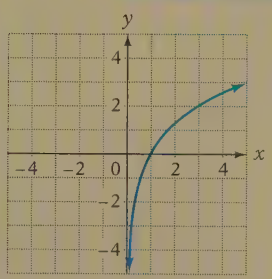
$f(x) = 2 \log_3 x$

$y = 2 \log_3 x$

$\frac{y}{2} = \log_3 x$

$3^{y/2} = x$ • Write the equivalent exponential equation.

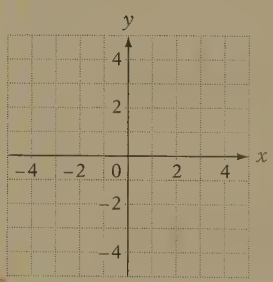
x	y
$\frac{1}{9}$	-4
$\frac{1}{3}$	-2
1	0
3	2



You Try It 2

Graph: $f(x) = \log_3 (2x)$

Your solution



Example 3

Graph: $f(x) = -\log_2 (x - 2)$

Solution

$f(x) = -\log_2 (x - 2)$

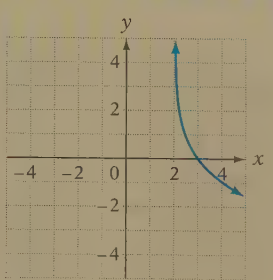
$y = -\log_2 (x - 2)$

$-y = \log_2 (x - 2)$

$2^{-y} = x - 2$

$2^{-y} + 2 = x$ • Write the equivalent exponential equation.

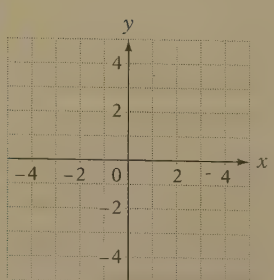
x	y
6	-2
4	-1
3	0
$\frac{5}{2}$	1
$\frac{9}{4}$	2
$\frac{17}{8}$	3



You Try It 3

Graph: $f(x) = -\log_3 (x + 1)$

Your solution



Solutions on p. S36

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12.3 Exercises

Objective A

1. Is the function $f(x) = \log x$ a 1–1 function? Why or why not?

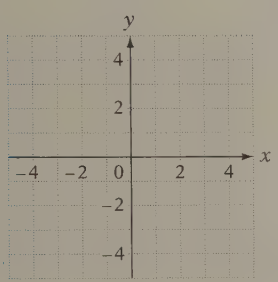
2. Name two characteristics of the graph of $y = \log_b x$, $b > 1$.

3. What is the relationship between the graph of $x = 3^y$ and that of $y = \log_3 x$?

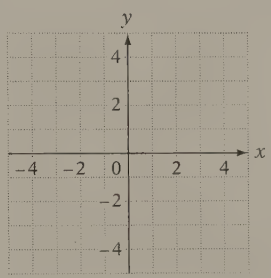
4. What is the relationship between the graph of $y = 3^x$ and that of $y = \log_3 x$?

Graph.

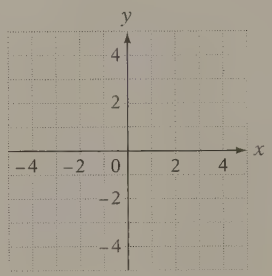
5. $f(x) = \log_4 x$



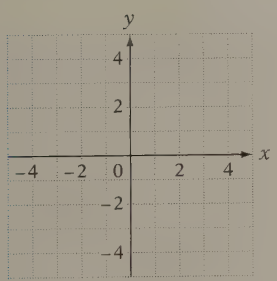
6. $f(x) = \log_2 (x + 1)$



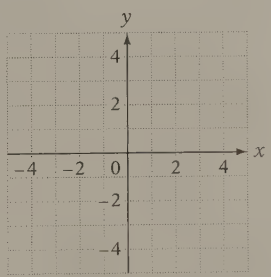
7. $f(x) = \log_3 (2x - 1)$



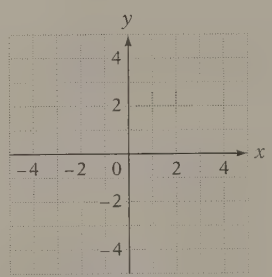
8. $f(x) = \log_2 \left(\frac{1}{2}x\right)$



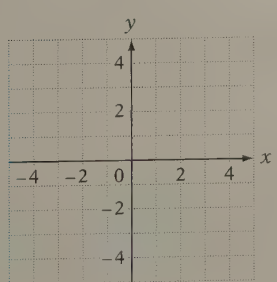
9. $f(x) = 3 \log_2 x$



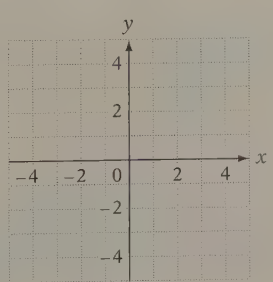
10. $f(x) = \frac{1}{2}\log_2 x$



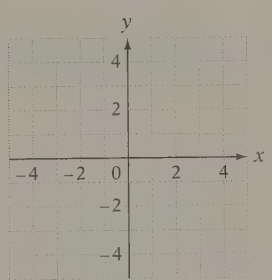
11. $f(x) = -\log_2 x$



12. $f(x) = -\log_3 x$

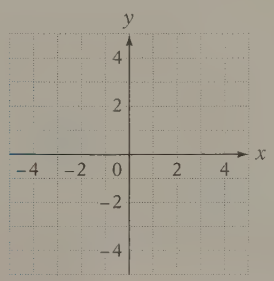


13. $f(x) = \log_2 (x - 1)$

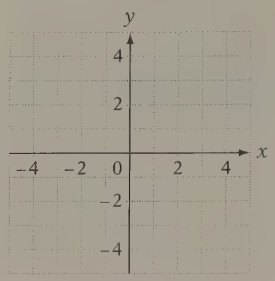


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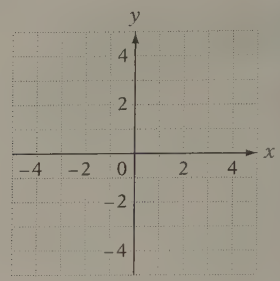
14. $f(x) = \log_3 (2 - x)$



15. $f(x) = -\log_2 (x - 1)$



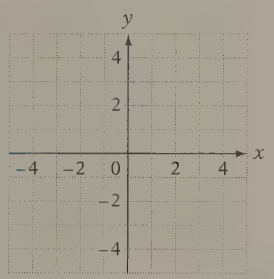
16. $f(x) = -\log_2 (1 - x)$



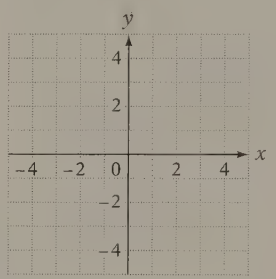
APPLYING THE CONCEPTS

Use a graphing calculator to graph the following.

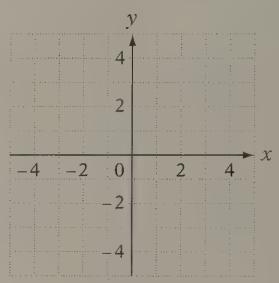
17. $f(x) = x - \log_2 (1 - x)$



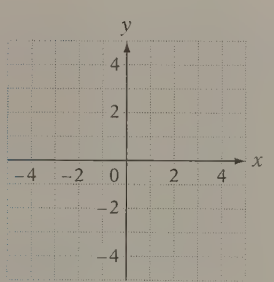
18. $f(x) = -\frac{1}{2}\log_2 x - 1$



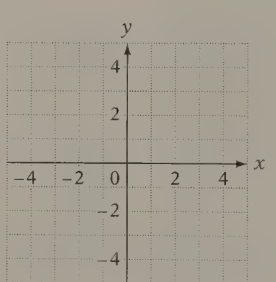
19. $f(x) = \frac{x}{2} - 2 \log_2 (x + 1)$



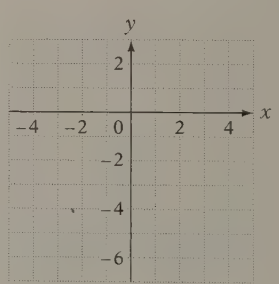
20. $f(x) = x + \log_3 (2 - x)$



21. $f(x) = x^2 - 10 \ln (x - 1)$



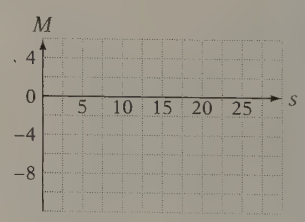
22. $f(x) = \frac{x}{3} - 3 \log_2 (x + 3)$



23. Astronomers use the *distance modulus* of a star as a method of determining the star's distance from Earth. The formula is $M = 5 \log s - 5$, where M is the distance modulus and s is the star's distance from the Earth in parsecs. (One parsec $\approx 1.9 \times 10^{13}$ mi.)

a. Graph the equation.

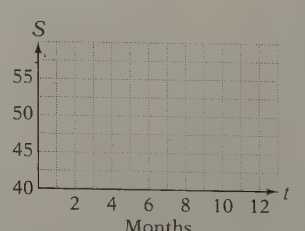
b. The point with coordinates (25.1, 2) is on the graph. Write a sentence that describes the meaning of this ordered pair.



24. Without practice, the proficiency of a typist decreases. The equation $S = 60 - .7 \ln (t + 1)$, where S is the typing speed in words per minute and t is the number of months without typing, approximates this decrease.

a. Graph the equation.

b. The point with coordinates (4, 49) is on the graph. Write a sentence that describes the meaning of this ordered pair.



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12.4 Solving Exponential and Logarithmic Equations

Objective A

To solve an exponential equation



An **exponential equation** is one in which the variable occurs in the exponent. The examples at the right are exponential equations.

$6^{2x+1} = 6^{3x-2}$

$4^x = 3$

$2^{x+1} = 7$

An exponential equation in which each side of the equation can be expressed in terms of the same base can be solved by using the One-to-One Property of Exponential Functions. Recall that the One-to-One Property of Exponential Functions states that

If $b^u = b^v$, then $u = v$.

In the two examples below, this property is used in solving exponential equations.

➡ Solve: $10^{3x+5} = 10^{x-3}$

$10^{3x+5} = 10^{x-3}$

$3x + 5 = x - 3$ • Use the 1–1 Property of Exponential Functions to equate the exponents.

$2x + 5 = -3$ • Solve the resulting equation.

$2x = -8$

$x = -4$

Check:

$10^{3x+5} = 10^{x-3}$

$10^{3(-4)+5}$	10^{-4-3}
10^{-12+5}	10^{-7}

$10^{-7} = 10^{-7}$

The solution is -4.

➡ Solve: $9^{x+1} = 27^{x-1}$

$9^{x+1} = 27^{x-1}$

$(3^2)^{x+1} = (3^3)^{x-1}$ • $3^2 = 9$; $3^3 = 27$

$3^{2x+2} = 3^{3x-3}$

$2x + 2 = 3x - 3$ • Use the 1–1 Property of Exponential Functions to equate the exponents.

$2 = x - 3$ • Solve for x.

$5 = x$

Check:

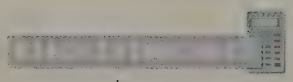
$9^{x+1} = 27^{x-1}$

9^{5+1}	27^{5-1}
9^6	27^4

$531{,}441 = 531{,}441$

The solution is 5.

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To evaluate $\frac{\log 7}{\log 4}$ on a scientific calculator, use the keystrokes

7 log ÷ 4 log =

The display should read 1.4036775.

When each side of an exponential equation cannot easily be expressed in terms of the same base, logarithms are used to solve the exponential equation.

Solve: $4^x = 7$

$$4^x = 7$$
$$\log 4^x = \log 7$$ • Take the common logarithm of each side of the equation.

$$x \log 4 = \log 7$$ • Rewrite the equation using the Properties of Logarithms.

$$x = \frac{\log 7}{\log 4} \approx 1.4037$$ • Solve for x.

The solution is 1.4037.

• Note that $\frac{\log 7}{\log 4} \neq \log 7 - \log 4$.

Solve: $3^{x+1} = 5$

$$3^{x+1} = 5$$
$$\log 3^{x+1} = \log 5$$ • Take the common logarithm of each side of the equation.

$$(x + 1)\log 3 = \log 5$$ • Rewrite the equation using the Properties of Logarithms.

$$x + 1 = \frac{\log 5}{\log 3}$$
$$x = \frac{\log 5}{\log 3} - 1$$ • Solve for x.
$$x \approx 0.4650$$

The solution is 0.4650.

Example 1

Solve for n: $(1.1)^n = 2$

Solution

$$(1.1)^n = 2$$
$$\log (1.1)^n = \log 2$$
$$n \log 1.1 = \log 2$$
$$n = \frac{\log 2}{\log 1.1}$$
$$n \approx 7.2725$$

The solution is 7.2725.

You Try It 1

Solve for n: $(1.06)^n = 1.5$

Your solution

Example 2

Solve for x: $3^{2x} = 4$

Solution

$$3^{2x} = 4$$
$$\log 3^{2x} = \log 4$$
$$2x \log 3 = \log 4$$
$$2x = \frac{\log 4}{\log 3}$$
$$x = \frac{\log 4}{2 \log 3}$$
$$x \approx 0.6309$$

The solution is 0.6309.

You Try It 2

Solve for x: $4^{3x} = 25$

Your solution

Solutions on p. S37

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Objective B **To solve a logarithmic equation** 

A logarithmic equation can be solved by using the Properties of Logarithms. Here are two examples.

➡ Solve: $\log_9 x + \log_9 (x - 8) = 1$

$$\log_9 x + \log_9 (x - 8) = 1$$

$$\log_9 [x(x - 8)] = 1$$ • Use the Logarithm Property of Products to rewrite the left side of the equation.

$$9^1 = x(x - 8)$$ • Write the equation in exponential form.

$$9 = x^2 - 8x$$ • Simplify.

$$0 = x^2 - 8x - 9$$ • Solve for x.

$$0 = (x - 9)(x + 1)$$ • Factor and use the Principle of Zero Products.

$$x - 9 = 0 \qquad x + 1 = 0$$

$$x = 9 \qquad x = -1$$

Replacing x by 9 in the original equation reveals that 9 checks as a solution. Replacing x by -1 in the original equation results in the expression $\log_9 (-1)$. Because the logarithm of a negative number is not a real number, -1 does not check as a solution.

The solution of the equation is 9.

➡ Solve: $\log_3 6 - \log_3 (2x + 3) = \log_3 (x + 1)$

$$\log_3 6 - \log_3 (2x + 3) = \log_3 (x + 1)$$

$$\log_3 \frac{6}{2x + 3} = \log_3 (x + 1)$$ • Use the Quotient Property of Logarithms to rewrite the left side of the equation.

$$\frac{6}{2x + 3} = x + 1$$ • Use the 1–1 Property of Logarithms.

$$6 = (2x + 3)(x + 1)$$ • Multiply each side by $2x + 3$.

$$6 = 2x^2 + 5x + 3$$ • Multiply the binomials.

$$0 = 2x^2 + 5x - 3$$ • Write the equation in standard form.

$$0 = (2x - 1)(x + 3)$$ • Solve for x.

$$2x - 1 = 0 \qquad x + 3 = 0$$

$$x = \frac{1}{2} \qquad x = -3$$

Replacing x by $\frac{1}{2}$ in the original equation reveals that $\frac{1}{2}$ checks as a solution. Replacing x by -3 in the original equation results in the expression $\log_3 (-2)$. Because the logarithm of a negative number is not a real number, -3 does not check as a solution.

The solution of the equation is $\frac{1}{2}$.

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Example 3

Solve for x: $\log_3 (2x - 1) = 2$

Solution

$\log_3 (2x - 1) = 2$

$3^2 = 2x - 1$ • Write in exponential form.

$9 = 2x - 1$

$10 = 2x$

$5 = x$

The solution is 5.

You Try It 3

Solve for x: $\log_4 (x^2 - 3x) = 1$

Your solution

Example 4

Solve for x: $\log_2 x - \log_2 (x - 1) = \log_2 2$

Solution

$\log_2 x - \log_2 (x - 1) = \log_2 2$

$\log_2 \left(\frac{x}{x - 1}\right) = \log_2 2$ • Use the Quotient Property of Logarithms.

$\frac{x}{x - 1} = 2$ • Use the 1–1 Property of Logarithms.

$(x - 1)\left(\frac{x}{x - 1}\right) = (x - 1)2$

$x = 2x - 2$

$-x = -2$

$x = 2$

The solution is 2.

You Try It 4

Solve for x: $\log_3 x + \log_3 (x + 3) = \log_3 4$

Your solution

Example 5

Solve for x:

$\log_2 (3x + 8) = \log_2 (2x + 2) + \log_2 (x - 2)$

Solution

$\log_2 (3x + 8) = \log_2 (2x + 2) + \log_2 (x - 2)$

$\log_2 (3x + 8) = \log_2 [(2x + 2)(x - 2)]$

$\log_2 (3x + 8) = \log_2 (2x^2 - 2x - 4)$

$3x + 8 = 2x^2 - 2x - 4$ • Use the 1–1 Property of Logarithms.

$0 = 2x^2 - 5x - 12$

$0 = (2x + 3)(x - 4)$

$2x + 3 = 0 \qquad x - 4 = 0$

$x = -\frac{3}{2} \qquad x = 4$

$-\frac{3}{2}$ does not check as a solution; 4 checks as a solution. The solution is 4.

You Try It 5

Solve for x: $\log_3 x + \log_3 (x + 6) = 3$

Your solution

Solutions on p. S37

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12.4 Exercises

Objective A

1. What is an exponential equation?

2. **a.** What does the One-to-One Property of Exponential Functions state?
   b. Provide an example of when you would use this property.

Solve for x. Round to the nearest ten-thousandth.

3. $5^{4x-1} = 5^{x-2}$

4. $7^{4x-3} = 7^{2x+1}$

5. $8^{x-4} = 8^{5x+8}$

6. $10^{4x-5} = 10^{x+4}$

7. $9^{x} = 3^{x+1}$

8. $2^{x-1} = 4^{x}$

9. $8^{x+2} = 16^{x}$

10. $9^{3x} = 81^{x-4}$

11. $16^{2-x} = 32^{2x}$

12. $27^{2x-3} = 81^{4-x}$

13. $25^{3-x} = 125^{2x-1}$

14. $8^{4x-7} = 64^{x-3}$

15. $5^{x} = 6$

16. $7^{x} = 10$

17. $e^{x} = 3$

18. $e^{x} = 2$

19. $10^{x} = 21$

20. $10^{x} = 37$

21. $2^{-x} = 7$

22. $3^{-x} = 14$

23. $2^{x-1} = 6$

24. $4^{x+1} = 9$

25. $3^{2x-1} = 4$

26. $4^{-x+2} = 12$

Objective B

27. What is a logarithmic equation?

28. What does the One-to-One Property of Logarithms state?

Solve for x.

29. $\log_2 (2x - 3) = 3$

30. $\log_4 (3x + 1) = 2$

31. $\log_2 (x^2 + 2x) = 3$

32. $\log_3 (x^2 + 6x) = 3$

33. $\log_5 \left(\dfrac{2x}{x - 1}\right) = 1$

34. $\log_6 \left(\dfrac{3x}{x + 1}\right) = 1$

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35. $\log x = \log (1 - x)$ **36.** $\ln (3x - 2) = \ln (x + 1)$ **37.** $\ln 5 = \ln (4x - 13)$

38. $\log_3 (x - 2) = \log_3 (2x)$ **39.** $\ln (3x + 2) = 4$ **40.** $\ln (2x + 3) = -1$

41. $\log_2 (8x) - \log_2 (x^2 - 1) = \log_2 3$ **42.** $\log_5 (3x) - \log_5 (x^2 - 1) = \log_5 2$

43. $\log_9 x + \log_9 (2x - 3) = \log_9 2$ **44.** $\log_6 x + \log_6 (3x - 5) = \log_6 2$

45. $\log_8 (6x) = \log_8 2 + \log_8 (x - 4)$ **46.** $\log_7 (5x) = \log_7 3 + \log_7 (2x + 1)$

APPLYING THE CONCEPTS

Solve each equation with a graphing calculator. Round answers to the nearest hundredth.

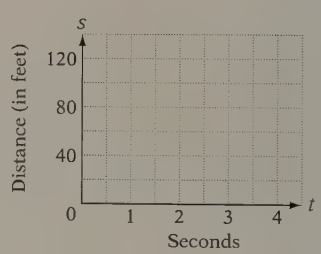
47. $3^x = -x$ **48.** $2^x = -x + 2$ **49.** $2^{-x} = x - 1$ **50.** $3^{-x} = 2x$

51. $\ln x = x^2$ **52.** $2 \ln x = -x + 1$ **53.** $\log_3 x = -2x - 2$ **54.** $\log_5 (2x) = -2x + 1$

55. A model for the distance s (in feet) that an object that is experiencing air resistance will fall in t seconds is given by $s = 312.5 \ln \left(\frac{e^{0.32t} + e^{-0.32t}}{2}\right)$.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 4.5, Ymin = 0, Ymax = 140, and Yscl = 20.

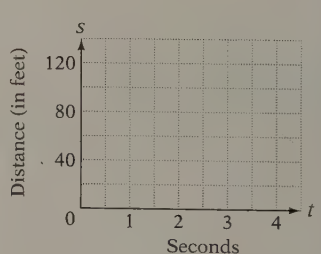
b. Determine, to the nearest hundredth of a second, the time it takes the object to travel 100 ft.



56. A model for the distance s (in feet) that an object that is experiencing air resistance will fall in t seconds is given by $s = 78 \ln \left(\frac{e^{0.8t} + e^{-0.8t}}{2}\right)$.

a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 4.5, Ymin = 0, Ymax = 140, and Yscl = 20.

b. Determine, to the nearest hundredth of a second, the time it takes the object to travel 125 ft.



57. The following "proof" shows that $0.5 < 0.25$. Explain the error.

$$\begin{aligned} 1 &< 2 \\ 1 \cdot \log 0.5 &< 2 \cdot \log 0.5 \\ \log 0.5 &< \log (0.5)^2 \\ 0.5 &< (0.5)^2 \\ 0.5 &< 0.25 \end{aligned}$$

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12.5 Applications of Exponential and Logarithmic Functions

Objective A

To solve application problems

VIDEO & DVD CD TUTOR WEB SSM

A biologist places one single-celled bacterium in a culture, and each hour that particular species of bacteria divides into two bacteria. After one hour there will be two bacteria. After two hours, each of the two bacteria will divide and there will be four bacteria. After three hours, each of the four bacteria will divide and there will be eight bacteria.

Point of Interest

Parkinson's Law (C. Northcote Parkinson) is sometimes stated as "A job will expand to fill the time allotted for the job." However, Parkinson actually said that in any new government administration, administrative employees will be added at the rate of about 5% to 6% per year. This is an example of exponential growth and means that a staff of 500 will grow to approximately 630 by the end of a 4-year term.

The table at the right shows the number of bacteria in the culture after various intervals of time, t, in hours. Values in this table could also be found by using the exponential equation $N = 2^t$.

Time, t	*Number of Bacteria, N*
0	1
1	2
2	4
3	8
4	16

The equation $N = 2^t$ is an example of an **exponential growth equation**. In general, any equation that can be written in the form $A = A_0 b^{kt}$, where A is the size at time t, A_0 is the initial size, $b > 1$, and k is a positive real number, is an exponential growth equation. These equations are important not only in population growth studies but also in physics, chemistry, psychology, and economics.

Recall that interest is the amount of money paid (or received) when borrowing (or investing) money. **Compound interest** is interest that is computed not only on the original principal, but also on the interest already earned. The compound interest formula is an exponential growth equation.

The **compound interest formula** is $P = A(1 + i)^n$, where A is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and P is the value of the investment after n periods.

An investment broker deposits \$1000 into an account that earns 12% annual interest compounded quarterly. What is the value of the investment after two years?

$$i = \frac{12\%}{4} = \frac{0.12}{4} = 0.03$$

- Find i, the interest rate per quarter. The quarterly rate is the annual rate divided by 4, the number of quarters in one year.

$$n = 4 \cdot 2 = 8$$

- Find n, the number of compounding periods. The investment is compounded quarterly, 4 times a year, for 2 years.

$$P = A(1 + i)^n$$

- Use the compound interest formula.

$$P = 1000(1 + 0.03)^8$$

- Replace A, i, and n by their values.

$$P \approx 1267$$

- Solve for P.

The value of the investment after two years is approximately \$1267.

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Exponential decay offers another example of an exponential equation. One of the most common illustrations of exponential decay is the decay of a radioactive substance. For instance, tritium, a radioactive nucleus of hydrogen that has been used in luminous watch dials, has a half-life of approximately 12 years. This means that one-half of any given amount of tritium will disintegrate in 12 years.

The table at the right indicates the amount of an initial 10-microgram sample of tritium that remains after various intervals of time, t, in years. Values in this table could also be found by using the exponential equation $A = 10(0.5)^{t/12}$.

Time, t	*Amount, A*
0	10
12	5
24	2.5
36	1.25
48	0.625

The equation $A = 10(0.5)^{t/12}$ is an example of an **exponential decay equation**. Comparing this equation to the exponential growth equation, note that for exponential growth, the base of the exponential equation is greater than 1, whereas for exponential decay, the base is between 0 and 1.



A method by which an archaeologist can measure the age of a bone is called carbon dating. Carbon dating is based on a radioactive isotope of carbon called carbon-14, which has a half-life of approximately 5570 years. The exponential decay equation is given by $A = A_0(0.5)^{t/5570}$, where A_0 is the original amount of carbon-14 present in the bone, t is the age of the bone, and A is the amount of carbon-14 present after t years.

➡ A bone that originally contained 100 mg of carbon-14 now has 70 mg of carbon-14. What is the approximate age of the bone?

$$A = A_0(0.5)^{t/5570}$$ • Use the exponential decay equation.

$$70 = 100(0.5)^{t/5570}$$ • Replace A by 70 and A_0 by 100 and solve for t.

$$0.7 = (0.5)^{t/5570}$$ • Divide each side by 100.

$$\log 0.7 = \log (0.5)^{t/5570}$$ • Take the common logarithm of each side of the equation. Then simplify.

$$\log 0.7 = \frac{t}{5570}\log (0.5)$$

$$\frac{5570 \log 0.7}{\log 0.5} = t$$

$$2866 \approx t$$

The bone is approximately 2866 years old.

Point of Interest

The pH scale was created by the Danish biochemist Søren Sørensen in 1909 to measure the acidity of water used in the brewing of beer. pH is an abbreviation for *pondus hydrogenii*, which translates as "potential hydrogen."

A chemist measures the acidity or alkalinity of a solution by measuring the concentration of hydrogen ions, H^+, in the solution using the formula $pH = -\log (H^+)$. A neutral solution such as distilled water has a pH of 7, acids have a pH less than 7, and alkaline solutions (also called basic solutions) have a pH greater than 7.

➡ Find the pH of orange juice that has a hydrogen ion concentration, H^+, of 2.9×10^{-4}. Round to the nearest tenth.

$$pH = -\log (H^+)$$

$$= -\log (2.9 \times 10^{-4})$$ • $H^+ = 2.9 \times 10^{-4}$

$$\approx 3.5376$$

The pH of the orange juice is approximately 3.5.

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Logarithmic functions are used to scale very large or very small numbers into numbers that are easier to comprehend. For instance, the *Richter scale magnitude* of an earthquake uses a logarithmic function to convert the intensity of shock waves I into a number M, which for most earthquakes is in the range of 0 to 10. The intensity I of an earthquake is often given in terms of the constant I_0, where I_0 is the intensity of the smallest earthquake, called a **zero-level earthquake,** that can be measured on a seismograph near the earthquake's epicenter. An earthquake with an intensity I has a Richter scale magnitude of $M = \log\left(\frac{I}{I_0}\right)$, where I_0 is the measure of a zero-level earthquake.

TAKE NOTE
Note that we do not need to know the value of I_0 in order to determine the Richter scale magnitude of the earthquake.

Find the Richter scale magnitude of the 1999 Joshua Tree, California, earthquake, which had an intensity I of $12{,}589{,}254I_0$. Round to the nearest tenth.

$$M = \log\left(\frac{I}{I_0}\right)$$
$$M = \log\left(\frac{12{,}589{,}254I_0}{I_0}\right) \quad \bullet\ I = 12{,}589{,}254I_0$$
$$M = \log 12{,}589{,}254 \quad \bullet\ \text{Divide the numerator and denominator by } I_0.$$
$$M \approx 7.1 \quad \bullet\ \text{Evaluate } \log 12{,}589{,}254.$$

The 1999 Joshua Tree earthquake had a Richter scale magnitude of 7.1.

If you know the Richter scale magnitude of an earthquake, you can determine the intensity of the earthquake.

Find the intensity of the 1999 Taiwan earthquake, which measured 7.6 on the Richter scale. Write the answer in terms of I_0.

Point of Interest
The Richter scale was created by seismologist Charles F. Richter in 1935. Note that a tenfold increase in the intensity level of an earthquake increases the Richter scale magnitude of the earthquake by only 1.

$$7.6 = \log\left(\frac{I}{I_0}\right) \quad \bullet\ \text{Replace } M \text{ in } M = \log\left(\frac{I}{I_0}\right) \text{ by } 7.6.$$
$$10^{7.6} = \frac{I}{I_0} \quad \bullet\ \text{Write in exponential form.}$$
$$10^{7.6}I_0 = I \quad \bullet\ \text{Multiply both sides by } I_0.$$
$$39{,}810{,}717I_0 \approx I \quad \bullet\ \text{Evaluate } 10^{7.6}.$$

The 1999 Taiwan earthquake had an intensity that was approximately 39,810,717 times the intensity of a zero-level earthquake.

The percent of light that will pass through a substance is given by $\log P = -kd$, where P is the percent of light passing through the substance, k is a constant depending on the substance, and d is the thickness of the substance in meters.

For certain parts of the ocean, $k = 0.03$. Using this value, at what depth will the percent of light be 50% of the light at the surface of the ocean? Round to the nearest meter.

$$\log P = -kd$$
$$\log(0.5) = -0.03d \quad \bullet\ \text{Replace } P \text{ by } 0.5\ (50\%) \text{ and } k \text{ by } 0.03.$$
$$\frac{\log(0.5)}{-0.03} = d \quad \bullet\ \text{Solve for } d.$$
$$10.0343 \approx d$$

At a depth of about 10 m, the light will be 50% of the light at the surface.

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Example 1

An investment of \$3000 is placed into an account that earns 12% annual interest compounded monthly. In approximately how many years will the investment be worth twice the original amount?

Strategy

To find the time, solve the compound interest formula for n. Use $P = 6000$, $A = 3000$, and $i = \frac{12\%}{12} = \frac{0.12}{12} = 0.01$.

Solution

$$\begin{aligned} P &= A(1 + i)^n \\ 6000 &= 3000(1 + 0.01)^n \\ 6000 &= 3000(1.01)^n \\ 2 &= (1.01)^n \\ \log 2 &= \log (1.01)^n \\ \log 2 &= n \log 1.01 \\ \frac{\log 2}{\log 1.01} &= n \\ 70 &\approx n \end{aligned}$$

70 months $\div$ 12 $\approx$ 5.8 years

In approximately 6 years, the investment will be worth \$6000.

You Try It 1

Find the hydrogen ion concentration, H^+, of vinegar that has a pH of 2.9. Round to the nearest hundred thousandth.

Your strategy

Your solution

Example 2

The number of words per minute that a student can type will increase with practice and can be approximated by the equation $N = 100[1 - (0.9)^t]$, where N is the number of words typed per minute after t days of instruction. Find the number of words a student will type per minute after 8 days of instruction.

Strategy

To find the number of words per minute, replace t by its given value in the equation and solve for N.

Solution

$$\begin{aligned} N &= 100[1 - (0.9)^t] \\ &= 100[1 - (0.9)^8] \\ &\approx 56.95 \end{aligned}$$

After 8 days of instruction, a student will type approximately 57 words per minute.

You Try It 2

On September 3, 2000, an earthquake measuring 5.2 on the Richter scale struck the Napa Valley, located 50 mi north of San Francisco. Find the intensity of the quake in terms of I_0.

Your strategy

Your solution

Solutions on p. S37

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12.5 Exercises

Objective A

For Exercises 1 to 4, use the compound interest formula $P = A(1 + i)^n$, where A is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and P is the value of the investment after n periods.

1. An investment broker deposits \$1000 into an account that earns 8% annual interest compounded quarterly. What is the value of the investment after 2 years? Round to the nearest dollar.

2. A financial advisor recommends that a client deposit \$2500 into a fund that earns 7.5% annual interest compounded monthly. What will be the value of the investment after 3 years? Round to the nearest cent.

3. To save for college tuition, the parents of a preschooler invest \$5000 in a bond fund that earns 6% annual interest compounded monthly. In approximately how many years will the investment be worth \$15,000?

4. A hospital administrator deposits \$10,000 into an account that earns 9% annual interest compounded monthly. In approximately how many years will the investment be worth \$15,000?

For Exercises 5 to 8, use the exponential decay equation $A = A_0\left(\frac{1}{2}\right)^{t/k}$, where A is the amount of a radioactive material present after time t, k is the half-life of the radioactive substance, and A_0 is the original amount of radioactive substance. Round to the nearest tenth.

5. An isotope of technetium is used to prepare images of internal body organs. This isotope has a half-life of approximately 6 h. A patient is injected with 30 mg of this isotope.
   a. What is the technetium level in the patient after 3 h?
   b. How long (in hours) will it take for the technetium level to reach 20 mg?

6. Iodine-131 is an isotope that is used to study the functioning of the thyroid gland. This isotope has a half-life of approximately 8 days. A patient is given an injection that contains 8 micrograms of iodine-131.
   a. What is the iodine level in the patient after 5 days?
   b. How long (in days) will it take for the iodine level to reach 5 micrograms?

7. A sample of promethium-147 (used in some luminous paints) weighs 25 mg. One year later, the sample weighs 18.95 mg. What is the half-life of promethium-147, in years?

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8. Francium-223 is a very rare radioactive isotope discovered in 1939 by Marguerite Percy. A 3-microgram sample of francium-223 decays to 2.54 micrograms in 5 min. What is the half-life of francium-223, in minutes?

9. Earth's atmospheric pressure changes as you rise above its surface. At an altitude of h kilometers, where $0 < h < 80$, the pressure P, in newtons per square centimeters (N/cm^2), is approximately modeled by the equation $P(h) = 10.13e^{-0.116h}$.

a. What is the approximate pressure at 40 km above Earth's surface?

b. What is the approximate pressure on Earth's surface?

c. Does atmospheric pressure increase or decrease as you rise above Earth's surface?

10. The U.S. Census Bureau provides information about various segments of the population in the United States. The following table gives the number of people, in millions, age 80 and older at the beginning of each decade from 1910 to 2000. An equation that approximately models the data is $y = 0.18808(1.0365)^x$, where $x = 0$ corresponds to 1900 and y is the population, in millions, of people age 80 and over.

Year	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Number of people age 80 and over (in millions)	0.3	0.4	0.5	0.8	1.1	1.6	2.3	2.9	3.9	9.3

a. According to the model, what is the predicted population of this age group in the year 2020? Round to the nearest tenth of a million. (*Hint*: You will need to determine the x-value for the year 2020.)

b. In what year does this model predict that the population of this age group will be 15 million? Round to the nearest year.

For Exercises 11 and 12, use the equation $pH = -\log(H^+)$, where H^+ is the hydrogen ion concentration of a solution. Round to the nearest tenth.

11. Find the pH of milk, which has a hydrogen ion concentration of 3.97×10^{-7}.

12. Find the pH of a baking soda solution for which the hydrogen ion concentration is 3.98×10^{-9}.



For Exercises 13 and 14, use the equation $\log P = -kd$, which gives the relationship between the percent P, as a decimal, of light passing through a substance of thickness d, in meters. Round to the nearest tenth.

13. The value of k for a swimming pool is approximately 0.05. At what depth, in meters, will the percent of light be 75% of the light at the surface of the pool?

14. The constant k for a piece of blue stained glass is 20. What percent of light will pass through a piece of this glass that is 0.005 m thick?

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For Exercises 15 and 16, use the equation $D = 10(\log I + 16)$, where D is the number of decibels of a sound and I is the power of the sound measured in watts. Round to the nearest whole number.

15. Find the number of decibels of normal conversation. The power of the sound of normal conversation is approximately 3.2×10^{-10} watts.

16. The loudest sound made by any animal is made by the blue whale and can be heard from more than 500 mi away. The power of the sound is 630 watts. Find the number of decibels of sound emitted by the blue whale.



17. During the 1980s and 1990s, the average time T that it took to play a major league baseball game increased each year. If the year 1981 is represented by $x = 1$, then the function $T(x) = 149.57 + 7.63 \ln x$ approximates the time T, in minutes, to play a major league baseball game for the years $x = 1$ to $x = 19$. By how many minutes did the average time of a major league baseball game increase from 1981 to 1999? Round to the nearest tenth.



18. In 1962, the cost of a first-class postage stamp was \$.04. In 2002, the cost was \$.37. The increase in cost can be modeled by the equation $C = 0.04e^{0.057t}$, where C is the cost and t is the number of years after 1962. According to this model, in what year did a first-class postage stamp cost \$.22?

19. The intensity I of an X-ray after it has passed through a material that is x centimeters thick is given by $I = I_0 e^{-kx}$, where I_0 is the initial intensity of the X-ray and k is a number that depends on the material. The constant k for copper is 3.2. Find the thickness of copper that is needed so that the intensity of an X-ray after passing through the copper is 25% of the original intensity. Round to the nearest tenth.

20. One model for the time it will take for the world's oil supply to be depleted is given by the equation $T = 14.29 \ln (0.00411r + 1)$, where r is the estimated world oil reserves in billions of barrels and T is the time, in years, before that amount of oil is depleted. Use this equation to determine how many barrels of oil are necessary to meet the world demand for 20 years. Round to the nearest tenth.

For Exercises 21 to 24, use the Richter scale equation $M = \log \dfrac{I}{I_0}$, where M is the magnitude of an earthquake, I is the intensity of the shock waves, and I_0 is the measure of the intensity of a zero-level earthquake.

21. On July 14, 2000, an earthquake struck the Kodiak Island region of Alaska. The earthquake had an intensity of $I = 6{,}309{,}573I_0$. Find the Richter scale magnitude of this earthquake. Round to the nearest tenth.

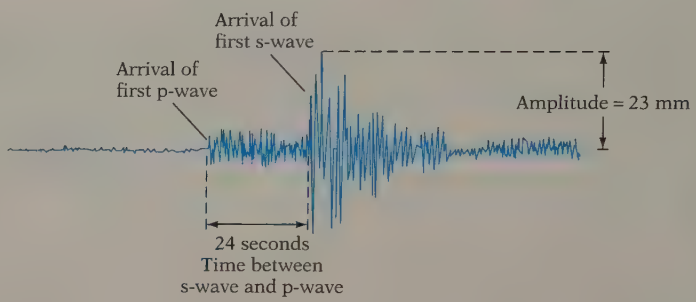
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22. The earthquake that occurred on January 26, 2000, near Gujarat, India, had an intensity of $I = 50{,}118{,}723I_0$. Find the Richter scale magnitude of this earthquake. Round to the nearest tenth.

23. An earthquake that occurred in Japan on March 2, 1933, measured 8.9 on the Richter scale. Find the intensity of this earthquake in terms of I_0. Round to the nearest whole number.

24. An earthquake that occurred in China in 1978 measured 8.2 on the Richter scale. Find the intensity of this earthquake in terms of I_0. Round to the nearest whole number.

Shown at the right is a **seismogram,** which is used to measure the magnitude of an earthquake. The magnitude is determined by the amplitude A of a shock wave and the difference in time t, in seconds, between the occurrences of two types of waves called primary waves and secondary waves. As you can see on the graph, a primary wave is abbreviated p-wave, and a secondary wave is abbreviated s-wave. The amplitude A of a wave is one-half the difference between its highest and lowest points. For this graph, A is 23 mm. The equation is $M = \log A + 3 \log 8t - 2.92$.



25. Determine the magnitude of the earthquake for the seismogram given in the figure. Round to the nearest tenth.

26. Find the magnitude of an earthquake that has a seismogram with an amplitude of 30 mm and for which t is 21 s.

27. Find the magnitude of an earthquake that has a seismogram with an amplitude of 28 mm and for which t is 28 s.

APPLYING THE CONCEPTS

28. The value of an investment in an account that earns an annual interest rate of 10% compounded daily grows according to the equation $A = A_0\left(1 + \frac{0.10}{365}\right)^{365t}$, where A_0 is the original value of an investment and t is the time, in years. Find the time for the investment to double in value. Round to the nearest year.

29. Some banks now use continuous compounding of an amount invested. In this case, the value of an initial investment of A dollars after t years at an annual interest rate of $r\%$ is given by the equation $P = Ae^{rt}$. Using this equation, find the value after 5 years of an investment of \$2500 in an account that earns 5% annual interest.

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Focus on Problem Solving

Proof by Contradiction

The four-step plan for solving problems that we have used before is restated here.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

One of the techniques that can be used in the second step is a method called *proof by contradiction*. In this method you assume that the conditions of the problem you are trying to solve can be met and then show that your assumption leads to a condition you already know is not true.

To illustrate this method, suppose we try to prove that $\sqrt{2}$ is a rational number. We begin by recalling that a rational number is one that can be written as the quotient of integers. Therefore, let a and b be two integers with no common factors. If $\sqrt{2}$ is a rational number, then

$$\sqrt{2} = \frac{a}{b}$$

$$(\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \quad \bullet \text{ Square each side.}$$

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2 \quad \bullet \text{ Multiply each side by } b^2.$$

From the last equation, a^2 is an even number. Because a^2 is an even number, a is an even number. Now divide each side of the last equation by 2.

$$2b^2 = a^2$$

$$2b^2 = a \cdot a$$

$$b^2 = a \cdot x \quad \bullet \ x = \frac{a}{2}$$

Because a is an even number, $a \cdot x$ is an even number. Because $a \cdot x$ is an even number, b^2 is an even number, and this in turn means that b is an even number. This result, however, contradicts the assumption that a and b are two integers with no common factors. Because this assumption is now known not to be true, we conclude that our original assumption, that $\sqrt{2}$ is a rational number, is false. This proves that $\sqrt{2}$ is an irrational number.

Try a proof by contradiction for the following problem: "Is it possible to write numbers using each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 exactly once such that the sum of the numbers is exactly 100?"* Here are some suggestions. First note that the sum of the 10 digits is 45. This means that some of the digits used must be tens digits. Let x be the sum of those digits.

1. What is the sum of the remaining units digits?
2. Express "the sum of the units digits and the tens digits equals 100" as an equation.
3. Solve the equation for x.
4. Explain why this result means that it is impossible to satisfy both conditions of the problem.

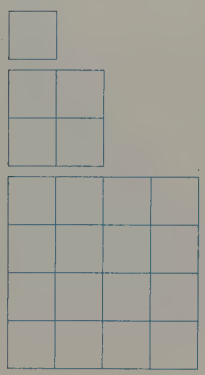
*G. Polya, *How to Solve It: A New Aspect of Mathematical Method*. Copyright © 1957 by Princeton University Press. Reprinted by permission of Princeton University Press.

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Projects and Group Activities

Fractals*

Fractals have a wide variety of applications. They have been used to create special effects for the *Star Wars* and *Star Trek* movies and to explain the behavior of some biological and economic systems. One aspect of fractals that has fascinated mathematicians is that they apparently have *fractional dimension*.



To understand the idea of fractional dimension, one must first understand the terms "scale factor" and "size." Consider a unit square (a square of length 1). By joining four of these squares, we can create another square, the length of which is 2, and the size of which is 4. (Here, size = number of square units.) Four of these larger squares can in turn be put together to make a third square of length 4 and size 16. This process of grouping together four squares can in theory be done an infinite number of times; yet at each step, the following quantities will be the same:

$$\text{Scale factor} = \frac{\text{new length}}{\text{old length}} \qquad \text{Size ratio} = \frac{\text{new size}}{\text{old size}}$$

Consider the unit square as Step 1, the four unit squares as Step 2, etc.

1. Calculate the scale factor going from **a.** Step 1 to Step 2, **b.** Step 2 to Step 3, and **c.** Step 3 to Step 4.
2. Calculate the size ratio going from **a.** Step 1 to Step 2, **b.** Step 2 to Step 3, and **c.** Step 3 to Step 4.
3. What is **a.** the scale factor and **b.** the size ratio going from Step n to Step $n + 1$?

Mathematicians have defined dimension using the formula $d = \dfrac{\log (\text{size ratio})}{\log (\text{scale factor})}$.

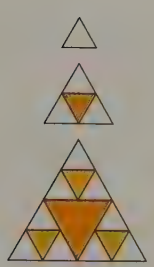
For the squares discussed above, $d = \dfrac{\log (\text{size ratio})}{\log (\text{scale factor})} = \dfrac{\log 4}{\log 2} = 2$.

So by this definition of dimension, squares are two-dimensional figures.

Now consider a unit cube (Step 1). Group eight unit cubes to form a cube that is 2 units on each side (Step 2). Group eight of the cubes from Step 2 to form a cube that is 4 units on each side (Step 3).

4. Calculate **a.** the scale factor and **b.** the size ratio for this process.
5. Show that the cubes are three-dimensional figures.

In each of the above examples, if the process is continued indefinitely, we still have a square or a cube. Consider a process that is more difficult to envision. Let Step 1 be an equilateral triangle whose base has length 1 unit, and let Step 2 be a grouping of three of these equilateral triangles, such that the space between them is another equilateral triangle with a base of length 1 unit. Three shapes from Step 2 are arranged with an equilateral triangle in their center, and so on. It is hard to imagine the result if this is done an infinite number of times, but mathematicians have shown that the result is a single figure of fractional dimension. (Similar processes have been used to create fascinating artistic patterns and to explain scientific phenomena.)



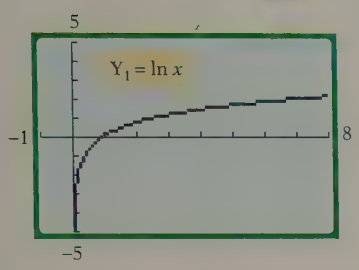
6. Show that for this process **a.** the scale factor is 2 and **b.** the size ratio is 3.
7. Calculate the dimension of the fractal. (Note that it is a *fractional* dimension!)

*Adapted with permission from "Student Math Notes," by Tami Martin, *News Bulletin*, November 1991.

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Solving Exponential and Logarithmic Equations Using a Graphing Calculator

A graphing calculator can be used to draw the graphs of logarithmic functions. Note that there are two logarithmic keys, LOG and LN, on a graphing calculator. The first key gives the values of common logarithms (base 10), and the second gives the values of natural logarithms (base e).



To graph $y = \ln x$, press the Y= key. Clear any equations already entered. Press LN X,T,θ,n) GRAPH. The graph is shown at the left with a viewing window of Xmin = −1, Xmax = 8, Ymin = −5, Ymax = 5.

Some exponential and logarithmic equations cannot be solved algebraically. In these cases, a graphical approach may be appropriate. Here is an example.

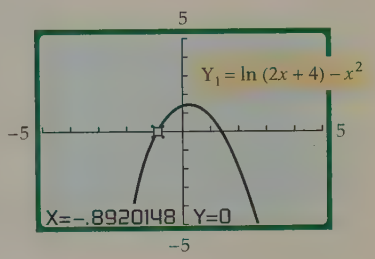
➡ Solve $\ln (2x + 4) = x^2$ for x. Round to the nearest hundredth.

Rewrite the equation by subtracting x^2 from each side of the equation.

$$\ln (2x + 4) = x^2$$
$$\ln (2x + 4) - x^2 = 0$$

The zeros of $f(x) = \ln (2x + 4) - x^2$ are the solutions of $\ln (2x + 4) - x^2 = 0$.

Graph f and use the zero feature of the graphing calculator to estimate the solutions to the nearest hundredth.



The zeros are approximately −0.89 and 1.38.

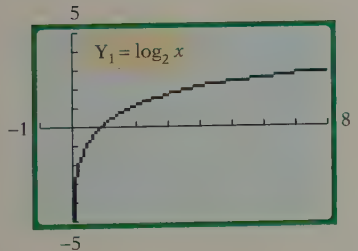
The solutions are −0.89 and 1.38.

Solve for x by graphing. Round to the nearest hundredth.

1. $2^x = 2x + 4$
2. $3^x = -x - 1$
3. $e^x = -2x - 2$
4. $e^x = 3x + 4$
5. $\log (2x - 1) = -x + 3$
6. $\log (x + 4) = -2x + 1$
7. $\ln (x + 2) = x^2 - 3$
8. $\ln x = -x^2 + 1$

To graph a logarithmic function with a base other than base 10 or base e, the Change-of-Base Formula is used. Here is the Change-of-Base Formula, which we discussed earlier.

$$\log_a N = \frac{\log_b N}{\log_b a}$$

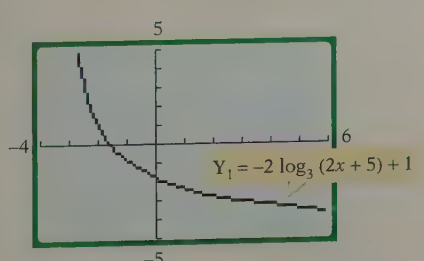


To graph $y = \log_2 x$, first change from base 2 logarithms to the equivalent base 10 logarithms or natural logarithms. Base 10 is shown here.

$$y = \log_2 x = \frac{\log x}{\log 2}$$

• $a = 2, b = 10, N = x$

Then graph the equivalent equation $y = \frac{\log x}{\log 2}$. The graph is shown at the left with a viewing window of Xmin = −1, Xmax = 8, Ymin = −5, Ymax = 5.



The graphs of more complicated functions involving logarithms are produced in much the same way as outlined above. To graph $y = -2 \log_3 (2x + 5) + 1$, enter into the calculator the equivalent equation $y = \frac{-2 \log (2x + 5)}{\log 3} + 1$.

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Chapter Summary

Key Words

A function of the form $f(x) = b^x$, where b is a positive real number not equal to 1, is an *exponential function*. The number b is the *base* of the exponential function. [p. 651]

The function defined by $f(x) = e^x$ is called the *natural exponential function.* [p. 652]

Because the exponential function is a 1–1 function, it has an inverse function that is called a *logarithm.* The defnition of logarithm is as follows: For $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $x = b^y$. [p. 659]

If $\log_b M = N$, then M is the *antilogarithm* base b of N. In exponential form, $M = b^N$. [p. 660]

Logarithms with base 10 are called *common logarithms.* The base, 10, is omitted when the common logarithm of a number is written. The decimal part of a common logarithm is called the *mantissa*; the integer part is called the *characteristic.* [p. 660]

When e (the base of the natural exponential function) is used as the base of a logarithm, the logarithm is referred to as the *natural logarithm* and is abbreviated $\ln x$. [p. 660]

An *exponential equation* is one in which a variable occurs in the exponent. [p. 673]

An *exponential growth equation* is an equation that can be written in the form $A = A_0 b^{kt}$, where A is the size at time t, A_0 is the initial size, $b > 1$, and k is a positive real number. In an *exponential decay equation,* the value of b is between 0 and 1. [pp. 679, 680]

Essential Rules

The One-to-One Property of Exponential Functions [p. 659]
For $b > 0$, $b \neq 1$, if $b^u = b^v$, then $u = v$.

The Logarithm Property of the Product of Two Numbers [p. 661]
For any positive real numbers x, y, and b, $b \neq 1$, $\log_b (xy) = \log_b x + \log_b y$.

The Logarithm Property of the Quotient of Two Numbers [p. 662]
For any positive real numbers x, y, and b, $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

The Logarithm Property of the Power of a Number [p. 662]
For any positive real numbers x and b, $b \neq 1$, and for any real number r, $\log_b x^r = r \log_b x$.

Additional Properties of Logarithms [p. 662]
For any positive real numbers x, y, and b, $b \neq 1$: $\log_b 1 = 0$, $\log_b b^x = x$

Change-of-Base Formula [p. 664]

$$\log_a N = \frac{\log_b N}{\log_b a}$$

To Solve Numerical Problems Using Logarithms [p. 662]
For any positive real numbers x, y, and b, $b \neq 1$, if $\log_b x = \log_b y$, then $x = y$.

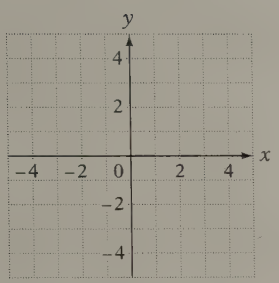
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Chapter Review

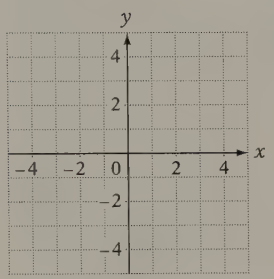
1. Evaluate $f(x) = e^{x-2}$ at $x = 2$.

2. Write $\log_5 25 = 2$ in exponential form.

3. Graph: $f(x) = 3^{-x} + 2$



4. Graph: $f(x) = \log_3 (x - 1)$



5. Write $\log_3 \sqrt[5]{x^2y^4}$ in expanded form.

6. Write $2 \log_3 x - 5 \log_3 y$ as a single logarithm with a coefficient of 1.

7. Solve: $27^{2x+4} = 81^{x-3}$

8. Solve: $\log_5 \frac{7x + 2}{3x} = 1$

9. Find $\log_6 22$. Round to the nearest ten-thousandth.

10. Solve: $\log_2 x = 5$

11. Solve: $\log_3 (x + 2) = 4$

12. Solve: $\log_{10} x = 3$

13. Write $\frac{1}{3}(\log_7 x + 4 \log_7 y)$ as a single logarithm with a coefficient of 1.

14. Write $\log_8 \sqrt{\frac{x^5}{y^3}}$ in expanded form.

15. Write $2^5 = 32$ in logarithmic form.

16. Find $\log_3 1.6$. Round to the nearest ten-thousandth.

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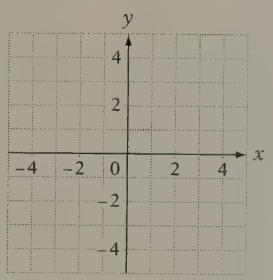
17. Solve $3^{x+2} = 5$ for x. Round to the nearest thousandth.

18. Evaluate $f(x) = \left(\frac{2}{3}\right)^{x+2}$ at $x = -3$.

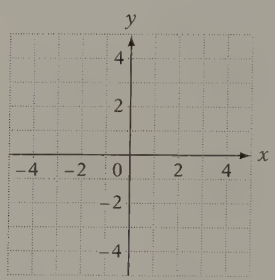
19. Solve: $\log_8 (x + 2) - \log_8 x = \log_8 4$

20. Solve: $\log_6 (2x) = \log_6 2 + \log_6 (3x - 4)$

21. Graph: $f(x) = \left(\frac{2}{3}\right)^{x+1}$



22. Graph: $f(x) = \log_2 (2x - 1)$



23. Use the compound interest formula $P = A(1 + i)^n$, where A is the original value of an investment, i is the interest rate per compounding period, and n is the number of compounding periods, to find the value of an investment after 2 years. The amount of the investment is \$4000, and it is invested at 8% compounded monthly. Round to the nearest dollar.

24. An earthquake off the coast of Central America in January, 2001, had an intensity of $I = 39{,}810{,}717I_0$. Find the Richter scale magnitude of the earthquake. Use the Richter scale equation $M = \log \frac{I}{I_0}$, where M is the magnitude of an earthquake, I is the intensity of the shock waves, and I_0 is the measure of the intensity of a zero-level earthquake. Round to the nearest tenth.

25. Use the exponential decay equation $A = A_0\left(\frac{1}{2}\right)^{t/k}$, where A is the amount of a radioactive material present after time t, k is the half-life, and A_0 is the original amount of radioactive material, to find the half-life of a material that decays from 25 mg to 15 mg in 20 days. Round to the nearest whole number.

26. The number of decibels, D, of a sound can be given by the equation $D = 10(\log I + 16)$, where I is the power of the sound measured in watts. Find the number of decibels of sound emitted from a busy street corner for which the power of the sound is 5×10^{-6} watts.

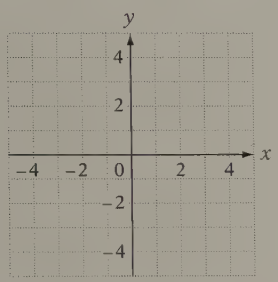
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Chapter Test

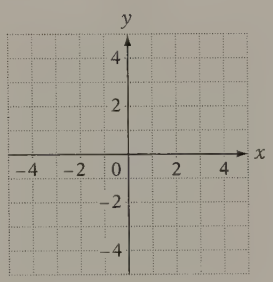
1. Evaluate $f(x) = \left(\frac{2}{3}\right)^x$ at $x = 0$.

2. Evaluate $f(x) = 3^{x+1}$ at $x = -2$.

3. Graph: $f(x) = 2^x - 3$



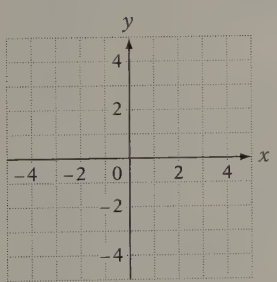
4. Graph: $f(x) = 2^x + 2$



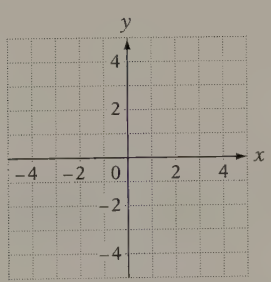
5. Evaluate: $\log_4 16$

6. Solve for x: $\log_3 x = -2$

7. Graph: $f(x) = \log_2 (2x)$



8. Graph: $f(x) = \log_3 (x + 1)$



9. Write $\log_6 \sqrt{xy^3}$ in expanded form.

10. Write $\frac{1}{2}(\log_3 x - \log_3 y)$ as a single logarithm with a coefficient of 1.

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11. Write $\ln\left(\frac{x}{\sqrt{z}}\right)$ in expanded form.

12. Write $3 \ln x - \ln y - \frac{1}{2} \ln z$ as a single logarithm with a coefficient of 1.

13. Solve for x: $3^{7x+1} = 3^{4x-5}$

14. Solve for x: $8^x = 2^{x-6}$

15. Solve for x: $3^x = 17$
Round to the nearest ten-thousandth.

16. Solve for x: $\log x + \log (x - 4) = \log 12$

17. Solve for x: $\log_6 x + \log_6 (x - 1) = 1$

18. Find $\log_5 9$.
Round to the nearest ten-thousandth.

19. Find $\log_3 19$.
Round to the nearest ten-thousandth.

20. Use the exponential decay equation $A = A_0\left(\frac{1}{2}\right)^{t/k}$, where A is the amount of a radioactive material present after time t, k is the half-life of the material, and A_0 is the original amount of radioactive material, to find the half-life of a material that decays from 10 mg to 9 mg in 5 h. Round to the nearest whole number.

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Cumulative Review

1. Solve: $4 - 2[x - 3(2 - 3x) - 4x] = 2x$

2. Find the equation of the line that contains the point $(2, -2)$ and is parallel to the line $2x - y = 5$.

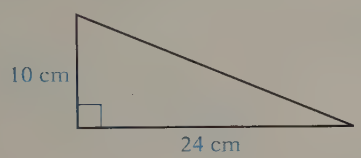
3. Factor: $4x^{2n} + 7x^n + 3$

4. Simplify: $\dfrac{1 - \dfrac{5}{x} + \dfrac{6}{x^2}}{1 + \dfrac{1}{x} - \dfrac{6}{x^2}}$

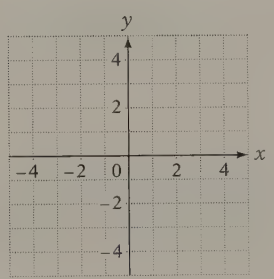
5. Simplify: $\dfrac{\sqrt{xy}}{\sqrt{x} - \sqrt{y}}$

6. Solve by completing the square: $x^2 - 4x - 6 = 0$

7. Find the unknown side of the triangle in the figure below.



8. Graph the solution set: $2x - y < 3$
$x + y < 1$



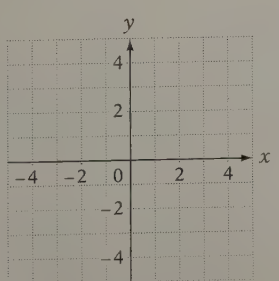
9. Solve by the addition method.
$3x - y + z = 3$
$x + y + 4z = 7$
$3x - 2y + 3z = 8$

10. Subtract: $\dfrac{x - 4}{2 - x} - \dfrac{1 - 6x}{2x^2 - 7x + 6}$

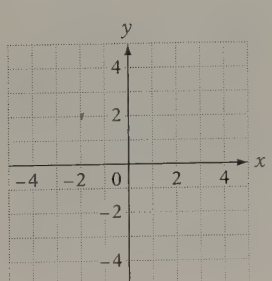
11. Solve: $x^2 + 4x - 5 \le 0$

12. Solve: $|2x - 5| \le 3$

13. Graph: $f(x) = \left(\dfrac{1}{2}\right)^x + 1$



14. Graph: $f(x) = \log_2 x - 1$



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15. New carpet is installed in a room measuring 18 ft by 14 ft. Find the area of the room in square yards. ($9 \text{ ft}^2 = 1 \text{ yd}^2$)

16. Solve for x: $\log_5 x = 3$

17. Write $3 \log_b x - 5 \log_b y$ as a single logarithm with a coefficient of 1.

18. Find $\log_3 7$. Round to the nearest ten-thousandth.

19. Solve for x: $4^{5x-2} = 4^{3x+2}$

20. Solve for x: $\log x + \log (2x + 3) = \log 2$

21. A bank offers two types of checking accounts. One account has a charge of \$5 per month plus 2 cents per check. The second account has a charge of \$2 per month plus 8 cents per check. How many checks can a customer who has the second type of account write if it is to cost the customer less than the first type of checking account?

22. Find the cost per pound of a mixture made from 16 lb of chocolate that costs \$4.00 per pound and 24 lb of chocolate that costs \$2.50 per pound.

23. A plane can fly at a rate of 225 mph in calm air. Traveling with the wind, the plane flew 1000 mi in the same amount of time that it took to fly 800 mi against the wind. Find the rate of the wind.

24. The distance (d) that a spring stretches varies directly as the force (f) used to stretch the spring. If a force of 20 lb stretches a spring 6 in., how far will a force of 34 lb stretch the spring?

25. A carpenter purchased 80 ft of redwood and 140 ft of fir for a total cost of \$67. A second purchase, at the same prices, included 140 ft of redwood and 100 ft of fir for a total cost of \$81. Find the cost of redwood and of fir.

26. The compound interest formula is $P = A(1 + i)^n$, where A is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and P is the value of the investment after n periods. Use the compound interest formula to find the number of years in which an investment of \$5000 will double in value. The investment earns 9% annual interest and is compounded semiannually. Round to the nearest whole number.

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Final Exam

1. Simplify:
$12 - 8[3 - (-2)]^2 \div 5 - 3$

2. Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

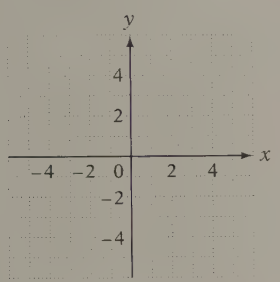
3. Simplify: $5 - 2[3x - 7(2 - x) - 5x]$

4. Solve: $\frac{3}{4}x - 2 = 4$

5. Solve: $8 - |5 - 3x| = 1$

6. Find the volume of a sphere with a diameter of 8 ft. Round to the nearest tenth.

7. Graph $2x - 3y = 9$ using the x- and y-intercepts.



8. Find the equation of the line containing the points $(3, -2)$ and $(1, 4)$.

9. Find the equation of the line that contains the point $(-2, 1)$ and is perpendicular to the line $3x - 2y = 6$.

10. Simplify: $2a[5 - a(2 - 3a) - 2a] + 3a^2$

11. Factor: $8 - x^3y^3$

12. Factor: $x - y - x^3 + x^2y$

13. Divide: $(2x^3 - 7x^2 + 4) \div (2x - 3)$

14. Divide: $\frac{x^2 - 3x}{2x^2 - 3x - 5} \div \frac{4x - 12}{4x^2 - 4}$

15. Subtract: $\frac{x - 2}{x + 2} - \frac{x + 3}{x - 3}$

16. Simplify: $\dfrac{\frac{3}{x} + \frac{1}{x + 4}}{\frac{1}{x} + \frac{3}{x + 4}}$

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17. Solve: $\frac{5}{x-2} - \frac{5}{x^2-4} = \frac{1}{x+2}$

18. Solve $a_n = a_1 + (n-1)d$ for d.

19. Simplify: $\left(\frac{4x^2y^{-1}}{3x^{-1}y}\right)^{-2}\left(\frac{2x^{-1}y^2}{9x^{-2}y^2}\right)^3$

20. Simplify: $\left(\frac{3x^{2/3}y^{1/2}}{6x^2y^{4/3}}\right)^6$

21. Subtract: $x\sqrt{18x^2y^3} - y\sqrt{50x^4y}$

22. Simplify: $\frac{\sqrt{16x^5y^4}}{\sqrt{32xy^7}}$

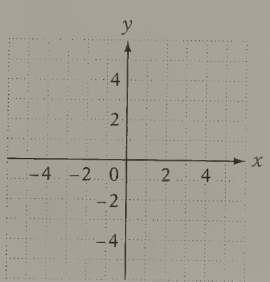
23. Simplify: $\frac{3}{2+i}$

24. Write a quadratic equation that has integer coefficients and has solution $-\frac{1}{2}$ and 2.

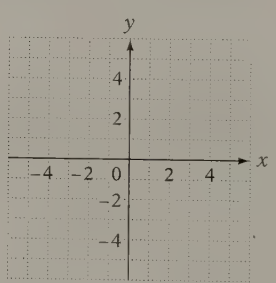
25. Solve by using the quadratic formula:
$2x^2 - 3x - 1 = 0$

26. Solve: $x^{2/3} - x^{1/3} - 6 = 0$

27. Graph: $f(x) = -x^2 + 4$



28. Graph: $\frac{x^2}{16} + \frac{y^2}{4} = 1$



29. Solve: $\frac{2}{x} - \frac{2}{2x+3} = 1$

30. Find the inverse of the function
$f(x) = \frac{2}{3}x - 4$.

31. Solve by the addition method:
$3x - 2y = 1$
$5x - 3y = 3$

32. Evaluate the determinant:

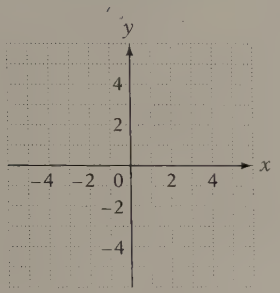
$$\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$$

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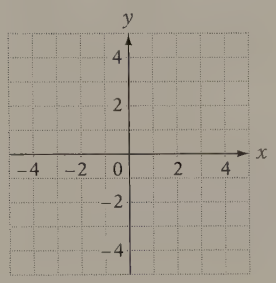
33. Solve: $2 - 3x < 6$ and $2x + 1 > 4$

34. Solve: $|2x + 5| < 3$

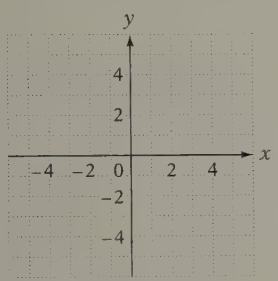
35. Graph the solution set: $3x + 2y > 6$



36. Graph: $f(x) = 3^{-x} - 2$



37. Graph: $f(x) = \log_2 (x + 1)$



38. Write $2(\log_2 a - \log_2 b)$ as a single logarithm with a coefficient of 1.

39. Solve for x: $\log_3 x - \log_3 (x - 3) = \log_3 2$

40. An average score of 70–79 in a history class receives a C grade. A student has grades of 64, 58, 82, and 77 on four history tests. Find the range of scores on the fifth test that will give the student a C grade for the course.

41. A jogger and a cyclist set out at 8 A.M. from the same point headed in the same direction. The average speed of the cyclist is two and a half times the average speed of the jogger. In 2 h, the cyclist is 24 mi ahead of the jogger. How far did the cyclist ride in that time?

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42. You have a total of \$12,000 invested in two simple interest accounts. On one account, a money market fund, the annual simple interest rate is 8.5%. On the other account, a tax-free bond fund, the annual simple interest rate is 6.4%. The total annual interest earned by the two accounts is \$936. How much do you have invested in each account?

43. The length of a rectangle is 1 ft less than three times the width. The area of the rectangle is 140 ft^2. Find the length and width of the rectangle.

44. Three hundred shares of a utility stock earn a yearly dividend of \$486. How many additional shares of the utility stock would give a total dividend income of \$810?

45. An account executive traveled 45 mi by car and then an additional 1050 mi by plane. The rate of the plane was seven times the rate of the car. The total time for the trip was $3\frac{1}{4}$ h. Find the rate of the plane.

46. An object is dropped from the top of a building. Find the distance the object has fallen when the speed reaches 75 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance. Round to the nearest whole number.

47. A small plane made a trip of 660 mi in 5 h. The plane traveled the first 360 mi at a constant rate before increasing its speed by 30 mph. Then it traveled another 300 mi at the increased speed. Find the rate of the plane for the first 360 mi.

48. The intensity (L) of a light source is inversely proportional to the square of the distance (d) from the source. If the intensity is 8 foot-candles at a distance of 20 ft, what is the intensity when the distance is 4 ft?

49. A motorboat traveling with the current can go 30 mi in 2 h. Against the current, it takes 3 h to go the same distance. Find the rate of the motorboat in calm water and the rate of the current.

50. An investor deposits \$4000 into an account that earns 9% annual interest compounded monthly. Use the compound interest formula $P = A(1 + i)^n$, where A is the original value of the investment, i is the interest rate per compounding period, n is the total number of compounding periods, and P is the value of the investment after n periods, to find the value of the investment after 2 years. Round to the nearest cent.

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Chapter R

Review of Introductory Algebra Topics

Objectives

Section R.1

A To evaluate a variable expression
B To simplify a variable expression

Section R.2

A To solve a first-degree equation in one variable
B To solve an inequality in one variable

Section R.3

A To graph points in a rectangular coordinate system
B To graph a linear equation in two variables
C To evaluate a function
D To find the equation of a line

Section R.4

A To multiply and divide monomials
B To add and subtract polynomials
C To multiply polynomials
D To divide polynomials
E To factor polynomials of the form $ax^2 + bx + c$



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R.1 Variable Expressions

Objective A To evaluate a variable expression

Whenever an expression contains more than one operation, the operations must be performed in a specified order, as listed below in the Order of Operations Agreement.

1.3A* Order of Operations Agreement

The Order of Operations Agreement

Step 1 Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, the absolute value symbol | |, and fraction bars.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

Example 1

Evaluate: $-2(7-3)^2+4-2(5-2)$

Solution

$-2(7-3)^2+4-2(5-2)$

$=-2(4)^2+4-2(3)$ • Perform operations inside parentheses.

$=-2(16)+4-2(3)$ • Simplify the exponential expression.

$=-32+4-2(3)$ • Do the multiplication and division from left to right.

$=-32+4-6$

$=-28-6$ • Do the addition and subtraction from left to right.

$=-28+(-6)=-34$

You Try It 1

Evaluate: $(-4)(6-8)^2-(-12\div 4)$

Your solution

Solution on p. S38

1.4A Evaluate variable expressions

A **variable** is a letter that represents a quantity that is unknown or that can change or vary. An expression that contains one or more variables is a **variable expression.** $3x-4y+7z$ is a variable expressions. It contains the variables x, y, and z.

Replacing a variable in a variable expression by a number and then simplifying the resulting numerical expression is called **evaluating the variable expression.** The number substituted for the variable is called the **value of the variable.** The result is called the **value of the variable expression.**

*Review this objective for more detailed coverage of this topic.

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Example 2

Evaluate $5ab^3 + 2a^2b^2 - 4$ when $a = 3$ and $b = -2$.

Solution

$5ab^3 + 2a^2b^2 - 4$

$5(3)(-2)^3 + 2(3)^2(-2)^2 - 4$ • Replace a by 3 and b by 2.

$= 5(3)(-8) + 2(9)(4) - 4$ • Use the Order of Operations Agreement to simplify the numerical expression.

$= -120 + 72 - 4$

$= -48 - 4$

$= -48 + (-4)$

$= -52$

You Try It 2

Evaluate $3xy^2 - 3x^2y$ when $x = -2$ and $y = 5$.

Your solution

Solution on p. S38

To simplify a variable expression

1.4B 1.4C Simplify variable expressions using the Properties of Addition and Multiplication

The Properties of Real Numbers are used to simplify variable expressions.

Example 3

Simplify: $-\frac{1}{3}(-3y)$

Solution

$-\frac{1}{3}(-3y)$

$= \left[-\frac{1}{3}(-3)\right]y$ • Use the Associative Property of Multiplication to regroup factors.

$= 1y$ • Use the Inverse Property of Multiplication.

$= y$ • Use the Multiplication Property of One.

You Try It 3

Simplify: $-5(-3a)$

Your solution

Solution on p. S38

A variable expression is shown at the right. The expression can be rewritten by writing subtraction as addition of the opposite. Note that the expression has four addends. The **terms** of a variable expression are the addends of the expression. The expression has four terms.

$3x^2 - 4xy + 5z - 2$

$3x^2 + (-4xy) + 5z + (-2)$

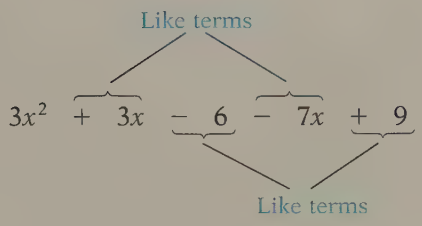
Four terms

$\overbrace{3x^2 - 4xy + 5z}^{\text{Variable terms}} \overbrace{- 2}^{\text{Constant term}}$

The terms $3x^2$, $-4xy$, and $5z$ are **variable terms.** The term -2 is a **constant term,** or simply a **constant.**

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Like terms of a variable expression are terms that have the same variable part. The terms $3x$ and $-7x$ are like terms. Constant terms are also like terms. Thus -6 and 9 are like terms.



By using the Commutative Property of Multiplication, we can rewrite the Distributive Property as $ba + ca = (b + c)a$. This form of the Distributive Property is used to **combine like terms** of a variable expression by adding their coefficients. For instance,

$$7x + 9x = (7 + 9)x \qquad \text{• Use the Distributive Property: } ba + ca = (b + c)a.$$
$$= 16x$$

Example 4

Simplify: $4x^2 + 5x - 6x^2 - 7x$

Solution

$4x^2 + 5x - 6x^2 - 7x$

$= 4x^2 - 6x^2 + 5x - 7x$ • Use the Associative and Commutative Properties of Addition to rearrange and group like terms.

$= (4x^2 - 6x^2) + (5x - 7x)$

$= -2x^2 + (-2x)$ • Use the Distributive Property to combine like terms.

$= -2x^2 - 2x$

You Try It 4

Simplify: $2z^2 - 5z - 3z^2 + 6z$

Your solution

Solution on p. S38

1.4D Simplify variable expressions using the Distributive Property

The Distributive Property also is used to remove parentheses from a variable expression. Here is an example.

$$4(2x + 5z) = 4(2x) + 4(5z) \qquad \text{• Use the Distributive Property: } a(b + c) = ab + ac.$$
$$= (4 \cdot 2)x + (4 \cdot 5)z \qquad \text{• Use the Associative Property of Multiplication to regroup factors.}$$
$$= 8x + 20z \qquad \text{• Multiply } 4 \cdot 2 \text{ and } 4 \cdot 5.$$

The Distributive Property can be extended to expressions containing more than two terms. For instance,

$$4(2x + 3y + 5z) = 4(2x) + 4(3y) + 4(5z)$$
$$= 8x + 12y + 20z$$

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Example 5

Simplify. **a.** $-3(2x + 4)$ **b.** $6(3x - 4y + z)$

Solution

a. $-3(2x + 4) = -3(2x) + (-3)(4)$
$= -6x - 12$

b. $6(3x - 4y + z)$
$= 6(3x) - 6(4y) + 6(z)$
$= 18x - 24y + 6z$

You Try It 5

Simplify. **a.** $-3(5y - 2)$ **b.** $-2(4x + 2y - 6z)$

Your solution

Solution on p. S38

TAKE NOTE

Recall that the Distributive Property states that if a, b, and c are real numbers, then

$$a(b + c) = ab + ac$$

To simplify the expression $5 + 3(4x - 2)$, use the Distributive Property to remove the parentheses.

$5 + 3(4x - 2) = 5 + 3(4x) - 3(2)$ • Use the Distributive Property.
$= 5 + 12x - 6$
$= 12x - 1$ • Add the like terms 5 and -6.

Example 6

Simplify.

a. $3(2x - 4) - 5(3x + 2)$

b. $3a - 2[7a - 2(2a + 1)]$

Solution

a. $3(2x - 4) - 5(3x + 2)$
$= 6x - 12 - 15x - 10$
$= -9x - 22$

b. $3a - 2[7a - 2(2a + 1)]$
$= 3a - 2[7a - 4a - 2]$
$= 3a - 2[3a - 2]$
$= 3a - 6a + 4$
$= -3a + 4$

You Try It 6

Simplify.

a. $7(-3x - 4y) - 3(3x + y)$

b. $2y - 3[5 - 3(3 + 2y)]$

Your solution

Solution on p. S38

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R.1 Exercises

Objective A

Evaluate the variable expression when $a = 2$, $b = 3$, and $c = -4$.

1. $a - 2c$

2. $-3a + 4b$

3. $3b - 3c$

4. $-3c + 4$

5. $16 \div (2c)$

6. $6b \div (-a)$

7. $3b - (a + c)^2$

8. $(a - b)^2 + 2c$

9. $(b - 3a)^2 + bc$

Evaluate the variable expression when $a = -1$, $b = 3$, $c = -2$, and $d = 4$.

10. $\dfrac{b - a}{d}$

11. $\dfrac{d - b}{a}$

12. $\dfrac{2d + b}{-a}$

13. $\dfrac{b - d}{c - a}$

14. $2(b + c) - 2a$

15. $3(b - a) - bc$

16. $\dfrac{-4bc}{2a + c}$

17. $\dfrac{abc}{b - d}$

18. $(d - a)^2 - (b - c)^2$

19. $(-b + d)^2 + (-a + c)^2$

20. $4ab + (2c)^2$

21. $3cd - (4a)^2$

Evaluate the variable expression when $a = 2.7$, $b = -1.6$, and $c = -0.8$.

22. $c^2 - ab$

23. $(a + b)^2 - c$

24. $\dfrac{b^3}{c} - 4a$

Objective B

Simplify each of the following.

25. $x + 7x$

26. $12y + 9y$

27. $8b - 5b$

28. $4y - 11y$

29. $-12a + 17a$

30. $-15xy + 7xy$

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31. $4x + 5x + 2x$

32. $-5x^2 - 10x^2 + x^2$

33. $6x - 2y + 9x$

34. $3x - 7y - 6x + 4x$

35. $5a + 6a - 2a$

36. $2a - 5a + 3a$

37. $12y^2 + 10y^2$

38. $2z^2 - 9z^2$

39. $\frac{3}{4}x - \frac{1}{4}x$

40. $\frac{2}{5}y - \frac{3}{5}y$

41. $-4(5x)$

42. $-2(-8y)$

43. $(6a)(-4)$

44. $-5(7x^2)$

45. $\frac{1}{4}(4x)$

46. $\frac{12x}{5}\left(\frac{5}{12}\right)$

47. $\frac{1}{3}(21x)$

48. $-\frac{5}{8}(24a^2)$

49. $(36y)\left(\frac{1}{12}\right)$

50. $-(z + 4)$

51. $-3(a + 5)$

52. $(4 - 3b)9$

53. $(-2x - 6)8$

54. $3(5x^2 + 2x)$

55. $-5(2y^2 - 1)$

56. $4(x^2 - 3x + 5)$

57. $6(3x^2 - 2xy - y^2)$

58. $5a - (4a + 6)$

59. $3 - (10 + 8y)$

60. $12(y - 2) + 3(7 - 4y)$

61. $-5[2x + 3(5 - x)]$

62. $-3[2x - (x + 7)]$

63. $-5a - 2[2a - 4(a + 7)]$

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R.2 Equations and Inequalities

Objective A To solve a first-degree equation in one variable

An **equation** expresses the equality of two mathematical expressions. Each of the equations below is a **first-degree equation in one variable.** *First degree* means that the variable has an exponent of 1.

$$x + 11 = 14$$
$$3a + 5 = 8a$$
$$2(6y - 1) = 3$$

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation.

3 is a solution of the equation $x + 4 = 7$ because $3 + 4 = 7$.
9 is not a solution of the equation $x + 4 = 7$ because $9 + 4 \neq 7$.

To **solve an equation** means to find a solution of the equation. In solving an equation, the goal is to rewrite the given equation with the variable alone on one side of the equation and a constant term on the other side of the equation; the constant term is the solution of the equation. The following properties of equations are used to rewrite equations in this form.

Properties of Equations

Addition Property of Equations

The same number can be added to each side of an equation without changing the solution of the equation. In symbols, the equation $a = b$ has the same solution as the equation $a + c = b + c$.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same nonzero number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

2.1B 2.1C Solving equations using the Addition and Multiplication Properties of Equations

TAKE NOTE

Subtraction is defined as addition of the opposite.

$a - b = a + (-b)$

The Addition Property of Equations is used to remove a term from one side of the equation by adding the opposite of that term to each side of the equation. Because subtraction is defined in terms of addition, the Addition Property of Equations also makes it possible to subtract the same number from each side of an equation without changing the solution of the equation.

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For example, to solve the equation $t + 9 = -4$, subtract the constant term (9) from each side of the equation.

$$\begin{aligned} t + 9 &= -4 \\ t + 9 - 9 &= -4 - 9 \\ t &= -13 \end{aligned}$$

TAKE NOTE
Division is defined as multiplication by the reciprocal.

$$a \div b = a \cdot \frac{1}{b}$$

Now the variable is alone on one side of the equation and a constant term (−13) is on the other side. The solution is the constant. The solution is −13.

The Multiplication Property of Equations is used to remove a coefficient by multiplying each side of the equation by the reciprocal of the coefficient. Because division is defined in terms of multiplication, each side of an equation can be divided by the same nonzero number without changing the solution of the equation.

TAKE NOTE
When using the Multiplication Property of Equations, multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction. Divide each side of the equation by the coefficient when the coefficient is an integer or decimal.

For example, to solve the equation $-5q = 120$, divide each side of the equation by the coefficient −5.

$$\begin{aligned} -5q &= 120 \\ \frac{-5q}{-5} &= \frac{120}{-5} \\ q &= -24 \end{aligned}$$

Now the variable is alone on one side of the equation and a constant (−24) is on the other side. The solution is the constant. The solution is −24.

2.2B Solve general **2.2C** equations

In solving more complicated first-degree equations in one variable, use the following sequence of steps.

Steps for Solving a First-Degree Equation in One Variable

1. Use the Distributive Property to remove parentheses.
2. Combine any like terms on the right side of the equation and any like terms on the left side of the equation.
3. Use the Addition Property to rewrite the equation with only one variable term.
4. Use the Addition Property to rewrite the equation with only one constant term.
5. Use the Multiplication Property to rewrite the equation with the variable alone on one side of the equation and a constant on the other side of the equation.

If one of these steps is not needed to solve a given equation, proceed to the next step.

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Example 1

Solve.

a. $5x + 9 = 23 - 2x$

b. $8x - 3(4x - 5) = -2x + 6$

Solution

a.
$$\begin{aligned} 5x + 9 &= 23 - 2x \\ 5x + 2x + 9 &= 23 - 2x + 2x && \text{• Step 3} \\ 7x + 9 &= 23 \\ 7x + 9 - 9 &= 23 - 9 && \text{• Step 4} \\ 7x &= 14 \\ \frac{7x}{7} &= \frac{14}{7} && \text{• Step 5} \\ x &= 2 \end{aligned}$$

The solution is 2.

b.
$$\begin{aligned} 8x - 3(4x - 5) &= -2x + 6 \\ 8x - 12x + 15 &= -2x + 6 && \text{• Step 1} \\ -4x + 15 &= -2x + 6 && \text{• Step 2} \\ -4x + 2x + 15 &= -2x + 2x + 6 && \text{• Step 3} \\ -2x + 15 &= 6 \\ -2x + 15 - 15 &= 6 - 15 && \text{• Step 4} \\ -2x &= -9 \\ \frac{-2x}{-2} &= \frac{-9}{-2} && \text{• Step 5} \\ x &= \frac{9}{2} \end{aligned}$$

The solution is $\frac{9}{2}$.

You Try It 1

Solve.

a. $4x + 3 = 7x + 9$

b. $4 - (5x - 8) = 4x + 3$

Your solution

Solution on p. S38

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Objective B **To solve an inequality in one variable**

An **inequality** contains the symbol $>$, $<$, $\geq$, or $\leq$. An inequality expresses the relative order of two mathematical expressions. Here are some examples of inequalities in one variable.

$$\left.\begin{aligned} 4x &\geq 12 \\ 2x + 7 &\leq 9 \\ x^2 + 1 &> 3x \end{aligned}\right\} \text{ Inequalities in one variable}$$

2.4A Solve an inequality in one variable

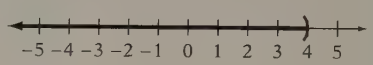
A **solution of an inequality in one variable** is a number that, when substituted for the variable, results in a true inequality. For the inequality $x < 4$ shown below, 3, 0, and -5 are solutions of the inequality because replacing the variable by these numbers results in a true inequality.

$x < 4$	$x < 4$	$x < 4$
$3 < 4$ True	$0 < 4$ True	$-5 < 4$ True

The number 7 is not a solution of the inequality $x < 4$ because $7 < 4$ is a false inequality.

Besides the numbers 3, 0, and -5, there are an infinite number of other solutions of the inequality $x < 4$. Any number less than 4 is a solution; for instance, -5.2, $\frac{5}{2}$, π, and 1 are also solutions of the inequality. The set of all the solutions of an inequality is called the **solution set of the inequality.** The solution set of the inequality $x < 4$ is written in set-builder notation as $\{x|x < 4\}$. This is read, "the set of all x such that x is less than 4."

The graph of the solution set of $x < 4$ is shown at the right.



In solving an inequality, the goal is to rewrite the given inequality in the form

$$\textit{variable} < \textit{constant} \quad \text{or} \quad \textit{variable} > \textit{constant}$$

The Addition Property of Inequalities is used to rewrite an inequality in this form.

TAKE NOTE
The Addition Property of Inequalities states that the same number can be added to each side of an inequality without changing the solution set of the inequality.

The Addition Property of Inequalities

If $a > b$ and c is a real number, then the inequalities $a > b$ and $a + c > b + c$ have the same solution set.

If $a < b$ and c is a real number, then the inequalities $a < b$ and $a + c < b + c$ have the same solution set.

The Addition Property of Inequalities is also true for the symbols $\leq$ and $\geq$.

The Addition Property of Inequalities is used to remove a term from one side of an inequality by adding the additive inverse of that term to each side of the inequality. Because subtraction is defined in terms of addition, the same number can be subtracted from each side of an inequality without changing the solution set of the inequality.

As shown in the example below, the Addition Property of Inequalities applies to variable terms as well as to constants.

➡ Solve $4x - 5 \leq 3x - 2$. Write the solution set in set-builder notation.

$$4x - 5 \leq 3x - 2$$
$$4x - 3x - 5 \leq 3x - 3x - 2 \quad \text{• Subtract } 3x \text{ from each side of the inequality.}$$
$$x - 5 \leq -2 \quad \text{• Simplify.}$$
$$x - 5 + 5 \leq -2 + 5 \quad \text{• Add 5 to each side of the inequality.}$$
$$x \leq 3 \quad \text{• Simplify.}$$

The solution set is $\{x|x \leq 3\}$.

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When multiplying or dividing an inequality by a number, the inequality symbol may be reversed, depending on whether the number is positive or negative. Look at the following two examples.

$3 < 5$
$2(3) < 2(5)$
$6 < 10$

- Multiply by positive 2. The inequality symbol remains the same.
- $6 < 10$ is a true statement.

$3 < 5$
$-2(3) > -2(5)$
$-6 > -10$

- Multiply by negative 2. The inequality symbol is reversed in order to make the inequality a true statement.

This is summarized in the Multiplication Property of Inequalities.

The Multiplication Property of Inequalities

Rule 1

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.

Rule 2

If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Rule 1 states that when each side of an inequality is multiplied by a positive number, the inequality symbol remains the same. Rule 2 states that when each side of an inequality is multiplied by a negative number, the inequality symbol must be reversed.

Here are a few more examples of this property.

Rule 1		**Rule 2**	
$-4 < -2$	$5 > -3$	$3 < 5$	$-2 > -6$
$-4(2) < -2(2)$	$5(3) > -3(3)$	$3(-2) > 5(-2)$	$-2(-3) < -6(-3)$
$-8 < -4$	$15 > -9$	$-6 > -10$	$6 < 18$

Use the Multiplication Property of Inequalities to remove a coefficient other than 1 from one side of an inequality so that the inequality can be rewritten with the variable alone on one side of the inequality and a constant term on the other side. The Multiplication Property of Inequalities is also true for the symbols $\leq$ and $\geq$.

Because division is defined in terms of multiplication, when each side of an inequality is divided by a positive number, the inequality symbol remains the same. When each side of an inequality is divided by a negative number, the inequality symbol must be reversed.

TAKE NOTE
Solving inequalities in one variable is similar to solving equations in one variable *except* that when you multiply or divide by a negative number, you must reverse the inequality symbol.

➡ Solve $-3x < 9$. Write the solution set in set-builder notation.

$-3x < 9$

$\frac{-3x}{-3} > \frac{9}{-3}$

- Divide each side of the inequality by the coefficient -3 and reverse the inequality symbol.

$x > -3$

- Simplify.

$\{x | x > -3\}$

- Write the answer in set-builder notation.

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Example 2

Solve $x + 3 > 4x + 6$. Write the solution set in set-builder notation.

Solution

$$\begin{aligned} x + 3 &> 4x + 6 \\ x - 4x + 3 &> 4x - 4x + 6 \\ -3x + 3 &> 6 \\ -3x + 3 - 3 &> 6 - 3 \\ -3x &> 3 \\ \frac{-3x}{-3} &< \frac{3}{-3} \\ x &< -1 \end{aligned}$$

$\{x | x < -1\}$

You Try It 2

Solve $3x - 1 \leq 5x - 7$. Write the solution set in set-builder notation.

Your solution

Solution on p. S38

When an inequality contains parentheses, often the first step in solving the inequality is to use the Distributive Property to remove the parentheses.

Example 3

Solve $-2(x - 7) > 3 - 4(2x - 3)$. Write the solution set in set-builder notation.

Solution

$$\begin{aligned} -2(x - 7) &> 3 - 4(2x - 3) \\ -2x + 14 &> 3 - 8x + 12 \\ -2x + 14 &> 15 - 8x \\ -2x + 8x + 14 &> 15 - 8x + 8x \\ 6x + 14 &> 15 \\ 6x + 14 - 14 &> 15 - 14 \\ 6x &> 1 \\ \frac{6x}{6} &> \frac{1}{6} \\ x &> \frac{1}{6} \end{aligned}$$

$\left\{x \middle| x > \frac{1}{6}\right\}$

You Try It 3

Solve $3 - 2(3x + 1) < 7 - 2x$. Write the solution set in set-builder notation.

Your solution

Solution on p. S38

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R.2 Exercises

Objective A

Solve.

1. $x + 7 = -5$

2. $9 + b = 21$

3. $-9 = z - 8$

4. $b - 11 = 11$

5. $-48 = 6z$

6. $-9a = -108$

7. $-\frac{3}{4}x = 15$

8. $\frac{5}{2}x = -10$

9. $-\frac{x}{4} = -2$

10. $\frac{2x}{5} = -8$

11. $4 - 2b = 2 - 4b$

12. $4y - 10 = 6 + 2y$

13. $5x - 3 = 9x - 7$

14. $3m + 5 = 2 - 6m$

15. $6a - 1 = 2 + 2a$

16. $5x + 7 = 8x + 5$

17. $2 - 6y = 5 - 7y$

18. $4b + 15 = 3 - 2b$

19. $2(x + 1) + 5x = 23$

20. $9n - 15 = 3(2n - 1)$

21. $7a - (3a - 4) = 12$

22. $5(3 - 2y) = 3 - 4y$

23. $9 - 7x = 4(1 - 3x)$

24. $2(3b + 5) - 1 = 10b + 1$

25. $2z - 2 = 5 - (9 - 6z)$

26. $4a + 3 = 7 - (5 - 8a)$

27. $5(6 - 2x) = 2(5 - 3x)$

28. $4(3y + 1) = 2(y - 8)$

29. $2(3b - 5) = 4(6b - 2)$

30. $3(x - 4) = 1 - (2x - 7)$

Objective B

Solve. Write the answer in set-builder notation.

31. $x - 5 > -2$

32. $5 + n \geq 4$

33. $-2 + n \geq 0$

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34. $x - 3 < 2$

35. $8x \leq -24$

36. $-4x < 8$

37. $3n > 0$

38. $-2n \leq -8$

39. $2x - 1 > 7$

40. $5x - 2 \leq 8$

41. $4 - 3x < 10$

42. $7 - 2x \geq 1$

43. $3x - 1 > 2x + 2$

44. $6x + 4 \leq 8 + 5x$

45. $8x + 1 \geq 2x + 13$

46. $6x + 3 > 4x - 1$

47. $-3 - 4x > -11$

48. $4x - 2 < x - 11$

49. $4x - 2 > 3x + 1$

50. $7x + 5 \leq 9 + 6x$

51. $9x + 2 \geq 3x + 14$

52. $8x + 1 > 6x - 3$

53. $-5 - 2x > -13$

54. $5x - 3 < x - 11$

55. $4(2x - 1) > 3x - 2(3x - 5)$

56. $2 - 5(x + 1) \geq 3(x - 1) - 8$

57. $3(4x + 3) \leq 7 - 4(x - 2)$

58. $3 + 2(x + 5) \geq x + 5(x + 1) + 1$

59. $3 - 4(x + 2) \leq 6 + 4(2x + 1)$

60. $12 - 2(3x - 2) \geq 5x - 2(5 - x)$

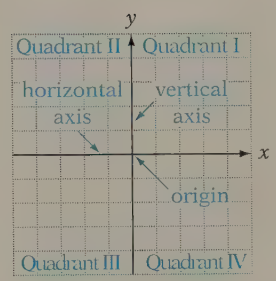
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R.3 Linear Equations in Two Variables

Objective A

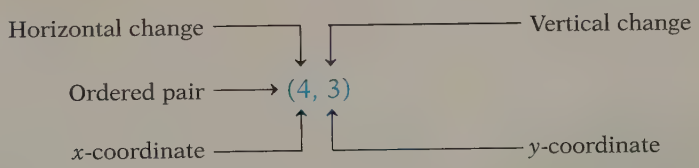
To graph points in a rectangular coordinate system

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical. The point of intersection is called the **origin.** The two axes are called the **coordinate axes** or simply the **axes.** Generally, the horizontal axis is labeled the x-axis, and the vertical axis is labeled the y-axis.

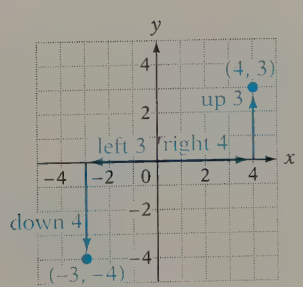


4.1A Points in the plane

Each point in the plane can be identified by a pair of numbers called an **ordered pair.** The first number of the ordered pair measures a horizontal change from the y-axis and is called the **abscissa** or ***x*-coordinate.** The second number of the pair measures a vertical change from the x-axis and is called the **ordinate** or ***y*-coordinate.** The ordered pair (x, y) associated with a point is also called the **coordinates** of the point.



To **graph,** or **plot,** a point means to place a dot at the coordinates of the point. For example, to graph the ordered pair $(4, 3)$, start at the origin. Move 4 units to the right and then 3 units up. Draw a dot. To graph $(-3, -4)$, start at the origin. Move 3 units to the left and then 4 units down. Draw a dot.

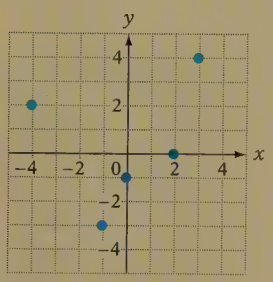


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Example 1

Graph the ordered pairs $(-4, 2)$, $(3, 4)$, $(0, -1)$, $(2, 0)$, and $(-1, -3)$.

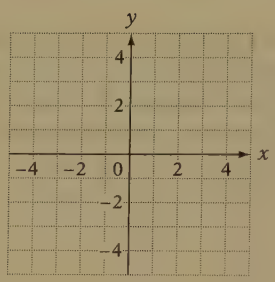
Solution



You Try It 1

Graph the ordered pairs $(-2, 4)$, $(4, 0)$, $(0, 3)$, $(-3, -4)$, and $(5, -1)$.

Your solution



Solution on p. S39

Objective B

To graph a linear equation in two variables

The equations below are examples of equations in two variables.

$$y = 3x - 4$$
$$2x - y = 7$$
$$y = x^2 + 1$$

4.1B Solutions of equations in two variables

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

Is the ordered pair $(-4, 9)$ a solution of the equation $y = -2x + 1$?

$$\begin{array}{r|l} y = -2x + 1 & \\ \hline 9 & -2(-4) + 1 \\ 9 & 8 + 1 \\ 9 = 9 & \end{array}$$

- Replace x by -4 and y by 9.
- Simplify the right side.
- Compare the results. If the resulting equation is true, the ordered pair is a solution of the equation. If it is not true, the ordered pair is not a solution of the equation.

Yes, the ordered pair $(-4, 9)$ is a solution of the equation $y = -2x + 1$.

Besides $(-4, 9)$, there are many other ordered pairs that are solutions of the equation $y = -2x + 1$. For example, $(0, 1)$, $(3, -5)$, and $\left(-\frac{3}{2}, 4\right)$ are also solutions. In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

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Find the ordered-pair solution of $y = \frac{2}{5}x - 4$ that corresponds to $x = 5$.

$$y = \frac{2}{5}x - 4$$

$$y = \frac{2}{5}(5) - 4$$ • We are given that $x = 5$. Replace x by 5 in the equation.

$$y = 2 - 4$$ • Simplify the right side.

$$y = -2$$ • When $x = 5$, $y = -2$.

The ordered-pair solution of $y = \frac{2}{5}x - 4$ in which $x = 5$ is $(5, -2)$.

4.3A Graph an equation of the form $y = mx + b$

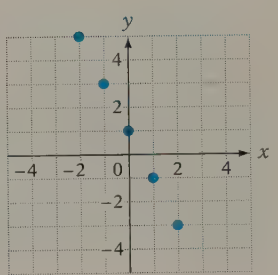
Solutions of an equation in two variables can be graphed in the rectangular coordinate system.

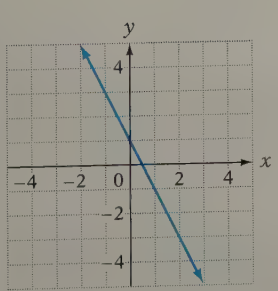
Graph the ordered-pair solutions of $y = -2x + 1$ for $x = -2, -1, 0, 1$, and 2.

x	$y = -2x + 1$	y	(x, y)
-2	$-2(-2) + 1$	5	$(-2, 5)$
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$

• Use the given values of x to determine ordered-pair solutions of the equation. It is convenient to record these in a table.

The ordered-pair solutions of $y = -2x + 1$ for $x = -2, -1, 0, 1$, and 2 are $(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$, and $(2, -3)$. These are graphed at the right.





The **graph of an equation in two variables** is a graph of the ordered-pair solutions of the equation. Consider the equation $y = -2x + 1$ above. The ordered-pair solutions $(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$, and $(2, -3)$ are graphed in the figure above. We can choose values of x that are not integers to produce more ordered pairs to graph, such as $\left(\frac{5}{2}, -4\right)$ and $\left(-\frac{1}{2}, 2\right)$. Choosing still other values of x would result in more and more ordered pairs being graphed. The result would be so many dots that the graph would appear as a straight line, as shown at the left. This is the graph of $y = -2x + 1$.

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The equation $y = -2x + 1$ is an example of a linear equation because its graph is a straight line. It is also called a first-degree equation in two variables because the exponent on each variable is 1.

TAKE NOTE

The equation $y = x^2 + 2x - 4$ is not a linear equation in two variables because there is a term with a variable squared. The equation $y = \frac{5}{x - 3}$ is not a linear equation because a variable occurs in the denominator of a fraction.

Linear Equation in Two Variables

An equation of the form $y = mx + b$, where m is the coefficient of x and b is a constant, is a linear equation in two variables. The graph of a linear equation in two variables is a straight line.

Examples of linear equations are shown at the right.

$y = 5x + 3 \quad (m = 5, b = 3)$

$y = x - 4 \quad (m = 1, b = -4)$

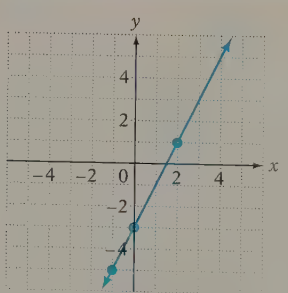
$y = -\frac{3}{4}x \quad \left(m = -\frac{3}{4}, b = 0\right)$

To graph a linear equation, find ordered-pair solutions of the equation. Do this by choosing any value of x and finding the corresponding value of y. Repeat this procedure, choosing different values for x, until you have found the number of solutions desired. Because the graph of a linear equation in two variables is a straight line, and a straight line is determined by two points, it is necessary to find only two solutions. However, it is recommended that at least three points be used to ensure accuracy.

Graph: $y = 2x - 3$

x	$y = 2x - 3$	y
0	$2(0) - 3$	-3
2	$2(2) - 3$	1
-1	$2(-1) - 3$	-5

- Choose any values of x. Then find the corresponding values of y. The numbers 0, 2, and -1 were chosen arbitrarily for x.



- Graph the ordered-pair solutions (0, −3), (2, 1), and (−1, −5). Draw a straight line through the points.

Remember that a graph is a drawing of the ordered-pair solutions of the equation. Therefore, every point on the graph is a solution of the equation, and every solution of the equation is a point on the graph.

When graphing an equation of the form $y = mx + b$, if m is a fraction, choose values of x that will simplify the evaluation. This is illustrated in Example 2. Note that the values of x chosen are multiples of the denominator, 2.

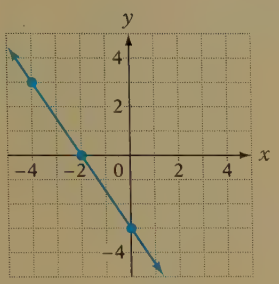
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Example 2

Graph $y = -\frac{3}{2}x - 3$.

Solution

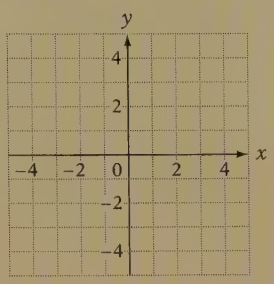
x	y
0	−3
−2	0
−4	3



You Try It 2

Graph $y = \frac{3}{5}x - 4$.

Your solution



Solution on p. S39

4.3B Graph an equation of the form $Ax + By = C$

An equation of the form $Ax + By = C$ is also a linear equation in two variables. Examples of these equations are shown below.

$3x + 4y = 12$ $\quad (A = 3, B = 4, C = 12)$
$x - 5y = -10$ $\quad (A = 1, B = -5, C = -10)$
$2x - y = 0$ $\quad (A = 2, B = -1, C = 0)$

One method of graphing an equation of the form $Ax + By = C$ involves first solving the equation for y and then following the same procedure used for graphing an equation of the form $y = mx + b$. To solve the equation for y means to rewrite the equation so that y is alone on one side of the equation and the term containing x and the constant are on the other side of the equation. The Addition and Multiplication Properties of Equations are used to rewrite an equation of the form $Ax + By = C$ in the form $y = mx + b$.

Graph $3x + 2y = 6$.

$3x + 2y = 6$
$2y = -3x + 6$

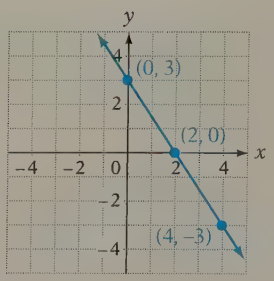
- The equation is in the form $Ax + By = C$.
- Solve the equation for y. Subtract $3x$ from each side of the equation.

$y = -\frac{3}{2}x + 3$

- Divide each side of the equation by 2. Note that each term on the right side is divided by 2.
- Find at least three solutions.

x	y
0	3
2	0
4	−3

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- **Graph the ordered pairs (0, 3), (2, 0), and (4, −3). Draw a straight line through the points.**

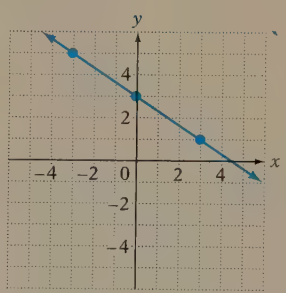
Example 3

Graph $2x + 3y = 9$.

Solution

$$2x + 3y = 9$$
$$3y = -2x + 9$$
$$y = -\frac{2}{3}x + 3$$

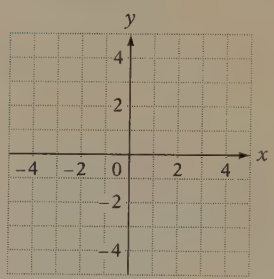
x	y
-3	5
0	3
3	1



You Try It 3

Graph $-3x + 2y = 4$.

Your solution



Solution on p. S39

4.4A Slope

The **slope** of a line is a measure of the slant of the line. The symbol for slope is m.

For an equation of the form $y = mx + b$, m is the slope. Here are a few examples:

The slope of the line $y = -2x + 5$ is -2.
The slope of the line $y = 8x$ is 8.
The slope of the line $y = \frac{3}{4}x - 1$ is $\frac{3}{4}$.

The slope of a line containing two points is the ratio of the change in the y values between the two points to the change in the x values.

TAKE NOTE

Slope $= m = \dfrac{\text{change in } y}{\text{change in } x}$

Slope Formula

The slope of the line containing the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

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Example 4

Find the slope of the line containing the points $(-4, -3)$ and $(-1, 1)$.

Solution

Let $(x_1, y_1) = (-4, -3)$ and $(x_2, y_2) = (-1, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-3)}{-1 - (-4)} = \frac{4}{3}$$

The slope is $\frac{4}{3}$.

You Try It 4

Find the slope of the line containing the points $(-2, 3)$ and $(1, -3)$.

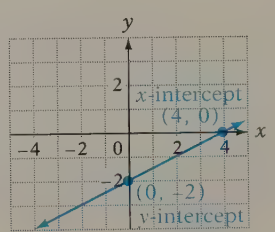
Your solution

Solution on p. S39

4.4B Slope-intercept form of a straight line

One important characteristic of the graph of a linear equation is its *intercepts*. An ***x*-intercept** is a point at which the graph crosses the x-axis. A ***y*-intercept** is a point at which the graph crosses the y-axis.

The graph of the equation $y = \frac{1}{2}x - 2$ is shown at the right. The x-intercept of the graph is $(4, 0)$. The y-intercept of the graph is $(0, -2)$.



Note that at an x-intercept, the y-coordinate is 0. At a y-intercept, the x-coordinate is 0.

We can find the y-intercept of a linear equation by letting $x = 0$.

➡ Find the y-intercept of the graph of the equation $y = 3x + 4$.

$y = 3x + 4$

$y = 3(0) + 4$ • To find the *y*-intercept, let $x = 0$.

$y = 0 + 4$

$y = 4$

The y-intercept is $(0, 4)$.

Note that the constant term of $y = mx + b$, the y-intercept is $(0, b)$.

In general, **for any equation of the form $y = mx + b$, the y-intercept is $(0, b)$.**

Because the slope and the y-intercept can be determined directly from the equation $y = mx + b$, this equation is called the slope-intercept form of a straight line.

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Slope-Intercept Form of a Straight Line

The equation $y = mx + b$ is called the **slope-intercept form of a straight line.** The slope of the line is m, the coefficient of x. The y-intercept is $(0, b)$.

The following equations are written in slope-intercept form.

$y = -4x + 3$ Slope $= -4$, y-intercept $= (0, 3)$

$y = \frac{2}{5}x - 1$ Slope $= \frac{2}{5}$, y-intercept $= (0, -1)$

$y = -x$ Slope $= -1$, y-intercept $= (0, 0)$

When an equation is in slope-intercept form, it is possible to quickly draw a graph of the function.

Graph $x + 2y = 4$ by using the slope and y-intercept.

Solve the equation for y.

$$x + 2y = 4$$
$$2y = -x + 4$$
$$y = -\frac{1}{2}x + 2$$

From the equation $y = -\frac{1}{2}x + 2$, the slope is $-\frac{1}{2}$ and the y-intercept is $(0, 2)$.

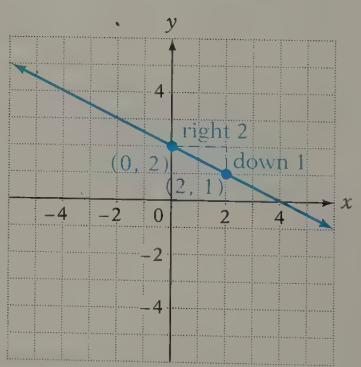
Rewrite the slope $-\frac{1}{2}$ as $\frac{-1}{2}$.

TAKE NOTE

Recall that slope $= m = \frac{\text{change in } y}{\text{change in } x}$. For the example at the right, $m = -\frac{1}{2} = \frac{-1}{2} = \frac{\text{change in } y}{\text{change in } x}$. Therefore, the change in y is -1 and the change in x is 2.

Beginning at the y-intercept, move right 2 units (change in x) and then down 1 unit (change in y).

The point whose coordinates are $(2, 1)$ is a second point on the graph. Draw a straight line through the points $(0, 2)$ and $(2, 1)$.



Example 5

Graph $y = -\frac{3}{2}x + 4$ by using the slope and the y-intercept.

You Try It 5

Graph $y = -\frac{2}{3}x + 2$ by using the slope and the y-intercept.

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Solution

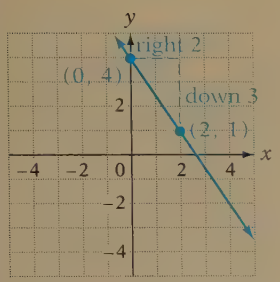
From the equation, the slope is $-\frac{3}{2}$ and the y-intercept is $(0, 4)$.

Rewrite the slope $-\frac{3}{2}$ as $\frac{-3}{2}$.

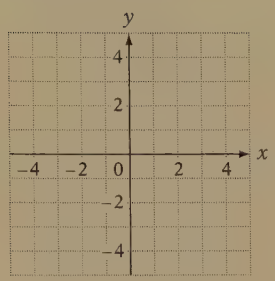
Place a dot at the y-intercept.

Starting at the y-intercept, move right 2 units (the change in x) and down 3 units (the change in y). Place a dot at that location.

Draw a line through the two points.



Your solution



Solution on p. S39

Objective C

To evaluate a function

4.2A Evaluate a function

A **relation** is a set of ordered pairs. A **function** is a relation in which no two ordered pairs have the same first coordinate and different second coordinates.

The relation $\{(0, 4), (1, 5), (1, 6), (2, 7)\}$ is not a function, because the ordered pairs $(1, 5)$, and $(1, 6)$ have the same first coordinate and different second coordinates.

The relation $\{(0, 4), (1, 5), (2, 6), (3, 7)\}$ is a function.

The phrase "y is a function of x," or a similar phrase with different variables, is used to describe those equations in two variables that define functions. To emphasize that the equation represents a function, functional notation is used. For example, the square function is written in functional notation as follows:

$$f(x) = x^2$$

TAKE NOTE

The symbol $f(x)$ is read "the value of f at x" or "f of x."

The process of determining $f(x)$ for a given value of x is called **evaluating the function.** For instance, to evaluate $f(x) = x^2$ when $x = 3$, replace x by 3 and simplify.

$$f(x) = x^2$$
$$f(3) = 3^2 = 9$$

The value of the function is 9 when $x = 3$. An ordered pair of the function is $(3, 9)$.

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Example 6

Evaluate $f(x) = 2x - 4$ when $x = 3$. Use your answer to write an ordered pair of the function.

Solution

$f(x) = 2x - 4$
$f(3) = 2(3) - 4$
$f(3) = 6 - 4$
$f(3) = 2$

An ordered pair of the function is (3, 2).

You Try It 6

Evaluate the function $f(x) = 4 - 2x$ at $x = -3$. Use your answer to write an ordered pair of the function.

Your solution

Solution on p. S39

To find the equation of a line

4.5A Find the equation of a line given a point and the slope

When the slope of a line and a point on the line are known, the equation of the line can be determined by using the point-slope formula.

Point-Slope Formula

Let m be the slope of a line, and let (x_1, y_1) be the coordinates of a point on the line. The equation of the line can be found using the point-slope formula:

$$y - y_1 = m(x - x_1)$$

Example 7

Find the equation of the line that contains the point (1, −3) and has slope −2.

Solution

$y - y_1 = m(x - x_1)$
$y - (-3) = -2(x - 1)$
$y + 3 = -2x + 2$
$y = -2x - 1$

The equation of the line is $y = -2x - 1$.

You Try It 7

Find the equation of the line that contains the point $(-2, 2)$ and has slope $-\frac{1}{2}$.

Your solution

Solution on p. S39

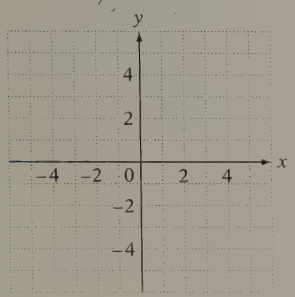
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R.3 Exercises

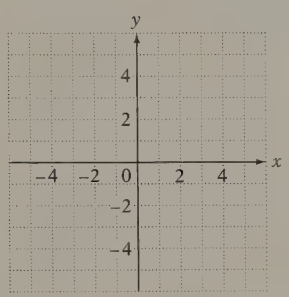
Objective A

Graph the ordered pairs.

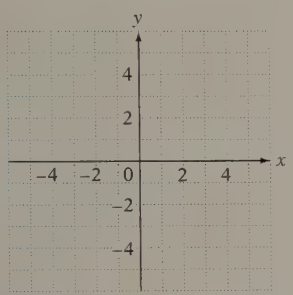
1. (2, 3), (4, 0), (−4, 1), (−2, −2)



2. (0, 2), (−4, −1), (2, 0), (1, −3)



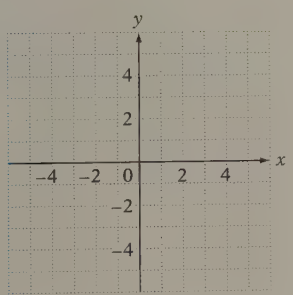
3. (−2, 5), (3, 4), (0, 0), (−3, −2)



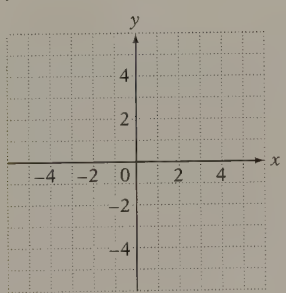
Objective B

Graph.

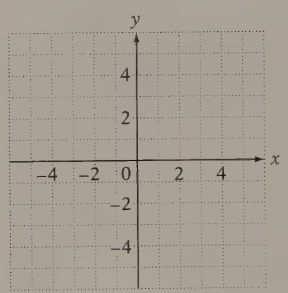
4. $y = 2x - 4$



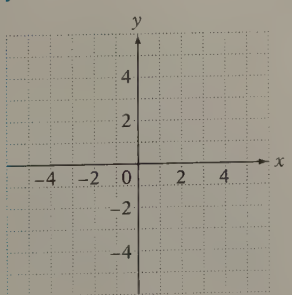
5. $y = 2x + 1$



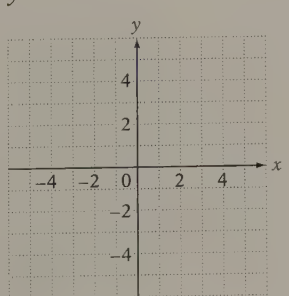
6. $y = -4x + 1$



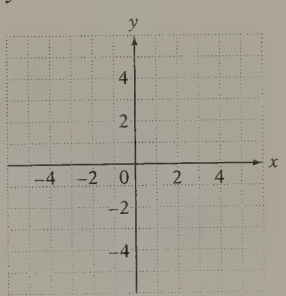
7. $y = -3x + 4$



8. $y = 2x$

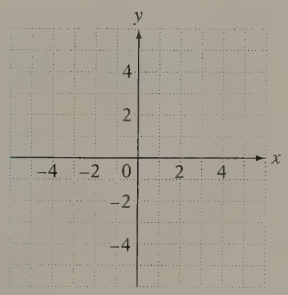


9. $y = 3x$

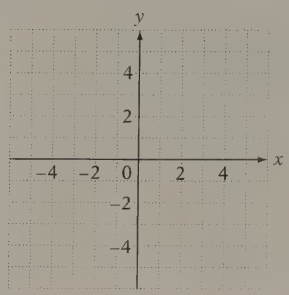


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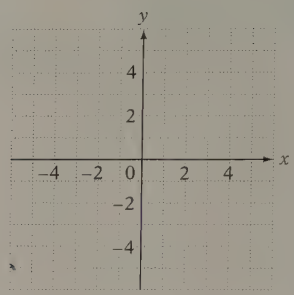
10. $y = -\frac{5}{2}x$



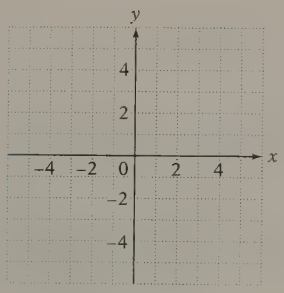
11. $y = -\frac{4}{3}x$



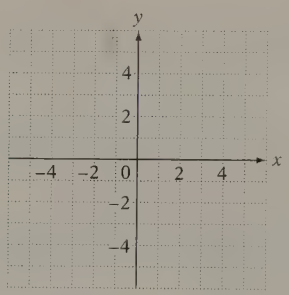
12. $y = \frac{2}{3}x + 1$



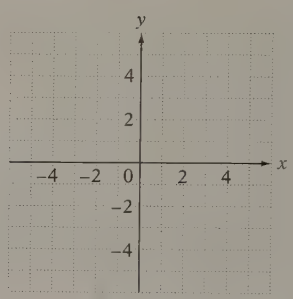
13. $y = \frac{3}{2}x - 1$



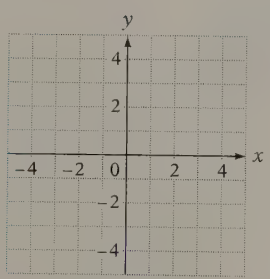
14. $y = -\frac{1}{2}x + 3$



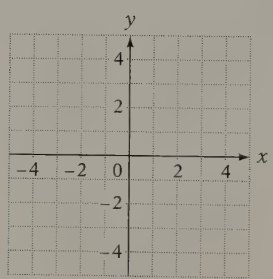
15. $y = -\frac{2}{3}x + 1$



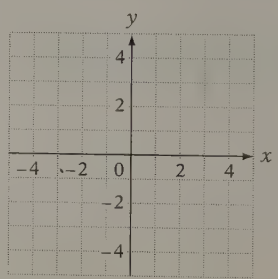
16. $2x - y = 3$



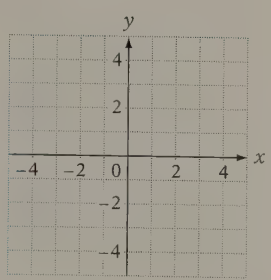
17. $2x + y = -3$



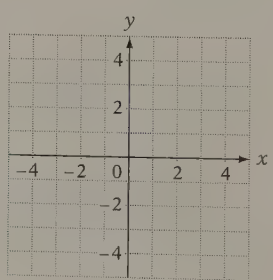
18. $2x + 5y = 10$



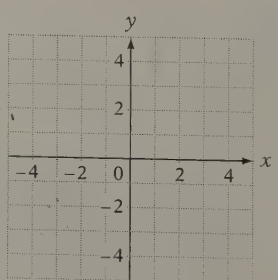
19. $x - 4y = 8$



20. $3x - y = -2$



21. $3x - 2y = 8$



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Find the slope of the line containing the given points.

22. $P_1(4, 2), P_2(3, 4)$

23. $P_1(2, 1), P_2(3, 4)$

24. $P_1(-1, 3), P_2(2, 4)$

25. $P_1(-2, 1), P_2(2, 2)$

26. $P_1(2, 4), P_2(4, -1)$

27. $P_1(1, 3), P_2(5, -3)$

28. $P_1(3, -4), P_2(3, 5)$

29. $P_1(-1, 2), P_2(-1, 3)$

30. $P_1(4, -2), P_2(3, -2)$

31. $P_1(5, 1), P_2(-2, 1)$

32. $P_1(0, -1), P_2(3, -2)$

33. $P_1(3, 0), P_2(2, -1)$

Give the slope and the y-intercept of the graph of the equation.

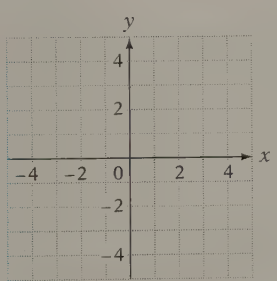
34. $y = -3x + 7$

35. $y = \frac{5}{2}x - 4$

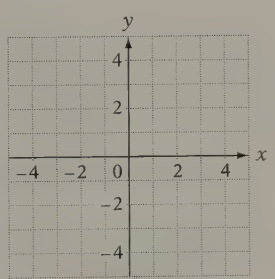
36. $y = x$

Graph by using the slope and the y-intercept.

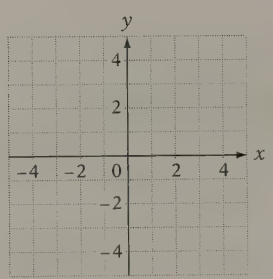
37. $y = \frac{1}{2}x + 2$



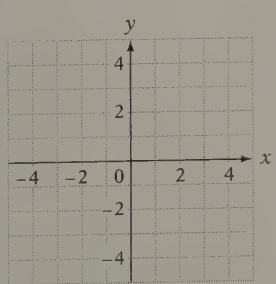
38. $y = \frac{2}{3}x - 3$



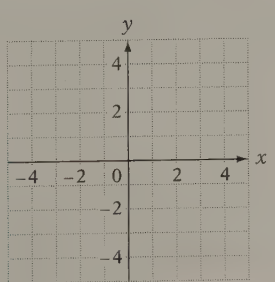
39. $y = -\frac{3}{2}x$



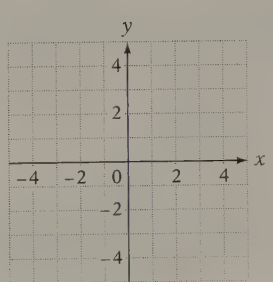
40. $y = \frac{3}{4}x$



41. $y = -\frac{1}{2}x + 2$

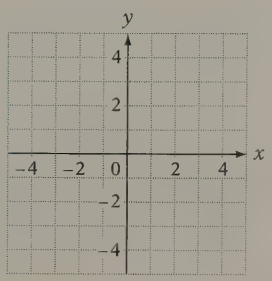


42. $y = \frac{2}{3}x - 1$

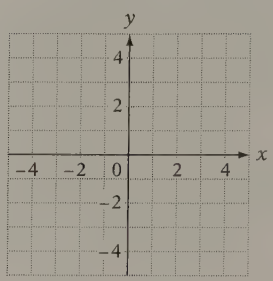


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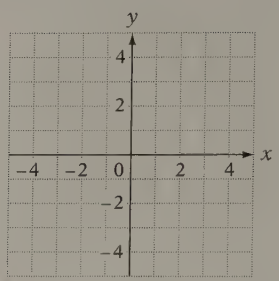
43. $x - 3y = 3$



44. $3x + 2y = 8$



45. $4x + y = 2$



Objective c

Evaluate the function for the given value of the variable. Use your answer to write an ordered pair of the function.

46. $f(x) = 2x + 7; x = -2$

47. $g(x) = -3x + 1; x = -4$

48. $h(x) = 4x - 2; x = 3$

49. $p(x) = 6 - 8x; x = -1$

50. $t(x) = 5 - 7x; x = 0$

51. $f(t) = t^2 - t - 3; t = 2$

52. $p(n) = n^2 - 4n - 7; n = -3$

53. $h(x) = -3x^2 + x - 1; x = -2$

54. $g(t) = 4t^3 - 2t; t = -1$

Objective b

Find the equation of the line that contains the given point and has the given slope.

55. Point $(-1, 2)$; $m = -3$

56. Point $(2, -3)$; $m = 3$

57. Point $(4, -5)$; $m = -2$

58. Point $(-3, 5)$; $m = 3$

59. Point $(5, -3)$; $m = -\frac{3}{5}$

60. Point $(3, 1)$; $m = \frac{1}{3}$

61. Point $(-3, -2)$; $m = -\frac{2}{3}$

62. Point $(4, -2)$; $m = \frac{3}{4}$

63. Point $(2, 3)$; $m = -\frac{1}{2}$

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R.4 Polynomials

Objective A To multiply and divide monomials

6.1A Multiply and divide monomials
6.1B

A **monomial** is a number, a variable, or a product of a number and variables. The following rules and definitions are used to multiply and divide monomials and to write monomials in simplest form.

Rule for Multiplying Exponential Expressions

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

Rule for Simplifying Powers of Exponential Expressions

If m and n are integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m, n, and p are integers, then $(x^m y^n)^p = x^{mp} y^{np}$.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

Rule for Simplifying Powers of Quotients

If m, n, and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is undefined.

Definition of Negative Exponents

If n is a positive integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

➨ Simplify: $(3x^4)^2(4x^3)$

$$\begin{aligned}(3x^4)^2(4x^3) &= (3^{1\cdot 2}x^{4\cdot 2})(4x^3) \\ &= (3^2x^8)(4x^3) \\ &= (9x^8)(4x^3) \\ &= (9\cdot 4)(x^8\cdot x^3) \\ &= 36x^{8+3} \\ &= 36x^{11}\end{aligned}$$

- Use the Rule for Simplifying Powers of Products to simplify $(3x^4)^2$.
- Use the Rule for Multiplying Exponential Expressions.

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An exponential expression is in simplest form when there are no negative exponents in the expression. For example, the expression y^{-7} is not in simplest form; use the Definition of Negative Exponents to rewrite the expression with a positive exponent: $y^{-7} = \frac{1}{y^7}$. The expression $\frac{1}{c^{-4}}$ is not in simplest form; use the Definition of Negative Exponents to rewrite the expression with a positive exponent: $\frac{1}{c^{-4}} = c^4$.

Simplify: $\frac{6x^2}{8x^9}$

$$\frac{6x^2}{8x^9} = \frac{3x^2}{4x^9} = \frac{3x^{2-9}}{4}$$

$$= \frac{3x^{-7}}{4} = \frac{3}{4} \cdot \frac{x^{-7}}{1} = \frac{3}{4} \cdot \frac{1}{x^7}$$

$$= \frac{3}{4x^7}$$

- Divide the coefficients by their common factors. Use the Rule for Dividing Exponential Expressions.
- Rewrite the expression with only positive exponents.

Simplify: $\left(\frac{a^4}{b^3}\right)^{-2}$

$$\left(\frac{a^4}{b^3}\right)^{-2} = \frac{a^{4(-2)}}{b^{3(-2)}} = \frac{a^{-8}}{b^{-6}}$$

$$= \frac{b^6}{a^8}$$

- Use the Rule for Simplifying Powers of Quotients.
- Rewrite the expression with positive exponents.

Example 1

Simplify.

a. $(-2x)(3x^{-2})^{-3}$

b. $\left(\frac{3a^2b^{-1}}{27a^{-3}b^{-4}}\right)^{-2}$

Solution

a. $(-2x)(3x^{-2})^{-3}$

$$= (-2x)(3^{-3}x^6)$$

- Use the Rule for Simplifying Powers of Products.

$$= \frac{-2x \cdot x^6}{3^3}$$

- Write the expression with positive exponents.

$$= -\frac{2x^7}{27}$$

- Use the Rule for Multiplying Exponential Expressions. Simplify 3^3.

You Try It 1

Simplify.

a. $(-2ab)(2a^3b^{-2})^{-3}$

b. $\left(\frac{2x^2y^{-4}}{4x^{-2}y^{-5}}\right)^{-3}$

Your solution

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b. Use the Rule for Simplifying Powers of Quotients. Then simplify the expression and write it with positive exponents.

$$\left(\frac{3a^2b^{-1}}{27a^{-3}b^{-4}}\right)^{-2}$$

$$= \left(\frac{a^2b^{-1}}{9a^{-3}b^{-4}}\right)^{-2}$$

$$= \frac{a^{2(-2)}b^{(-1)(-2)}}{9^{1(-2)}a^{(-3)(-2)}b^{(-4)(-2)}}$$

$$= \frac{a^{-4}b^2}{9^{-2}a^6b^8}$$

$$= 9^2a^{-4-6}b^{2-8}$$

$$= 81a^{-10}b^{-6} = \frac{81}{a^{10}b^6}$$

Solution on p. S39

Objective B To add and subtract polynomials

A **polynomial** is a variable expression in which the terms are monomials. The polynomial $15t^2 - 2t + 3$ has three terms: $15t^2$, $-2t$, and 3. Note that each of these three terms is a monomial.

A polynomial of *one* term is a **monomial.** $-7x^2$ is a monomial.
A polynomial of *two* terms is a **binomial.** $4y + 3$ is a binomial.
A polynomial of *three* terms is a **trinomial.** $6b^2 + 5b - 8$ is a trinomial.

6.2B Add polynomials

Polynomials can be added by combining like terms. This is illustrated in Example 2 below.

Example 2
Add: $(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$.

Solution
$(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$
$= (8x^2 + 2x^2) + (-4x + 9x) + (-9 - 9)$
$= 10x^2 + 5x - 18$

You Try It 2
Add: $(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)$.

Your solution

Solution on p. S39

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6.2B Subtract polynomials

The additive inverse of the polynomial $(3x^2 - 7x + 8)$ is $-(3x^2 - 7x + 8)$.

To find the additive inverse of a polynomial, change the sign of each term inside the parentheses. $-(3x^2 - 7x + 8) = -3x^2 + 7x - 8$

To subtract two polynomials, add the additive inverse of the second polynomial to the first.

Simplify: $(5a^2 - a + 2) - (-2a^3 + 3a - 3)$

$(5a^2 - a + 2) - (-2a^3 + 3a - 3)$

$= (5a^2 - a + 2) + (2a^3 - 3a + 3)$ • Rewrite subtraction as addition of the additive inverse.

$= 2a^3 + 5a^2 - 4a + 5$ • Combine like terms.

Example 3

Subtract: $(7c^2 - 9c - 12) - (9c^2 + 5c - 8)$

Solution

$(7c^2 - 9c - 12) - (9c^2 + 5c - 8)$

$= (7c^2 - 9c - 12) + (-9c^2 - 5c + 8)$

$= -2c^2 - 14c - 4$

You Try It 3

Subtract:

$(-4w^3 + 8w - 8) - (3w^3 - 4w^2 + 2w - 1)$

Your solution

Solution on p. S40

To multiply polynomials

6.3A Multiply a polynomial by a monomial

The Distributive Property is used to multiply a polynomial by a monomial. Each term of the polynomial is multiplied by the monomial.

Multiply: $3x^3(4x^4 - 2x + 5)$

$3x^3(4x^4 - 2x + 5)$

$= 3x^3(4x^4) - 3x^3(2x) + 3x^3(5)$ • Use the Distributive Property. Multiply each term of the polynomial by $3x^3$.

$= 12x^7 - 6x^4 + 15x^3$ • Use the Rule for Multiplying Exponential Expressions.

Example 4

Multiply: $2xy(3x^2 - xy + 2y^2)$

Solution

$2xy(3x^2 - xy + 2y^2)$

$= 2xy(3x^2) - 2xy(xy) + 2xy(2y^2)$

$= 6x^3y - 2x^2y^2 + 4xy^3$

You Try It 4

Multiply: $3mn^2(2m^2 - 3mn - 1)$

Your solution

Solution on p. S40

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6.3B Multiply two polynomials

A vertical format similar to that used for multiplication of whole numbers is used to multiply two polynomials. The product $(2y - 3)(y^2 + 2y + 5)$ is shown below.

$$\begin{array}{rl} & y^2 + 2y + 5 \\ & 2y - 3 \\ \hline & -3y^2 - 6y - 15 \quad \text{This is } -3(y^2 + 2y + 5). \\ 2y^3 + 4y^2 + 10y & \quad \text{This is } 2y(y^2 + 2y + 5). \text{ Like terms are placed in the same columns.} \\ \hline 2y^3 + y^2 + 4y - 15 & \quad \text{Add the terms in each column.} \end{array}$$

Example 5

Multiply: $(2a^2 + 4a - 5)(3a + 5)$

Solution

$$\begin{array}{rl} & 2a^2 + 4a - 5 \\ & 3a + 5 \\ \hline & 10a^2 + 20a - 25 \quad \bullet \text{ Align like terms in the same column.} \\ 6a^3 + 12a^2 - 15a & \\ \hline 6a^3 + 22a^2 + 5a - 25 & \end{array}$$

You Try It 5

Multiply: $(3c^2 - 4c + 5)(2c - 3)$

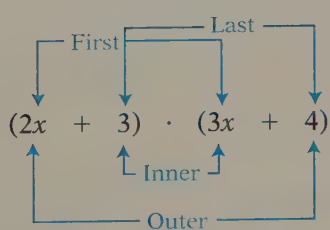
Your solution

Solution on p. S40

6.3C Multiply two binomials

It is frequently necessary to multiply two binomials. The product is computed by using a method called FOIL. The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast. The FOIL method is based on the Distributive Property and involves adding the products of the first terms, the outer terms, the inner terms, and the last terms.

The product $(2x + 3)(3x + 4)$ is shown below using FOIL.



$$\begin{array}{ll} & \text{First terms} \quad \text{Outer terms} \quad \text{Inner terms} \quad \text{Last terms} \\ = & (2x)(3x) + (2x)(4) + (3)(3x) + (3)(4) \\ = & 6x^2 + 8x + 9x + 12 \\ = & 6x^2 + 17x + 12 \end{array}$$

Example 6

Multiply: $(4x - 3)(2x + 5)$

Solution

$$\begin{aligned} &(4x - 3)(2x + 5) \\ &= (4x)(2x) + (4x)(5) + (-3)(2x) + (-3)(5) \\ &= 8x^2 + 20x + (-6x) + (-15) \\ &= 8x^2 + 14x - 15 \end{aligned}$$

You Try It 6

Multiply: $(4y - 7)(3y - 5)$

Your solution

Solution on p. S40

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Objective D

To divide polynomials

6.4A Divide polynomials

To divide two polynomials, use a method similar to that used for division of whole numbers.

To divide $(x^2 - 5x + 8) \div (x - 3)$:

Step 1

$$\begin{array}{r} x \\ x - 3 \overline{)\, x^2 - 5x + 8} \\ \underline{x^2 - 3x} \quad \downarrow \\ -2x + 8 \end{array}$$

Think: $x \overline{)x^2} = \dfrac{x^2}{x} = x$

Multiply: $x(x - 3) = x^2 - 3x$

Subtract: $(x^2 - 5x) - (x^2 - 3x) = -2x$

Bring down the $+8$.

Step 2

$$\begin{array}{r} x - 2 \\ x - 3 \overline{)\, x^2 - 5x + 8} \\ \underline{x^2 - 3x} \\ -2x + 8 \\ \underline{-2x + 6} \\ 2 \end{array}$$

Think: $x \overline{)-2x} = \dfrac{-2x}{x} = -2$

Multiply: $-2(x - 3) = -2x + 6$

Subtract: $(-2x + 8) - (-2x + 6) = 2$

The remainder is 2.

Check: (Quotient $\times$ Divisor) + Remainder = Dividend

$$(x - 2)(x - 3) + 2 = x^2 - 3x - 2x + 6 + 2 = x^2 - 5x + 8$$

$$(x^2 - 5x + 8) \div (x - 3) = x - 2 + \frac{2}{x - 3}$$

Example 7

Divide: $(6x + 2x^3 + 26) \div (x + 2)$

Solution

Arrange the terms of the dividend in descending order. There is no x^2 term in $2x^3 + 6x + 26$. Insert $0x^2$ for the missing term so that like terms will be in columns.

$$\begin{array}{r} 2x^2 - 4x + 14 \\ x + 2 \overline{)\, 2x^3 + 0x^2 + 6x + 26} \\ \underline{2x^3 + 4x^2} \\ -4x^2 + 6x \\ \underline{-4x^2 - 8x} \\ 14x + 26 \\ \underline{14x + 28} \\ -2 \end{array}$$

Check: $(x + 2)(2x^2 - 4x + 14) - 2$

$= 2x^3 + 6x + 28 - 2 = 2x^3 + 6x + 26$

$$(6x + 2x^3 + 26) \div (x + 2) = 2x^2 - 4x + 14 - \frac{2}{x + 2}$$

You Try It 7

Divide: $(x^3 - 7 - 2x) \div (x - 2)$

Your solution

Solution on p. S40

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Objective E

To factor polynomials of the form $ax^2 + bx + c$

7.1A Factor a monomial from a polynomial

A polynomial is in **factored form** when it is written as a product of other polynomials. It can be thought of as the reverse of multiplication.

Polynomial	Factored Form
$2x^3 + 6x^2 - 10x$	$= 2x(x^2 + 3x - 5)$
$x^2 - 3x - 28$	$= (x + 4)(x - 7)$

To factor out a common monomial from the terms of a polynomial, first find the greatest common factor (GCF) of the terms.

The GCF of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

$10a^3b = \mathbf{2} \cdot 5 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot a \cdot \boldsymbol{b}$
$4a^2b^2 = \mathbf{2} \cdot 2 \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} \cdot b$
$\text{GCF} = \mathbf{2} \cdot \boldsymbol{a} \cdot \boldsymbol{a} \cdot \boldsymbol{b} = 2a^2b$

Note that the exponent of each variable in the GCF is the same as the smallest exponent of that variable in either of the monomials.

The GCF of $10a^3b$ and $4a^2b^2$ is $2a^2b$.

Factor: $5x^3 - 35x^2 + 10x$

The GCF is $5x$.

- Find the GCF of the terms $5x^3$, $-35x^2$, and $10x$.

$$\frac{5x^3 - 35x^2 + 10x}{5x} = x^2 - 7x + 2$$

- Divide each term of the polynomial by the GCF.

$$5x^3 - 35x^2 + 10x = 5x(x^2 - 7x + 2)$$

- Write the polynomial as the product of the GCF and the quotient found above.

$$5x(x^2 - 7x + 2) = 5x^3 - 35x^2 + 10x$$

- Check the factorization by multiplying.

Example 8

Factor: $16x^4y^5 + 8x^4y^2 - 12x^3y$

Solution

The GCF is $4x^3y$.

$16x^4y^5 + 8x^4y^2 - 12x^3y = 4x^3y(4xy^4 + 2xy - 3)$

You Try It 8

Factor: $6x^4y^2 - 9x^3y^2 + 12x^2y^4$

Your solution

Solution on p. S40

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A **quadratic trinomial** is a trinomial of the form $ax^2 + bx + c$, where a and b are coefficients and c is a constant. Examples of quadratic trinomials are shown below.

$x^2 + 9x + 14 \quad a = 1, b = 9, c = 14$
$x^2 - 2x - 15 \quad a = 1, b = -2, c = -15$
$3x^2 - x + 4 \quad a = 3, b = -1, c = 4$

To **factor a quadratic trinomial** means to express the trinomial as the product of two binomials. For example,

Trinomial		Factored Form
$2x^2 - x - 1$	=	$(2x + 1)(x - 1)$
$y^2 - 3y + 2$	=	$(y - 1)(y - 2)$

7.2A Factor a trinomial of the form $x^2 + bx + c$

We will begin by factoring trinomials of the form $x^2 + bx + c$, where the coefficient of x^2 is 1.

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant term of the binomials and the terms of the trinomial.

Sum of the binomial constants

Product of the binomial constants

$(x + 4)(x + 5) = x \cdot x + 5x + 4x + 4 \cdot 5 \quad = x^2 + 9x + 20$
$(x - 6)(x + 8) = x \cdot x + 8x - 6x + (-6)(8) \quad = x^2 + 2x - 48$
$(x - 3)(x - 2) = x \cdot x - 2x - 3x + (-3)(-2) \quad = x^2 - 5x + 6$

TAKE NOTE
Once the correct factors are found, it is not necessary to try the remaining factors.

TAKE NOTE
Always check the proposed factorization by multiplying the factors.

Factor: $x^2 - 7x + 10$

Find two integers whose product is 10 and whose sum is -7.

Negative Factors of 10	Sum	
$-1, -10$	-11	
$-2, -5$	**-7**	• These are the correct factors.

$x^2 - 7x + 10 = (x - 2)(x - 5)$ • Write the trinomial as a product of factors.

Check:
$(x - 2)(x - 5) = x^2 - 5x - 2x + 10$
$= x^2 - 7x + 10$

Example 9
Factor: $x^2 + 6x - 27$

Solution
Two factors of -27 whose sum is 6 are -3 and 9.

$x^2 + 6x - 27 = (x - 3)(x + 9)$

You Try It 9
Factor: $x^2 - 8x + 15$

Your solution

Solution on p. S40

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7.3A Factor a trinomial of the form $ax^2 + bx + c$ by using trial factors

To use the trial factor method to factor a trinomial of the form $ax^2 + bx + c$ where $a \neq 1$, use the factors of a and the factors of c to write all of the possible binomial factors of the trinomial. Then use FOIL to determine the correct factorization. To reduce the number of trial factors that must be considered, remember the following guidelines.

Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials will have different signs.

➨ Factor: $2x^2 - 7x + 3$

Because the constant term is positive (+3) and the coefficient of x is negative (−7), the binomial constants will be negative.

Positive Factors of 2 ($a = 2$)	**Negative Factors of 3 ($c = 3$)**
1, 2	−1, −3

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-7x$, of the trinomial.

Trial Factors	**Middle Term**	
$(x - 1)(2x - 3)$	$-3x - 2x = -5x$	
$\mathbf{(x - 3)(2x - 1)}$	$\mathbf{-x - 6x = -7x}$	• $-7x$ is the middle term.

$2x^2 - 7x + 3 = (x - 3)(2x - 1)$

Check: $(x - 3)(2x - 1) = 2x^2 - x - 6x + 3 = 2x^2 - 7x + 3$

➨ Factor: $5x^2 + 22x - 15$

The constant term is negative (−15). The binomial constants will have different signs.

Positive Factors of 5 ($a = 5$)	**Factors of -15 ($c = -15$)**
1, 5	−1, 15
	1, −15
	−3, 5
	3, −5

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $22x$, of the trinomial.

Trial Factors	**Middle Term**	
$(x - 1)(5x + 15)$	common factor	
$(x + 15)(5x - 1)$	$-x + 75x = 74x$	
$(x + 1)(5x - 15)$	common factor	
$(x - 15)(5x + 1)$	$x - 75x = -74x$	
$(x - 3)(5x + 5)$	common factor	
$\mathbf{(x + 5)(5x - 3)}$	$\mathbf{-3x + 25x = 22x}$	• $22x$ is the middle term.
$(x + 3)(5x - 5)$	common factor	
$(x - 5)(5x + 3)$	$3x - 25x = -22x$	

$5x^2 + 22x - 15 = (x + 5)(5x - 3)$

Check: $(x + 5)(5x - 3) = 5x^2 - 3x + 25x - 15 = 5x^2 + 22x - 15$

TAKE NOTE

It is not necessary to test trial factors that have a common factor. If the trinomial does not have a common factor, then its factors cannot have a common factor.

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Example 10

Factor: $2x^2 + 3x - 5$

Solution

Positive Factors of 2	Factors of −5
1, 2	1, −5
	−1, 5

Trial Factors	Middle Term
$(x + 1)(2x - 5)$	$-5x + 2x = -3x$
$(x - 5)(2x + 1)$	$x - 10x = -9x$
$(x - 1)(2x + 5)$	$5x - 2x = 3x$
$(x + 5)(2x - 1)$	$-x + 10x = 9x$

$2x^2 + 3x - 5 = (x - 1)(2x + 5)$

Check: $(x - 1)(2x + 5) = 2x^2 + 5x - 2x - 5$
$= 2x^2 + 3x - 5$

You Try It 10

Factor: $3x^2 - x - 2$

Your solution

Solution on p. S40

7.4D Factor completely

A polynomial is factored completely when it is written as a product of factors that are nonfactorable over the integers.

The first step in any factoring problem is to determine whether the terms of the polynomial have a common factor. If they do, factor it out first.

Example 11

Factor: $5x^2y + 60xy + 100y$

Solution

There is a common factor, $5y$.
Factor out the GCF.

$5x^2y + 60xy + 100y = 5y(x^2 + 12x + 20)$

Factor $x^2 + 12x + 20$. The two factors of 20 whose sum is 12 are 2 and 10.

$5y(x^2 + 12x + 20) = 5y(x + 2)(x + 10)$

$5x^2y + 60xy + 100y = 5y(x + 2)(x + 10)$

Check:
$5y(x + 2)(x + 10) = (5xy + 10y)(x + 10)$
$= 5x^2y + 50xy + 10xy + 100y$
$= 5x^2y + 60xy + 100y$

You Try It 11

Factor: $4a^3 - 4a^2 - 24a$

Your solution

Solution on p. S40

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R.4 Exercises

Objective 4

Simplify.

1. $z^3 \cdot z \cdot z^4$
2. $b \cdot b^2 \cdot b^6$
3. $(x^3)^5$
4. $(b^2)^4$
5. $(x^2y^3)^6$
6. $(m^4n^2)^3$
7. $\dfrac{a^8}{a^2}$
8. $\dfrac{c^{12}}{c^5}$
9. $(-m^3n)(m^6n^2)$
10. $(-r^4t^3)(r^2t^9)$
11. $(-2a^3bc^2)^3$
12. $(-4xy^3z^2)^2$
13. $\dfrac{m^4n^7}{m^3n^5}$
14. $\dfrac{a^5b^6}{a^3b^2}$
15. $\dfrac{-16a^7}{24a^6}$
16. $\dfrac{18b^5}{-45b^4}$
17. $(9mn^4p)(-3mp^2)$
18. $(-3v^2wz)(-4vz^4)$
19. $(-2n^2)(-3n^4)^3$
20. $(-3m^3n)(-2m^2n^3)^3$
21. $\dfrac{14x^4y^6z^2}{16x^3y^9z}$
22. $\dfrac{25x^4y^7z^2}{20x^5y^9z^{11}}$
23. $(-2x^3y^2)^3(-xy^2)^4$
24. $(-m^4n^2)^5(-2m^3n^3)^3$
25. $4x^{-7}$
26. $-6y^{-1}$
27. $d^{-4}d^{-6}$
28. $x^{-3}x^{-5}$
29. $\dfrac{x^{-3}}{x^2}$
30. $\dfrac{x^4}{x^{-5}}$
31. $\dfrac{1}{3x^{-2}}$
32. $\dfrac{2}{5c^{-6}}$
33. $(x^2y^{-4})^3$
34. $(x^3y^5)^{-4}$
35. $(3x^{-1}y^{-2})^2$
36. $(5xy^{-3})^{-2}$

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37. $(2x^{-1})(x^{-3})$

38. $(-2x^{-5})(x^{7})$

39. $\dfrac{3x^{-2}y^{2}}{6xy^{2}}$

40. $\dfrac{2x^{-2}y}{8xy}$

41. $\dfrac{2x^{-1}y^{-4}}{4xy^{2}}$

42. $\dfrac{3a^{-2}b}{ab}$

43. $(x^{-2}y)^{2}(xy)^{-2}$

44. $(x^{-1}y^{2})^{-3}(x^{2}y^{-4})^{-3}$

45. $\left(\dfrac{x^{2}y^{-1}}{xy}\right)^{-4}$

46. $\left(\dfrac{x^{-2}y^{-4}}{x^{-2}y}\right)^{-2}$

47. $\left(\dfrac{4a^{-2}b}{8a^{3}b^{-4}}\right)^{2}$

48. $\left(\dfrac{6ab^{-2}}{3a^{-2}b}\right)^{-2}$

Objective 5

Add or subtract.

49. $(4b^{2}-5b)+(3b^{2}+6b-4)$

50. $(2c^{2}-4)+(6c^{2}-2c+4)$

51. $(2a^{2}-7a+10)+(a^{2}+4a+7)$

52. $(-6x^{2}+7x+3)+(3x^{2}+x+3)$

53. $(x^{2}-2x+1)-(x^{2}+5x+8)$

54. $(3x^{2}+2x-2)-(5x^{2}-5x+6)$

55. $(-2x^{3}+x-1)-(-x^{2}+x-3)$

56. $(2x^{2}+5x-3)-(3x^{3}+2x-5)$

57. $(x^{3}-7x+4)+(2x^{2}+x-10)$

58. $(3y^{3}+y^{2}+1)+(-4y^{3}-6y-3)$

59. $(5x^{3}+7x-7)+(10x^{2}-8x+3)$

60. $(3y^{3}+4y+9)+(2y^{2}+4y-21)$

61. $(2y^{3}+6y-2)-(y^{3}+y^{2}+4)$

62. $(-2x^{2}-x+4)-(-x^{3}+3x-2)$

63. $(4y^{3}-y-1)-(2y^{2}-3y+3)$

64. $(3x^{2}-2x-3)-(2x^{3}-2x^{2}+4)$

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Objective C

Multiply.

65. $4b(3b^3 - 12b^2 - 6)$

66. $-2a^2(3a^2 - 2a + 3)$

67. $3b(3b^4 - 3b^2 + 8)$

68. $-2x^2(2x^2 - 3x - 7)$

69. $-2x^2y(x^2 - 3xy + 2y^2)$

70. $3ab^2(3a^2 - 2ab + 4b^2)$

71. $(x^2 + 3x + 2)(x + 1)$

72. $(x^2 - 2x + 7)(x - 2)$

73. $(a - 3)(a^2 - 3a + 4)$

74. $(2x - 3)(x^2 - 3x + 5)$

75. $(-2b^2 - 3b + 4)(b - 5)$

76. $(-a^2 + 3a - 2)(2a - 1)$

77. $(x^3 - 3x + 2)(x - 4)$

78. $(y^3 + 4y^2 - 8)(2y - 1)$

79. $(y + 2)(y^3 + 2y^2 - 3y + 1)$

80. $(2a - 3)(2a^3 - 3a^2 + 2a - 1)$

81. $(a - 3)(a + 4)$

82. $(b - 6)(b + 3)$

83. $(y - 7)(y - 3)$

84. $(a - 8)(a - 9)$

85. $(2x + 1)(x + 7)$

86. $(y + 2)(5y + 1)$

87. $(3x - 1)(x + 4)$

88. $(7x - 2)(x + 4)$

89. $(4x - 3)(x - 7)$

90. $(2x - 3)(4x - 7)$

91. $(3y - 8)(y + 2)$

92. $(5y - 9)(y + 5)$

93. $(7a - 16)(3a - 5)$

94. $(5a - 12)(3a - 7)$

95. $(x + y)(2x + y)$

96. $(2a + b)(a + 3b)$

97. $(3x - 4y)(x - 2y)$

98. $(2a - b)(3a + 2b)$

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99. $(5a - 3b)(2a + 4b)$ **100.** $(2x + 3)(2x - 3)$ **101.** $(4x - 7)(4x + 7)$

Objective D

Divide.

102. $(b^2 - 14 + 49) \div (b - 7)$ **103.** $(x^2 - x - 6) \div (x - 3)$ **104.** $(2x^2 + 5x + 2) \div (x + 2)$

105. $(2y^2 - 13y + 21) \div (y - 3)$ **106.** $(x^2 + 1) \div (x - 1)$ **107.** $(x^2 + 4) \div (x + 2)$

108. $(6x^2 - 7x) \div (3x - 2)$ **109.** $(6y^2 + 2y) \div (2y + 4)$ **110.** $(a^2 + 5a + 10) \div (a + 2)$

111. $(b^2 - 8b - 9) \div (b - 3)$ **112.** $(2y^2 - 9y + 8) \div (2y + 3)$ **113.** $(3x^2 + 5x - 4) \div (x - 4)$

114. $(8x + 3 + 4x^2) \div (2x - 1)$ **115.** $(10 + 21y + 10y^2) \div (2y + 3)$

116. $(x^3 + 3x^2 + 5x + 3) \div (x + 1)$ **117.** $(x^3 - 6x^2 + 7x - 2) \div (x - 1)$

118. $(x^4 - x^2 - 6) \div (x^2 + 2)$ **119.** $(x^4 + 3x^2 - 10) \div (x^2 - 2)$

Objective E

Factor.

120. $8x + 12$ **121.** $12y^2 - 5y$ **122.** $10x^4 - 12x^2$

123. $10x^2yz^2 + 15xy^3z$ **124.** $x^3 - 3x^2 - x$ **125.** $5x^2 - 15x + 35$

126. $3x^3 + 6x^2 + 9x$ **127.** $3y^4 - 9y^3 - 6y^2$ **128.** $2x^3 + 6x^2 - 14x$

129. $x^4y^4 - 3x^3y^3 + 6x^2y^2$ **130.** $4x^5y^5 - 8x^4y^4 + x^3y^3$ **131.** $16x^2y - 8x^3y^4 - 48x^2y^2$

132. $x^2 + 5x + 6$ **133.** $x^2 + x - 2$ **134.** $x^2 + x - 6$

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135. $a^2 + a - 12$

136. $a^2 - 2a - 35$

137. $a^2 - 3a + 2$

138. $a^2 - 5a + 4$

139. $b^2 + 7b - 8$

140. $y^2 + 6y - 55$

141. $z^2 - 4z - 45$

142. $y^2 - 8y + 15$

143. $z^2 - 14z + 45$

144. $p^2 + 12p + 27$

145. $b^2 + 9b + 20$

146. $y^2 - 8y + 32$

147. $y^2 - 9y + 81$

148. $p^2 + 24p + 63$

149. $x^2 - 15x + 56$

150. $5x^2 + 6x + 1$

151. $2y^2 + 7y + 3$

152. $2a^2 - 3a + 1$

153. $3a^2 - 4a + 1$

154. $4x^2 - 3x - 1$

155. $2x^2 - 5x - 3$

156. $6t^2 - 11t + 4$

157. $10t^2 + 11t + 3$

158. $8x^2 + 33x + 4$

159. $10z^2 + 3z - 4$

160. $3x^2 + 14x - 5$

161. $3z^2 + 95z + 10$

162. $8z^2 - 36z + 1$

163. $2t^2 - t - 10$

164. $2t^2 + 5t - 12$

165. $12y^2 + 19y + 5$

166. $5y^2 - 22y + 8$

167. $11a^2 - 54a - 5$

168. $4z^2 + 11z + 6$

169. $6b^2 - 13b + 6$

170. $6x^2 + 35x - 6$

171. $3x^2 + 15x + 18$

172. $3a^2 + 3a - 18$

173. $ab^2 + 7ab - 8a$

174. $3y^3 - 15y^2 + 18y$

175. $2y^4 - 26y^3 - 96y^2$

176. $3y^4 + 54y^3 + 135y^2$

177. $2x^3 - 11x^2 + 5x$

178. $2x^3 + 3x^2 - 5x$

179. $10t^2 - 5t - 50$

180. $16t^2 + 40t - 96$

181. $6p^3 + 5p^2 + p$

182. $12x^2y - 36xy + 27y$

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Appendix

Calculator Guide for the TI-83 and TI-83Plus

Basic Operations

To evaluate an expression

a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression to be evaluated. For example, to input the expression $-3a^2b - 4c$, use the following keystrokes:

[(−)] 3 [ALPHA] A [X²] [ALPHA] B [−] 4 [ALPHA] C [2nd] QUIT

Note the difference between the keys for a negative sign [(−)] and a minus sign [−].

c. Store the value of each variable that will be used in the expression. For example, to evaluate the expression above when $a = 3$, $b = -2$, and $c = -4$, use the following keystrokes:

3 [STO▷] [ALPHA] A [ENTER] [(−)] 2 [STO▷] [ALPHA] B [ENTER] [(−)] 4 [STO▷] [ALPHA] C [ENTER]

These steps store the value of each variable.

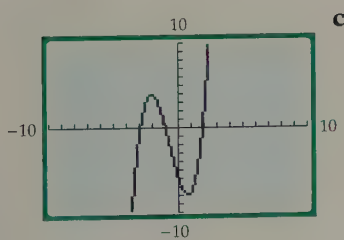
d. Press [VARS] [▷] [1] [1] [ENTER]. The value for the expression, Y1, for the given values is displayed; in this case, Y1 = 70.

To graph a function

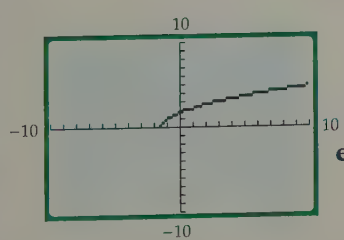
a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression for each function that is to be graphed. Press [X,T,θ,n] to input x. For example, to input $y = x^3 + 2x^2 - 5x - 6$, use the following keystrokes:

[X,T,θ,n] [^] 3 [+] 2 [X,T,θ,n] [X²] [−] 5 [X,T,θ,n] [−] 6



c. Set the viewing window by pressing [WINDOW]. Enter the values for the minimum x-value (Xmin), the maximum x-value (Xmax), the distance between tick marks on the x-axis (Xscl), the minimum y-value (Ymin), the maximum y-value (Ymax), and the distance between tick marks on the y-axis (Yscl). Now press [GRAPH]. For the graph shown at the left, Xmin = −10, Xmax = 10, Xscl = 1, Ymin = −10, Ymax = 10, and Yscl = 1. This is called the standard viewing window. Pressing [ZOOM] [6] is a quick way to set the calculator to the standard viewing window. *Note:* This will also immediately graph the function in that window.



d. Press the [Y=] key. The equal sign has a black rectangle around it. This indicates that the function is active and will be graphed when the [GRAPH] key is pressed. A function is deactivated by using the arrow keys. Move the cursor over the equal sign and press [ENTER]. When the cursor is moved to the right, the black rectangle will not be present and that equation will not be active.

e. Graphing some radical equations requires special care. To graph the equation $y = \sqrt{2x + 3}$ shown at the left, enter the following keystrokes:

[Y=] [CLEAR] [2nd] [√] 2 [X,T,θ,n] [+] 3 [)] [GRAPH]

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To display the *x*-coordinates of rectangular coordinates as integers

a. Set the viewing window as follows: Xmin = −47, Xmax = 47, Xscl = 10, Ymin = −31, Ymax = 31, Yscl = 10.

b. Graph the function and use the TRACE feature. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To display the *x*-coordinates of rectangular coordinates in tenths

a. Set the viewing window as follows: [ZOOM] [4]

b. Graph the function. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To evaluate a function for a given value of *x*, or to produce ordered pairs of a function

a. Input the equation; for example, input $Y_1 = 2x^3 - 3x + 2$.

b. Press [2nd] QUIT.

c. To evaluate the function when $x = 3$, press [VARS] [▷] [1] [1] [(] 3 [)] [ENTER]. The value for the function for the given x-value is displayed, in this case, $Y_1(3) = 47$. An ordered pair of the function is (3, 47).

d. Repeat step **c.** to produce as many pairs as desired.

The TABLE feature can also be used to determine ordered pairs.

To use a table

a. Press [2nd] TBLSET to activate the table setup menu.

b. TblStart is the beginning number for the table; ΔTbl is the difference between any two successive x-values in the table.

c. The portion of the table that appears as Indpnt: Auto Ask / Depend: Auto Ask allows you to choose between having the calculator automatically produce the results (Auto) or having the calculator ask you for values of x. You can choose Ask by using the arrow keys.

d. Once a table has been set up, enter an expression for Y_1. Now select TABLE by pressing [2nd] TABLE. A table showing ordered pair solutions of the equation will be displayed on the screen.

Zoom Features

To zoom in or out on a graph

Here are two methods of using ZOOM.

a. The first method uses the built-in features of the calculator. Press [TRACE] and then move the cursor to a point on the graph that is of interest. Press [ZOOM]. The ZOOM menu will appear. Press [2] [ENTER] to zoom in on the graph by the amount shown under the SET FACTORS menu. The center of the new graph is the location at which you placed the cursor. Press [ZOOM] [3] [ENTER] to zoom out on the graph by the amount under the SET FACTORS menu. (The SET FACTORS menu is accessed by pressing [ZOOM] [▷] [4].)

b. The second method uses the ZBOX option under the ZOOM menu. To use this method, press [ZOOM] [1]. A cursor will appear on the graph. Use the arrow keys to move the cursor to a portion of the graph that is of interest. Press [ENTER]. Now

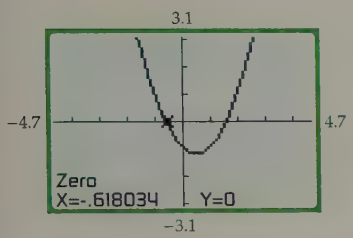
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use the arrow keys to draw a box around the portion of the graph you wish to zoom in on. Press [ENTER]. The portion of the graph defined by the box will be drawn.

c. Pressing [ZOOM] [6] resets the window to the standard viewing window.

Solving Equations

This discussion is based on the fact that the real number solutions of an equation are related to the x-intercepts of a graph. For instance, the real solutions of the equation $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.



To solve $x^2 = x + 1$, rewrite the equation with all terms on one side: $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.

a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercepts are visible.

c. Press [2nd] CALC [2].

d. Move the cursor to a point on the curve that is to the left of an x-intercept. Press [ENTER].

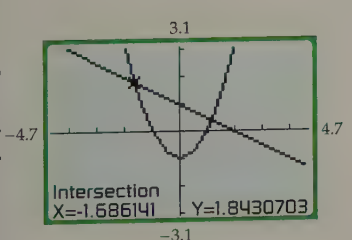
e. Move the cursor to a point on the curve that is to the right of the same x-intercept. Press [ENTER] [ENTER].

f. The root is shown as the x-coordinate on the bottom of the screen; in this case, the root is approximately -0.618034. To find the next intercept, repeat steps **c.** through **e.** The SOLVER feature under the MATH menu can also be used to find solutions of equations.

Solving Systems of Equations in Two Variables

To solve a system of equations

To solve $\begin{aligned} y &= x^2 - 1 \\ \frac{1}{2}x + y &= 1 \end{aligned}$,



a. If necessary, solve one or both of the equations for y.

b. Enter the first equation as Y_1: $Y_1 = x^2 - 1$.

c. Enter the second equation as Y_2: $Y_2 = 1 - .5x$.

d. Graph both equations. (*Note:* The point of intersection must appear on the screen. It may be necessary to adjust the viewing window so that the points of intersection are displayed.)

e. Press [2nd] CALC [5].

f. Move the cursor close to the first point of intersection. Press [ENTER] [ENTER] [ENTER].

g. The first point of intersection is $(-1.686141, 1.8430703)$.

h. Repeat steps **e.** and **f.** for each point of intersection.

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Finding Minimum or Maximum Values of a Function

a. Enter the function into Y1. The equation $y = x^2 - x - 1$ is used here.

b. Graph the equation. You may need to adjust the viewing window so that the maximum or minimum points are visible.

c. Press [2nd] CALC [3] to determine a minimum value or press [2nd] CALC [4] to determine a maximum value.

d. Move the cursor to a point on the curve that is to the left of the minimum (maximum). Press [ENTER].

e. Move the cursor to a point on the curve that is to the right of the minimum (maximum). Press [ENTER] [ENTER].

f. The minimum (maximum) is shown as the y-coordinate on the bottom of the screen; in this case the minimum value is -1.25.

Statistics

To calculate a linear regression equation

a. Press [STAT] to access the statistics menu. Press 1 to Edit or enter a new list of data. To delete data already in a list, press the up arrow to highlight the list name. For instance, to delete data in L1, highlight L1. Then press [CLEAR] and [ENTER]. Now enter each value of the independent variable in L1. Enter each value of the dependent variable in L2.

b. When all the data has been entered, press [STAT] [▷] 4 [ENTER]. The values of the slope and y-intercept of the linear regression equation will be displayed on the screen.

c. To graph the linear regression equation, modify step **b.** with these keystrokes: [STAT] [▷] 4 [VARS] [▷] 1 1 [ENTER]. This will store the regression equation in Y1. Now press [GRAPH]. It may be necessary to adjust the viewing window.

d. To evaluate the regression equation for a value of x, complete step **c.** but do not graph the equation. Now press [VARS] [▷] 1 1 [(] A [)] [ENTER], where A is replaced by the number at which you want to evaluate the expression.

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Proofs and Tables

Proofs of Logarithmic Properties

In each of the following proofs of logarithmic properties, it is assumed that the Properties of Exponents are true for all real number exponents.

The Logarithm Property of the Product of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b xy = \log_b x + \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.	$x = b^m \qquad y = b^n$
Use substitution and the Properties of Exponents.	$xy = b^m b^n$
	$xy = b^{m+n}$
Write the equation in its equivalent logarithmic form.	$\log_b xy = m + n$
Substitute $\log_b x$ for m and $\log_b y$ for n.	$\log_b xy = \log_b x + \log_b y$

The Logarithm Property of the Quotient of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.	$x = b^m \qquad y = b^n$
Use substitution and the Properties of Exponents.	$\frac{x}{y} = \frac{b^m}{b^n}$
	$\frac{x}{y} = b^{m-n}$
Write the equation in its equivalent logarithmic form.	$\log_b \frac{x}{y} = m - n$
Substitute $\log_b x$ for m and $\log_b y$ for n.	$\log_b \frac{x}{y} = \log_b x - \log_b y$

The Logarithm Property of the Power of a Number

For any real numbers x, r, and b, $b \neq 1$, $\log_b x^r = r \log_b x$.

Proof: Let $\log_b x = m$.

Write the equation in its equivalent exponential form.	$x = b^m$
Raise both sides to the r power.	$x^r = (b^m)^r$
	$x^r = b^{mr}$
Write the equation in its equivalent logarithmic form.	$\log_b x^r = mr$
Substitute $\log_b x$ for m.	$\log_b x^r = r \log_b x$

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Table of Symbols

$+$	add
$-$	subtract
$\cdot, \times, (a)(b)$	multiply
$\frac{a}{b}, \div, a\overline{)b}$	divide
$( )$	parentheses, a grouping symbol
$[ ]$	brackets, a grouping symbol
π	pi, a number approximately equal to $\frac{22}{7}$ or 3.14
$-a$	the opposite, or additive inverse, of a
$\frac{1}{a}$	the reciprocal, or multiplicative inverse, of a
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
(a, b)	an ordered pair whose first component is a and whose second component is b
$^\circ$	degree (for angles)
$\sqrt{a}$	the principal square root of a
$\varnothing, \{\}$	the empty set
$\lvert a \rvert$	the absolute value of a
$\cup$	union of two sets
$\cap$	intersection of two sets
$\in$	is an element of (for sets)
$\notin$	is not an element of (for sets)

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Solutions to Chapter 1 "You Try It"

SECTION 1.1

You Try It 1 Replace y by each of the elements of the set and determine whether the inequality is true.

$y > -1$
$-5 > -1$ False
$-1 > -1$ False
$5 > -1$ True

The inequality is true for 5.

You Try It 2 Replace z by each element of the set and determine the value of the expression.

$-z$	$\lvert z\rvert$
$-(-11) = 11$	$\lvert -11\rvert = 11$
$-(0) = 0$	$\lvert 0\rvert = 0$
$-(8) = -8$	$\lvert 8\rvert = 8$

You Try It 3 $100 + (-43) = 57$

You Try It 4
$$\begin{aligned}(-51) + 42 + 17 + (-102) &= -9 + 17 + (-102)\\ &= 8 + (-102)\\ &= -94\end{aligned}$$

You Try It 5
$$\begin{aligned}19 - (-32) &= 19 + 32\\ &= 51\end{aligned}$$

You Try It 6
$$\begin{aligned}-9 - (-12) - 17 - 4 &= -9 + 12 + (-17) + (-4)\\ &= 3 + (-17) + (-4)\\ &= -14 + (-4)\\ &= -18\end{aligned}$$

You Try It 7
$$\begin{aligned}8(-9)10 &= -72(10)\\ &= -720\end{aligned}$$

You Try It 8
$$\begin{aligned}(-2)3(-8)7 &= -6(-8)7\\ &= 48(7)\\ &= 336\end{aligned}$$

You Try It 9 $(-135) \div (-9) = 15$

You Try It 10 $\dfrac{-72}{4} = -18$

You Try It 11
$$\begin{aligned}-\frac{36}{-12} &= -(-3)\\ &= 3\end{aligned}$$

You Try It 12

Strategy To find the average daily low temperature:

- Add the seven temperature readings.
- Divide the sum by 7.

Solution

$$\begin{aligned}&-6 + (-7) + 0 + (-5) + (-8) + (-1) + (-1)\\ &= -28 - 28 \div 7 = -4\end{aligned}$$

The average daily low temperature was $-4°C$.

SECTION 1.2

You Try It 1
$$\begin{array}{r} 0.444\\ 9\overline{)4.000}\\ \underline{-36}\\ 40\\ \underline{-36}\\ 40\\ \underline{-36}\\ 4\end{array}$$

$\dfrac{4}{9} = 0.\overline{4}$

You Try It 2
$$125\% = 125\left(\frac{1}{100}\right) = \frac{125}{100} = \frac{5}{4}$$
$$125\% = 125(0.01) = 1.25$$

You Try It 3
$$\begin{aligned}\frac{1}{3} &= \frac{1}{3}(100\%)\\ &= \frac{100}{3}\% = 33\frac{1}{3}\%\end{aligned}$$

You Try It 4 $0.043 = 0.043(100\%) = 4.3\%$

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You Try It 5 The LCM of 8, 6, and 4 is 24.

$$-\frac{7}{8}-\frac{5}{6}+\frac{3}{4}=-\frac{21}{24}-\frac{20}{24}+\frac{18}{24}$$
$$=\frac{-21}{24}+\frac{-20}{24}+\frac{18}{24}$$
$$=\frac{-21-20+18}{24}$$
$$=\frac{-23}{24}=-\frac{23}{24}$$

You Try It 6
$$16.127-67.91$$
$$=16.127+(-67.91)$$
$$=-51.783$$

You Try It 7
The quotient is positive.
$$-\frac{3}{8}\div\left(-\frac{5}{12}\right)=\frac{3}{8}\div\frac{5}{12}=\frac{3}{8}\cdot\frac{12}{5}$$
$$=\frac{3\cdot 12}{8\cdot 5}$$
$$=\frac{3\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 3}{2\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 5}=\frac{9}{10}$$

You Try It 8
$$\begin{array}{r} 5.44 \\ \times\ 3.8 \\ \hline 4352 \\ 1632 \\ \hline 20.672 \end{array}$$
$$-5.44(3.8)=-20.672$$

You Try It 9
$$-6^3=-(6\cdot 6\cdot 6)=-216$$

You Try It 10
$$(-3)^4=(-3)(-3)(-3)(-3)=81$$

You Try It 11
$$(3^3)(-2)^3=(3)(3)(3)\cdot(-2)(-2)(-2)$$
$$=27(-8)=-216$$

You Try It 12
$$\left(-\frac{2}{5}\right)^2=\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)=\frac{4}{25}$$

You Try It 13
$$-3(0.3)^3=-3(0.3)(0.3)(0.3)$$
$$=-0.9(0.3)(0.3)$$
$$=-0.27(0.3)=-0.081$$

You Try It 14
$$-5\sqrt{32}=-5\sqrt{16\cdot 2}=-5\sqrt{16}\sqrt{2}$$
$$=-5\cdot 4\sqrt{2}=-20\sqrt{2}$$

You Try It 15 $\sqrt{216}=\sqrt{36\cdot 6}=\sqrt{36}\sqrt{6}=6\sqrt{6}$

You Try It 16

Strategy To find the average monthly net income for the first quarter of 2000, divide Friendly's net income for the quarter (−18.510 million) by the number of months in the quarter (3).

Solution $-18.510\div 3=-6.170$

The average monthly net income was −$6.17 million.

SECTION 1.3

You Try It 1
$$18-5[8-2(2-5)]\div 10$$
$$=18-5[8-2(-3)]\div 10$$
$$=18-5[8+6]\div 10$$
$$=18-5[14]\div 10$$
$$=18-70\div 10$$
$$=18-7$$
$$=11$$

You Try It 2
$$36\div(8-5)^2-(-3)^2\cdot 2$$
$$=36\div(3)^2-(-3)^2\cdot 2$$
$$=36\div 9-9\cdot 2$$
$$=4-9\cdot 2$$
$$=4-18$$
$$=-14$$

You Try It 3
$$(6.97-4.72)^2\cdot 4.5\div 0.05$$
$$=(2.25)^2\cdot 4.5\div 0.05$$
$$=5.0625\cdot 4.5\div 0.05$$
$$=22.78125\div 0.05$$
$$=455.625$$

SECTION 1.4

You Try It 1
$$\frac{a^2+b^2}{a+b}$$
$$\frac{5^2+(-3)^2}{5+(-3)}=\frac{25+9}{5+(-3)}$$
$$=\frac{34}{2}$$
$$=17$$

You Try It 2
$$x^3-2(x+y)+z^2$$
$$(2)^3-2[2+(-4)]+(-3)^2$$
$$=(2)^3-2(-2)+(-3)^2$$
$$=8-2(-2)+9$$
$$=8+4+9$$
$$=12+9$$
$$=21$$

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You Try It 3 $3a - 2b - 5a + 6b = -2a + 4b$

You Try It 4 $-3y^2 + 7 + 8y^2 - 14 = 5y^2 - 7$

You Try It 5 $-5(4y^2) = -20y^2$

You Try It 6 $-7(-2a) = 14a$

You Try It 7 $(-5x)(-2) = 10x$

You Try It 8 $-8(-2a + 7b) = 16a - 56b$

You Try It 9 $(3a - 1)5 = 15a - 5$

You Try It 10 $2(x^2 - x + 7) = 2x^2 - 2x + 14$

You Try It 11
$$\begin{aligned} 3y - 2(y - 7x) &= 3y - 2y + 14x \\ &= 14x + y \end{aligned}$$

You Try It 12
$$\begin{aligned} -2(x - 2y) - (-x + 3y) &= -2x + 4y + x - 3y \\ &= -x + y \end{aligned}$$

You Try It 13
$$\begin{aligned} 3y - 2[x - 4(2 - 3y)] &= 3y - 2[x - 8 + 12y] \\ &= 3y - 2x + 16 - 24y \\ &= -2x - 21y + 16 \end{aligned}$$

You Try It 14 the unknown number: x
the difference between the number and sixty: $x - 60$

$5(x - 60)$; $5x - 300$

You Try It 15 the speed of the older model: s
the speed of the new model: $2s$

You Try It 16 the length of the longer piece: L
the length of the shorter piece: $6 - L$

SECTION 1.5

You Try It 1 $A = \{-9, -7, -5, -3, -1\}$

You Try It 2 $A = \{1, 3, 5, \ldots\}$

You Try It 3 $A \cup B = \{-2, -1, 0, 1, 2, 3, 4\}$

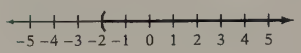
You Try It 4 $C \cap D = \{10, 16\}$

You Try It 5 $A \cap B = \varnothing$

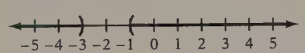
You Try It 6 $\{x | x < 59, x \in$ positive even integers$\}$

You Try It 7 $\{x | x > -3, x \in$ real numbers$\}$

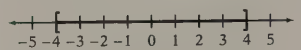
You Try It 8 The graph is the numbers greater than -2.



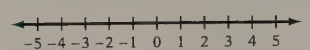
You Try It 9 The graph is the numbers greater than -1 and the numbers less than -3.



You Try It 10 The graph is the numbers less than or equal to 4 and greater than or equal to -4.



You Try It 11 The graph is the real numbers.



Solutions to Chapter 2 "You Try It"

SECTION 2.1

You Try It 1

$5 - 4x = 8x + 2$	
$5 - 4\left(\frac{1}{4}\right)$	$8\left(\frac{1}{4}\right) + 2$
$5 - 1$	$2 + 2$
$4 = 4$	

Yes, $\frac{1}{4}$ is a solution.

You Try It 2

$10x - x^2 = 3x - 10$	
$10(5) - (5)^2$	$3(5) - 10$
$50 - 25$	$15 - 10$
$25 \neq 5$	

No, 5 is not a solution.

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You Try It 3

$$\frac{5}{6} = y - \frac{3}{8}$$
$$\frac{5}{6} + \frac{3}{8} = y - \frac{3}{8} + \frac{3}{8}$$
$$\frac{29}{24} = y$$

The solution is $\frac{29}{24}$.

You Try It 4

$$-\frac{2x}{5} = 6$$
$$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}x\right) = \left(-\frac{5}{2}\right)(6)$$
$$x = -15$$

The solution is -15.

You Try It 5

$$4x - 8x = 16$$
$$-4x = 16$$
$$\frac{-4x}{-4} = \frac{16}{-4}$$
$$x = -4$$

The solution is -4.

You Try It 6

$$P \cdot B = A$$
$$\frac{1}{6}B = 18 \qquad \bullet\ 16\frac{2}{3}\% = \frac{1}{6}$$
$$6 \cdot \frac{1}{6}B = 6 \cdot 18$$
$$B = 108$$

18 is $16\frac{2}{3}\%$ of 108.

You Try It 7

Strategy To find the percent, use the basic percent equation. $B = 158.3$ million; $A = 22.45$ million; P is unknown.

Solution

$$P \cdot B = A$$
$$P(158.3) = 22.45$$
$$\frac{P(158.3)}{158.3} = \frac{22.45}{158.3}$$
$$P \approx 0.142 = 14.2\%$$

The Big Ten Conference received approximately 14.2% of the total revenue.

SECTION 2.2

You Try It 1

$$5x + 7 = 10$$
$$5x + 7 - 7 = 10 - 7$$
$$5x = 3$$
$$\frac{5x}{5} = \frac{3}{5}$$
$$x = \frac{3}{5}$$

The solution is $\frac{3}{5}$.

You Try It 2

$$2 = 11 + 3x$$
$$2 - 11 = 11 - 11 + 3x$$
$$-9 = 3x$$
$$\frac{-9}{3} = \frac{3x}{3}$$
$$-3 = x$$

The solution is -3.

You Try It 3

$$x - 5 + 4x = 25$$
$$5x - 5 = 25$$
$$5x - 5 + 5 = 25 + 5$$
$$5x = 30$$
$$\frac{5x}{5} = \frac{30}{5}$$
$$x = 6$$

The solution is 6.

You Try It 4

$$5x + 4 = 6 + 10x$$
$$5x - 10x + 4 = 6 + 10x - 10x$$
$$-5x + 4 = 6$$
$$-5x + 4 - 4 = 6 - 4$$
$$-5x = 2$$
$$\frac{-5x}{-5} = \frac{2}{-5}$$
$$x = -\frac{2}{5}$$

The solution is $-\frac{2}{5}$.

You Try It 5

$$5x - 10 - 3x = 6 - 4x$$
$$2x - 10 = 6 - 4x$$
$$2x + 4x - 10 = 6 - 4x + 4x$$
$$6x - 10 = 6$$
$$6x - 10 + 10 = 6 + 10$$
$$6x = 16$$
$$\frac{6x}{6} = \frac{16}{6}$$
$$x = \frac{8}{3}$$

The solution is $\frac{8}{3}$.

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You Try It 6

$$5x - 4(3 - 2x) = 2(3x - 2) + 6$$
$$5x - 12 + 8x = 6x - 4 + 6$$
$$13x - 12 = 6x + 2$$
$$13x - 6x - 12 = 6x - 6x + 2$$
$$7x - 12 = 2$$
$$7x - 12 + 12 = 2 + 12$$
$$7x = 14$$
$$\frac{7x}{7} = \frac{14}{7}$$
$$x = 2$$

The solution is 2.

You Try It 7

$$-2[3x - 5(2x - 3)] = 3x - 8$$
$$-2[3x - 10x + 15] = 3x - 8$$
$$-2[-7x + 15] = 3x - 8$$
$$14x - 30 = 3x - 8$$
$$14x - 3x - 30 = 3x - 3x - 8$$
$$11x - 30 = -8$$
$$11x - 30 + 30 = -8 + 30$$
$$11x = 22$$
$$\frac{11x}{11} = \frac{22}{11}$$
$$x = 2$$

The solution is 2.

You Try It 8

the smaller number: n
the larger number: $14 - n$

one more than three times the smaller number	equals	the sum of the larger number and three

$$3n + 1 = (14 - n) + 3$$
$$3n + 1 = 17 - n$$
$$3n + n + 1 = 17 - n + n$$
$$4n + 1 = 17$$
$$4n + 1 - 1 = 17 - 1$$
$$4n = 16$$
$$\frac{4n}{4} = \frac{16}{4}$$
$$n = 4$$

$$14 - n = 14 - 4 = 10$$

These numbers check as solutions.

The smaller number is 4.
The larger number is 10.

SECTION 2.3

You Try It 1

Strategy

- Pounds of \$.55 fertilizer: x

	Amount	*Cost*	*Value*
\$.80 fertilizer	20	0.80	0.80(20)
\$.55 fertilizer	x	0.55	$0.55x$
\$.75 fertilizer	$20 + x$	0.75	$0.75(20 + x)$

- The sum of the values before mixing equals the value after mixing.

Solution

$$0.80(20) + 0.55x = 0.75(20 + x)$$
$$16 + 0.55x = 15 + 0.75x$$
$$16 - 0.20x = 15$$
$$-0.20x = -1$$
$$x = 5$$

5 lb of the \$.55 fertilizer must be added.

You Try It 2

Strategy

- Liters of the 6% solution: x

	Amount	*Percent*	*Quantity*
6%	x	0.06	$0.06x$
12%	5	0.12	5(0.12)
8%	$x + 5$	0.08	$0.08(x + 5)$

- The sum of the quantities before mixing equals the quantity after mixing.

Solution

$$0.06x + 5(0.12) = 0.08(x + 5)$$
$$0.06x + 0.60 = 0.08x + 0.40$$
$$0.06x + 0.20 = 0.08x$$
$$0.20 = 0.02x$$
$$10 = x$$

The pharmacist adds 10 L of the 6% solution to the 12% solution to get an 8% solution.

You Try It 3

Strategy

- Additional amount: x

	Principal	*Rate*	*Interest*
8%	5000	0.08	0.08(5000)
11%	x	0.11	$0.11x$
9%	$5000 + x$	0.09	$0.09(5000 + x)$

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- The sum of the interest earned by the two investments equals 9% of the total investment.

Solution

$$0.08(5000) + 0.11x = 0.09(5000 + x)$$
$$400 + 0.11x = 450 + 0.09x$$
$$400 + 0.02x = 450$$
$$0.02x = 50$$
$$x = 2500$$

$2500 more must be invested at 11%.

You Try It 4

Strategy

- Rate of the first train: r
  Rate of the second train: $2r$

	Rate	*Time*	*Distance*
1st train	r	3	$3r$
2nd train	$2r$	3	$3(2r)$

- The sum of the distances traveled by the two trains equals 288 mi.

Solution

$$3r + 3(2r) = 288$$
$$3r + 6r = 288$$
$$9r = 288$$
$$r = 32$$

$2r = 2(32) = 64$

The first train is traveling at 32 mph.
The second train is traveling at 64 mph.

You Try It 5

Strategy

- Time spent flying out: t
  Time spent flying back: $5 - t$

	Rate	*Time*	*Distance*
Out	150	t	$150t$
Back	100	$5 - t$	$100(5 - t)$

- The distance out equals the distance back.

Solution

$$150t = 100(5 - t)$$
$$150t = 500 - 100t$$
$$250t = 500$$
$$t = 2 \text{ (The time out was 2 h.)}$$

The distance out $= 150t = 150(2) =$ 300 mi.

The parcel of land was 300 mi away.

SECTION 2.4

You Try It 1

$$2x - 1 < 6x + 7$$
$$-4x - 1 < 7$$
$$-4x < 8$$
$$\frac{-4x}{-4} > \frac{8}{-4}$$
$$x > -2$$

$\{x | x > -2\}$

You Try It 2

$$5x - 2 \le 4 - 3(x - 2)$$
$$5x - 2 \le 4 - 3x + 6$$
$$5x - 2 \le 10 - 3x$$
$$8x - 2 \le 10$$
$$8x \le 12$$
$$\frac{8x}{8} \le \frac{12}{8}$$
$$x \le \frac{3}{2}$$

$\left\{x | x \le \frac{3}{2}\right\}$

You Try It 3

$$-2 \le 5x + 3 \le 13$$
$$-2 - 3 \le 5x + 3 - 3 \le 13 - 3$$
$$-5 \le 5x \le 10$$
$$\frac{-5}{5} \le \frac{5x}{5} \le \frac{10}{5}$$
$$-1 \le x \le 2$$

$\{x | -1 \le x \le 2\}$

You Try It 4

$2 - 3x > 11$ or $5 + 2x > 7$

$-3x > 9$ $\qquad 2x > 2$

$x < -3$ $\qquad x > 1$

$\{x | x < -3\}$ $\qquad \{x | x > 1\}$

$\{x | x < -3\} \cup \{x | x > 1\}$
$= \{x | x < -3 \text{ or } x > 1\}$

You Try It 5

Strategy

To find the maximum number of miles:

- Write an expression for the cost of each car, using x to represent the number of miles driven during the week.
- Write and solve an inequality.

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Solution

Cost of a Company A car	is less than	Cost of a Company B car

$$\begin{aligned}
24(7) + 0.10x &< 30(7) + 0.08x \\
168 + 0.10x &< 210 + 0.08x \\
168 + 0.10x - 0.08x &< 210 + 0.08x - 0.08x \\
168 + 0.02x &< 210 \\
168 - 168 + 0.02x &< 210 - 168 \\
0.02x &< 42 \\
\frac{0.02x}{0.02} &< \frac{42}{0.02} \\
x &< 2100
\end{aligned}$$

The maximum number of miles is 2099.

SECTION 2.5

You Try It 1

$$\begin{aligned}
|2x - 3| &= 5 \\
2x - 3 = 5 \quad & \quad 2x - 3 = -5 \\
2x = 8 \quad & \quad 2x = -2 \\
x = 4 \quad & \quad x = -1
\end{aligned}$$

The solutions are 4 and -1.

You Try It 2

$$\begin{aligned}
5 - |3x + 5| &= 3 \\
-|3x + 5| &= -2 \\
|3x + 5| &= 2 \\
3x + 5 = 2 \quad & \quad 3x + 5 = -2 \\
3x = -3 \quad & \quad 3x = -7 \\
x = -1 \quad & \quad x = -\frac{7}{3}
\end{aligned}$$

The solutions are -1 and $-\frac{7}{3}$.

You Try It 3

$$\begin{aligned}
|3x + 2| &< 8 \\
-8 &< 3x + 2 < 8 \\
-8 - 2 &< 3x + 2 - 2 < 8 - 2 \\
-10 &< 3x < 6 \\
\frac{-10}{3} &< \frac{3x}{3} < \frac{6}{3} \\
-\frac{10}{3} &< x < 2
\end{aligned}$$

$$\left\{x \left| -\frac{10}{3} < x < 2 \right.\right\}$$

You Try It 4

$$|5x + 3| > 8$$

$5x + 3 < -8$ or $5x + 3 > 8$

$5x < -11$ $\qquad$ $5x > 5$

$x < -\frac{11}{5}$ $\qquad$ $x > 1$

$$\left\{x \left| x < -\frac{11}{5} \right.\right\} \qquad \{x | x > 1\}$$

$$\left\{x \left| x < -\frac{11}{5} \right.\right\} \cup \{x | x > 1\}$$

$$= \left\{x \left| x < -\frac{11}{5} \text{ or } x > 1 \right.\right\}$$

You Try It 5

Strategy Let b represent the desired diameter of the bushing, T the tolerance, and d the actual diameter. Solve the absolute value inequality $|d - b| \le T$ for d.

Solution

$$\begin{aligned}
|d - b| &\le T \\
|d - 2.55| &\le 0.003 \\
-0.003 \le d - 2.55 &\le 0.003 \\
-0.003 + 2.55 \le d - 2.55 + 2.55 &\le 0.003 + 2.55 \\
2.547 \le d &\le 2.553
\end{aligned}$$

The lower and upper limits of the diameter of the bushing are 2.547 in. and 2.553 in.

Solutions to Chapter 3 "You Try It"

SECTION 3.1

You Try It 1

$$\begin{aligned}
QR + RS + ST &= QT \\
24 + RS + 17 &= 62 \\
41 + RS &= 62 \\
RS &= 21
\end{aligned}$$

$RS = 21$ cm

You Try It 2

$$AC = AB + BC$$

$$AC = \frac{1}{4}(BC) + BC$$

$$AC = \frac{1}{4}(16) + 16$$

$$AC = 4 + 16$$

$$AC = 20$$

$AC = 20$ ft

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You Try It 3

Strategy Supplementary angles are two angles whose sum is 180°. To find the supplement, let x represent the supplement of a 129° angle. Write an equation and solve for x.

Solution
$$x + 129° = 180°$$
$$x = 51°$$

The supplement of a 129° angle is a 51° angle.

You Try It 4

Strategy To find the measure of $\angle a$, write an equation using the fact that the sum of the measure of $\angle a$ and 68° is 118°. Solve for $\angle a$.

Solution
$$\angle a + 68° = 118°$$
$$\angle a = 50°$$

The measure of $\angle a$ is 50°.

You Try It 5

Strategy The angles labeled are adjacent angles of intersecting lines and are, therefore, supplementary angles. To find x, write an equation and solve for x.

Solution
$$(x + 16°) + 3x = 180°$$
$$4x + 16° = 180°$$
$$4x = 164°$$
$$x = 41°$$

You Try It 6

Strategy $3x = y$ because corresponding angles have the same measure. $y + (x + 40°) = 180°$ because adjacent angles of intersecting lines are supplementary angles. Substitute $3x$ for y and solve for x.

Solution
$$3x + (x + 40°) = 180°$$
$$4x + 40° = 180°$$
$$4x = 140°$$
$$x = 35°$$

You Try It 7

Strategy
- To find the measure of angle b, use the fact that $\angle b$ and $\angle x$ are supplementary angles.
- To find the measure of angle c, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
- To find the measure of angle y, use the fact that $\angle c$ and $\angle y$ are vertical angles.

Solution
$$\angle b + \angle x = 180°$$
$$\angle b + 100° = 180°$$
$$\angle b = 80°$$

$$\angle a + \angle b + \angle c = 180°$$
$$45° + 80° + \angle c = 180°$$
$$125° + \angle c = 180°$$
$$\angle c = 55°$$

$$\angle y = \angle c = 55°$$

You Try It 8

Strategy To find the measure of the third angle, use the facts that the measure of a right angle is 90° and the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution
$$x + 90° + 34° = 180°$$
$$x + 124° = 180°$$
$$x = 56°$$

The measure of the third angle is 56°.

SECTION 3.2

You Try It 1

Strategy To find the perimeter, use the formula for the perimeter of a square. Substitute 60 for s and solve for P.

Solution
$$P = 4s$$
$$P = 4(60)$$
$$P = 240$$

The perimeter of the infield is 240 ft.

You Try It 2

Strategy To find the perimeter, use the formula for the perimeter of a rectangle.

Substitute 11 for L and $8\frac{1}{2}$ for W and solve for P.

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Solution

$P = 2L + 2W$

$P = 2(11) + 2\left(8\frac{1}{2}\right)$

$P = 2(11) + 2\left(\frac{17}{2}\right)$

$P = 22 + 17$

$P = 39$

The perimeter of a standard piece of typing paper is 39 in.

You Try It 3

Strategy To find the circumference, use the circumference formula that involves the diameter. Leave the answer in terms of π.

Solution

$C = \pi d$

$C = \pi(9)$

$C = 9\pi$

The circumference is 9π in.

You Try It 4

Strategy To find the number of rolls of wallpaper to be purchased:

- Use the formula for the area of a rectangle to find the area of one wall.
- Multiply the area of one wall by the number of walls to be covered (2).
- Divide the area of wall to be covered by the area that one roll of wallpaper will cover (30).

Solution

$A = LW$

$A = 12 \cdot 8 = 96$ The area of one wall is 96 ft^2.

$2(96) = 192$ The area of the two walls is 192 ft^2.

$192 \div 30 = 6.4$

Because a portion of a seventh roll is needed, 7 rolls of wallpaper should be purchased.

You Try It 5

Strategy To find the area, use the formula for the area of a circle. An approximation is asked for; use the π key on a calculator. $r = 11$.

Solution

$A = \pi r^2$

$A = \pi(11)^2$

$A = 121\pi$

$A \approx 380.13$

The area is approximately 380.13 cm^2.

SECTION 3.3

You Try It 1

Strategy To find the volume, use the formula for the volume of a cube. $s = 2.5$.

Solution

$V = s^3$

$V = (2.5)^3 = 15.625$

The volume of the cube is 15.625 m^3.

You Try It 2

Strategy To find the volume:

- Find the radius of the base of the cylinder. $d = 8$.
- Use the formula for the volume of a cylinder. Leave the answer in terms of π.

Solution

$r = \frac{1}{2}d = \frac{1}{2}(8) = 4$

$V = \pi r^2 h = \pi(4)^2(22) = \pi(16)(22)$
$= 352\pi$

The volume of the cylinder is 352π ft^3.

You Try It 3

Strategy To find the surface area of the cylinder:

- Find the radius of the base of the cylinder. $d = 6$.
- Use the formula for the surface area of a cylinder. An approximation is asked for; use the π key on a calculator.

Solution

$r = \frac{1}{2}d = \frac{1}{2}(6) = 3$

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(3)^2 + 2\pi(3)(8)$

$SA = 2\pi(9) + 2\pi(3)(8)$

$SA = 18\pi + 48\pi$

$SA = 66\pi$

$SA \approx 207.35$

The surface area of the cylinder is approximately 207.35 ft^2.

You Try It 4

Strategy To find which solid has the larger surface area:

- Use the formula for the surface area of a cube to find the surface area of the cube. $s = 10$.
- Find the radius of the sphere. $d = 8$.

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- Use the formula for the surface area of a sphere to find the surface area of the sphere. Because this number is to be compared to another number, use the π key on a calculator to approximate the surface area.
- Compare the two numbers.

Solution

$SA = 6s^2$

$SA = 6(10)^2 = 6(100) = 600$

The surface area of the cube is 600 cm^2.

$r = \frac{1}{2}d = \frac{1}{2}(8) = 4$

$SA = 4\pi r^2$

$SA = 4\pi(4)^2 = 4\pi(16) = 64\pi \approx 201.06$

The surface area of the sphere is approximately 201.06 cm^2.

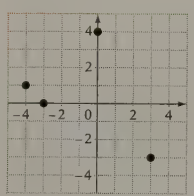
$600 > 201.06$

The cube has a larger surface area than the sphere.

Solutions to Chapter 4 "You Try It"

SECTION 4.1

You Try It 1



You Try It 2

The coordinates of A are $(4, -2)$.
The coordinates of B are $(-2, 4)$.
The abscissa of D is 0.
The ordinate of C is 0.

You Try It 3

$x - 3y = -14$	
$-2 - 3(4)$	-14
$-2 - 12$	-14
$-14 = -14$	

Yes, $(-2, 4)$ is a solution of $x - 3y = -14$.

You Try It 4

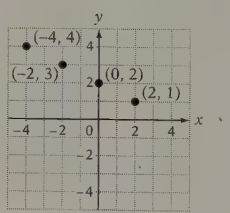
Replace x by -2 and solve for y.

$$y = \frac{3x}{x+1} = \frac{3(-2)}{-2+1} = \frac{-6}{-1} = 6$$

The ordered-pair solution is $(-2, 6)$.

You Try It 5

x	$y = -\frac{1}{2}x + 2$	y	(x, y)
-4	$-\frac{1}{2}(-4) + 2$	4	$(-4, 4)$
-2	$-\frac{1}{2}(-2) + 2$	3	$(-2, 3)$
0	$-\frac{1}{2}(0) + 2$	2	$(0, 2)$
2	$-\frac{1}{2}(2) + 2$	1	$(2, 1)$



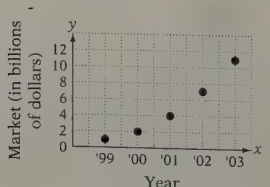
You Try It 6

Strategy

To draw a scatter diagram:

- Draw a coordinate grid with the horizontal axis representing the year and the vertical axis representing the software rental market, in billions of dollars.
- Graph the ordered pairs (1999, 1), (2000, 2), (2001, 4), (2002, 7), and (2003, 11).

Solution



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SECTION 4.2

You Try It 1 Domain: $\{-1, 3, 4, 6\}$
Range: $\{5\}$

You Try It 2
$$G(x) = \frac{3x}{x+2}$$
$$G(-4) = \frac{3(-4)}{-4+2} = \frac{-12}{-2} = 6$$

You Try It 3
$$f(x) = x^2 - 11$$
$$f(3h) = (3h)^2 - 11$$
$$= 9h^2 - 11$$

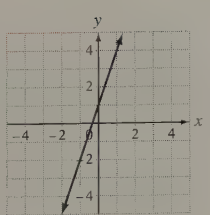
You Try It 4
$$h(z) = 3z + 1$$
$$h(0) = 3(0) + 1 = 1$$
$$h\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 1 = 2$$
$$h\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 1 = 3$$
$$h(1) = 3(1) + 1 = 4$$
The range is $\{1, 2, 3, 4\}$.

You Try It 5
$$f(x) = \frac{2}{x-5}$$
For $x = 5$, $f(5) = \frac{2}{5-5} = \frac{2}{0}$, which is not a real number.

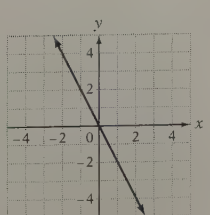
5 is excluded from the domain of the function.

SECTION 4.3

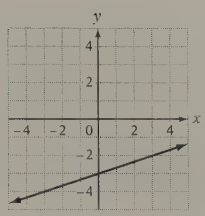
You Try It 1



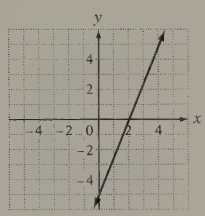
You Try It 2



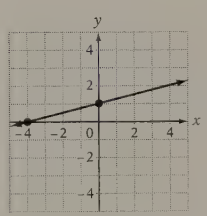
You Try It 3



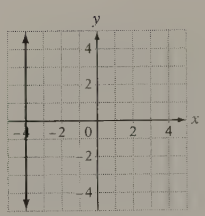
You Try It 4
$$5x - 2y = 10$$
$$-2y = -5x + 10$$
$$y = \frac{5}{2}x - 5$$



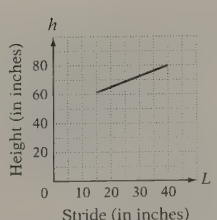
You Try It 5 x-intercept: $x - 4y = -4$
$$x - 4(0) = -4$$
$$x = -4$$
$(-4, 0)$
y-intercept: $x - 4y = -4$
$$0 - 4y = -4$$
$$-4y = -4$$
$$y = 1$$
$(0, 1)$



You Try It 6



You Try It 7



The ordered pair (32, 74) means that a person with a stride of 32 in. is 74 in. tall.

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SECTION 4.4

You Try It 1 Let $P_1 = (4, -3)$ and $P_2 = (2, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-3)}{2 - 4} = \frac{10}{-2} = -5$$

The slope is -5.

You Try It 2 Let $P_1 = (6, -1)$ and $P_2 = (6, 7)$.

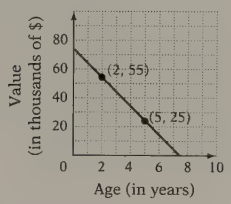
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{6 - 6} = \frac{8}{0}$$

Division by zero is not defined.

The slope of the line is undefined.

You Try It 3

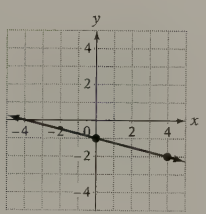
$$m = \frac{55{,}000 - 25{,}000}{2 - 5} = \frac{30{,}000}{-3} = -10{,}000$$



A slope of $-10{,}000$ means that the value of the printing press is decreasing by \$10,000 per year.

You Try It 4 y-intercept $= (0, b) = (0, -1)$

$$m = -\frac{1}{4}$$

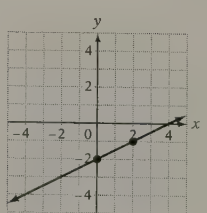


You Try It 5 Solve the equation for y.

$$x - 2y = 4$$
$$-2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$

y-intercept $= (0, b) = (0, -2)$

$$m = \frac{1}{2}$$



SECTION 4.5

You Try It 1 $m = -\frac{1}{3}$ $(x_1, y_1) = (-3, -2)$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{3}[x - (-3)]$$
$$y + 2 = -\frac{1}{3}(x + 3)$$
$$y + 2 = -\frac{1}{3}x - 1$$
$$y = -\frac{1}{3}x - 1 - 2$$
$$y = -\frac{1}{3}x - 3$$

The equation of the line is $y = -\frac{1}{3}x - 3$.

You Try It 2 $m = -3$ $(x_1, y_1) = (4, -3)$

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -3(x - 4)$$
$$y + 3 = -3(x - 4)$$
$$y + 3 = -3x + 12$$
$$y = -3x + 12 - 3$$
$$y = -3x + 9$$

The equation of the line is $y = -3x + 9$.

You Try It 3 Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{5 - 2} = \frac{3}{3} = 1$$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - 2)$$
$$y = 1(x - 2)$$
$$y = x - 2$$

The equation of the line is $y = x - 2$.

You Try It 4 Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (-5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{-5 - 2} = \frac{0}{-7} = 0$$

The line has zero slope.

The line is a horizontal line. All points on the line have an ordinate of 3.

The equation of the line is $y = 3$.

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You Try It 5

Strategy

- To find a linear equation that models the number of trucks that would be sold:
  Let y represent the number of trucks that would be sold.
  Then $y = 40{,}000$ when $x = 18{,}000$, and $y = 44{,}000$ when $x = 17{,}500$.
  Find the slope of the line through the points (18,000, 40,000) and (17,500, 44,000).
  Use the point-slope formula to write the linear equation.
- To find the number of trucks that would be sold at a price of \$16,500, substitute 16,500 for x in the equation and solve for y.

Solution

$(x_1, y_1) = (18{,}000, 40{,}000)$
$(x_2, y_2) = (17{,}500, 44{,}000)$

$$m = \frac{44{,}000 - 40{,}000}{17{,}500 - 18{,}000} = \frac{4000}{-500} = -8$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 40{,}000 &= -8(x - 18{,}000) \\ y - 40{,}000 &= -8x + 144{,}000 \\ y &= -8x + 184{,}000 \end{aligned}$$

A linear equation that models the number of trucks that would be sold is $y = -8x + 184{,}000$.

$$\begin{aligned} y &= -8x + 184{,}000 \\ y &= -8(16{,}500) + 184{,}000 \\ y &= -132{,}000 + 184{,}000 \\ y &= 52{,}000 \end{aligned}$$

At a price of \$16,500, 52,000 trucks would be sold.

SECTION 4.6

You Try It 1

$$m_1 = \frac{1 - (-3)}{7 - (-2)} = \frac{4}{9}$$

$$m_2 = \frac{-5 - 1}{6 - 4} = \frac{-6}{2} = -3$$

$$m_1 \cdot m_2 = \frac{4}{9} \cdot -3 = -\frac{4}{3}$$

No, the lines are not perpendicular.

You Try It 2

$$\begin{aligned} 5x + 2y &= 2 \\ 2y &= -5x + 2 \\ y &= -\frac{5}{2}x + 1 \end{aligned}$$

$$m_1 = -\frac{5}{2}$$

$$\begin{aligned} 5x + 2y &= -6 \\ 2y &= -5x - 6 \\ y &= -\frac{5}{2}x - 3 \end{aligned}$$

$$m_2 = -\frac{5}{2}$$

$$m_1 = m_2 = -\frac{5}{2}$$

Yes, the lines are parallel.

You Try It 3

$$y = \frac{1}{4}x - 3$$

$$m_1 = \frac{1}{4}$$

$$m_1 \cdot m_2 = -1$$

$$\frac{1}{4} \cdot m_2 = -1$$

$$m_2 = -4$$

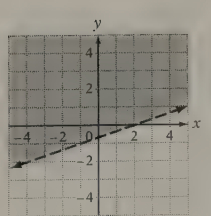
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -4[x - (-2)] \\ y - 2 &= -4(x + 2) \\ y - 2 &= -4x - 8 \\ y &= -4x - 6 \end{aligned}$$

The equation of the line is $y = -4x - 6$.

SECTION 4.7

You Try It 1

$$\begin{aligned} x - 3y &< 2 \\ x - x - 3y &< -x + 2 \\ -3y &< -x + 2 \\ \frac{-3y}{-3} &> \frac{-x + 2}{-3} \\ y &> \frac{1}{3}x - \frac{2}{3} \end{aligned}$$

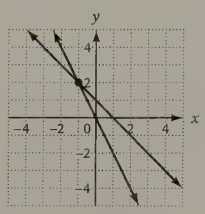


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Solutions to Chapter 5 "You Try It"

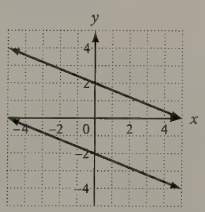
SECTION 5.1

You Try It 1



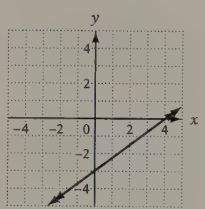
The solution is $(-1, 2)$.

You Try It 2



The lines are parallel and therefore do not intersect. The system of equations has no solution. The system of equations is inconsistent.

You Try It 3



The two equations represent the same line. The system of equations is dependent. The solutions are the ordered pairs $\left(x, \frac{3}{4}x - 3\right)$.

You Try It 4

(1) $3x - y = 3$
(2) $6x + 3y = -4$

Solve Equation (1) for y.

$$\begin{aligned} 3x - y &= 3 \\ -y &= -3x + 3 \\ y &= 3x - 3 \end{aligned}$$

Substitute into Equation (2).

$$\begin{aligned} 6x + 3y &= -4 \\ 6x + 3(3x - 3) &= -4 \\ 6x + 9x - 9 &= -4 \\ 15x - 9 &= -4 \\ 15x &= 5 \\ x &= \frac{5}{15} = \frac{1}{3} \end{aligned}$$

Substitute the value of x into Equation (1).

$$\begin{aligned} 3x - y &= 3 \\ 3\left(\frac{1}{3}\right) - y &= 3 \\ 1 - y &= 3 \\ -y &= 2 \\ y &= -2 \end{aligned}$$

The solution is $\left(\frac{1}{3}, -2\right)$.

You Try It 5

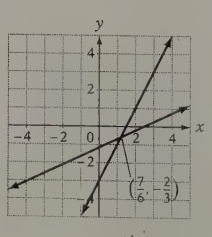
(1) $y = 2x - 3$
(2) $2x - 4y = 5$

$$\begin{aligned} 2x - 4y &= 5 \\ 2x - 4(2x - 3) &= 5 \\ 2x - 8x + 12 &= 5 \\ -6x + 12 &= 5 \\ -6x &= -7 \\ x &= \frac{-7}{-6} = \frac{7}{6} \end{aligned}$$

Substitute the value of x into Equation (1).

$$\begin{aligned} y &= 2x - 3 \\ y &= 2\left(\frac{7}{6}\right) - 3 \\ y &= \frac{7}{3} - 3 \\ y &= -\frac{2}{3} \end{aligned}$$

The solution is $\left(\frac{7}{6}, -\frac{2}{3}\right)$.



You Try It 6

(1) $6x - 3y = 6$
(2) $2x - y = 2$

Solve Equation (2) for y.

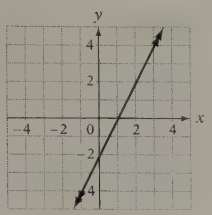
$$\begin{aligned} 2x - y &= 2 \\ -y &= -2x + 2 \\ y &= 2x - 2 \end{aligned}$$

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Substitute into Equation (1).

$$\begin{aligned} 6x - 3y &= 6 \\ 6x - 3(2x - 2) &= 6 \\ 6x - 6x + 6 &= 6 \\ 6 &= 6 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs $(x, 2x - 2)$.



SECTION 5.2

You Try It 1

(1) $2x + 5y = 6$
(2) $3x - 2y = 6x + 2$

Write Equation (2) in the form $Ax + By = C$.

$$\begin{aligned} 3x - 2y &= 6x + 2 \\ -3x - 2y &= 2 \end{aligned}$$

Solve the system: $2x + 5y = 6$, $-3x - 2y = 2$

Eliminate y.

$$\begin{aligned} 2(2x + 5y) &= 2(6) \\ 5(-3x - 2y) &= 5(2) \end{aligned}$$

$$\begin{aligned} 4x + 10y &= 12 \\ -15x - 10y &= 10 \end{aligned}$$

Add the equations.

$$\begin{aligned} -11x &= 22 \\ x &= -2 \end{aligned}$$

Replace x in Equation (1).

$$\begin{aligned} 2x + 5y &= 6 \\ 2(-2) + 5y &= 6 \\ -4 + 5y &= 6 \\ 5y &= 10 \\ y &= 2 \end{aligned}$$

The solution is $(-2, 2)$.

You Try It 2

$$\begin{aligned} 2x + y &= 5 \\ 4x + 2y &= 6 \end{aligned}$$

Eliminate y.

$$\begin{aligned} -2(2x + y) &= -2(5) \\ 4x + 2y &= 6 \end{aligned}$$

$$\begin{aligned} -4x - 2y &= -10 \\ 4x + 2y &= 6 \end{aligned}$$

Add the equations.

$$\begin{aligned} 0x + 0y &= -4 \\ 0 &= -4 \end{aligned}$$

This is not a true equation. The system is inconsistent and therefore has no solution.

You Try It 3

(1) $x - y + z = 6$
(2) $2x + 3y - z = 1$
(3) $x + 2y + 2z = 5$

Eliminate z. Add Equations (1) and (2).

$$\begin{aligned} x - y + z &= 6 \\ 2x + 3y - z &= 1 \\ (4)\quad 3x + 2y &= 7 \end{aligned}$$

Multiply Equation (2) by 2 and add to Equation (3).

$$\begin{aligned} 4x + 6y - 2z &= 2 \\ x + 2y + 2z &= 5 \\ (5)\quad 5x + 8y &= 7 \end{aligned}$$

Solve the system of two equations.

(4) $3x + 2y = 7$
(5) $5x + 8y = 7$

Multiply Equation (4) by -4 and add to Equation (5).

$$\begin{aligned} -12x - 8y &= -28 \\ 5x + 8y &= 7 \\ -7x &= -21 \\ x &= 3 \end{aligned}$$

Replace x by 3 in Equation (4).

$$\begin{aligned} 3x + 2y &= 7 \\ 3(3) + 2y &= 7 \\ 9 + 2y &= 7 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

Replace x by 3 and y by -1 in Equation (1).

$$\begin{aligned} x - y + z &= 6 \\ 3 - (-1) + z &= 6 \\ 4 + z &= 6 \\ z &= 2 \end{aligned}$$

The solution is $(3, -1, 2)$.

SECTION 5.3

You Try It 1

$$\begin{vmatrix} -1 & -4 \\ 3 & -5 \end{vmatrix} = -1(-5) - 3(-4) = 5 + 12 = 17$$

The value of the determinant is 17.

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You Try It 2 Expand by cofactors of the first row.

$$\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} + (-2)\begin{vmatrix} 3 & 1 \\ 0 & -2 \end{vmatrix}$$

$$= 1(2 + 2) - 4(6 - 0) - 2(-6 - 0)$$
$$= 4 - 24 + 12$$
$$= -8$$

The value of the determinant is -8.

You Try It 3

$$\begin{vmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \\ -2 & 1 & 3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} + 0\begin{vmatrix} 1 & 4 \\ -2 & 1 \end{vmatrix}$$

$$= 3(12 - 2) + 2(3 + 4) + 0$$
$$= 3(10) + 2(7)$$
$$= 30 + 14$$
$$= 44$$

The value of the determinant is 44.

You Try It 4

$$3x - y = 4$$
$$6x - 2y = 5$$

$$D = \begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix} = -6 + 6 = 0$$

Because $D = 0$, $\frac{D_x}{D}$ is undefined. Therefore, the system of equations is dependent or inconsistent.

It is not possible to solve this system by using Cramer's Rule.

You Try It 5

$$2x - y + z = -1$$
$$3x + 2y - z = 3$$
$$x + 3y + z = -2$$

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 21$$

$$D_x = \begin{vmatrix} -1 & -1 & 1 \\ 3 & 2 & -1 \\ -2 & 3 & 1 \end{vmatrix} = 9$$

$$D_y = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix} = -3$$

$$D_z = \begin{vmatrix} 2 & -1 & -1 \\ 3 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix} = -42$$

$$x = \frac{D_x}{D} = \frac{9}{21} = \frac{3}{7}$$

$$y = \frac{D_y}{D} = \frac{-3}{21} = -\frac{1}{7}$$

$$z = \frac{D_z}{D} = \frac{-42}{21} = -2$$

The solution is $\left(\frac{3}{7}, -\frac{1}{7}, -2\right)$.

SECTION 5.4

You Try It 1

Strategy

- Rate of the rowing team in calm water: t
  Rate of the current: c

	Rate	*Time*	*Distance*
With current	$t + c$	2	$2(t + c)$
Against current	$t - c$	2	$2(t - c)$

- The distance traveled with the current is 18 mi.
  The distance traveled against the current is 10 mi.

Solution

$$2(t + c) = 18 \qquad \frac{1}{2} \cdot 2(t + c) = \frac{1}{2} \cdot 18$$

$$2(t - c) = 10 \qquad \frac{1}{2} \cdot 2(t - c) = \frac{1}{2} \cdot 10$$

$$t + c = 9$$
$$t - c = 5$$
$$2t = 14$$
$$t = 7$$

$$t + c = 9$$
$$7 + c = 9$$
$$c = 2$$

The rate of the rowing team in calm water is 7 mph.

The rate of the current is 2 mph.

You Try It 2

Strategy

- Amount invested at 8%: x
  Amount invested at 7%: y

Investment	*Principal*	*Interest Rate*	*Interest Earned*
8%	x	0.08	$0.08x$
7%	y	0.07	$0.07y$

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- The total amount invested was \$15,000.
  The total amount of interest earned was \$1170.

Solution

$$\begin{aligned} x + y &= 15{,}000 \\ 0.08x + 0.07y &= 1170 \end{aligned}$$

$$\begin{aligned} -8x - 8y &= -120{,}000 \\ 8x + 7y &= 117{,}000 \\ -y &= -3000 \\ y &= 3000 \end{aligned}$$

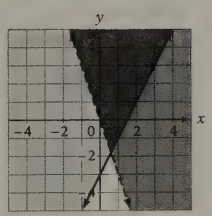
$$\begin{aligned} x + y &= 15{,}000 \\ x + 3000 &= 15{,}000 \\ x &= 12{,}000 \end{aligned}$$

\$12,000 was invested in the 8% account.
\$3000 was invested in the 7% account.

SECTION 5.5

You Try It 1 Shade above the solid line $y = 2x - 3$.
Shade above the dashed line $y = -3x$.

The solution set of the system is the intersection of the solution sets of the individual inequalities.



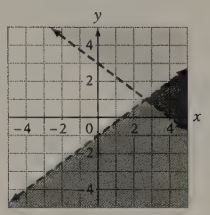
You Try It 2

$$\begin{aligned} 3x + 4y &> 12 \\ 4y &> -3x + 12 \\ y &> -\frac{3}{4}x + 3 \end{aligned}$$

Shade above the dashed line $y = -\frac{3}{4}x + 3$.

Shade below the dashed line $y = \frac{3}{4}x - 1$.

The solution set of the system is the intersection of the solution sets of the individual inequalities.



Solutions to Chapter 6 "You Try It"

SECTION 6.1

You Try It 1

$$\begin{aligned} (-3a^2b^4)(-2ab^3)^4 &= (-3a^2b^4)[(-2)^4a^4b^{12}] \\ &= (-3a^2b^4)(16a^4b^{12}) \\ &= -48a^6b^{16} \end{aligned}$$

You Try It 2 $(y^{n-3})^2 = y^{(n-3)2} = y^{2n-6}$

You Try It 3 $[(ab^3)^3]^4 = [a^3b^9]^4 = a^{12}b^{36}$

You Try It 4

$$\begin{aligned} \frac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}} &= -\frac{4 \cdot 5r^{-2-(-3)}s^2t^{-5}}{4 \cdot 4} \\ &= -\frac{5rs^2}{4t^5} \end{aligned}$$

You Try It 5

$$\begin{aligned} \frac{(9u^{-6}v^4)^{-1}}{(6u^{-3}v^{-2})^{-2}} &= \frac{9^{-1}u^6v^{-4}}{6^{-2}u^6v^4} \\ &= 9^{-1} \cdot 6^2u^0v^{-8} \\ &= \frac{36}{9v^8} \\ &= \frac{4}{v^8} \end{aligned}$$

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You Try It 6 $\dfrac{a^{2n+1}}{a^{n+3}} = a^{2n+1-(n+3)} = a^{2n+1-n-3} = a^{n-2}$

You Try It 7 $942{,}000{,}000 = 9.42 \times 10^8$

You Try It 8 $2.7 \times 10^{-5} = 0.000027$

You Try It 9

$$\frac{5{,}600{,}000 \times 0.000000081}{900 \times 0.000000028} = \frac{5.6 \times 10^6 \times 8.1 \times 10^{-8}}{9 \times 10^2 \times 2.8 \times 10^{-8}} = \frac{(5.6)(8.1) \times 10^{6+(-8)-2-(-8)}}{(9)(2.8)} = 1.8 \times 10^4 = 18{,}000$$

You Try It 10

Strategy To find the number of arithmetic operations:

- Find the reciprocal of 1×10^{-7}, which is the number of operations performed in one second.
- Write the number of seconds in one minute (60) in scientific notation.
- Multiply the number of arithmetic operations per second by the number of seconds in one minute.

Solution $\dfrac{1}{1 \times 10^{-7}} = 10^7$

$60 = 6 \times 10$

$6 \times 10 \times 10^7 = 6 \times 10^8$

The computer can perform 6×10^8 operations in one minute.

SECTION 6.2

You Try It 1

$$R(x) = -2x^4 - 5x^3 + 2x - 8$$
$$R(2) = -2(2)^4 - 5(2)^3 + 2(2) - 8 = -2(16) - 5(8) + 2(2) - 8 = -32 - 40 + 4 - 8 = -76$$

You Try It 2 The leading coefficient is -3, the constant term is -12, and the degree is 4.

You Try It 3

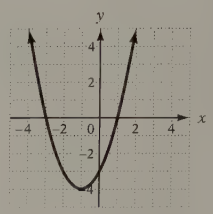
a. This is a polynomial function.

b. This is not a polynomial function. A polynomial function does not have a variable expression raised to a negative power.

c. This is not a polynomial function. A polynomial function does not have a variable expression within a radical.

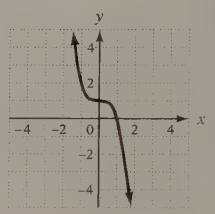
You Try It 4

x	$y = f(x)$
-3	0
-2	-3
-1	-4
0	-3
1	0
2	5



You Try It 5

x	$y = f(x)$
-3	28
-2	9
-1	2
0	1
1	0
2	-7
3	-26



You Try It 6

$$\begin{array}{r} -3x^2 - 4x + 9 \\ -5x^2 - 7x + 1 \\ \hline -8x^2 - 11x + 10 \end{array}$$

You Try It 7 Add the additive inverse of $6x^2 + 3x - 7$ to $-5x^2 + 2x - 3$.

$$\begin{array}{r} -5x^2 + 2x - 3 \\ -6x^2 - 3x + 7 \\ \hline -11x^2 - x + 4 \end{array}$$

You Try It 8

$$S(x) = P(x) + R(x) = (4x^3 - 3x^2 + 2) + (-2x^2 + 2x - 3) = 4x^3 - 5x^2 + 2x - 1$$

$$S(-1) = 4(-1)^3 - 5(-1)^2 + 2(-1) - 1 = 4(-1) - 5(1) + 2(-1) - 1 = -4 - 5 - 2 - 1 = -12$$

You Try It 9

$$D(x) = P(x) - R(x)$$

$$D(x) = (5x^{2n} - 3x^n - 7) - (-2x^{2n} - 5x^n + 8) = (5x^{2n} - 3x^n - 7) + (2x^{2n} + 5x^n - 8) = 7x^{2n} + 2x^n - 15$$

SECTION 6.3

You Try It 1 $(-2y + 3)(-4y) = 8y^2 - 12y$

You Try It 2 $-a^2(3a^2 + 2a - 7) = -3a^4 - 2a^3 + 7a^2$

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You Try It 3

$$\begin{array}{r} 2y^3 + 2y^2 \quad - 3 \\ \hline 3y - 1 \\ \hline -2y^3 - 2y^2 \quad + 3 \\ 6y^4 + 6y^3 \quad - 9y \\ \hline 6y^4 + 4y^3 - 2y^2 - 9y + 3 \end{array}$$

You Try It 4

$$\begin{aligned}(4y - 5)(2y - 3) &= 8y^2 - 12y - 10y + 15 \\ &= 8y^2 - 22y + 15\end{aligned}$$

You Try It 5

$$\begin{aligned}(3b + 2)(3b - 5) &= 9b^2 - 15b + 6b - 10 \\ &= 9b^2 - 9b - 10\end{aligned}$$

You Try It 6 $(2a + 5c)(2a - 5c) = 4a^2 - 25c^2$

You Try It 7 $(3x + 2y)^2 = 9x^2 + 12xy + 4y^2$

You Try It 8

Strategy To find the area, replace the variables b and h in the equation $A = \frac{1}{2}bh$ by the given values and solve for A.

Solution $A = \frac{1}{2}bh$

$$A = \frac{1}{2}(2x + 6)(x - 4)$$
$$A = (x + 3)(x - 4)$$
$$A = x^2 - 4x + 3x - 12$$
$$A = x^2 - x - 12$$

The area is $(x^2 - x - 12)$ ft^2.

SECTION 6.4

You Try It 1

$$\begin{array}{r} 5x - 1 \\ 3x + 4\overline{)15x^2 + 17x - 20} \\ 15x^2 + 20x \\ \hline -3x - 20 \\ -3x - 4 \\ \hline -16 \end{array}$$

$$\frac{15x^2 + 17x - 20}{3x + 4} = 5x - 1 - \frac{16}{3x + 4}$$

You Try It 2

$$\begin{array}{r} x^2 + 3x - 1 \\ 3x - 1\overline{)3x^3 + 8x^2 - 6x + 2} \\ 3x^3 - x^2 \\ \hline 9x^2 - 6x \\ 9x^2 - 3x \\ \hline -3x + 2 \\ -3x + 1 \\ \hline 1 \end{array}$$

$$\frac{3x^3 + 8x^2 - 6x + 2}{3x - 1} = x^2 + 3x - 1 + \frac{1}{3x - 1}$$

You Try It 3

$$\begin{array}{r} 3x^2 - 2x + 4 \\ x^2 - 3x + 2\overline{)3x^4 - 11x^3 + 16x^2 - 16x + 8} \\ 3x^4 - 9x^3 + 6x^2 \\ \hline -2x^3 + 10x^2 - 16x \\ -2x^3 + 6x^2 - 4x \\ \hline 4x^2 - 12x + 8 \\ 4x^2 - 12x + 8 \\ \hline 0 \end{array}$$

$$\frac{3x^4 - 11x^3 + 16x^2 - 16x + 8}{x^2 - 3x + 2} = 3x^2 - 2x + 4$$

You Try It 4

$$\begin{array}{r|rrr} -2 & 6 & 8 & -5 \\ & & -12 & 8 \\ \hline & 6 & -4 & 3 \end{array}$$

$$(6x^2 + 8x - 5) \div (x + 2) = 6x - 4 + \frac{3}{x + 2}$$

You Try It 5

$$\begin{array}{r|rrrr} 2 & 5 & -12 & -8 & 16 \\ & & 10 & -4 & -24 \\ \hline & 5 & -2 & -12 & -8 \end{array}$$

$$(5x^3 - 12x^2 - 8x + 16) \div (x - 2)$$
$$= 5x^2 - 2x - 12 - \frac{8}{x - 2}$$

You Try It 6

$$\begin{array}{r|rrrrr} 3 & 2 & -3 & -8 & 0 & -2 \\ & & 6 & 9 & 3 & 9 \\ \hline & 2 & 3 & 1 & 3 & 7 \end{array}$$

$$(2x^4 - 3x^3 - 8x^2 - 2) \div (x - 3)$$
$$= 2x^3 + 3x^2 + x + 3 + \frac{7}{x - 3}$$

You Try It 7

$$\begin{array}{r|rrrr} -3 & 2 & -5 & 0 & 7 \\ & & -6 & 33 & -99 \\ \hline & 2 & -11 & 33 & -92 \end{array}$$

$P(-3) = -92$

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Solutions to Chapter 7 "You Try It"

SECTION 7.1

You Try It 1
The GCF is $7a^2$.

$$14a^2 - 21a^4b = 7a^2(2) + 7a^2(-3a^2b)$$
$$= 7a^2(2 - 3a^2b)$$

You Try It 2
The GCF is 9.

$$27b^2 + 18b + 9 = 9(3b^2) + 9(2b) + 9(1)$$
$$= 9(3b^2 + 2b + 1)$$

You Try It 3
The GCF is $3x^2y^2$.

$$6x^4y^2 - 9x^3y^2 + 12x^2y^4$$
$$= 3x^2y^2(2x^2) + 3x^2y^2(-3x) + 3x^2y^2(4y^2)$$
$$= 3x^2y^2(2x^2 - 3x + 4y^2)$$

You Try It 4
The GCF is a^2.

$$a^{n+4} + a^2 = a^2(a^{n+2}) + a^2(1)$$
$$= a^2(a^{n+2} + 1)$$

You Try It 5
$$2y(5x - 2) - 3(2 - 5x)$$
$$= 2y(5x - 2) + 3(5x - 2)$$
$$= (5x - 2)(2y + 3)$$

You Try It 6
$$a^2 - 3a + 2ab - 6b$$
$$= (a^2 - 3a) + (2ab - 6b)$$
$$= a(a - 3) + 2b(a - 3)$$
$$= (a - 3)(a + 2b)$$

You Try It 7
$$2mn^2 - n + 8mn - 4 = (2mn^2 - n) + (8mn - 4)$$
$$= n(2mn - 1) + 4(2mn - 1)$$
$$= (2mn - 1)(n + 4)$$

You Try It 8
$$3xy - 9y - 12 + 4x$$
$$= (3xy - 6y) - (12 - 4x)$$
$$= 3y(x - 3) - 4(3 - x)$$
$$= 3y(x - 3) + 4(x - 3)$$
$$= (x - 3)(3y + 4)$$

SECTION 7.2

You Try It 1

Factors	*Sum*
1, 20	21
2, 10	12
4, 5	9

• Find the positive factors of 20 whose sum is 9.

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

You Try It 2

Factors	*Sum*
+1, −18	−17
−1, +18	17
+2, −9	−7
−2, +9	7
+3, −6	−3
−3, +6	+3

• Find the factors of −18 whose sum is 7.

$$x^2 + 7x - 18 = (x + 9)(x - 2)$$

You Try It 3
The GCF is $-2x$.

$$-2x^3 + 14x^2 - 12x = -2x(x^2 - 7x + 6)$$

Factor the trinomial $x^2 - 7x + 6$. Find two negative factors of 6 whose sum is −7.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$$-2x^3 + 14x^2 - 12x = -2x(x - 6)(x - 1)$$

You Try It 4
The GCF is 3.

$$3x^2 - 9xy - 12y^2 = 3(x^2 - 3xy - 4y^2)$$

Factor the trinomial. Find the factors of −4 whose sum is −3.

Factors	*Sum*
+1, −4	−3
−1, +4	3
+2, −2	0

$$3x^2 - 9xy - 12y^2 = 3(x + y)(x - 4y)$$

SECTION 7.3

You Try It 1
Factor the trinomial $2x^2 - x - 3$.

Positive factors of 2: 1, 2

Factors of −3: 1, −3; −1, 3

Trial Factors	*Middle Term*
$(x + 1)(2x - 3)$	$-3x + 2x = -x$
$(x - 3)(2x + 1)$	$x - 6x = -5x$
$(x - 1)(2x + 3)$	$3x - 2x = x$
$(x + 3)(2x - 1)$	$-x + 6x = 5x$

$$2x^2 - x - 3 = (x + 1)(2x - 3)$$

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You Try It 2
The GCF is $-3y$.

$$-45y^3 + 12y^2 + 12y = -3y(15y^2 - 4y - 4)$$

Factor the trinomial $15y^2 - 4y - 4$.

Positive factors of 15:	Factors of -4:
1, 15	1, -4
	-1, 4
3, 5	2, -2

Trial Factors	*Middle Term*
$(y + 1)(15y - 4)$	$-4y + 15y = 11y$
$(y - 4)(15y + 1)$	$y - 60y = -59y$
$(y - 1)(15y + 4)$	$4y - 15y = -11y$
$(y + 4)(15y - 1)$	$-y + 60y = 59y$
$(y + 2)(15y - 2)$	$-2y + 30y = 28y$
$(y - 2)(15y + 2)$	$2y - 30y = -28y$
$(3y + 1)(5y - 4)$	$-12y + 5y = -7y$
$(3y - 4)(5y + 1)$	$3y - 20y = -17y$
$(3y - 1)(5y + 4)$	$12y - 5y = 7y$
$(3y + 4)(5y - 1)$	$-3y + 20y = 17y$
$(3y + 2)(5y - 2)$	$-6y + 10y = 4y$
$(3y - 2)(5y + 2)$	$6y - 10y = -4y$

$$-45y^3 + 12y^2 + 12y = -3y(3y - 2)(5y + 2)$$

You Try It 3

Factors of -14 $[2(-7)]$	*Sum*
1, -14	-13
-1, 14	13
2, -7	-5
-2, 7	5

$$\begin{aligned} 2a^2 + 13a - 7 &= 2a^2 - a + 14a - 7 \\ &= (2a^2 - a) + (14a - 7) \\ &= a(2a - 1) + 7(2a - 1) \\ &= (2a - 1)(a + 7) \end{aligned}$$

$$2a^2 + 13a - 7 = (2a - 1)(a + 7)$$

You Try It 4
The GCF is $5x$.

$$15x^3 + 40x^2 - 80x = 5x(3x^2 + 8x - 16)$$

Factors of -48 $[3(-16)]$	*Sum*
1, -48	-47
-1, 48	47
2, -24	-22
-2, 24	22
3, -16	-13
-3, 16	13
4, -12	-8
-4, 12	8

$$\begin{aligned} 3x^2 + 8x - 16 &= 3x^2 - 4x + 12x - 16 \\ &= (3x^2 - 4x) + (12x - 16) \\ &= x(3x - 4) + 4(3x - 4) \\ &= (3x - 4)(x + 4) \end{aligned}$$

$$\begin{aligned} 15x^3 + 40x^2 - 80x &= 5x(3x^2 + 8x - 16) \\ &= 5x(3x - 4)(x + 4) \end{aligned}$$

SECTION 7.4

You Try It 1
$$\begin{aligned} x^2 - 36y^4 &= x^2 - (6y^2)^2 \\ &= (x + 6y^2)(x - 6y^2) \end{aligned}$$

You Try It 2
$$9x^2 + 12x + 4 = (3x + 2)^2$$

You Try It 3
$$\begin{aligned} &(a + b)^2 - (a - b)^2 \\ &= [(a + b) + (a - b)][(a + b) - (a - b)] \\ &= (a + b + a - b)(a + b - a + b) \\ &= (2a)(2b) = 4ab \end{aligned}$$

You Try It 4
$$\begin{aligned} 8x^3 + y^3z^3 &= (2x)^3 + (yz)^3 \\ &= (2x + yz)(4x^2 - 2xyz + y^2z^2) \end{aligned}$$

You Try It 5
$$\begin{aligned} &(x - y)^3 + (x + y)^3 \\ &= [(x - y) + (x + y)][(x - y)^2 - (x - y)(x + y) + (x + y)^2] \\ &= 2x[x^2 - 2xy + y^2 - (x^2 - y^2) + x^2 + 2xy + y^2] \\ &= 2x(x^2 - 2xy + y^2 - x^2 + y^2 + x^2 + 2xy + y^2) \\ &= 2x(x^2 + 3y^2) \end{aligned}$$

You Try It 6
Let $u = xy$.

$$\begin{aligned} 6x^2y^2 - 19xy + 10 &= 6u^2 - 19u + 10 \\ &= (2u - 5)(3u - 2) \\ &= (2xy - 5)(3xy - 2) \end{aligned}$$

You Try It 7
Let $u = x^2$.

$$\begin{aligned} 3x^4 + 4x^2 - 4 &= 3u^2 + 4u - 4 \\ &= (u + 2)(3u - 2) \\ &= (x^2 + 2)(3x^2 - 2) \end{aligned}$$

You Try It 8
Let $u = a^2b^2$.

$$\begin{aligned} a^4b^4 + 6a^2b^2 - 7 &= u^2 + 6u - 7 \\ &= (u + 7)(u - 1) \\ &= (a^2b^2 + 7)(a^2b^2 - 1) \end{aligned}$$

You Try It 9
$$\begin{aligned} 18x^3 - 6x^2 - 60x &= 6x(3x^2 - x - 10) \\ &= 6x(3x + 5)(x - 2) \end{aligned}$$

You Try It 10
$$\begin{aligned} &4x - 4y - x^3 + x^2y \\ &= (4x - 4y) - (x^3 - x^2y) \\ &= 4(x - y) - x^2(x - y) \\ &= (x - y)(4 - x^2) \\ &= (x - y)(2 + x)(2 - x) \end{aligned}$$

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You Try It 11

$$\begin{aligned} x^{4n} - x^{2n}y^{2n} &= x^{2n+2n} - x^{2n}y^{2n} \\ &= x^{2n}(x^{2n} - y^{2n}) \\ &= x^{2n}[(x^n)^2 - (y^n)^2] \\ &= x^{2n}(x^n + y^n)(x^n - y^n) \end{aligned}$$

You Try It 12

$$\begin{aligned} ax^5 - ax^2y^6 &= ax^2(x^3 - y^6) \\ &= ax^2(x - y^2)(x^2 + xy^2 + y^4) \end{aligned}$$

SECTION 7.5

You Try It 1

$$2x(x + 7) = 0$$

$$2x = 0 \qquad x + 7 = 0$$

$$x = 0 \qquad x = -7$$

The solutions are 0 and −7.

You Try It 2

$$4x^2 - 9 = 0$$

$$(2x - 3)(2x + 3) = 0$$

$$2x - 3 = 0 \qquad 2x + 3 = 0$$

$$2x = 3 \qquad 2x = -3$$

$$x = \frac{3}{2} \qquad x = -\frac{3}{2}$$

The solutions are $\frac{3}{2}$ and $-\frac{3}{2}$.

You Try It 3

$$(x + 2)(x - 7) = 52$$

$$x^2 - 5x - 14 = 52$$

$$x^2 - 5x - 66 = 0$$

$$(x + 6)(x - 11) = 0$$

$$x + 6 = 0 \qquad x - 11 = 0$$

$$x = -6 \qquad x = 11$$

The solutions are −6 and 11.

You Try It 4

Strategy First positive consecutive integer: n
Second positive consecutive integer: $n + 1$

The sum of the squares of the two positive consecutive integers is 61.

Solution

$$n^2 + (n + 1)^2 = 61$$

$$n^2 + n^2 + 2n + 1 = 61$$

$$2n^2 + 2n + 1 = 61$$

$$2n^2 + 2n - 60 = 0$$

$$2(n^2 + n - 30) = 0$$

$$n^2 + n - 30 = 0$$

$$(n - 5)(n + 6) = 0$$

$$n - 5 = 0 \qquad n + 6 = 0$$

$$n = 5 \qquad n = -6$$

Because −6 is not a positive integer, it is not a solution.

$$n = 5$$

$$n + 1 = 5 + 1 = 6$$

The two integers are 5 and 6.

You Try It 5

Strategy Width = x
Length = $2x + 4$

The area of the rectangle is 96 in^2. Use the equation $A = L \cdot W$.

Solution

$$A = L \cdot W$$

$$96 = (2x + 4)x$$

$$96 = 2x^2 + 4x$$

$$0 = 2x^2 + 4x - 96$$

$$0 = 2(x^2 + 2x - 48)$$

$$0 = x^2 + 2x - 48$$

$$0 = (x + 8)(x - 6)$$

$$x + 8 = 0 \qquad x - 6 = 0$$

$$x = -8 \qquad x = 6$$

Because the width cannot be a negative number, −8 is not a solution.

$$x = 6$$

$$2x + 4 = 2(6) + 4 = 12 + 4 = 16$$

The width is 6 in.
The length is 16 in.

Solutions to Chapter 8 "You Try It"

SECTION 8.1

You Try It 1

$$\frac{6x^5y}{12x^2y^3} = \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot x^5y}{\underset{1}{2} \cdot 2 \cdot \underset{1}{3} \cdot x^2y^3} = \frac{x^3}{2y^2}$$

You Try It 2

$$\frac{x^2 + 2x - 24}{16 - x^2} = \frac{x^2 + 2x - 24}{-(x^2 - 16)} = \frac{\overset{1}{(x - 4)}(x + 6)}{-\underset{1}{(x - 4)}(x + 4)}$$

$$= -\frac{x + 6}{x + 4}$$

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You Try It 3 $\dfrac{x^2+4x-12}{x^2-3x+2} = \dfrac{\cancel{(x-2)}^{1}(x+6)}{(x-1)\cancel{(x-2)}_{1}} = \dfrac{x+6}{x-1}$

You Try It 4

$$\frac{12x^2+3x}{10x-15}\cdot\frac{8x-12}{9x+18} = \frac{3x(4x+1)}{5(2x-3)}\cdot\frac{4(2x-3)}{9(x+2)}$$

$$= \frac{\overset{1}{\cancel{3}x(4x+1)\cdot 2\cdot 2}\overset{1}{\cancel{(2x-3)}}}{5\underset{1}{\cancel{(2x-3)}}\cdot\underset{1}{\cancel{3}}\cdot 3(x+2)}$$

$$= \frac{4x(4x+1)}{15(x+2)}$$

You Try It 5

$$\frac{x^2+2x-15}{9-x^2}\cdot\frac{x^2-3x-18}{x^2-7x+6}$$

$$= \frac{(x-3)(x+5)}{(3-x)(3+x)}\cdot\frac{(x+3)(x-6)}{(x-1)(x-6)}$$

$$= \frac{\overset{-1}{\cancel{(x-3)}}(x+5)\overset{1}{\cancel{(x+3)}}\overset{1}{\cancel{(x-6)}}}{\underset{1}{\cancel{(3-x)}}\underset{1}{\cancel{(3+x)}}(x-1)\underset{1}{\cancel{(x-6)}}} = -\frac{x+5}{x-1}$$

You Try It 6

$$\frac{a^2}{4bc^2-2b^2c}\div\frac{a}{6bc-3b^2} = \frac{a^2}{4bc^2-2b^2c}\cdot\frac{6bc-3b^2}{a}$$

$$= \frac{a^2\cdot 3b\overset{1}{\cancel{(2c-b)}}}{2bc\underset{1}{\cancel{(2c-b)}}\cdot a} = \frac{3a}{2c}$$

You Try It 7

$$\frac{3x^2+26x+16}{3x^2-7x-6}\div\frac{2x^2+9x-5}{x^2+2x-15}$$

$$= \frac{3x^2+26x+16}{3x^2-7x-6}\cdot\frac{x^2+2x-15}{2x^2+9x-5}$$

$$= \frac{\overset{1}{\cancel{(3x+2)}}(x+8)\cdot\overset{1}{\cancel{(x+5)}}\overset{1}{\cancel{(x-3)}}}{\underset{1}{\cancel{(3x+2)}}\underset{1}{\cancel{(x-3)}}\cdot(2x-1)\underset{1}{\cancel{(x+5)}}} = \frac{x+8}{2x-1}$$

SECTION 8.2

You Try It 1

$8uv^2 = 2\cdot 2\cdot 2\cdot u\cdot v\cdot v$

$12uw = 2\cdot 2\cdot 3\cdot u\cdot w$

LCM $= 2\cdot 2\cdot 2\cdot 3\cdot u\cdot v\cdot v\cdot w = 24uv^2w$

You Try It 2

$m^2-6m+9 = (m-3)(m-3)$

$m^2-2m-3 = (m+1)(m-3)$

LCM $= (m-3)(m-3)(m+1)$

You Try It 3

The LCM is $36xy^2z$.

$$\frac{x-3}{4xy^2} = \frac{x-3}{4xy^2}\cdot\frac{9z}{9z} = \frac{9xz-27z}{36xy^2z}$$

$$\frac{2x+1}{9y^2z} = \frac{2x+1}{9y^2z}\cdot\frac{4x}{4x} = \frac{8x^2+4x}{36xy^2z}$$

You Try It 4

The LCM is $(x+2)(x-5)(x+5)$

$$\frac{x+4}{x^2-3x-10} = \frac{x+4}{(x+2)(x-5)}\cdot\frac{x+5}{x+5}$$

$$= \frac{x^2+9x+20}{(x+2)(x-5)(x+5)}$$

$$\frac{2x}{25-x^2} = \frac{2x}{-(x^2-25)} = -\frac{2x}{(x-5)(x+5)}\cdot\frac{x+2}{x+2}$$

$$= -\frac{2x^2+4x}{(x+2)(x-5)(x+5)}$$

You Try It 5 $\dfrac{3}{xy}+\dfrac{12}{xy} = \dfrac{3+12}{xy} = \dfrac{15}{xy}$

You Try It 6

$$\frac{2x^2}{x^2-x-12}-\frac{7x+4}{x^2-x-12}$$

$$= \frac{2x^2-(7x+4)}{x^2-x-12} = \frac{2x^2-7x-4}{x^2-x-12}$$

$$= \frac{(2x+1)\overset{1}{\cancel{(x-4)}}}{(x+3)\underset{1}{\cancel{(x-4)}}} = \frac{2x+1}{x+3}$$

You Try It 7

$$\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}$$

$$= \frac{(x^2-1)-(2x+1)+x}{x^2-8x+12} = \frac{x^2-1-2x-1+x}{x^2-8x+12}$$

$$= \frac{x^2-x-2}{x^2-8x+12} = \frac{(x+1)\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{(x-2)}}(x-6)} = \frac{x+1}{x-6}$$

You Try It 8

The LCM of the denominators is $24y$.

$$\frac{z}{8y}-\frac{4z}{3y}+\frac{5z}{4y} = \frac{z}{8y}\cdot\frac{3}{3}-\frac{4z}{3y}\cdot\frac{8}{8}+\frac{5z}{4y}\cdot\frac{6}{6}$$

$$= \frac{3z}{24y}-\frac{32z}{24y}+\frac{30z}{24y}$$

$$= \frac{3z-32z+30z}{24y} = \frac{z}{24y}$$

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You Try It 9

$2 - x = -(x - 2)$

Therefore, $\frac{3}{2-x} = \frac{-3}{x-2}$.

The LCM is $x - 2$.

$$\frac{5x}{x-2} - \frac{3}{2-x} = \frac{5x}{x-2} - \frac{-3}{x-2}$$
$$= \frac{5x-(-3)}{x-2} = \frac{5x+3}{x-2}$$

You Try It 10

The LCM is $(3x - 1)(x + 4)$.

$$\frac{4x}{3x-1} - \frac{9}{x+4} = \frac{4x}{3x-1}\cdot\frac{x+4}{x+4} - \frac{9}{x+4}\cdot\frac{3x-1}{3x-1}$$
$$= \frac{4x^2+16x}{(3x-1)(x+4)} - \frac{27x-9}{(3x-1)(x+4)}$$
$$= \frac{(4x^2+16x)-(27x-9)}{(3x-1)(x+4)}$$
$$= \frac{4x^2+16x-27x+9}{(3x-1)(x+4)}$$
$$= \frac{4x^2-11x+9}{(3x-1)(x+4)}$$

You Try It 11

The LCM is $x - 3$.

$$2 - \frac{1}{x-3} = \frac{2}{1}\cdot\frac{x-3}{x-3} - \frac{1}{x-3}$$
$$= \frac{2x-6}{x-3} - \frac{1}{x-3}$$
$$= \frac{2x-6-1}{x-3}$$
$$= \frac{2x-7}{x-3}$$

You Try It 12

The LCM is $a(a - 5)(a + 5)$.

$$\frac{a-3}{a^2-5a} + \frac{a-9}{a^2-25}$$
$$= \frac{a-3}{a(a-5)}\cdot\frac{a+5}{a+5} + \frac{a-9}{(a-5)(a+5)}\cdot\frac{a}{a}$$
$$= \frac{(a-3)(a+5)+a(a-9)}{a(a-5)(a+5)}$$
$$= \frac{(a^2+2a-15)+(a^2-9a)}{a(a-5)(a+5)}$$
$$= \frac{a^2+2a-15+a^2-9a}{a(a-5)(a+5)}$$
$$= \frac{2a^2-7a-15}{a(a-5)(a+5)} = \frac{(2a+3)(a-5)}{a(a-5)(a+5)}$$
$$= \frac{(2a+3)\cancel{(a-5)}}{a\cancel{(a-5)}(a+5)} = \frac{2a+3}{a(a+5)}$$

You Try It 13

The LCM is $(3x + 2)(x - 1)$.

$$\frac{2x-3}{3x^2-x-2} + \frac{5}{3x+2} - \frac{1}{x-1}$$
$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5}{3x+2}\cdot\frac{x-1}{x-1} - \frac{1}{x-1}\cdot\frac{3x+2}{3x+2}$$
$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5x-5}{(3x+2)(x-1)} - \frac{3x+2}{(3x+2)(x-1)}$$
$$= \frac{(2x-3)+(5x-5)-(3x+2)}{(3x+2)(x-1)}$$
$$= \frac{2x-3+5x-5-3x-2}{(3x+2)(x-1)}$$
$$= \frac{4x-10}{(3x+2)(x-1)} = \frac{2(2x-5)}{(3x+2)(x-1)}$$

SECTION 8.3

You Try It 1

The LCM of 3, x, 9, and x^2 is $9x^2$.

$$\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}} = \frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}\cdot\frac{9x^2}{9x^2} = \frac{\frac{1}{3}\cdot 9x^2 - \frac{1}{x}\cdot 9x^2}{\frac{1}{9}\cdot 9x^2 - \frac{1}{x^2}\cdot 9x^2}$$
$$= \frac{3x^2-9x}{x^2-9} = \frac{3x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{3x}{x+3}$$

You Try It 2

The LCM is $x - 5$.

$$\frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x+5}} = \frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}\cdot\frac{x-5}{x-5}$$
$$= \frac{x(x-5)+3(x-5)-\frac{20}{x-5}\cdot(x-5)}{x(x-5)+8(x-5)+\frac{30}{x-5}\cdot(x-5)}$$
$$= \frac{x^2-5x+3x-15-20}{x^2-5x+8x-40+30}$$
$$= \frac{x^2-2x-35}{x^2+3x-10}$$
$$= \frac{\cancel{(x+5)}(x-7)}{(x-2)\cancel{(x+5)}} = \frac{x-7}{x-2}$$

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SECTION 8.4

You Try It 1

$$\frac{x}{x+6} = \frac{3}{x} \quad \text{The LCM is } x(x+6).$$

$$\frac{\cancel{x(x+6)}}{1} \cdot \frac{x}{\cancel{x+6}} = \frac{x(x+6)}{1} \cdot \frac{3}{x}$$

$$x^2 = (x+6)3$$
$$x^2 = 3x + 18$$
$$x^2 - 3x - 18 = 0$$
$$(x+3)(x-6) = 0$$

$$x + 3 = 0 \qquad x - 6 = 0$$
$$x = -3 \qquad x = 6$$

Both -3 and 6 check as solutions.
The solutions are -3 and 6.

You Try It 2

$$\frac{5x}{x+2} = 3 - \frac{10}{x+2} \quad \text{The LCM is } x+2.$$

$$\frac{(x+2)}{1} \cdot \frac{5x}{x+2} = \frac{(x+2)}{1}\left(3 - \frac{10}{x+2}\right)$$

$$\frac{\cancel{x+2}}{1} \cdot \frac{5x}{\cancel{x+2}} = (x+2)3 - \frac{\cancel{(x+2)}}{1} \cdot \frac{10}{\cancel{x+2}}$$

$$5x = (x+2)3 - 10$$
$$5x = 3x + 6 - 10$$
$$5x = 3x - 4$$
$$2x = -4$$
$$x = -2$$

-2 does not check as a solution.
The equation has no solution.

You Try It 3

$$\frac{2}{x+3} = \frac{6}{5x+5}$$

$$\frac{(x+3)(5x+5)}{1} \cdot \frac{2}{x+3} = \frac{(x+3)(5x+5)}{1} \cdot \frac{6}{5x+5}$$

$$\frac{\cancel{(x+3)}(5x+5)}{1} \cdot \frac{2}{\cancel{x+3}} = \frac{(x+3)\cancel{(5x+5)}}{1} \cdot \frac{6}{\cancel{5x+5}}$$

$$(5x+5)2 = (x+3)6$$
$$10x + 10 = 6x + 18$$
$$4x + 10 = 18$$
$$4x = 8$$
$$x = 2$$

The solution is 2.

You Try It 4

Strategy To find the area of triangle AOB:

- Solve a proportion to find the length of AO (the height of triangle AOB).
- Use the formula for the area of a triangle. AB is the base and AO is the height.

Solution

$$\frac{CD}{AB} = \frac{DO}{AO}$$
$$\frac{4}{10} = \frac{3}{AO}$$
$$10 \cdot AO \cdot \frac{4}{10} = 10 \cdot AO \cdot \frac{3}{AO}$$
$$4(AO) = 30$$
$$AO = 7.5$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(7.5)$$
$$= 37.5$$

The area of triangle AOB is 37.5 cm^2.

You Try It 5

Strategy To find the total area that 256 ceramic tiles will cover, write and solve a proportion, using x to represent the number of square feet that 256 tiles will cover.

Solution

$$\frac{9}{16} = \frac{x}{256}$$
$$256\left(\frac{9}{16}\right) = 256\left(\frac{x}{256}\right)$$
$$144 = x$$

A 144-square-foot area can be tiled using 256 ceramic tiles.

You Try It 6

Strategy To find the additional amount of medication required for a 200-pound adult, write and solve a proportion, using x to represent the additional medication. Then $3 + x$ is the total amount required for a 200-pound adult.

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Solution

$$\frac{150}{3} = \frac{200}{3+x}$$

$$\frac{50}{1} = \frac{200}{3+x}$$

$$(3+x)\cdot 50 = (3+x)\cdot\frac{200}{3+x}$$

$$150 + 50x = 200$$

$$50x = 50$$

$$x = 1$$

One additional ounce is required for a 200-pound adult.

SECTION 8.5

You Try It 1

$$A = P + Prt$$

$$A - P = P - P + Prt$$

$$A - P = Prt$$

$$\frac{A-P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A-P}{Pt} = r$$

You Try It 2

$$s = \frac{A+L}{2}$$

$$2\cdot s = 2\left(\frac{A+L}{2}\right)$$

$$2s = A + L$$

$$2s - A = A - A + L$$

$$2s - A = L$$

You Try It 3

$$s = a + (n-1)d$$

$$s = a + nd - d$$

$$s - a = a - a + nd - d$$

$$s - a = nd - d$$

$$s - a + d = nd - d + d$$

$$s - a + d = nd$$

$$\frac{s-a+d}{d} = \frac{nd}{d}$$

$$\frac{s-a+d}{d} = n$$

You Try It 4

$$S = C + rC$$

$$S = (1+r)C$$

$$\frac{S}{1+r} = \frac{(1+r)C}{1+r}$$

$$\frac{S}{1+r} = C$$

SECTION 8.6

You Try It 1

Strategy

- Time for one printer to complete the job: t

	Rate	Time	Part
1st printer	$\frac{1}{t}$	2	$\frac{2}{t}$
2nd printer	$\frac{1}{t}$	5	$\frac{5}{t}$

- The sum of the parts of the task completed must equal 1.

Solution

$$\frac{2}{t} + \frac{5}{t} = 1$$

$$t\left(\frac{2}{t} + \frac{5}{t}\right) = t\cdot 1$$

$$2 + 5 = t$$

$$7 = t$$

Working alone, one printer takes 7 h to print the payroll.

You Try It 2

Strategy

- Rate sailing across the lake: r
  Rate sailing back: $3r$

	Distance	Rate	Time
Across	6	r	$\frac{6}{r}$
Back	6	$3r$	$\frac{6}{3r}$

- The total time for the trip was 2 h.

Solution

$$\frac{6}{r} + \frac{6}{3r} = 2$$

$$3r\left(\frac{6}{r} + \frac{6}{3r}\right) = 3r(2)$$

$$3r\cdot\frac{6}{r} + 3r\cdot\frac{6}{3r} = 6r$$

$$18 + 6 = 6r$$

$$24 = 6r$$

$$4 = r$$

The rate across the lake was 4 km/h.

SECTION 8.7

You Try It 1

Strategy To find the distance:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.

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- Write the direct variation equation, replacing k by its value. Substitute 5 for t and solve for s.

Solution

$$s = kt^2$$
$$64 = k(2)^2$$
$$64 = k \cdot 4$$
$$16 = k$$
$$s = 16t^2 = 16(5)^2 = 400$$

The object will fall 400 ft in 5 s.

You Try It 2

Strategy To find the resistance:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 0.02 for d and solve for R.

Solution

$$R = \frac{k}{d^2}$$
$$0.5 = \frac{k}{(0.01)^2}$$
$$0.5 = \frac{k}{0.0001}$$
$$0.00005 = k$$
$$R = \frac{0.00005}{d^2} = \frac{0.00005}{(0.02)^2} = 0.125$$

The resistance is 0.125 ohm.

You Try It 3

Strategy To find the strength:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 4 for w, 8 for d, and 16 for L, and solve for s.

Solution

$$s = \frac{kwd^2}{L}$$
$$1200 = \frac{k \cdot 2 \cdot (12)^2}{12}$$
$$1200 = \frac{k \cdot 2 \cdot 144}{12}$$
$$1200 = 24k$$
$$50 = k$$
$$s = \frac{50wd^2}{L} = \frac{50 \cdot 4 \cdot (8)^2}{16} = 800$$

The strength is 800 lb.

Solutions to Chapter 9 "You Try It"

SECTION 9.1

You Try It 1

$$16^{-3/4} = (2^4)^{-3/4}$$
$$= 2^{-3}$$
$$= \frac{1}{2^3} = \frac{1}{8}$$

You Try It 2 $(-81)^{3/4}$

The base of the exponential expression is negative, and the denominator of the exponent is a positive even number.

Therefore, $(-81)^{3/4}$ is not a real number.

You Try It 3

$$(x^{3/4}y^{1/2}z^{-2/3})^{-4/3} = x^{-1}y^{-2/3}z^{8/9}$$
$$= \frac{z^{8/9}}{xy^{2/3}}$$

You Try It 4

$$\left(\frac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2} = \left(\frac{2^4a^{-6}b^2}{3^2}\right)^{-1/2}$$
$$= \frac{2^{-2}a^3b^{-1}}{3^{-1}}$$
$$= \frac{3a^3}{2^2b} = \frac{3a^3}{4b}$$

You Try It 5

$$(2x^3)^{3/4} = \sqrt[4]{(2x^3)^3}$$
$$= \sqrt[4]{8x^9}$$

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You Try It 6 $-5a^{5/6} = -5(a^5)^{1/6}$
$= -5\sqrt[6]{a^5}$

You Try It 7 $\sqrt[3]{3ab} = (3ab)^{1/3}$

You Try It 8 $\sqrt[4]{x^4 + y^4} = (x^4 + y^4)^{1/4}$

You Try It 9 $\sqrt[3]{-8x^{12}y^3} = -2x^4y$

You Try It 10 $-\sqrt[4]{81x^{12}y^8} = -3x^3y^2$

SECTION 9.2

You Try It 1
$$\sqrt[5]{x^7} = \sqrt[5]{x^5 \cdot x^2} = \sqrt[5]{x^5}\,\sqrt[5]{x^2} = x\sqrt[5]{x^2}$$

You Try It 2
$$\sqrt[3]{-64x^8y^{18}} = \sqrt[3]{(-4)^3x^8y^{18}} = \sqrt[3]{(-4)^3x^6y^{18}(x^2)} = \sqrt[3]{(-4)^3x^6y^{18}}\,\sqrt[3]{x^2} = -4x^2y^6\sqrt[3]{x^2}$$

You Try It 3
$$3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4} = 3xy\sqrt[3]{27x^3}\,\sqrt[3]{3x^2y} - \sqrt[3]{64x^6y^3}\,\sqrt[3]{3x^2y} = 3xy \cdot 3x\sqrt[3]{3x^2y} - 4x^2y\sqrt[3]{3x^2y} = 9x^2y\sqrt[3]{3x^2y} - 4x^2y\sqrt[3]{3x^2y} = 5x^2y\sqrt[3]{3x^2y}$$

You Try It 4
$$\sqrt{5b}(\sqrt{3b} - \sqrt{10}) = \sqrt{15b^2} - \sqrt{50b} = \sqrt{15b^2} - \sqrt{25 \cdot 2b} = \sqrt{b^2}\sqrt{15} - \sqrt{25}\sqrt{2b} = b\sqrt{15} - 5\sqrt{2b}$$

You Try It 5
$$(2\sqrt[3]{2x} - 3)(\sqrt[3]{2x} - 5) = 2\sqrt[3]{4x^2} - 10\sqrt[3]{2x} - 3\sqrt[3]{2x} + 15 = 2\sqrt[3]{4x^2} - 13\sqrt[3]{2x} + 15$$

You Try It 6
$$(\sqrt{a} - 3\sqrt{y})(\sqrt{a} + 3\sqrt{y}) = (\sqrt{a})^2 - (3\sqrt{y})^2 = a - 9y$$

You Try It 7
$$\frac{y}{\sqrt{3y}} = \frac{y}{\sqrt{3y}} \cdot \frac{\sqrt{3y}}{\sqrt{3y}} = \frac{y\sqrt{3y}}{(\sqrt{3y})^2} = \frac{y\sqrt{3y}}{3y} = \frac{\sqrt{3y}}{3}$$

You Try It 8
$$\frac{3x}{\sqrt[3]{3x^2}} = \frac{3x}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{9x}}{\sqrt[3]{9x}} = \frac{3x\sqrt[3]{9x}}{\sqrt[3]{27x^3}} = \frac{3x\sqrt[3]{9x}}{3} = \sqrt[3]{9x}$$

You Try It 9
$$\frac{3 + \sqrt{6}}{2 - \sqrt{6}} = \frac{3 + \sqrt{6}}{2 - \sqrt{6}} \cdot \frac{2 + \sqrt{6}}{2 + \sqrt{6}} = \frac{6 + 3\sqrt{6} + 2\sqrt{6} + (\sqrt{6})^2}{2^2 - (\sqrt{6})^2} = \frac{6 + 5\sqrt{6} + 6}{4 - 6} = \frac{12 + 5\sqrt{6}}{-2} = -\frac{12 + 5\sqrt{6}}{2}$$

SECTION 9.3

You Try It 1 $\sqrt{-45} = i\sqrt{45} = i\sqrt{9 \cdot 5} = 3i\sqrt{5}$

You Try It 2
$$\sqrt{98} - \sqrt{-60} = \sqrt{98} - i\sqrt{60} = \sqrt{49 \cdot 2} - i\sqrt{4 \cdot 15} = 7\sqrt{2} - 2i\sqrt{15}$$

You Try It 3 $(-4 + 2i) - (6 - 8i) = -10 + 10i$

You Try It 4
$$(16 - \sqrt{-45}) - (3 + \sqrt{-20}) = (16 - i\sqrt{45}) - (3 + i\sqrt{20}) = (16 - i\sqrt{9 \cdot 5}) - (3 + i\sqrt{4 \cdot 5}) = (16 - 3i\sqrt{5}) - (3 + 2i\sqrt{5}) = 13 - 5i\sqrt{5}$$

You Try It 5 $(3 - 2i) + (-3 + 2i) = 0 + 0i = 0$

You Try It 6 $(-3i)(-10i) = 30i^2 = 30(-1) = -30$

You Try It 7
$$-\sqrt{-8} \cdot \sqrt{-5} = -i\sqrt{8} \cdot i\sqrt{5} = -i^2\sqrt{40} = -(-1)\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

You Try It 8
$$-6i(3 + 4i) = -18i - 24i^2 = -18i - 24(-1) = 24 - 18i$$

You Try It 9
$$\sqrt{-3}(\sqrt{27} - \sqrt{-6}) = i\sqrt{3}(\sqrt{27} - i\sqrt{6}) = i\sqrt{81} - i^2\sqrt{18} = i\sqrt{81} - (-1)\sqrt{9 \cdot 2} = 9i + 3\sqrt{2} = 3\sqrt{2} + 9i$$

You Try It 10
$$(4 - 3i)(2 - i) = 8 - 4i - 6i + 3i^2 = 8 - 10i + 3i^2 = 8 - 10i + 3(-1) = 5 - 10i$$

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You Try It 11 $(3 + 6i)(3 - 6i) = 3^2 + 6^2$
$= 9 + 36$
$= 45$

You Try It 12
$$(3 - i)\left(\frac{3}{10} + \frac{1}{10}i\right) = \frac{9}{10} + \frac{3}{10}i - \frac{3}{10}i - \frac{1}{10}i^2$$
$$= \frac{9}{10} - \frac{1}{10}i^2 = \frac{9}{10} - \frac{1}{10}(-1)$$
$$= \frac{9}{10} + \frac{1}{10} = 1$$

You Try It 13
$$\frac{2 - 3i}{4i} = \frac{2 - 3i}{4i} \cdot \frac{i}{i}$$
$$= \frac{2i - 3i^2}{4i^2}$$
$$= \frac{2i - 3(-1)}{4(-1)}$$
$$= \frac{3 + 2i}{-4} = -\frac{3}{4} - \frac{1}{2}i$$

You Try It 14
$$\frac{2 + 5i}{3 - 2i} = \frac{2 + 5i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i} = \frac{6 + 4i + 15i + 10i^2}{3^2 + 2^2}$$
$$= \frac{6 + 19i + 10(-1)}{13} = \frac{-4 + 19i}{13}$$
$$= -\frac{4}{13} + \frac{19}{13}i$$

SECTION 9.4

You Try It 1
$\sqrt[4]{x - 8} = 3$
$(\sqrt[4]{x - 8})^4 = 3^4$
$x - 8 = 81$
$x = 89$

Check:
$\sqrt[4]{x - 8} = 3$
$\sqrt[4]{89 - 8} \mid 3$
$\sqrt[4]{81} \mid 3$
$3 = 3$

The solution is 89.

You Try It 2
$\sqrt{x} - \sqrt{x + 5} = 1$
$\sqrt{x} = 1 + \sqrt{x + 5}$
$(\sqrt{x})^2 = (1 + \sqrt{x + 5})^2$
$x = 1 + 2\sqrt{x + 5} + x + 5$
$0 = 6 + 2\sqrt{x + 5}$
$-6 = 2\sqrt{x + 5}$
$-3 = \sqrt{x + 5}$
$(-3)^2 = (\sqrt{x + 5})^2$
$9 = x + 5$
$4 = x$

4 does not check as a solution. The equation has no solution.

You Try It 3

Strategy To find the diagonal, use the Pythagorean Theorem. One leg is the length of the rectangle. The second leg is the width of the rectangle. The hypotenuse is the diagonal of the rectangle.

Solution
$c^2 = a^2 + b^2$
$c^2 = (6)^2 + (3)^2$
$c^2 = 36 + 9$
$c^2 = 45$
$(c^2)^{1/2} = (45)^{1/2}$
$c = \sqrt{45}$
$c \approx 6.7$

The diagonal is 6.7 cm.

You Try It 4

Strategy To find the height, replace d in the equation with the given value and solve for h.

Solution
$d = \sqrt{1.5h}$
$5.5 = \sqrt{1.5h}$
$(5.5)^2 = (\sqrt{1.5h})^2$
$30.25 = 1.5h$
$20.17 \approx h$

The periscope must be approximately 20.17 ft above the water.

You Try It 5

Strategy To find the distance, replace the variables v and a in the equation by their given values and solve for s.

Solution
$v = \sqrt{2as}$
$88 = \sqrt{2 \cdot 22s}$
$88 = \sqrt{44s}$
$(88)^2 = (\sqrt{44s})^2$
$7744 = 44s$
$176 = s$

The distance required is 176 ft.

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Solutions to Chapter 10 "You Try It"

SECTION 10.1

You Try It 1

$$2x^2 = 7x - 3$$
$$2x^2 - 7x + 3 = 0$$
$$(2x - 1)(x - 3) = 0$$

$2x - 1 = 0$ $\qquad$ $x - 3 = 0$
$2x = 1$ $\qquad$ $x = 3$
$x = \frac{1}{2}$

The solutions are $\frac{1}{2}$ and 3.

You Try It 2

$$x^2 - 3ax - 4a^2 = 0$$
$$(x + a)(x - 4a) = 0$$

$x + a = 0$ $\qquad$ $x - 4a = 0$
$x = -a$ $\qquad$ $x = 4a$

The solutions are $-a$ and $4a$.

You Try It 3

$$(x - r_1)(x - r_2) = 0$$
$$(x - 3)\left[x - \left(-\frac{1}{2}\right)\right] = 0$$
$$(x - 3)\left(x + \frac{1}{2}\right) = 0$$
$$x^2 - \frac{5}{2}x - \frac{3}{2} = 0$$
$$2\left(x^2 - \frac{5}{2}x - \frac{3}{2}\right) = 2 \cdot 0$$
$$2x^2 - 5x - 3 = 0$$

You Try It 4

$$2(x + 1)^2 - 24 = 0$$
$$2(x + 1)^2 = 24$$
$$(x + 1)^2 = 12$$
$$\sqrt{(x + 1)^2} = \sqrt{12}$$
$$x + 1 = \pm 2\sqrt{3}$$

$x + 1 = 2\sqrt{3}$ $\qquad$ $x + 1 = -2\sqrt{3}$
$x = -1 + 2\sqrt{3}$ $\qquad$ $x = -1 - 2\sqrt{3}$

The solutions are
$-1 + 2\sqrt{3}$ and $-1 - 2\sqrt{3}$.

SECTION 10.2

You Try It 1

$$4x^2 - 4x - 1 = 0$$
$$4x^2 - 4x = 1$$
$$\frac{1}{4}(4x^2 - 4x) = \frac{1}{4} \cdot 1$$
$$x^2 - x = \frac{1}{4}$$

Complete the square.

$$x^2 - x + \frac{1}{4} = \frac{1}{4} + \frac{1}{4}$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{2}{4}$$
$$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{2}{4}}$$
$$x - \frac{1}{2} = \pm\frac{\sqrt{2}}{2}$$

$x - \frac{1}{2} = \frac{\sqrt{2}}{2}$ $\qquad$ $x - \frac{1}{2} = -\frac{\sqrt{2}}{2}$
$x = \frac{1}{2} + \frac{\sqrt{2}}{2}$ $\qquad$ $x = \frac{1}{2} - \frac{\sqrt{2}}{2}$

The solutions are $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$.

You Try It 2

$$2x^2 + x - 5 = 0$$
$$2x^2 + x = 5$$
$$\frac{1}{2}(2x^2 + x) = \frac{1}{2} \cdot 5$$
$$x^2 + \frac{1}{2}x = \frac{5}{2}$$

Complete the square.

$$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{5}{2} + \frac{1}{16}$$
$$\left(x + \frac{1}{4}\right)^2 = \frac{41}{16}$$
$$\sqrt{\left(x + \frac{1}{4}\right)^2} = \sqrt{\frac{41}{16}}$$
$$x + \frac{1}{4} = \pm\frac{\sqrt{41}}{4}$$

$x + \frac{1}{4} = \frac{\sqrt{41}}{4}$ $\qquad$ $x + \frac{1}{4} = -\frac{\sqrt{41}}{4}$
$x = -\frac{1}{4} + \frac{\sqrt{41}}{4}$ $\qquad$ $x = -\frac{1}{4} - \frac{\sqrt{41}}{4}$

The solutions are $\frac{-1 + \sqrt{41}}{4}$ and $\frac{-1 - \sqrt{41}}{4}$.

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SECTION 10.3

You Try It 1

$$x^2 - 2x + 10 = 0$$
$$a = 1, b = -2, c = 10$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2 \cdot 1}$$
$$= \frac{2 \pm \sqrt{4 - 40}}{2} = \frac{2 \pm \sqrt{-36}}{2}$$
$$= \frac{2 \pm 6i}{2} = 1 \pm 3i$$

The solutions are $1 + 3i$ and $1 - 3i$.

You Try It 2

$$4x^2 = 4x - 1$$
$$4x^2 - 4x + 1 = 0$$
$$a = 4, b = -4, c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2 \cdot 4}$$
$$= \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4 \pm \sqrt{0}}{8}$$
$$= \frac{4}{8} = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 3

$$3x^2 - x - 1 = 0$$
$$a = 3, b = -1, c = -1$$
$$b^2 - 4ac = (-1)^2 - 4(3)(-1)$$
$$= 1 + 12 = 13$$
$$13 > 0$$

Because the discriminant is greater than zero, the equation has two real number solutions.

SECTION 10.4

You Try It 1

$$x - 5x^{1/2} + 6 = 0$$
$$(x^{1/2})^2 - 5(x^{1/2}) + 6 = 0$$
$$u^2 - 5u + 6 = 0$$
$$(u - 2)(u - 3) = 0$$

$$u - 2 = 0 \qquad u - 3 = 0$$
$$u = 2 \qquad u = 3$$

Replace u by $x^{1/2}$.

$$x^{1/2} = 2 \qquad x^{1/2} = 3$$
$$\sqrt{x} = 2 \qquad \sqrt{x} = 3$$
$$(\sqrt{x})^2 = 2^2 \qquad (\sqrt{x})^2 = 3^2$$
$$x = 4 \qquad x = 9$$

The solutions are 4 and 9.

You Try It 2

$$\sqrt{2x + 1} + x = 7$$
$$\sqrt{2x + 1} = 7 - x$$
$$(\sqrt{2x + 1})^2 = (7 - x)^2$$
$$2x + 1 = 49 - 14x + x^2$$
$$0 = x^2 - 16x + 48$$
$$0 = (x - 4)(x - 12)$$

$$x - 4 = 0 \qquad x - 12 = 0$$
$$x = 4 \qquad x = 12$$

4 checks as a solution.
12 does not check as a solution.

The solution is 4.

You Try It 3

$$\sqrt{2x - 1} + \sqrt{x} = 2$$

Solve for one of the radical expressions.

$$\sqrt{2x - 1} = 2 - \sqrt{x}$$
$$(\sqrt{2x - 1})^2 = (2 - \sqrt{x})^2$$
$$2x - 1 = 4 - 4\sqrt{x} + x$$
$$x - 5 = -4\sqrt{x}$$

Square each side of the equation.

$$(x - 5)^2 = (-4\sqrt{x})^2$$
$$x^2 - 10x + 25 = 16x$$
$$x^2 - 26x + 25 = 0$$
$$(x - 1)(x - 25) = 0$$

$$x - 1 = 0 \qquad x - 25 = 0$$
$$x = 1 \qquad x = 25$$

1 checks as a solution.
25 does not check as a solution.

The solution is 1.

You Try It 4

$$3y + \frac{25}{3y - 2} = -8$$
$$(3y - 2)\left(3y + \frac{25}{3y - 2}\right) = (3y - 2)(-8)$$
$$(3y - 2)(3y) + (3y - 2)\left(\frac{25}{3y - 2}\right) = (3y - 2)(-8)$$
$$9y^2 - 6y + 25 = -24y + 16$$
$$9y^2 + 18y + 9 = 0$$
$$9(y^2 + 2y + 1) = 0$$
$$y^2 + 2y + 1 = 0$$
$$(y + 1)(y + 1) = 0$$

$$y + 1 = 0 \qquad y + 1 = 0$$
$$y = -1 \qquad y = -1$$

The solution is -1.

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SECTION 10.5

You Try It 1

Strategy

- This is a geometry problem.
- Width of the rectangle: W
  Length of the rectangle: $W + 3$
- Use the equation $A = L \cdot W$.

Solution

$A = L \cdot W$
$54 = (W + 3)(W)$
$54 = W^2 + 3W$
$0 = W^2 + 3W - 54$
$0 = (W + 9)(W - 6)$

$W + 9 = 0 \qquad W - 6 = 0$
$W = -9 \qquad W = 6$

The solution -9 is not possible.

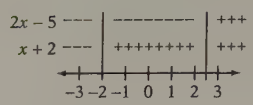
$W + 3 = 6 + 3 = 9$

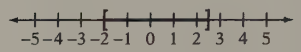
The length is 9 m.

SECTION 10.6

You Try It 1

$2x^2 - x - 10 \le 0$
$(2x - 5)(x + 2) \le 0$





$\left\{x \mid -2 \le x \le \frac{5}{2}\right\}$

Solutions to Chapter 11 "You Try It"

SECTION 11.1

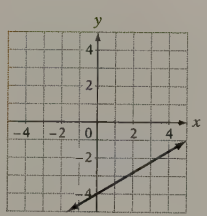
You Try It 1

$f(x) = \frac{3}{5}x - 4$

$b = -4$

y-intercept: $(0, -4)$

$m = \frac{3}{5}$



You Try It 2

Strategy

- Select the independent and dependent variables. The function is to predict the Celsius temperature, so that quantity is the dependent variable, y. The Fahrenheit variable is the independent variable, x.
- From the given data, two ordered pairs are (212, 100) and (32, 0). Use these ordered pairs to determine the linear function.

Solution

Let $(x_1, y_1) = (32, 0)$ and $(x_2, y_2) = (212, 100)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{100 - 0}{212 - 32} = \frac{100}{180} = \frac{5}{9}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{5}{9}(x - 32)$

$y = \frac{5}{9}(x - 32)$, or $C = \frac{5}{9}(F - 32)$.

The linear function is $f(F) = \frac{5}{9}(F - 32)$.

SECTION 11.2

You Try It 1

x-coordinate of vertex:

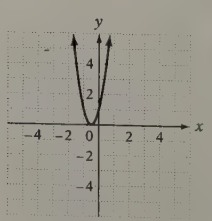
$-\frac{b}{2a} = -\frac{4}{2(4)} = -\frac{1}{2}$

y-coordinate of vertex:

$y = 4x^2 + 4x + 1$
$= 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 1$
$= 1 - 2 + 1$
$= 0$

vertex: $\left(-\frac{1}{2}, 0\right)$

axis of symmetry: $x = -\frac{1}{2}$



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You Try It 2

$y = x^2 + 3x + 4$

$0 = x^2 + 3x + 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(4)}}{2 \cdot 1}$$ • $a = 1, b = 3, c = 4$

$$= \frac{-3 \pm \sqrt{-7}}{2}$$

$$= \frac{-3 \pm i\sqrt{7}}{2} = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}i$$

The equation has no real number solutions. There are no x-intercepts.

You Try It 3

$y = x^2 - x - 6$

$a = 1, b = -1, c = -6$

$b^2 - 4ac$

$(-1)^2 - 4(1)(-6) = 1 + 24 = 25$

Because the discriminant is greater than zero, the parabola has two x-intercepts.

You Try It 4

$f(x) = -3x^2 + 4x - 1$

$$x = -\frac{b}{2a} = -\frac{4}{2(-3)} = \frac{2}{3}$$

$f(x) = -3x^2 + 4x - 1$

$$f\left(\frac{2}{3}\right) = -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) - 1$$

$$= -\frac{4}{3} + \frac{8}{3} - 1 = \frac{1}{3}$$

Because a is negative, the function has a maximum value.

The maximum value of the function is $\frac{1}{3}$.

You Try It 5

Strategy

- To find the time it takes the ball to reach its maximum height, find the t-coordinate of the vertex.
- To find the maximum height, evaluate the function at the t-coordinate of the vertex.

Solution

$$t = -\frac{b}{2a} = -\frac{64}{2(-16)} = 2$$

The ball reaches its maximum height in 2 s.

$s(t) = -16t^2 + 64t$

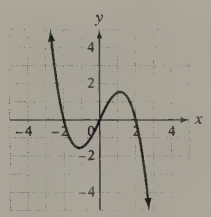
$s(2) = -16(2)^2 + 64(2) = -64 + 128 = 64$

The maximum height is 64 ft.

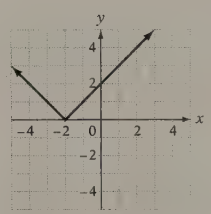
SECTION 11.3

You Try It 1 Any vertical line intersects the graph only once. The graph is the graph of a function.

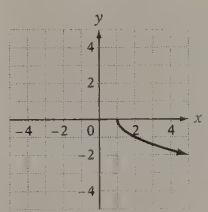
You Try It 2 domain: $\{x | x \in \text{real numbers}\}$
range: $\{y | y \in \text{real numbers}\}$



You Try It 3 domain: $\{x | x \in \text{real numbers}\}$
range: $\{y | y \geq 0\}$



You Try It 4 domain: $\{x | x \geq 1\}$
range: $\{y | y \leq 0\}$



SECTION 11.4

You Try It 1

$$(f + g)(-2) = f(-2) + g(-2)$$
$$= [(-2)^2 + 2(-2)] + [5(-2) - 2]$$
$$= (4 - 4) + (-10 - 2)$$
$$= -12$$

$(f + g)(-2) = -12$

You Try It 2

$$(f \cdot g)(3) = f(3) \cdot g(3)$$
$$= (4 - 3^2) \cdot [3(3) - 4]$$
$$= (4 - 9) \cdot (9 - 4)$$
$$= (-5)(5)$$
$$= -25$$

$(f \cdot g)(3) = -25$

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You Try It 3

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)}$$
$$= \frac{4^2 - 4}{4^2 + 2 \cdot 4 + 1}$$
$$= \frac{16 - 4}{16 + 8 + 1}$$
$$= \frac{12}{25}$$
$$\left(\frac{f}{g}\right)(4) = \frac{12}{25}$$

You Try It 4

$$g(x) = x^2$$
$$g(-1) = (-1)^2 = 1$$
$$f(x) = 1 - 2x$$
$$f[g(-1)] = f(1) = 1 - 2(1) = -1$$

You Try It 5

$$M(s) = s^3 + 1$$
$$M[L(s)] = M(s + 1) = (s + 1)^3 + 1$$
$$= s^3 + 3s^2 + 3s + 1 + 1$$
$$= s^3 + 3s^2 + 3s + 2$$

SECTION 11.5

You Try It 1 Because any horizontal line intersects the graph at most once, the graph is the graph of a 1–1 function.

You Try It 2

$$f(x) = \frac{1}{2}x + 4$$
$$y = \frac{1}{2}x + 4$$
$$x = \frac{1}{2}y + 4$$
$$x - 4 = \frac{1}{2}y$$
$$2x - 8 = y$$
$$f^{-1}(x) = 2x - 8$$

The inverse of the function is given by $f^{-1}(x) = 2x - 8$.

You Try It 3

$$f[g(x)] = 2\left(\frac{1}{2}x - 3\right) - 6$$
$$= x - 6 - 6 = x - 12$$

No, $g(x)$ is not the inverse of $f(x)$.

SECTION 11.6

You Try It 1

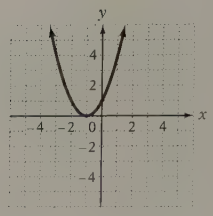
$y = x^2 + 2x + 1$

$$-\frac{b}{2a} = -\frac{2}{2(1)} = -1$$

axis of symmetry:
$x = -1$

$$y = (-1)^2 + 2(-1) + 1$$
$$= 0$$

vertex: $(-1, 0)$



You Try It 2

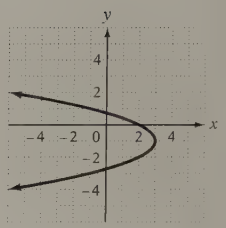
$x = -y^2 - 2y + 2$

$$-\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$$

axis of symmetry:
$y = -1$

$$x = -(-1)^2 - 2(-1) + 2$$
$$= 3$$

vertex: $(3, -1)$



You Try It 3

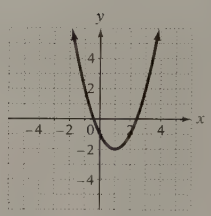
$y = x^2 - 2x - 1$

$$-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

axis of symmetry:
$x = 1$

$$y = 1^2 - 2(1) - 1$$
$$= -2$$

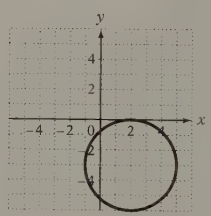
vertex: $(1, -2)$



You Try It 4

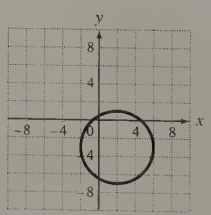
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 2)^2 + [y - (-3)]^2 = 3^2$$

center: $(h, k) = (2, -3)$
radius: $r = 3$



You Try It 5

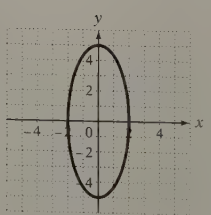
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 2)^2 + [y - (-3)]^2 = 4^2$$
$$(x - 2)^2 + (y + 3)^2 = 16$$



You Try It 6

x-intercepts:
$(2, 0)$ and $(-2, 0)$

y-intercepts:
$(0, 5)$ and $(0, -5)$



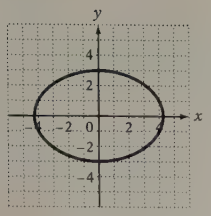
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You Try It 7

x-intercepts
$(3\sqrt{2}, 0)$ and $(-3\sqrt{2}, 0)$

y-intercepts:
$(0, 3)$ and $(0, -3)$

$\left(3\sqrt{2} \approx 4\frac{1}{4}\right)$

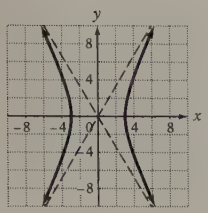


You Try It 8

axis of symmetry:
x-axis

vertices:
$(3, 0)$ and $(-3, 0)$

asymptotes:
$y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$

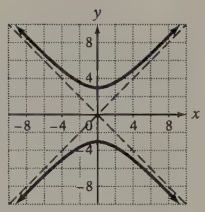


You Try It 9

axis of symmetry:
y-axis

vertices:
$(0, 3)$ and $(0, -3)$

asymptotes:
$y = x$ and $y = -x$



Solutions to Chapter 12 "You Try It"

SECTION 12.1

You Try It 1

$$f(x) = \left(\frac{2}{3}\right)^x$$

$$f(3) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$f(-2) = \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

You Try It 2

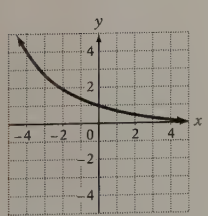
$$f(x) = 2^{2x+1}$$
$$f(0) = 2^{2(0)+1} = 2^1 = 2$$
$$f(-2) = 2^{2(-2)+1} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

You Try It 3

$$f(x) = e^{2x-1}$$
$$f(2) = e^{2\cdot 2-1} = e^3 \approx 20.0855$$
$$f(-2) = e^{2(-2)-1} = e^{-5} \approx 0.0067$$

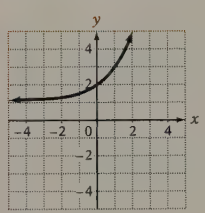
You Try It 4

x	y
-4	4
-2	2
0	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$



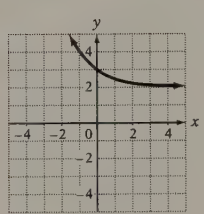
You Try It 5

x	y
-2	$\frac{5}{4}$
-1	$\frac{3}{2}$
0	2
1	3
2	5



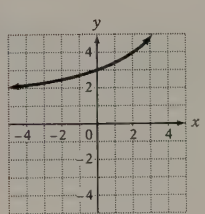
You Try It 6

x	y
-2	6
-1	4
0	3
1	$\frac{5}{2}$
2	$\frac{9}{4}$



You Try It 7

x	y
-4	$\frac{9}{4}$
-2	$\frac{5}{2}$
0	3
2	4
4	6



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SECTION 12.2

You Try It 1

$$\log_4 64 = x$$
$$64 = 4^x$$
$$4^3 = 4^x$$
$$3 = x$$
$$\log_4 64 = 3$$

You Try It 2

$$\log_2 x = -4$$
$$2^{-4} = x$$
$$\frac{1}{2^4} = x$$
$$\frac{1}{16} = x$$

The solution is $\frac{1}{16}$.

You Try It 3

$$\ln x = 3$$
$$e^3 = x$$
$$20.0855 \approx x$$

You Try It 4

$$\log_8 \sqrt[3]{xy^2} = \log_8 (xy^2)^{1/3} = \frac{1}{3}\log_8 (xy^2)$$
$$= \frac{1}{3}(\log_8 x + \log_8 y^2)$$
$$= \frac{1}{3}(\log_8 x + 2\log_8 y)$$
$$= \frac{1}{3}\log_8 x + \frac{2}{3}\log_8 y$$

You Try It 5

$$\frac{1}{3}(\log_4 x - 2\log_4 y + \log_4 z)$$
$$= \frac{1}{3}(\log_4 x - \log_4 y^2 + \log_4 z)$$
$$= \frac{1}{3}\left(\log_4 \frac{x}{y^2} + \log_4 z\right) = \frac{1}{3}\left(\log_4 \frac{xz}{y^2}\right)$$
$$= \log_4 \left(\frac{xz}{y^2}\right)^{1/3} = \log_4 \sqrt[3]{\frac{xz}{y^2}}$$

You Try It 6 Because $\log_b 1 = 0$, $\log_9 1 = 0$

You Try It 7

$$\log_7 6.45 = \frac{\log 6.45}{\log 7} \approx 0.9579$$

You Try It 8

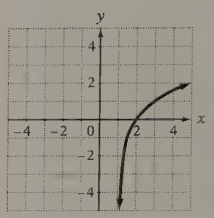
$$\log_3 0.834 = \frac{\ln 0.834}{\ln 3} \approx -0.1652$$

SECTION 12.3

You Try It 1

$$f(x) = \log_2 (x - 1)$$
$$y = \log_2 (x - 1)$$
$$2^y = x - 1$$
$$2^y + 1 = x$$

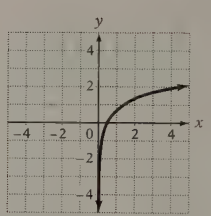
x	y
$\frac{5}{4}$	-2
$\frac{3}{2}$	-1
2	0
3	1
5	2



You Try It 2

$$f(x) = \log_3 (2x)$$
$$y = \log_3 (2x)$$
$$3^y = 2x$$
$$\frac{3^y}{2} = x$$

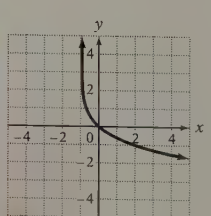
x	y
$\frac{1}{18}$	-2
$\frac{1}{6}$	-1
$\frac{1}{2}$	0
$\frac{3}{2}$	1
$\frac{9}{2}$	2



You Try It 3

$$f(x) = -\log_3 (x + 1)$$
$$y = -\log_3 (x + 1)$$
$$-y = \log_3 (x + 1)$$
$$3^{-y} = x + 1$$
$$3^{-y} - 1 = x$$

x	y
8	-2
2	-1
0	0
$-\frac{2}{3}$	1
$-\frac{8}{9}$	2



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SECTION 12.4

You Try It 1

$$(1.06)^n = 1.5$$
$$\log (1.06)^n = \log 1.5$$
$$n \log 1.06 = \log 1.5$$
$$n = \frac{\log 1.5}{\log 1.06}$$
$$n \approx 6.9585$$

The solution is 6.9585.

You Try It 2

$$4^{3x} = 25$$
$$\log 4^{3x} = \log 25$$
$$3x \log 4 = \log 25$$
$$3x = \frac{\log 25}{\log 4}$$
$$3x \approx 2.3219$$
$$x \approx 0.7740$$

The solution is 0.7740.

You Try It 3

$\log_4 (x^2 - 3x) = 1$

Rewrite in exponential form.

$$4^1 = x^2 - 3x$$
$$4 = x^2 - 3x$$
$$0 = x^2 - 3x - 4$$
$$0 = (x + 1)(x - 4)$$

$x + 1 = 0$ $\qquad$ $x - 4 = 0$
$x = -1$ $\qquad$ $x = 4$

The solutions are -1 and 4.

You Try It 4

$$\log_3 x + \log_3 (x + 3) = \log_3 4$$
$$\log_3 [x(x + 3)] = \log_3 4$$

Use the 1–1 Property of Logarithms.

$$x(x + 3) = 4$$
$$x^2 + 3x = 4$$
$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$

$x + 4 = 0$ $\qquad$ $x - 1 = 0$
$x = -4$ $\qquad$ $x = 1$

-4 does not check as a solution. The solution is 1.

You Try It 5

$$\log_3 x + \log_3 (x + 6) = 3$$
$$\log_3 [x(x + 6)] = 3$$

$$x(x + 6) = 3^3$$
$$x^2 + 6x = 27$$
$$x^2 + 6x - 27 = 0$$
$$(x + 9)(x - 3) = 0$$

$x + 9 = 0$ $\qquad$ $x - 3 = 0$
$x = -9$ $\qquad$ $x = 3$

-9 does not check as a solution. The solution is 3.

SECTION 12.5

You Try It 1

Strategy To find the hydrogen ion concentration, replace pH by 2.9 in the equation $\text{pH} = -\log (\text{H}^+)$ and solve for H^+.

Solution

$$\text{pH} = -\log (\text{H}^+)$$
$$2.9 = -\log (\text{H}^+)$$
$$-2.9 = \log (\text{H}^+)$$
$$10^{-2.9} = \text{H}^+$$
$$0.00126 \approx \text{H}^+$$

The hydrogen ion concentration is approximately 0.00126.

You Try It 2

Strategy To find the intensity, use the equation for the Richter scale magnitude of an earthquake, $M = \log \left(\frac{I}{I_0}\right)$. Replace M by 5.2 and solve for I.

Solution

$$M = \log \left(\frac{I}{I_0}\right)$$
$$5.2 = \log \frac{I}{I_0}$$ • Replace M by 5.2.
$$10^{5.2} = \frac{I}{I_0}$$ • Write in exponential form.
$$10^{5.2} I_0 = I$$
$$158{,}489 I_0 \approx I$$

The earthquake's intensity was approximately 158,489 times that of a zero-level earthquake.

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Solutions to Chapter R "You Try It"

SECTION R.1

You Try It 1

$$\begin{aligned}(-4)(6-8)^2-(-12\div 4) &= -4(-2)^2-(-3)\\ &= -4(4)-(-3)\\ &= -16-(-3)\\ &= -16+3\\ &= -13\end{aligned}$$

You Try It 2

$3xy^2-3x^2y$

$$\begin{aligned}3(-2)(5)^2-3(-2)^2(5) &= 3(-2)(25)-3(4)(5)\\ &= -6(25)-3(4)(5)\\ &= -150-3(4)(5)\\ &= -150-12(5)\\ &= -150-60\\ &= -150+(-60)\\ &= -210\end{aligned}$$

You Try It 3

$$\begin{aligned}-5(-3a) &= [-5(-3)]a\\ &= 15a\end{aligned}$$

You Try It 4

$$\begin{aligned}2z^2-5z-3z^2+6z &= 2z^2-3z^2-5z+6z\\ &= (2z^2-3z^2)+(-5z+6z)\\ &= -1z^2+z\\ &= -z^2+z\end{aligned}$$

You Try It 5

a.

$$\begin{aligned}-3(5y-2) &= -3(5y)-(-3)(2)\\ &= -15y+6\end{aligned}$$

b.

$$\begin{aligned}-2(4x+2y-6z) &= -2(4x)+(-2)(2y)-(-2)(6z)\\ &= -8x-4y+12z\end{aligned}$$

You Try It 6

a.

$$\begin{aligned}7(-3x-4y)-3(3x+y) &= -21x-28y-9x-3y\\ &= -30x-31y\end{aligned}$$

b.

$$\begin{aligned}2y-3[5-3(3+2y)] &= 2y-3[5-9-6y]\\ &= 2y-3[-4-6y]\\ &= 2y+12+18y\\ &= 20y+12\end{aligned}$$

SECTION R.2

You Try It 1

a.

$$\begin{aligned}4x+3 &= 7x+9\\ 4x-7x+3 &= 7x-7x+9\\ -3x+3 &= 9\\ -3x+3-3 &= 9-3\\ -3x &= 6\\ \frac{-3x}{-3} &= \frac{6}{-3}\\ x &= -2\end{aligned}$$

The solution is -2.

b.

$$\begin{aligned}4-(5x-8) &= 4x+3\\ 4-5x+8 &= 4x+3\\ -5x+12 &= 4x+3\\ -5x-4x+12 &= 4x-4x+3\\ -9x+12 &= 3\\ -9x+12-12 &= 3-12\\ -9x &= -9\\ \frac{-9x}{-9} &= \frac{-9}{-9}\\ x &= 1\end{aligned}$$

The solution is 1.

You Try It 2

$$\begin{aligned}3x-1 &\le 5x-7\\ 3x-5x-1 &\le 5x-5x-7\\ -2x-1 &\le -7\\ -2x-1+1 &\le -7+1\\ -2x &\le -6\\ \frac{-2x}{-2} &\ge \frac{-6}{-2}\\ x &\ge 3\end{aligned}$$

$\{x \mid x \ge 3\}$

You Try It 3

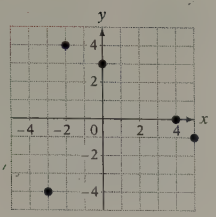
$$\begin{aligned}3-2(3x+1) &< 7-2x\\ 3-6x-2 &< 7-2x\\ 1-6x &< 7-2x\\ 1-6x+2x &< 7-2x+2x\\ 1-4x &< 7\\ 1-1-4x &< 7-1\\ -4x &< 6\\ \frac{-4x}{-4} &> \frac{6}{-4}\\ x &> -\frac{3}{2}\end{aligned}$$

$\left\{x \mid x > -\frac{3}{2}\right\}$

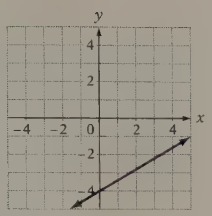
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SECTION R.3

You Try It 1

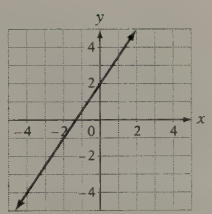


You Try It 2 $y = \frac{3}{5}x - 4$



You Try It 3

$$-3x + 2y = 4$$
$$2y = 3x + 4$$
$$y = \frac{3}{2}x + 2$$



You Try It 4 Let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (1, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{1 - (-2)} = \frac{-6}{3} = -2$$

The slope is -2.

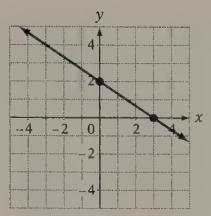
You Try It 5

$$y = -\frac{2}{3}x + 2$$

$$m = -\frac{2}{3} = \frac{-2}{3}$$

y-intercept $= (0, 2)$

Place a dot at the y-intercept.
Starting at the y-intercept, move to the right 3 units (the change in x) and down 2 units (the change in y).
Place a dot at that location.
Draw a line through the two points.



You Try It 6

$$f(x) = 4 - 2x$$
$$f(-3) = 4 - 2(-3)$$
$$f(-3) = 4 + 6$$
$$f(-3) = 10$$

An ordered pair of the function is $(-3, 10)$.

You Try It 7 $(x_1, y_1) = (-2, 2), m = -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{1}{2}[x - (-2)]$$
$$y - 2 = -\frac{1}{2}(x + 2)$$
$$y - 2 = -\frac{1}{2}x - 1$$
$$y = -\frac{1}{2}x + 1$$

SECTION R.4

You Try It 1

a.
$$(-2ab)(2a^3b^{-2})^{-3} = (-2ab)(2^{-3}a^{-9}b^6)$$
$$= -(2^{-2})a^{-8}b^7$$
$$= -\frac{b^7}{2^2a^8}$$
$$= -\frac{b^7}{4a^8}$$

b.
$$\left(\frac{2x^2y^{-4}}{4x^{-2}y^{-5}}\right)^{-3} = \left(\frac{x^2y^{-4}}{2x^{-2}y^{-5}}\right)^{-3}$$
$$= \frac{x^{2(-3)}y^{(-4)(-3)}}{2^{1(-3)}x^{(-2)(-3)}y^{(-5)(-3)}}$$
$$= \frac{x^{-6}y^{12}}{2^{-3}x^6y^{15}}$$
$$= 2^3x^{-6-6}y^{12-15}$$
$$= 2^3x^{-12}y^{-3}$$
$$= \frac{8}{x^{12}y^3}$$

You Try It 2

$$(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)$$
$$= (-4x^3 + 4x^3) + (2x^2 + 6x^2) + (-7x) + (-8 + 5)$$
$$= 8x^2 - 7x - 3$$

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You Try It 3

$$\begin{aligned}&(-4w^3 + 8w - 8) - (3w^3 - 4w^2 + 2w - 1)\\ &= (-4w^3 + 8w - 8) + (-3w^3 + 4w^2 - 2w + 1)\\ &= (-4w^3 - 3w^3) + 4w^2 + (8w - 2w) + (-8 + 1)\\ &= -7w^3 + 4w^2 + 6w - 7\end{aligned}$$

You Try It 4

$$\begin{aligned}&3mn^2(2m^2 - 3mn - 1)\\ &= 3mn^2(2m^2) - 3mn^2(3mn) - 3mn^2(1)\\ &= 6m^3n^2 - 9m^2n^3 - 3mn^2\end{aligned}$$

You Try It 5

$$\begin{array}{r} 3c^2 - 4c + 5 \\ \underline{\qquad\qquad 2c - 3} \\ -9c^2 + 12c - 15 \\ \underline{6c^3 - 8c^2 + 10c} \\ 6c^3 - 17c^2 + 22c - 15 \end{array}$$

You Try It 6

$$\begin{aligned}&(4y - 7)(3y - 5)\\ &= (4y)(3y) + (4y)(-5) + (-7)(3y) + (-7)(-5)\\ &= 12y^2 - 20y - 21y + 35\\ &= 12y^2 - 41y + 35\end{aligned}$$

You Try It 7

$(x^3 - 7 - 2x) \div (x - 2)$

$$\begin{array}{r} x^2 + 2x + 2 \\ x - 2\overline{)\,x^3 + 0x^2 - 2x - 7} \\ \underline{x^3 - 2x^2} \\ 2x^2 - 2x \\ \underline{2x^2 - 4x} \\ 2x - 7 \\ \underline{2x - 4} \\ -3 \end{array}$$

Check: $\begin{aligned}&(x - 2)(x^2 + 2x + 2) - 3\\ &= x^3 - 2x - 4 - 3\\ &= x^3 - 2x - 7\end{aligned}$

$$\begin{aligned}&(x^3 - 7 - 2x) \div (x - 2)\\ &= x^2 + 2x + 2 - \frac{3}{x - 2}\end{aligned}$$

You Try It 8 The GCF is $3x^2y^2$.

$$\begin{aligned}&6x^4y^2 - 9x^3y^2 + 12x^2y^4\\ &= 3x^2y^2(2x^2 - 3x + 4y^2)\end{aligned}$$

You Try It 9 Two factors of 15 whose sum is -8 are -3 and -5.

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

You Try It 10

$3x^2 - x - 2$

Positive Factors of 3	*Factors of −2*
1, 3	1, −2 −1, 2

Trial Factors	*Middle Term*
$(x + 1)(3x - 2)$	$-2x + 3x = x$
$(x - 2)(3x + 1)$	$x - 6x = -5x$
$(x - 1)(3x + 2)$	$2x - 3x = -x$
$(x + 2)(3x - 1)$	$-x + 6x = 5x$

$3x^2 - x - 2 = (x - 1)(3x + 2)$

Check: $\begin{aligned}(x - 1)(3x + 2) &= 3x^2 + 2x - 3x - 2\\ &= 3x^2 - x - 2\end{aligned}$

You Try It 11

$4a^3 - 4a^2 - 24a$

There is a common factor, $4a$. Factor out the GCF.

$4a^3 - 4a^2 - 24a = 4a(a^2 - a - 6)$

Factor $a^2 - a - 6$. The two factors of -6 whose sum is -1 are 2 and -3.

$4a(a^2 - a - 6) = 4a(a + 2)(a - 3)$

$4a^3 - 4a^2 - 24a = 4a(a + 2)(a - 3)$

Check: $\begin{aligned}4a(a + 2)(a - 3) &= (4a^2 + 8a)(a - 3)\\ &= 4a^3 - 12a^2 + 8a^2 - 24a\\ &= 4a^3 - 4a^2 - 24a\end{aligned}$

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Answers to Chapter 1 Selected Exercises

PREP TEST

1. 127.16 **2.** 55.107 **3.** 4517 **4.** 11,396 **5.** 24 **6.** 24 **7.** 4 **8.** $3 \cdot 7$ **9.** $\frac{2}{5}$

SECTION 1.1

1. $8 > -6$ **3.** $-12 < 1$ **5.** $42 > 19$ **7.** $0 > -31$ **9.** $53 > -46$ **11.** false **13.** true **15.** false **17.** true **19.** false **21.** $-23, -18$ **23.** 21, 37 **25.** -23 **27.** -4 **29.** 9 **31.** 28 **33.** 14 **35.** -77 **37.** 0 **39.** 74 **41.** -82 **43.** -81 **45.** $|-83| > |58|$ **47.** $|43| < |-52|$ **49.** $|-68| > |-42|$ **51.** $|-45| < |-61|$ **53.** 19, 0, -28 **55.** -45, 0, -17 **59.** -11 **61.** -5 **63.** -83 **65.** -46 **67.** 0 **69.** -5 **71.** 9 **73.** 1 **75.** 8 **77.** -7 **79.** -9 **81.** 9 **83.** -3 **85.** 18 **87.** -10 **89.** -41 **91.** -12 **93.** 0 **95.** -9 **97.** 11 **99.** -18 **101.** 0 **103.** 2 **105.** -138 **107.** -8 **109.** -12 **111.** -20 **115.** 42 **117.** -28 **119.** 60 **121.** -253 **123.** -238 **125.** -114 **127.** -2 **129.** 8 **131.** -7 **133.** -12 **135.** -6 **137.** -7 **139.** 11 **141.** -14 **143.** 15 **145.** -16 **147.** 0 **149.** -29 **151.** undefined **153.** -11 **155.** undefined **157.** -105 **159.** 252 **161.** -240 **163.** 96 **165.** -216 **167.** -315 **169.** 420 **171.** 2880 **173.** -2772 **175.** 0 **177.** The difference in elevation is 7046 m. **179.** The difference between the highest and lowest elevations is greatest in Asia. **181.** The difference is 5°C. **183.** The student's score is 93 points. **185.** The difference between the average temperatures is 0°. **187.** The difference is 29°F.

SECTION 1.2

1. 0.125 **3.** $0.\overline{2}$ **5.** $0.1\overline{6}$ **7.** 0.5625 **9.** $0.58\overline{3}$ **11.** 0.24 **13.** 0.225 **15.** $0.\overline{45}$ **19.** $\frac{2}{5}$, 0.40 **21.** $\frac{22}{25}$, 0.88 **23.** $\frac{8}{5}$, 1.6 **25.** $\frac{87}{100}$, 0.87 **27.** $\frac{9}{2}$, 4.50 **29.** $\frac{3}{70}$ **31.** $\frac{3}{8}$ **33.** $\frac{1}{400}$ **35.** $\frac{1}{16}$ **37.** $\frac{23}{400}$ **39.** 0.091 **41.** 0.167 **43.** 0.009 **45.** 0.0915 **47.** 0.1823 **49.** 37% **51.** 2% **53.** 12.5% **55.** 136% **57.** 0.4% **59.** 83% **61.** $37\frac{1}{2}\%$ **63.** $44\frac{4}{9}\%$ **65.** 45% **67.** 250% **69.** $\frac{5}{26}$ **71.** $\frac{11}{8}$ **73.** $\frac{1}{12}$ **75.** $\frac{7}{24}$ **77.** 0 **79.** $\frac{3}{8}$ **81.** $-\frac{7}{60}$ **83.** $-\frac{1}{16}$ **85.** -1.06 **87.** -23.845 **89.** -10.7893 **91.** -37.19 **93.** -17.5 **95.** 19.61 **97.** $-\frac{3}{8}$ **99.** $\frac{1}{10}$ **101.** $-\frac{4}{9}$ **103.** $-\frac{7}{30}$ **105.** $\frac{15}{64}$ **107.** $-\frac{10}{9}$ **109.** $-\frac{147}{32}$ **111.** $\frac{25}{8}$ **113.** $\frac{2}{3}$ **115.** 4.164 **117.** 4.347 **119.** -4.028 **121.** -2.22 **123.** -1.104 **125.** -2.59 **127.** -5.11 **129.** -2060.55 **131.** 2401 **133.** -64 **135.** -8 **137.** -125 **139.** $-\frac{27}{64}$ **141.** 3.375 **143.** -1 **145.** -8 **147.** -6750 **149.** -144 **151.** -18 **153.** 4 **155.** 7 **157.** $4\sqrt{2}$ **159.** $2\sqrt{2}$ **161.** $18\sqrt{2}$ **163.** $10\sqrt{10}$ **165.** $\sqrt{15}$ **167.** $\sqrt{29}$ **169.** $-54\sqrt{2}$ **171.** $3\sqrt{5}$ **173.** 0 **175.** $48\sqrt{2}$ **177.** 15.492 **179.** 16.971 **181.** 16 **183.** 16.583 **185.** 15.652 **187.** 18.762 **189a.** The 2001 annual net income would be $-\$103.408$ million. **b.** The average monthly net income was $-\$43.584$ million. **c.** The difference was \$30.481 million. **191.** 31% of the owners responded that the Internet had helped business. **193.** 3, 4, 5, 6, 7, 8, 9

SECTION 1.3

3. 0 **5.** -11 **7.** 20 **9.** -10 **11.** 20 **13.** 29 **15.** 11 **17.** 7 **19.** -11 **21.** 6 **23.** 15 **25.** 4 **27.** 5 **29.** -1 **31.** 4 **33.** 0.51 **35.** 1.7 **37.** Row 1: $-\frac{1}{6}$, 0; Row 2: $-\frac{1}{2}$; Row 3: $\frac{1}{3}, \frac{1}{2}$ **39.** Your savings on gasoline would pay for the increased cost of the car in 32 to 65 months.

SECTION 1.4

1. -9 **3.** 41 **5.** -7 **7.** 13 **9.** -15 **11.** 41 **13.** 1 **15.** 5 **17.** 1 **19.** 57 **21.** 5 **23.** 8

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25. -3 **27.** -2 **29.** -4 **31.** 10 **33.** -25 **35.** $25x$ **37.** $9a$ **39.** $2y$ **41.** $-12y - 3$ **43.** $9a$

45. $6ab$ **47.** $-12xy$ **49.** 0 **51.** $-\frac{1}{10}y$ **53.** $\frac{2}{9}y^2$ **55.** $20x$ **57.** $-4a$ **59.** $-2y^2$ **61.** $-2x + 8y$ **63.** $8x$

65. $19a - 12b$ **67.** $-12x - 2y$ **69.** $-7x^2 - 5x$ **71.** $60x$ **73.** $-10a$ **75.** $30y$ **77.** $72x$ **79.** $-28a$

81. $108b$ **83.** $-56x^2$ **85.** x^2 **87.** x **89.** a **91.** b **93.** x **95.** n **97.** $2x$ **99.** $-2x$ **101.** $-15a^2$

103. $6y$ **105.** $3y$ **107.** $-2x$ **109.** $-9y$ **111.** $-x - 7$ **113.** $10x - 35$ **115.** $-5a - 80$ **117.** $-15y + 35$

119. $20 - 14b$ **121.** $-4x + 2y$ **123.** $18x^2 + 12x$ **125.** $10x - 35$ **127.** $-14x + 49$ **129.** $-30x^2 - 15$

131. $-24y^2 + 96$ **133.** $5x^2 + 5y^2$ **135.** $-\frac{1}{2}x + 2y$ **137.** $3x^2 + 6x - 18$ **139.** $-2y^2 + 4y - 8$

141. $-2x + 3y - \frac{1}{3}$ **143.** $10x^2 + 15x - 35$ **145.** $6x^2 + 3xy - 9y^2$ **147.** $-3a^2 - 5a + 4$ **149.** $-2x - 16$

151. $-12y - 9$ **153.** $7n - 7$ **155.** $-2x + 41$ **157.** $3y - 3$ **159.** $2a - 4b$ **161.** $-4x + 24$ **163.** $-2x - 16$

165. $-3x + 21$ **167.** $-4x + 12$ **169.** $-x + 50$ **171.** $\frac{x}{18}$ **173.** $x + 20$ **175.** $10(x - 50)$; $10x - 500$

177. $\frac{5}{8}x + 6$ **179.** $x - (x + 3)$; -3 **181.** $4(x + 19)$; $4x + 76$ **183.** $\frac{15}{x + 12}$ **185.** $\frac{2}{3}(x + 7)$; $\frac{2}{3}x + \frac{14}{3}$

187. $40 - \frac{x}{20}$ **189.** $x^2 + 2x$ **191.** $(x + 8) + \frac{1}{3}x$; $\frac{4}{3}x + 8$ **193.** $x + (x + 9)$; $2x + 9$ **195.** $x - (8 - x)$; $2x - 8$

197. $\frac{1}{3}x - \frac{5}{8}x$; $-\frac{7}{24}x$ **199.** $(x + 5) + 2$; $x + 7$ **201.** $2(6x + 7)$; $12x + 14$ **203.** Let t be the number of tornadoes in 2000; $t + 235$ **205.** Let g be the amount of oil in one container: g, $20 - g$ **207.** Let p be the pounds of pecans produced in Texas: $\frac{1}{2}p$ **209.** Let d be the diameter of a baseball: $4d$ **211.** yes **213.** $2x$ **215.** $\frac{1}{4}x$ **217.** $\frac{3}{5}x$

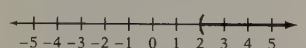
SECTION 1.5

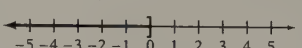
1. $A = \{16, 17, 18, 19, 20, 21\}$ **3.** $A = \{9, 11, 13, 15, 17\}$ **5.** $A = \{b, c\}$ **9.** $A \cup B = \{3, 4, 5, 6\}$

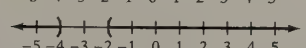
11. $A \cup B = \{-10, -9, -8, 8, 9, 10\}$ **13.** $A \cup B = \{a, b, c, d, e, f\}$ **15.** $A \cup B = \{1, 3, 7, 9, 11, 13\}$

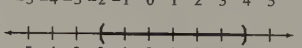
17. $A \cap B = \{4, 5\}$ **19.** $A \cap B = \varnothing$ **21.** $A \cap B = \{c, d, e\}$ **23.** $\{x | x > -5, x \in$ negative integers$\}$

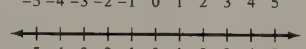
25. $\{x | x > 30, x \in$ integers$\}$ **27.** $\{x | x > 5, x \in$ even integers$\}$ **29.** $\{x | x > 8, x \in$ real numbers$\}$

31. 

33. 

35. 

37. 

39. 

41a. never true **b.** always true **c.** always true **43a.** yes **b.** yes

CHAPTER REVIEW*

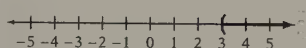
1. $-4, 0$ [1.1A] **2.** 4 [1.1B] **3.** -5 [1.1B] **4.** -13 [1.1C] **5.** 1 [1.1C] **6.** -42 [1.1D]

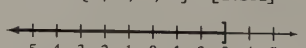
7. -20 [1.1D] **8.** 0.28 [1.2A] **9.** 0.062 [1.2B] **10.** 62.5% [1.2B] **11.** $\frac{7}{12}$ [1.2C] **12.** -1.068 [1.2C]

13. $-\frac{72}{85}$ [1.2D] **14.** -4.6224 [1.2D] **15.** $\frac{16}{81}$ [1.2E] **16.** 12 [1.2F] **17.** $-6\sqrt{30}$ [1.2F]

18. 31 [1.3A] **19.** 29 [1.4A] **20.** $8a - 4b$ [1.4B] **21.** $36y$ [1.4C] **22.** $10x - 35$ [1.4D]

23. $7x + 46$ [1.4D] **24.** $-90x + 25$ [1.4D] **25.** $\{1, 3, 5, 7\}$ [1.5A] **26.** $A \cap B = \{1, 5, 9\}$ [1.5A]

27. 

[1.5C] **28.** 

[1.5C] **29.** The student's score was 98. [1.1E] **30.** 50.8% of the candy consumed was chocolate. [1.2G] **31.** $2x - \frac{1}{2}x$; $\frac{3}{2}x$ [1.4E]

*The numbers in brackets following the answers in the Chapter Review and Chapter Test are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

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32. Let A be the number of American League playing cards; $5A$ [1.4E] **33.** Let T be the number of ten-dollar bills; $35 - T$ [1.4E]

CHAPTER TEST*

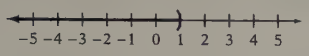
1. $-2 > -40$ [1.1A] **2.** 4 [1.1B] **3.** -4 [1.1B] **4.** -14 [1.1C] **5.** -16 [1.1C] **6.** 4 [1.1C]

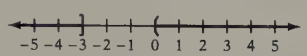
7. 17 [1.1D] **8.** $0.\overline{7}$ [1.2A] **9.** $\frac{9}{20}$, 0.45 [1.2B] **10.** $\frac{1}{15}$ [1.2C] **11.** -5.3578 [1.2D] **12.** $-\frac{1}{2}$ [1.2D]

13. 12 [1.2E] **14.** $-6\sqrt{5}$ [1.2F] **15.** 17 [1.3A] **16.** 22 [1.4A] **17.** $5x$ [1.4B] **18.** $2x$ [1.4C]

19. $-6x^2 + 21y^2$ [1.4D] **20.** $-x + 6$ [1.4D] **21.** $-7x + 33$ [1.4D] **22.** $\{-2, -1, 0, 1, 2, 3\}$ [1.5A]

23. $\{x | x < -3, x \in \text{real numbers}\}$ [1.5B] **24.** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ [1.5A]

25.  [1.5C]

26.  [1.5C]

27. $10(x - 3)$; $10x - 30$ [1.4E] **28.** Let s be the speed of the catcher's return throw; $2s$ [1.4E]

29a. The balance of trade increased from the previous year in 1981, 1988, 1989, 1990, 1991, and 1995. [1.2G] **b.** The difference was $288.6 billion. [1.2G] **c.** The difference was greatest between 1999 and 2000. [1.2G] **d.** The trade balance was approximately 4 times greater in 1990 than in 1980. [1.2G] **e.** The average trade balance per quarter for the year 2000 was $-$92.425 billion. [1.2G] **30.** The difference is 395.45°C. [1.2G]

Answers to Chapter 2 Selected Exercises

PREP TEST

1. -4 [1.1C] **2.** -6 [1.1C] **3.** 3 [1.1D] **4.** 1 [1.2D] **5.** $-\frac{1}{2}$ [1.2D] **6.** $10x - 5$ [1.4B]

7. $6x - 9$ [1.4D] **8.** $3n + 6$ [1.4B] **9.** $0.03x + 20$ [1.4D] **10.** $20 - n$ [1.4E]

SECTION 2.1

3. yes **5.** no **7.** yes **9.** no **11.** no **13.** yes **15.** yes **17.** no **21.** 6 **23.** 16 **25.** 7 **27.** 0

29. 3 **31.** -10 **33.** -3 **35.** -14 **37.** 2 **39.** 11 **41.** -9 **43.** -1 **45.** -14 **47.** -5 **49.** $-\frac{1}{2}$

51. $-\frac{3}{4}$ **53.** $\frac{1}{12}$ **55.** $-\frac{7}{12}$ **57.** 0.6529 **59.** -0.283 **61.** 9.257 **65.** -3 **67.** 0 **69.** -2 **71.** 9

73. 80 **75.** -4 **77.** 0 **79.** 8 **81.** -7 **83.** 12 **85.** -18 **87.** 15 **89.** -20 **91.** 0 **93.** 15

95. 75 **97.** $\frac{8}{3}$ **99.** $\frac{1}{3}$ **101.** $-\frac{1}{2}$ **103.** $-\frac{3}{2}$ **105.** $\frac{15}{7}$ **107.** 4 **109.** 3 **111.** 4.745 **113.** 2.06

115. -2.13 **119.** 28 **121.** 0.72 **123.** 64 **125.** 24% **127.** 7.2 **129.** 400 **131.** 200% **133.** 400

135. 7.7 **137.** 200 **139.** 400 **141.** 20 **143.** 80.34% **145.** 19% of the students are in the fine arts college.

147. Approximately 46.8% of the deaths were due to traffic accidents. **149.** Approximately 1.7% of the coupons were redeemed by customers. **151.** Western Europe's share is $675 billion more than Latin America's share.

153. Approximately 11.2% of the children were enrolled in private school. **155.** $x = \frac{b}{a}, a \neq 0$ **157a.** $\frac{5}{3}$ **b.** -1 **c.** $\frac{6}{5}$

159. The new value is 2 times the original value.

SECTION 2.2

1. 3 **3.** 6 **5.** -1 **7.** -3 **9.** 2 **11.** 2 **13.** 5 **15.** -3 **17.** 6 **19.** 3 **21.** 1 **23.** 6 **25.** -7

27. 0 **29.** $\frac{3}{4}$ **31.** $\frac{4}{9}$ **33.** $\frac{1}{3}$ **35.** $-\frac{1}{2}$ **37.** $-\frac{3}{4}$ **39.** $\frac{1}{3}$ **41.** $-\frac{1}{6}$ **43.** 1 **45.** 1 **47.** 0 **49.** $\frac{13}{10}$

51. $\frac{2}{5}$ **53.** $-\frac{4}{3}$ **55.** $-\frac{3}{2}$ **57.** 18 **59.** 8 **61.** -16 **63.** 25 **65.** 21 **67.** 15 **69.** -16 **71.** -21

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73. $\frac{15}{2}$ **75.** $-\frac{18}{5}$ **77.** 2 **79.** 3 **81.** 1 **83.** -2 **85.** 2 **87.** 3 **89.** -1 **91.** 2 **93.** -2 **95.** -3

97. 0 **99.** -1 **101.** -3 **103.** -1 **105.** 4 **107.** $\frac{2}{3}$ **109.** $\frac{5}{6}$ **111.** $\frac{3}{4}$ **113.** 1 **115.** 4 **117.** -1

119. -1 **121.** $-\frac{2}{3}$ **123.** $\frac{4}{3}$ **125.** $\frac{1}{2}$ **127.** $-\frac{1}{3}$ **129.** $\frac{10}{3}$ **131.** $-\frac{1}{4}$ **133.** $x + 12 = 20$; 8

135. $\frac{3}{5}x = -30$; -50 **137.** $3x + 4 = 13$; 3 **139.** $9x - 6 = 12$; 2 **141.** $x + 2x = 9$; 3 **143.** $5x - 17 = 2$; $\frac{19}{5}$

145. $6x + 7 = 3x - 8$; -5 **147.** $30 = 7x - 9$; $\frac{39}{7}$ **149.** $2x = (21 - x) + 3$; 8, 13 **151.** $23 - x = 2x + 5$; 6, 17

153. The approximate length of the humerus is 28.5 in. **155.** A car will slide 136 ft on black ice when the outside temperature is $-11°C$. **157.** -14 **159.** b **161.** no solution **163.** -21

SECTION 2.3

1. 20 oz of herbs should be used. **3.** The cost of the mixture is \$3.44 per pound. **5.** 3 lb of caramel is needed. **7.** 2 c of olive oil and 8 c of vinegar are used. **9.** The cost is \$3.00 per ounce. **11.** 16 oz of the \$400 alloy should be used. **13.** 37 lb of almonds and 63 lb of walnuts were used. **15.** 228 adult tickets were sold. **17.** The cost is \$.70 per pound. **19.** The resulting gold alloy is 24% gold. **21.** 20 gal of the 15% acid solution are used. **23.** 30 lb of the yarn that is 25% wool is used. **25.** 6.25 gal of the plant food that is 9% nitrogen is used. **27.** The resulting mixture is 19% sugar. **29.** 20 lb of the coffee that is 40% java beans are used. **31.** 100 ml of the 7% solution and 200 ml of the 4% solution are used. **33.** 150 oz of pure chocolate must be added. **35.** The resulting alloy is 50% silver. **37.** There must be an additional \$5000 added. **39.** There was \$9000 invested at 7% and \$6000 at 6.5%. **41.** There was \$2500 deposited in the mutual fund. **43.** The university deposited \$200,000 at 10% and \$100,000 at 8.5%. **45.** The mechanic must invest \$3000 in additional bonds. **47.** \$40,500 was invested at 8%. \$13,500 was invested at 12%. **49.** The total amount invested was \$650,000. **51.** The total amount invested was \$500,000. **53.** The rate of the first plane is 105 mph. The rate of the second plane is 130 mph. **55.** They will be 3000 km apart at 11 A.M. **57.** The cabin cruiser will be alongside the motorboat 2 h after the cabin cruiser leaves. **59.** The distance from the airport to the corporate offices is 120 mi. **61.** The rate of the car is 68 mph. **63.** The distance between the two airports is 300 mi. **65.** The two planes will pass each other 2.5 h after the plane leaves Seattle. **67.** They will meet 1.5 h after they begin. **69.** The bus overtakes the car 180 mi from the starting point. **71.** 3.75 gal must be drained from the radiator and replaced by pure antifreeze. **73.** The cyclist's average speed for the trip was $13\frac{1}{3}$ mph.

SECTION 2.4

3. a, c **5.** $\{x|x < 5\}$ **7.** $\{x|x \le 2\}$ **9.** $\{x|x < -4\}$ **11.** $\{x|x > 3\}$ **13.** $\{x|x > 4\}$ **15.** $\{x|x \le 2\}$

17. $\{x|x > -2\}$ **19.** $\{x|x \ge 2\}$ **21.** $\{x|x > -2\}$ **23.** $\{x|x \le 3\}$ **25.** $\{x|x < 2\}$ **27.** $\{x|x < -3\}$ **29.** $\{x|x \le 5\}$

31. $\left\{x|x \ge -\frac{1}{2}\right\}$ **33.** $\left\{x|x < \frac{23}{16}\right\}$ **35.** $\left\{x|x < \frac{8}{3}\right\}$ **37.** $\{x|x > 1\}$ **39.** $\left\{x|x > \frac{14}{11}\right\}$ **41.** $\{x|x \le 1\}$

43. $\left\{x|x \le \frac{7}{4}\right\}$ **45.** $\{x|x \le 2\}$ **49.** $\{x|-2 \le x \le 4\}$ **51.** $\{x|x < 3 \text{ or } x > 5\}$ **53.** $\{x|-4 < x < 2\}$

55. $\{x|x > 6 \text{ or } x < -4\}$ **57.** $\{x|x < -3\}$ **59.** $\{x|-3 < x < 2\}$ **61.** $\{x|-2 < x < 1\}$ **63.** $\{x|x < -2 \text{ or } x > 2\}$

65. $\{x|2 < x < 6\}$ **67.** $\{x|-3 < x < -2\}$ **69.** $\left\{x|x > 5 \text{ or } x < -\frac{5}{3}\right\}$ **71.** $\{x|x < 3\}$ **73.** The solution set is the empty set. **75.** $\{x|x \in \text{real numbers}\}$ **77.** $\left\{x\left|\frac{17}{7} \le x \le \frac{45}{7}\right.\right\}$ **79.** $\left\{x|-5 < x < \frac{17}{3}\right\}$ **81.** $\{x|x \in \text{real numbers}\}$

83. The smallest number is -12. **85.** The maximum width as an integer is 11 cm. **87.** The TopPage plan is less expensive for more than 460 pages per month. **89.** The call must be 7 min or less. **91.** The temperature range for the week was between 32°F and 86°F. **93.** The amount of sales must be \$44,000 or more. **95.** The business writes

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more than 200 checks monthly. **97.** The range of scores is $58 \le N \le 100$. **99.** The ski area is more than 38 mi away. **101.** $\{1, 2\}$ **103.** $\varnothing$

SECTION 2.5

1. yes **3.** yes **5.** 7, −7 **7.** 4, −4 **9.** 6, −6 **11.** 7, −7 **13.** There is no solution. **15.** 1, −5 **17.** 8, 2 **19.** There is no solution. **21.** $-\frac{3}{2}, 3$ **23.** $\frac{3}{2}$ **25.** There is no solution. **27.** 7, −3 **29.** $2, -\frac{10}{3}$ **31.** 1, 3 **33.** $\frac{3}{2}$ **35.** There is no solution. **37.** $\frac{11}{6}, -\frac{1}{6}$ **39.** $-\frac{1}{3}, -1$ **41.** There is no solution. **43.** 3, 0 **45.** There is no solution. **47.** $1, \frac{13}{3}$ **49.** There is no solution. **51.** $\frac{7}{3}, \frac{1}{3}$ **53.** $-\frac{1}{2}$ **55.** $-\frac{1}{2}, -\frac{7}{2}$ **57.** $-\frac{8}{3}, \frac{10}{3}$ **59.** There is no solution. **61.** $\{x|x > 3 \text{ or } x < -3\}$ **63.** $\{x|x > 1 \text{ or } x < -3\}$ **65.** $\{x|4 \le x \le 6\}$ **67.** $\{x|x \ge 5 \text{ or } x \le -1\}$ **69.** $\{x|-3 < x < 2\}$ **71.** $\left\{x|x > 2 \text{ or } x < -\frac{14}{5}\right\}$ **73.** $\varnothing$ **75.** The solution set is the set of real numbers. **77.** $\left\{x|x \le -\frac{1}{3} \text{ or } x \ge 3\right\}$ **79.** $\left\{x|-2 \le x \le \frac{9}{2}\right\}$ **81.** $\{x|x = 2\}$ **83.** $\left\{x|x < -2 \text{ or } x > \frac{22}{9}\right\}$ **85.** $\left\{x|-\frac{3}{2} < x < \frac{9}{2}\right\}$ **87.** $\left\{x|x < 0 \text{ or } x > \frac{4}{5}\right\}$ **89.** $\{x|x > 5 \text{ or } x < 0\}$ **91.** The lower limit is 1.742 in. The upper limit is 1.758 in. **93.** The lower limit is 195 volts. The upper limit is 245 volts. **95.** The lower limit is $9\frac{19}{32}$ in. The upper limit is $9\frac{21}{32}$ in. **97.** The lower limit is 28,420 ohms. The upper limit is 29,580 ohms. **99.** The lower limit is 23,750 ohms. The upper limit is 26,250 ohms. **101.a.** $\{x|x \ge -3\}$ **b.** $\{a|a \le 4\}$ **103.a.** $\le$ **b.** $\ge$ **c.** $\ge$ **d.** $=$ **e.** $=$

CHAPTER REVIEW

1. no [2.1A] **2.** 21 [2.1B] **3.** 20 [2.1C] **4.** −3 [2.2A] **5.** $-\frac{6}{7}$ [2.2A] **6.** $\frac{1}{3}$ [2.2B] **7.** 4 [2.2B] **8.** −2 [2.2C] **9.** 10 [2.2C] **10.** $\left\{x|x > \frac{5}{3}\right\}$ [2.4A] **11.** $\{x|x > -1\}$ [2.4A] **12.** $\left\{x|-3 < x < \frac{4}{3}\right\}$ [2.4B] **13.** $\{x|x \in \text{real numbers}\}$ [2.4B] **14.** $-\frac{9}{5}, 3$ [2.5A] **15.** 9, −1 [2.5A] **16.** $\{x|1 < x < 4\}$ [2.5B] **17.** $\left\{x|x \ge 2 \text{ or } x \le \frac{1}{2}\right\}$ [2.5B] **18.** 30 is 250% of 12. [2.1D] **19.** $\frac{1}{2}$% of 1600 is 8. [2.1D] **20.** $5x - 4 = 16; x = 4$ [2.2D] **21.** $3x = 2(21 - x) - 2$; 8, 13 [2.2D] **22.** The airline would sell 177 tickets. [2.1D] **23.** The rebate is 6.1% of the cost. [2.1D] **24.** 2 qt of cranberry juice and 8 qt of apple juice were used. [2.3A] **25.** The cost is $3 per ounce. [2.3A] **26.** The percent concentration of butterfat is 14%. [2.3B] **27.** 375 lb of the alloy containing 30% tin and 125 lb of the alloy containing 70% tin were used. [2.3B] **28.** The total amount invested was $450,000. [2.3C] **29.** $1400 was deposited at 6.75%. $1000 was deposited at 9.45%. [2.3C] **30.** The jet overtakes the propeller-driven plane 600 mi from the starting point. [2.3D] **31.** The average speed on the winding road was 32 mph. [2.3D] **32.** The range of scores is $82 \le x \le 100$. [2.4C] **33.** The lower limit is 1.75 cc. The upper limit is 2.25 cc. [2.5C]

CHAPTER TEST

1. −5 [2.2B] **2.** −5 [2.1B] **3.** −3 [2.2A] **4.** 2 [2.2C] **5.** no [2.1A] **6.** 5 [2.2A] **7.** 0.04 [2.1D] **8.** $-\frac{1}{3}$ [2.2C] **9.** 2 [2.2B] **10.** −12 [2.1C] **11.** $\{x|x \le -3\}$ [2.4A] **12.** $\{x|x > -1\}$ [2.4A] **13.** $\{x|x > -2\}$ [2.4B] **14.** $\varnothing$ [2.4B] **15.** $3, -\frac{9}{5}$ [2.5A] **16.** 7, −2 [2.5A] **17.** $\left\{x\left|\frac{1}{3} \le x \le 3\right.\right\}$ [2.5B] **18.** $\left\{x|x > 2 \text{ or } x < -\frac{1}{2}\right\}$ [2.5B] **19.** 4, 11 [2.2D] **20.** The cost of the hamburger mixture is $2.70 per

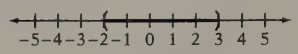
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pound. [2.3A] **21.** 1.25 gallons of water must be used. [2.3B] **22.** $5000 is invested at 3% and $15,000 is invested at 7%. [2.3C] **23.** The rate of the snowmobile is 6 mph. [2.3D] **24.** You can drive the Gambelli Agency car less than 120 mi per day. [2.4C] **25.** The lower limit is 2.648 in. The upper limit is 2.652 in. [2.5C]

CUMULATIVE REVIEW

1. 6 [1.1C] **2.** -48 [1.1D] **3.** $-\frac{19}{48}$ [1.2C] **4.** 54 [1.2E] **5.** $\frac{49}{40}$ [1.3A] **6.** 6 [1.4A] **7.** $-17x$ [1.4B]

8. $-5a - 4b$ [1.4B] **9.** $2x$ [1.4C] **10.** $36y$ [1.4C] **11.** $2x^2 + 6x - 4$ [1.4D] **12.** $-4x + 14$ [1.4D]

13. $6x - 34$ [1.4D] **14.** $A \cap B = \{-4, 0\}$ [1.5B] **15.**  [1.5C]

16. yes [2.1A]

17. -25 [2.1C] **18.** -3 [2.2A] **19.** 3 [2.2A] **20.** -3 [2.2B] **21.** $\frac{1}{2}$ [2.2B] **22.** 13 [2.2C]

23. $\{x \mid x \le 1\}$ [2.4A] **24.** $\{x \mid -4 \le x \le 1\}$ [2.4B] **25.** -1 and 4 [2.5A] **26.** $\left\{x \mid x > 2 \text{ or } x < -\frac{4}{3}\right\}$ [2.5B]

27. $\frac{11}{20}$ [1.2B] **28.** 103% [1.2B] **29.** 25% of 120 is 30. [2.1D] **30.** $6x + 13 = 3x - 5$; $x = -6$ [2.2D]

31. 20 lb of oat flour must be used. [2.3A] **32.** 25 g of pure gold must be added. [2.3B] **33.** The length of the track is 120 m. [2.3D]

Answers to Chapter 3 Selected Exercises

PREP TEST

1. 43 [2.1B] **2.** 51 [2.1B] **3.** 56 [1.3A] **4.** 56.52 [1.4A] **5.** 113.04 [1.4A] **6.** 120 [1.4A]

SECTION 3.1

1. 40°; acute **3.** 115°; obtuse **5.** 90°; right **7.** The complement is 28°. **9.** The supplement is 18°.
11. The length of BC is 14 cm. **13.** The length of QS is 28 ft. **15.** The length of EG is 30 m. **17.** The measure of $\angle MON$ is 86°. **19.** 71° **21.** 30° **23.** 36° **25.** 127° **27.** 116° **29.** 20° **31.** 20° **33.** 20°
35. 141° **37.** 106° **39.** 11° **41.** $\angle a = 38°$, $\angle b = 142°$ **43.** $\angle a = 47°$, $\angle b = 133°$ **45.** 20° **47.** 47°
49. $\angle x = 155°$, $\angle y = 70°$ **51.** $\angle a = 45°$, $\angle b = 135°$ **53.** $90° - x$ **55.** The measure of the third angle is 60°.
57. The measure of the third angle is 35°. **59.** The measure of the third angle is 102°. **61.a.** 1° **b.** 179° **65.** 360°

SECTION 3.2

1. hexagon **3.** pentagon **5.** scalene **7.** equilateral **9.** obtuse **11.** acute **13.** 56 in. **15.** 14 ft
17. 47 mi **19.** 8π cm or approximately 25.13 cm **21.** 11π mi or approximately 34.56 mi **23.** 17π ft or approximately 53.41 ft **25.** The perimeter is 17.4 cm. **27.** The perimeter is 8 cm. **29.** The perimeter is 24 m.
31. The perimeter is 48.8 cm. **33.** The perimeter is 17.5 in. **35.** The length of a diameter is 8.4 cm.
37. The circumference is 1.5π in. **39.** The circumference is 226.19 cm. **41.** 60 ft of fencing should be purchased.
43. The carpet must be nailed down along 44 ft. **45.** The length is 120 ft. **47.** The length of the third side is 10 in.
49. The length of each side is 12 in. **51.** The length of a diameter is 2.55 cm. **53.** The length is 13.19 ft.
55. The bicycle travels 50.27 ft. **57.** The circumference is 39,935.93 km. **59.** 60 ft^2 **61.** 20.25 in^2
63. 546 ft^2 **65.** 16π cm^2 or approximately 50.27 cm^2 **67.** 30.25π mi^2 or approximately 95.03 mi^2 **69.** 72.25π ft^2 or approximately 226.98 ft^2 **71.** The area is 156.25 cm^2. **73.** The area is 570 in^2. **75.** The area is 192 in^2.
77. The area is 13.5 ft^2. **79.** The area is 330 cm^2. **81.** The area is 25π in^2. **83.** The area is 9.08 ft^2.
85. The area is $10{,}000\pi$ in^2. **87.** The area is 126 ft^2. **89.** 7500 yd^2 must be purchased. **91.** The width is 10 in.

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93. The length of the base is 20 m. **95.** You should buy 2 qt. **97.** It will cost \$98. **99.** The increase in area is 113.10 in^2. **101.** The cost will be \$878. **103.** The area is 216 m^2. **105.** The cost is \$1600. **109.** $A = \frac{\pi d^2}{4}$

SECTION 3.3

1. 840 in^3 **3.** 15 ft^3 **5.** 4.5π cm^3 or approximately 14.14 cm^3 **7.** The volume is 34 m^3. **9.** The volume is 42.875 in^3. **11.** The volume is 36π ft^3. **13.** The volume is 8143.01 cm^3. **15.** The volume is 75π in^3. **17.** The volume is 120 in^3. **19.** The width is 2.5 ft. **21.** The radius of the base is 4.00 in. **23.** The length is 5 in. The width is 5 in. **25.** There are 75.40 m^3 in the tank. **27.** 94 m^2 **29.** 56 m^2 **31.** 96π in^2 or approximately 301.59 in^2 **33.** The surface area is 184 ft^2. **35.** The surface area is 69.36 m^2. **37.** The surface area is 225π cm^2. **39.** The surface area is 402.12 in^2. **41.** The surface area is 6π ft^2. **43.** The surface area is 297 in^2. **45.** The width is 3 cm. **47.** 11 cans of paint should be purchased. **49.** 456 in^2 of glass are needed. **51.** The surface area of the pyramid is 22.53 cm^2 larger. **53.a.** always true **b.** never true **c.** sometimes true

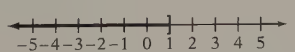
CHAPTER REVIEW

1. $\angle x = 22°, \angle y = 158°$ [3.1C] **2.** $\angle x = 68°$ [3.1B] **3.** The length of AC is 44 cm. [3.1A] **4.** 19° [3.1A] **5.** The volume is 96 cm^3. [3.3A] **6.** $\angle a = 138°, \angle b = 42°$ [3.1B] **7.** The surface area is 220 ft^2. [3.3B] **8.** The supplement is 148°. [3.1A] **9.** The area is 78 cm^2. [3.2B] **10.** The area is 63 m^2. [3.2B] **11.** The volume is 39 ft^3. [3.3A] **12.** The measure of the third angle is 95°. [3.1C] **13.** The length of the base is 8 cm. [3.2B] **14.** The volume is 288π mm^3. [3.3A] **15.** The volume is $\frac{784\pi}{3}$ cm^3. [3.3A] **16.** Each side measures 21.5 cm. [3.2A] **17.** 4 cans of paint should be purchased. [3.3B] **18.** 208 yd of fencing are needed. [3.2A] **19.** The area is 90.25 m^2. [3.2B] **20.** The area is 276 m^2. [3.2B]

CHAPTER TEST

1. The radius is 0.75 m. [3.2A] **2.** The circumference is 31.42 cm. [3.2A] **3.** The perimeter is 26 ft. [3.2A] **4.** $BC = 3$ [3.1A] **5.** The volume is 268.08 ft^3. [3.3A] **6.** The area is 63.62 cm^2. [3.2B] **7.** $a = 100°, b = 80°$ [3.1B] **8.** 75° [3.1A] **9.** $a = 135°, b = 45°$ [3.1B] **10.** The area is 55 m^2. [3.2B] **11.** The volume is 169.65 m^3. [3.3A] **12.** The perimeter is 6.8 m. [3.2A] **13.** 58° [3.1A] **14.** The surface area is 164.93 ft^2. [3.3B] **15.** There are 113.10 in^2 more in the pizza. [3.2B] **16.** The measures of the other two angles are 58° and 90°. [3.1C] **17.** The bicycle travels 73.3 ft. [3.2A] **18.** The area of the room is 28 yd^2. [3.2B] **19.** The volume of the silo is 1145.11 ft^3. [3.3A] **20.** The area is 11 m^2. [3.2B]

CUMULATIVE REVIEW

1. -3, 0, and 1 [1.1A] **2.** 0.089 [1.2B] **3.** 35% [1.2B] **4.** $-\frac{2}{3}$ [1.2D] **5.** -24.51 [1.2D] **6.** $-5\sqrt{5}$ [1.2F] **7.** -28 [1.3A] **8.** -8 [1.4A] **9.** $-3m + 3n$ [1.4B] **10.** $21y$ [1.4C] **11.** $7x + 9$ [1.4D] **12.** $\{-2, -1\}$ [1.5A] **13.** $\{-10, 0, 10, 20, 30\}$ [1.5A] **14.**  [1.5C] **15.** 5 [2.2B]

16. $\frac{1}{2}$ [2.2C] **17.** $\{y \mid y \leq -4\}$ [2.4A] **18.** $\{x \mid x \geq 2\}$ [2.4A] **19.** $\{x \mid x < -3 \text{ or } x > 4\}$ [2.4B] **20.** $\{x \mid 2 \leq x \leq 6\}$ [2.4B] **21.** $1, -\frac{1}{3}$ [2.5A] **22.** $\{x \mid 6 \leq x \leq 10\}$ [2.5B] **23.** $\angle x = 131°$ [3.1B] **24.** $4x - 10 = 2; x = 3$ [2.2D] **25.** The third angle measures 122°. [3.1C] **26.** \$5000 should be deposited in the 9.5% account. [2.3C] **27.** The third side measures 4.5 m. [3.2A] **28.** The women's median annual earnings are 70.8% of the men's median annual earnings. [1.2G] **29.** The area is 20.25π cm^2. [3.2B] **30.** The height of the box is 3 ft. [3.3A]

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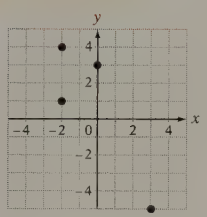
Answers to Chapter 4 Selected Exercises

PREP TEST

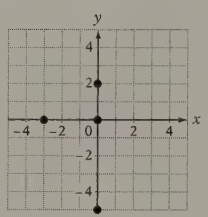
1. $-4x + 12$ [1.4D] **2.** 10 [1.2F] **3.** -2 [1.3A] **4.** 11 [1.4A] **5.** 2.5 [1.4A] **6.** 5 [1.4A]
7. 1 [1.4A] **8.** 4 [2.2A]

SECTION 4.1

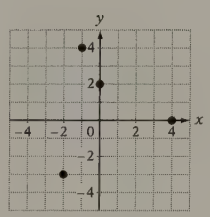
1.



3.



5.

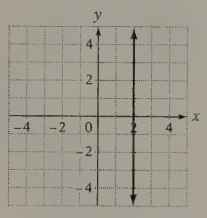


7. The coordinates of A are (2, 3). The coordinates of B are (4, 0). The coordinates of C are (−4, 1). The coordinates of D are (2, −2).

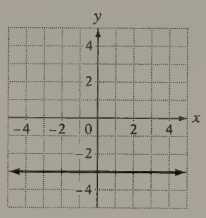
9. The coordinates of A are (−2, 5). The coordinates of B are (3, 4). The coordinates of C are (0, 0). The coordinates of D are (−3, −2).

11. The coordinates of A are (2, 4). The coordinates of B are (0, 1). The coordinates of C are (−4, 0). The coordinates of D are (3, −3).

13.

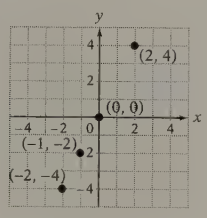


15.

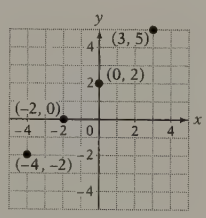


17. yes **19.** no **21.** no **23.** no **25.** (3, 7) **27.** (6, 3) **29.** (0, 1) **31.** (−5, 0)

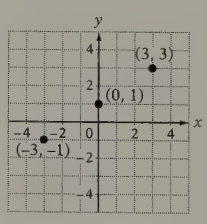
33.



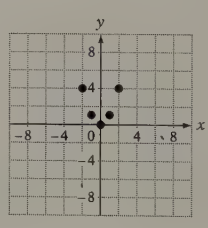
35.



37.

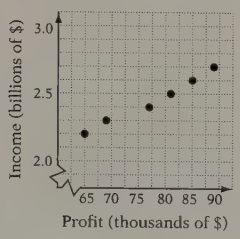


39.



41a. The temperature of the reaction after 20 min was 280°F. **b.** The temperature was 160°F in 50 min.

43.



SECTION 4.2

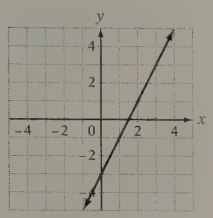
9. Function **11.** Function **13.** Function **15.** Not a function **17.** Yes. $29.62 **19.** $f(3) = 11$ **21.** $f(0) = -4$ **23.** $G(0) = 4$ **25.** $G(-2) = 10$ **27.** $q(3) = 5$ **29.** $q(-2) = 0$ **31.** $F(4) = 24$ **33.** $F(-3) = -4$ **35.** $H(1) = 1$ **37.** $H(t) = \dfrac{3t}{t+2}$ **39.** $s(-1) = 6$ **41.** $s(a) = a^3 - 3a + 4$ **43.** $P(-2 + h) - P(-2) = 4h$ **45a.** The packaging cost is $4.75 per game. **b.** The packaging cost is $4.00 per game. **47a.** The appraiser's fee is $3000. **b.** The appraiser's

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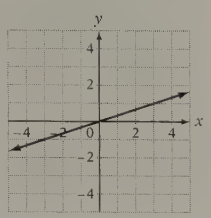
fee is $950. **49.** Domain: {1, 2, 3, 4, 5}; range: {1, 4, 7, 10, 13} **51.** Domain: {0, 2, 4, 6}; range: {1, 2, 3, 4} **53.** Domain: {1, 3, 5, 7, 9}; range: {0} **55.** Domain: {−2, −1, 0, 1, 2}; range: {0, 1, 2} **57.** Domain: {−2, −1, 0, 1, 2}; range: {−3, 3, 6, 7, 9} **59.** 1 **61.** −8 **63.** None **65.** None **67.** 0 **69.** None **71.** None **73.** −2 **75.** None **77.** Range: {−3, 1, 5, 9} **79.** Range: {−23, −13, −8, −3, 7} **81.** {0, 1, 4} **83.** {2, 14, 26, 42} **85.** $\left\{-5, \frac{5}{3}, 5\right\}$ **87.** $\left\{-1, -\frac{1}{2}, -\frac{1}{3}, 1\right\}$ **89.** {−38, −8, 2} **93.** {(−2, −8), (−1, −1), (0, 0), (1, 1), (2, 8)}. Yes, the set defines a function. **95.** 50.625 watts of power will be produced. **97a.** After 5 s the parachutist is falling at a rate of 22 ft/s. **b.** After 15 s the parachutist is falling at a rate of 30 ft/s. **99a.** After 5 h the temperature will be 64°F. **b.** After 15 h the temperature will be 52°F.

SECTION 4.3

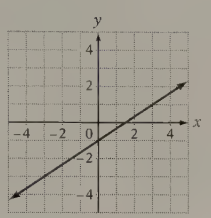
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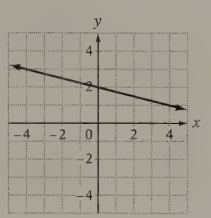
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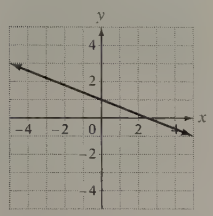
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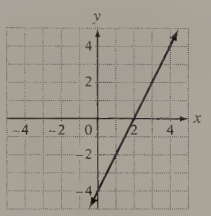
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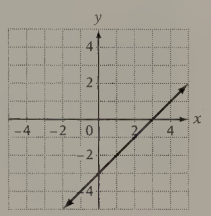
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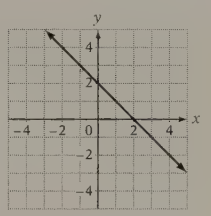
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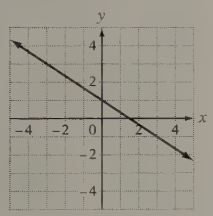
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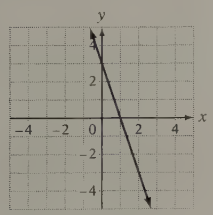


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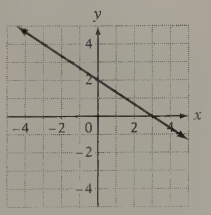


19. (3, 0), (0, −3) **21.** (10, 0), (0, −2) **23.** (0, 0), (0, 0)

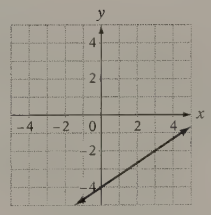
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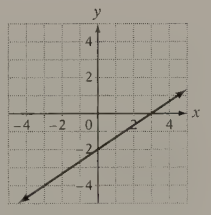
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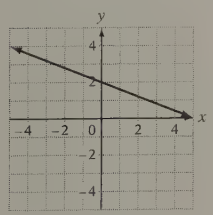
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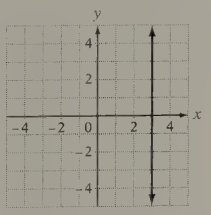
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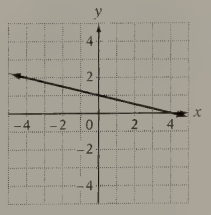
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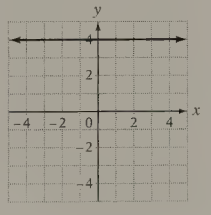
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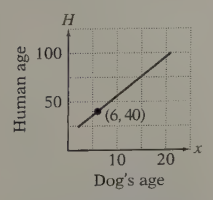
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39.

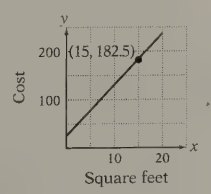


41.



A dog 6 years old is equivalent in age to a human 40 years old.

43.



It costs $182.50 for a custom sign 15 ft² in area.

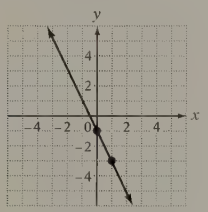
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SECTION 4.4

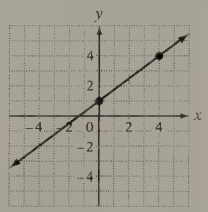
1. -1 **3.** $\frac{1}{3}$ **5.** $-\frac{2}{3}$ **7.** $-\frac{3}{4}$ **9.** The slope is undefined. **11.** $\frac{7}{5}$ **13.** The line has zero slope.

15. $-\frac{1}{2}$ **17.** The slope is undefined. **19.** The slope is 40. The slope is the average speed of the motorist in miles per hour. **21.** The slope is -0.05. For each mile the car is driven, approximately 0.05 gal of fuel is used. **23.** The slope is 385.5. The slope is the average speed of the runner in meters per minute. **25.** no **29.** yes

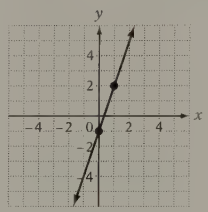
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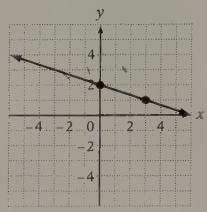
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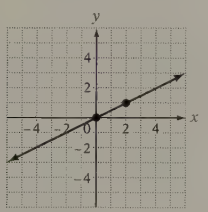
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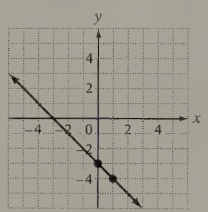
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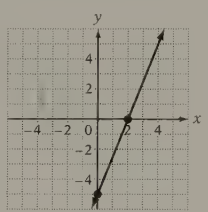
39.



41.



43.



45. i and D; ii and C; iii and B; iv and F; v and E; vi and A **47.** $k = 10$ **49.** $k = -4$

SECTION 4.5

3. $y = 2x + 5$ **5.** $y = \frac{1}{2}x + 2$ **7.** $y = \frac{1}{2}x + 4$ **9.** $y = -\frac{5}{3}x + 5$ **11.** $y = -3x + 9$ **13.** $y = -3x + 4$

15. $y = \frac{1}{3}x$ **17.** $y = \frac{1}{2}x$ **19.** $y = 3x - 9$ **21.** $y = -\frac{2}{3}x + 7$ **23.** $y = -x - 3$ **25.** $y = \frac{7}{5}x - \frac{27}{5}$

27. $y = -\frac{2}{5}x + \frac{3}{5}$ **29.** $x = 3$ **31.** $y = \frac{1}{5}x - 2$ **33.** $y = -3$ **35.** $y = -2x + 3$ **37.** $x = -5$ **39.** $y = x + 2$

41. $y = -2x - 3$ **43.** $y = \frac{1}{2}x + 2$ **45.** $y = -x + 5$ **47.** $y = -2x - 3$ **49.** $y = -\frac{3}{2}x + 3$ **51.** $y = -1$

53. $y = x - 1$ **55.** $y = -x + 1$ **57.** $y = -\frac{8}{3}x + \frac{25}{3}$ **59.** $y = \frac{1}{2}x - 1$ **61.** $y = -4$ **63.** $y = \frac{3}{4}x$

65. $y = -\frac{4}{3}x + \frac{5}{3}$ **67.** $x = -2$ **69.** $y = x - 1$ **71.** $y = \frac{4}{3}x + \frac{7}{3}$ **73.** $y = -x + 3$ **75.** $y = 1200x$. The height of the plane 11 min after takeoff is 13,200 ft. **77.** $y = -0.032x + 16$. After the driver has driven 150 mi, there are 11.2 gal in the tank. **83.** Answers will vary. Possible answers are $\{(0, 4), (3, 2), (9, -2)\}$. **85.** $f(4) = 0$ **87.** $y = -2x + 1$

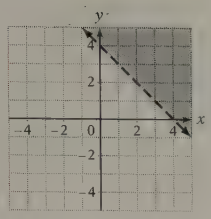
SECTION 4.6

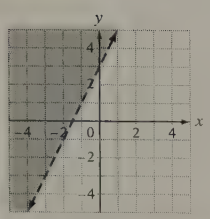
3. -5 **5.** $-\frac{1}{4}$ **7.** The lines are perpendicular. **9.** No, the lines are not parallel. **11.** No, the lines are not parallel. **13.** The lines are perpendicular. **15.** The lines are parallel. **17.** The lines are perpendicular. **19.** No, the lines are not parallel. **21.** The lines are perpendicular. **23.** $y = \frac{2}{3}x - \frac{8}{3}$ **25.** $y = \frac{1}{3}x - \frac{1}{3}$

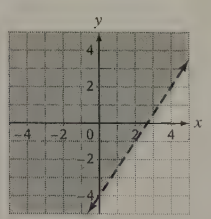
27. $y = -\frac{5}{3}x - \frac{14}{3}$ **29.** $y = -2x + 15$ **31.** $\frac{A_1}{B_1} = -\frac{B_2}{A_2}$ **33.** Any equation of the form $y = 2x + b$ where $b \neq -13$ or of the form $y = -\frac{3}{2}x + c$ where $c \neq 8$.

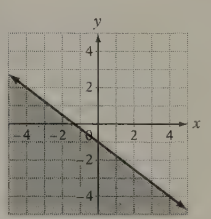
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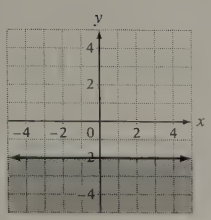
SECTION 4.7

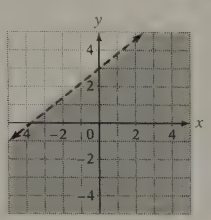
3. yes **5.** no **7.** 

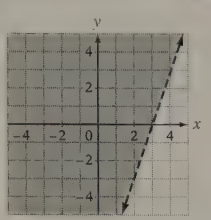
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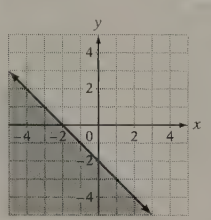
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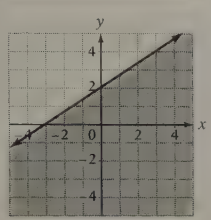
13. 

15. 

17. 

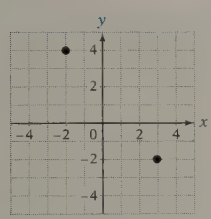
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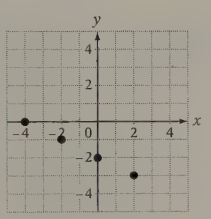
21. 

23. 

25. The inequality is $y \geq 2x + 2$.

CHAPTER REVIEW

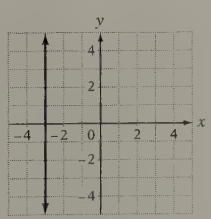
1a.  [4.1A]

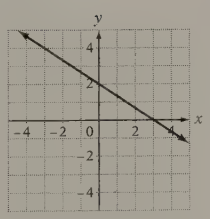
2.  [4.1B]

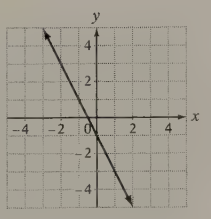
3. (8, 0) and (0, −12) [4.3B] **4.** −1 [4.2A]

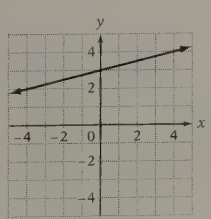
b. The abscissa of point A is −2.

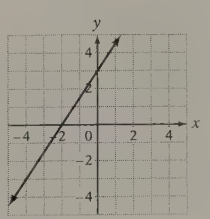
c. The ordinate of point B is −4.

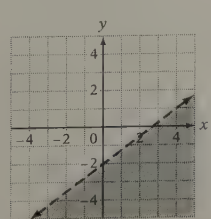
5.  [4.3B]

6.  [4.4B]

7.  [4.3A]

8.  [4.3A]

9. 0 [4.4A] **10.** $\frac{7}{11}$ [4.4A] **11.** 0 [4.2A] **12.**  [4.3B]

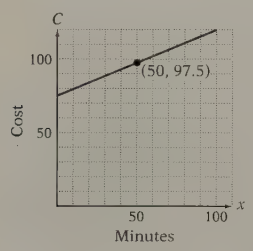
13.  [4.7A]

14. $y = -\frac{8}{3}x - \frac{1}{3}$ [4.5B] **15.** $y = -\frac{5}{2}x + 16$ [4.5A] **16.** $y = 2x$ [4.6A]

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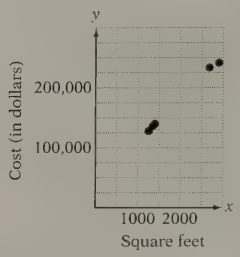
17. $y = 2x + 1$ [4.6A] **18.** $y = 80x + 25{,}000$. It will cost \$185,000 to build a house that contains 2000 ft². [4.5C]

19.



The cost of 50 min of access time for one month is \$97.50. [4.3C]

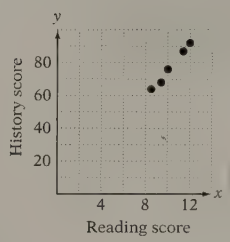
20.



[4.1C]

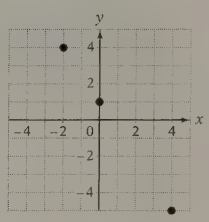
CHAPTER TEST

1. (3, −3) [4.1B] **2.** 6 [4.2A] **3.**



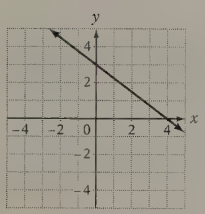
[4.1C]

4.



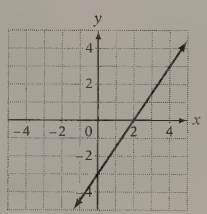
[4.1B]

5.



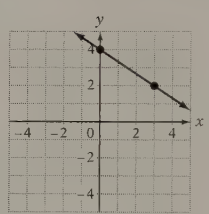
[4.3A]

6.



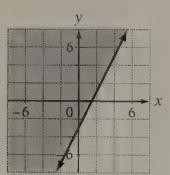
[4.3B]

7.



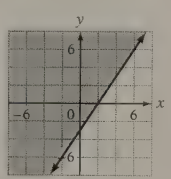
[4.4B]

8.



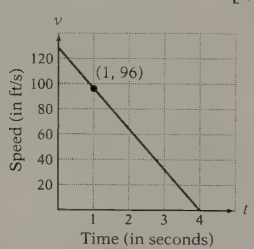
[4.7A]

9.



[4.7A]

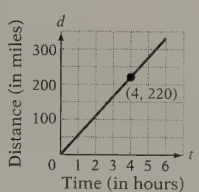
10.



After 1 s, the ball is traveling 96 ft/s. [4.3C]

11. $y = -\frac{3}{10}x + 175$. When the tuition is \$300, the enrollment will be 85 students. [4.5C]

12.



After 4 h, the car has traveled 220 mi. [4.3C]

13. (2, 0) and (0, −3) [4.3B] **14.** 2 [4.4A]

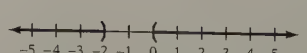
15. The slope is undefined. [4.4A] **16.** $-\frac{2}{3}$ [4.3B] **17.** $y = \frac{2}{3}x + 3$ [4.5A] **18.** $y = -\frac{2}{7}x - \frac{4}{7}$ [4.5B]

19. $y = -3x + 7$ [4.6A] **20.** $y = \frac{3}{2}x + 2$ [4.6A]

CUMULATIVE REVIEW

1. −5, −3 [1.1A] **2.** 0.85 [1.2B] **3.** $9\sqrt{5}$ [1.2F] **4.** −12 [1.3A] **5.** $-\frac{5}{8}$ [1.4A]

6. $-4d - 9$ [1.4B] **7.** $-32z$ [1.4C] **8.** $-13x + 7y$ [1.4D] **9.**

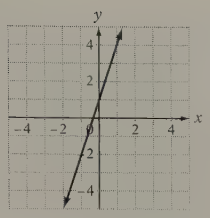


[1.5C]

10. $\frac{3}{2}$ [2.2A] **11.** 1 [2.2C] **12.** $\{x \mid x > -1\}$ [2.4A] **13.** $\varnothing$ [2.4B] **14.** $\left\{x \mid 0 < x < \frac{10}{3}\right\}$ [2.5B]

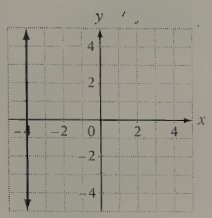
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15. $f(2) = 6$ [4.2A] **16.** The slope is 2. [4.4A] **17.**



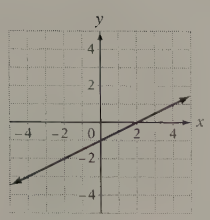
[4.3A]

18.



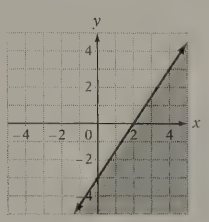
[4.3B]

19.



[4.4B]

20.



[4.7A]

21. $y = -\frac{1}{3}x - 2$ [4.5B] **22.** $y = -\frac{3}{2}x + 7$ [4.6A] **23.** The first plane is traveling at 400 mph. The second plane is traveling at 200 mph. [2.3D] **24.** 20 lb of \$6 coffee and 40 lb of \$9 coffee should be used. [2.3A]

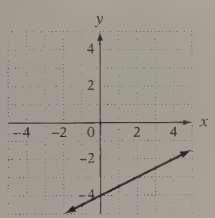
25. The slope is $-\frac{10{,}000}{3}$. The value of the house decreases by \$3333.33 each year. [4.4A]

Answers to Chapter 5 Selected Exercises

PREP TEST

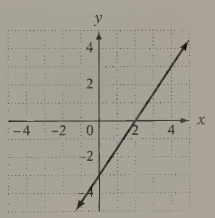
1. $6x + 5y$ [1.4D] **2.** 7 [1.4A] **3.** 0 [2.2A] **4.** -3 [2.2C] **5.** 1000 [2.2C]

6.



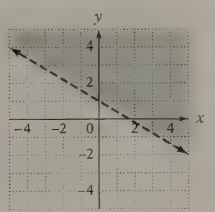
[4.3A]

7.



[4.3B]

8.

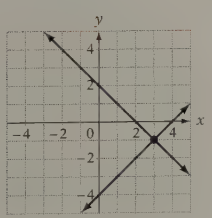


[4.7A]

SECTION 5.1

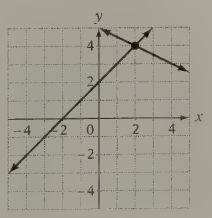
1. no **3.** yes **5.** no **7.** yes **9.** independent **11.** inconsistent

13.



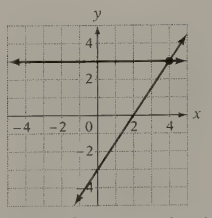
The solution is $(3, -1)$.

15.



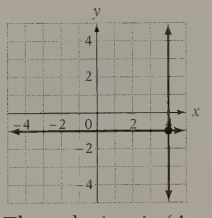
The solution is $(2, 4)$.

17.



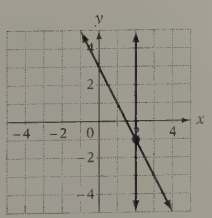
The solution is $(4, 3)$.

19.



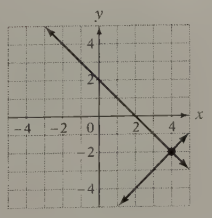
The solution is $(4, -1)$.

21.



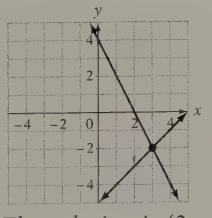
The solution is $(2, -1)$.

23.



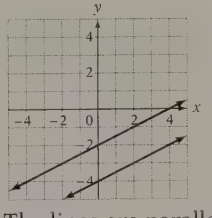
The solution is $(4, -2)$.

25.



The solution is $(3, -2)$.

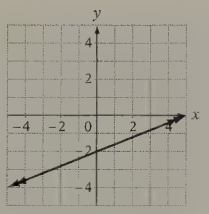
27.



The lines are parallel and therefore do not intersect. The system of equations has no solution.

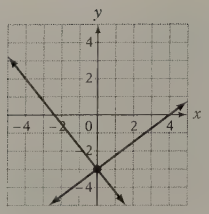
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29.



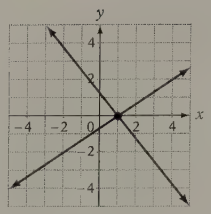
The equations are dependent. The solutions are the ordered pairs $\left(x, \frac{2}{5}x - 2\right)$.

31.



The solution is $(0, -3)$.

33.



The solution is $(1, 0)$.

35. $(4, 3)$ **37.** $(2, 4)$ **39.** $(3, 2)$ **41.** inconsistent **43.** $(x, 2 - x)$ **45.** $(-5, -7)$ **47.** $\left(\frac{17}{5}, -\frac{7}{5}\right)$

49. $\left(-\frac{6}{11}, \frac{31}{11}\right)$ **51.** $(2, 3)$ **53.** $(0, 0)$ **55.** $(x, -3x + 4)$ **57.** $(6, -1)$ **59.** $(5, 2)$ **61.** $(-17, -8)$

63. $\left(-\frac{5}{7}, \frac{13}{7}\right)$ **65.** 1 **67.** The assertion is false. The solution is $(0, 2)$. **69.** $(-2, -0.4)$ **71.** $(0.54, 1.03)$

SECTION 5.2

1. $(6, 1)$ **3.** $(1, 1)$ **5.** $(2, 1)$ **7.** $(-2, 1)$ **9.** The equations are dependent. The solutions are the ordered pairs $(x, 3x - 4)$. **11.** $\left(-\frac{1}{2}, 2\right)$ **13.** The system is inconsistent and therefore has no solution. **15.** $(-1, -2)$

17. $(-5, 4)$ **19.** $(2, 5)$ **21.** $\left(\frac{1}{2}, \frac{3}{4}\right)$ **23.** $(0, 0)$ **25.** $(-1, 3)$ **27.** $\left(\frac{2}{3}, -\frac{2}{3}\right)$ **29.** $(-2, 3)$ **31.** $(2, -1)$

33. $(10, -5)$ **35.** $\left(-\frac{1}{2}, \frac{2}{3}\right)$ **37.** $\left(\frac{5}{3}, \frac{1}{3}\right)$ **39.** The system is inconsistent and therefore has no solution.

41. $(1, -1)$ **43.** $(2, 1, 3)$ **45.** $(1, -1, 2)$ **47.** $(1, 2, 4)$ **49.** $(2, -1, -2)$ **51.** $(-2, -1, 3)$ **53.** The system is inconsistent and therefore has no solution. **55.** $(1, 4, 1)$ **57.** $(1, 3, 2)$ **59.** $(1, -1, 3)$ **61.** $(0, 2, 0)$ **63.** $(1, 5, 2)$

65. $(-2, 1, 1)$ **67.** $A = 3, B = -3$ **69a.** 14 **b.** $\frac{2}{3}$ **c.** $\frac{1}{2}$ **71.** $A = 1$

SECTION 5.3

3. 11 **5.** 18 **7.** 0 **9.** 15 **11.** -30 **13.** 0 **15.** $(3, -4)$ **17.** $(4, -1)$ **19.** $\left(\frac{11}{14}, \frac{17}{21}\right)$ **21.** $\left(\frac{1}{2}, 1\right)$

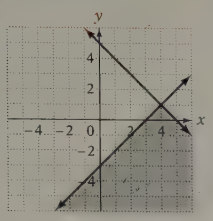
23. The system of equations is not independent. **25.** $(-1, 0)$ **27.** $(1, -1, 2)$ **29.** $(2, -2, 3)$ **31.** The system of equations is not independent. **33.** $\left(\frac{68}{25}, \frac{56}{25}, -\frac{8}{25}\right)$ **35.** $\left(\frac{2}{3}, -1, \frac{1}{2}\right)$ **37.** The area is 239 ft^2. **39a.** 0 **b.** 0

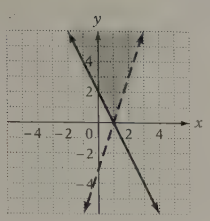
SECTION 5.4

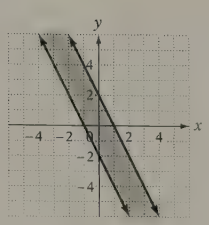
1. The rate of the boat is 15 mph. The rate of the current is 3 mph. **3.** The rate of the plane is 502.5 mph. The rate of the wind is 47.5 mph. **5.** The rate of the team is 8 km/h. The rate of the current is 2 km/h. **7.** The rate of the plane is 180 mph. The rate of the wind is 20 mph. **9.** The rate of the plane is 110 mph. The rate of the wind is 10 mph. **11.** The rate of the motorboat is 13 mph. The rate of the current is 3 mph. **13.** The rate of the cabin cruiser is 15 mph. The rate of the current is 3 mph. **15.** The cost of the cinnamon tea is \$2.50/lb. The cost of the spice tea is \$3/lb. **17.** The cost of the wool carpet is \$52/yd. **19.** The company plans to manufacture 25 mountain bikes. **21.** You have \$1500 in the 6% account and \$3500 in the 8% account. **23.** The measure of the smaller angle is 9°. The measure of the larger angle is 81°. **25.** The manufacturer charges \$4000 for each model VI. **27.** There were 190 regular tickets, 210 student tickets, and 350 member-discount tickets sold. **29.** The original number is 84.

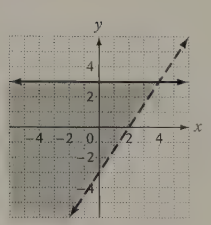
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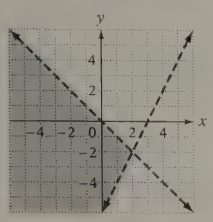
SECTION 5.5

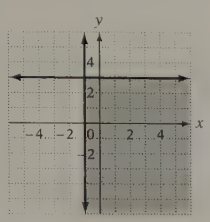
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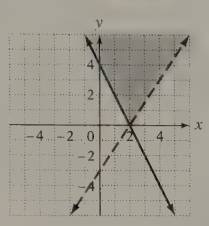
3. 

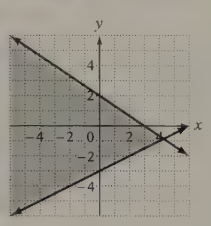
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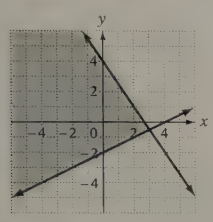
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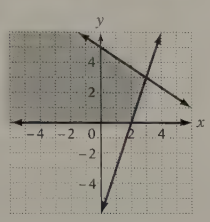
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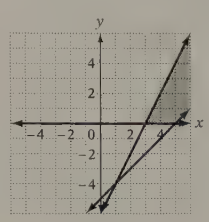
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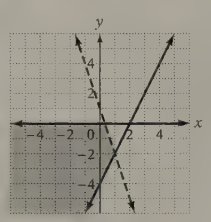
13. 

15. 

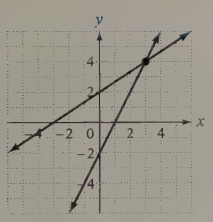
17. 

19. 

21. 

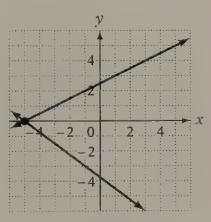
23. 

CHAPTER REVIEW

1. 

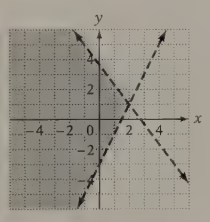
[5.1A]

The solution is (3, 4).

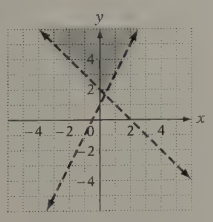
2. 

[5.1A]

The solution is (−5, 0).

3. 

[5.5A]

4. 

[5.5A]

5. $\left(\frac{3}{4}, \frac{7}{8}\right)$ [5.1B] **6.** (−3, −4) [5.1B] **7.** (2, −1) [5.1B] **8.** (−2, 1) [5.2A]

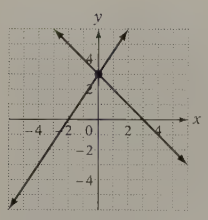
9. The system is inconsistent and therefore has no solution. [5.2A] **10.** (1, 1) [5.2A] **11.** The system is inconsistent and therefore has no solution. [5.2B] **12.** (2, −1, −2) [5.2B] **13.** 10 [5.3A] **14.** −32 [5.3A]

15. $\left(-\frac{1}{3}, -\frac{10}{3}\right)$ [5.3B] **16.** $\left(\frac{59}{19}, -\frac{62}{19}\right)$ [5.3B] **17.** $\left(\frac{1}{5}, -\frac{6}{5}, \frac{3}{5}\right)$ [5.3B] **18.** (0, −2, 3) [5.3B] **19.** The rate of the plane is 150 mph. The rate of the wind is 25 mph. [5.4A] **20.** The cost of the cotton is \$18/yd. The cost of the wool is \$28/yd. [5.4B]

CHAPTER TEST

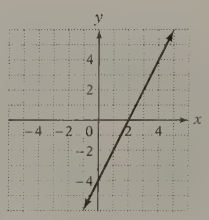
1. $\left(6, -\frac{1}{2}\right)$ [5.1B] **2.** (−4, 7) [5.2A]

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3. 

[5.1A]

The solution is (0, 3).

4. 

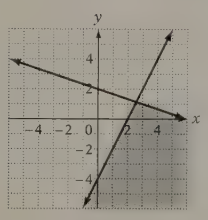
[5.1A]

The equations are dependent. The solutions are the ordered pairs $(x, 2x - 4)$.

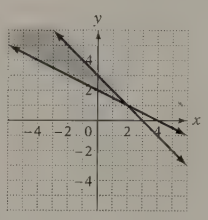
5. $\left(x, -\frac{1}{4}x + \frac{3}{2}\right)$ [5.1B] **6.** $\left(x, \frac{1}{3}x - 2\right)$ [5.2A]

7. $(3, -1, -2)$ [5.2B] **8.** $(5, -2, 3)$ [5.2B] **9.** 28 [5.3A] **10.** 0 [5.3A] **11.** $(3, -1)$ [5.3B]

12. $\left(\frac{110}{23}, \frac{25}{23}\right)$ [5.3B] **13.** $(-1, -3, 4)$ [5.3B] **14.** $(2, 3, -5)$ [5.3B]

15. 

[5.5A]

16. 

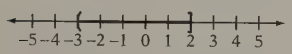
[5.5A]

17. (2, 1) [5.1B] **18.** The rate of the cabin cruiser in calm water is 16 mph. The rate of the current is 4 mph. [5.4A] **19.** The rate of the plane in calm air is 175 mph. The rate of the wind is 25 mph. [5.4A] **20.** On Friday evening, 100 children attended the movie theater. [5.4B]

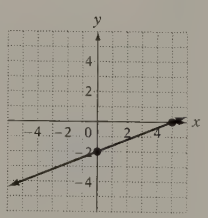
CUMULATIVE REVIEW

1. $-6\sqrt{10}$ [1.2F] **2.** 22 [2.2C] **3.** $3x - 24$ [1.4D] **4.** -4 [1.4A] **5.** $\{x|x < 6\}$ [2.4B]

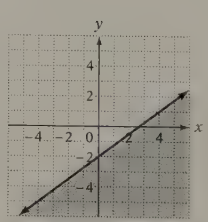
6. $\{x| -4 < x < 8\}$ [2.5B] **7.** $\{x|x > 4 \text{ or } x < -1\}$ [2.5B] **8.** $F(2) = 1$ [4.2A]

9. 

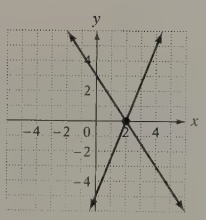
[1.5C] **10.** $y = -\frac{2}{3}x + \frac{5}{3}$ [4.5A] **11.** $y = 5x - 11$ [4.5B]

12. $y = -\frac{3}{2}x - 1$ [4.6A] **13.** 

[4.4B]

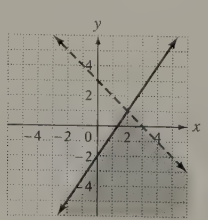
14. 

[4.7A]

15. 

[5.1A]

The solution is (2, 0).

16. 

[5.5A]

17. (1, 0, −1) [5.2B] **18.** 3 [5.3A] **19.** (2, −3) [5.3B] **20.** (−5, −11) [5.1B] **21.** 60 ml of pure water must be used. [2.3B] **22.** The rate of the wind is 12.5 mph. [5.4A] **23.** One pound of steak costs $10. [5.4B] **24.** The lower limit is 10,200 ohms. The upper limit is 13,800 ohms. [2.5C] **25.** The slope is 40. The slope represents the monthly income (in dollars) per thousand dollars in sales. [4.4A]

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Answers to Chapter 6 Selected Exercises

PREP TEST

1. 1 [1.1C] **2.** -18 [1.1D] **3.** $\frac{2}{3}$ [1.1D] **4.** $-12y$ [1.4C] **5.** -8 [1.2E] **6.** $3a - 8b$ [1.4B]

7. $11x - 2y - 2$, [1.4D] **8.** 0 [1.4B] **9.** No [1.4B]

SECTION 6.1

1. **a** is the multiplication of two exponential expressions. **b** is the power of an exponential expression. **3.** a^4b^4

5. $-18x^3y^4$ **7.** x^8y^{16} **9.** $81x^8y^{12}$ **11.** $729a^{10}b^6$ **13.** x^5y^{11} **15.** $729x^6$ **17.** $a^{18}b^{18}$ **19.** $4096x^{12}y^{12}$

21. $64a^{24}b^{18}$ **23.** x^{2n+1} **25.** y^{6n-2} **27.** a^{2n^2-6n} **29.** x^{15n+10} **31.** $-6x^5y^5z^4$ **33.** $-12a^2b^9c^2$

35. $-6x^4y^4z^5$ **37.** $-432a^7b^{11}$ **39.** $54a^{13}b^{17}$ **41.** 243 **43.** y^3 **45.** $\frac{a^3b^2}{4}$ **47.** $\frac{x}{y^4}$ **49.** $\frac{1}{2}$ **51.** $-\frac{1}{9}$ **53.** $\frac{1}{y^8}$

55. $\frac{1}{x^6y^{10}}$ **57.** x^5y^5 **59.** $\frac{a^8}{b^9}$ **61.** $\frac{1}{2187a}$ **63.** $\frac{y^6}{x^3}$ **65.** $\frac{y^4}{x^3}$ **67.** $-\frac{1}{2x^2}$ **69.** $-\frac{1}{243a^5b^{10}}$ **71.** $\frac{16x^8}{81y^4z^{12}}$

73. $\frac{3a^4}{4b^3}$ **75.** $-\frac{9a}{8b^6}$ **77.** $\frac{16x^2}{y^6}$ **79.** $\frac{1}{b^{4n}}$ **81.** $-\frac{1}{y^{6n}}$ **83.** y^{n-2} **85.** $\frac{1}{y^{2n}}$ **87.** $\frac{x^{n-5}}{y^6}$ **89.** $\frac{8b^{15}}{3a^{18}}$

91. 4.67×10^{-6} **93.** 1.7×10^{-10} **95.** 2×10^{11} **97.** 0.000000123 **99.** 8,200,000,000,000,000 **101.** 0.039

103. 150,000 **105.** 20,800,000 **107.** 0.000000015 **109.** 0.0000000000178 **111.** 140,000,000 **113.** 11,000,000

115. 0.000008 **117.** Light travels 2.592×10^{10} km in one day. **119.** The signals traveled at $1.0\overline{81} \times 10^7$ mi/min.

121. The centrifuge makes one revolution in $6.\overline{6} \times 10^{-7}$ s. **123.** The sun is 3.38983×10^5 times heavier than Earth.

125. One orchid seed weighs 3.225806×10^{-8} oz. **127.** The Coma cluster is 1.6423949×10^{21} mi from Earth.

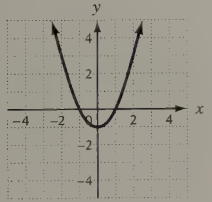
SECTION 6.2

1. $P(3) = 13$ **3.** $R(2) = 10$ **5.** $f(-1) = -11$ **7.a.** -1 **b.** 8 **c.** 2

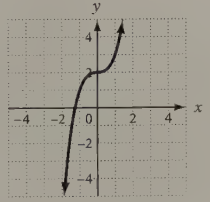
9. The expression is not a polynomial function. **11.** The expression is not a polynomial function.

13.a. 3 **b.** π **c.** 5 **15.a.** -5 **b.** 2 **c.** 3 **17.a.** 14 **b.** 14 **c.** 0

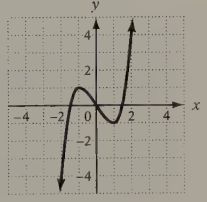
19.



21.



23.



25. $6x^2 - 6x + 5$

27. $-x^2 + 1$ **29.** $5y^2 - 15y + 2$ **31.** $7a^2 - a + 2$ **33.** $P(x) + R(x) = 3x^2 - 3x - 2$

35. $P(x) - R(x) = 8x^2 - 2x + 5$ **37.** $S(x) = 3x^4 - 8x^2 + 2x$; $S(2) = 20$ **39.a.** $k = 8$ **b.** $k = -4$

SECTION 6.3

1. $x^2 - 2x$ **3.** $-x^2 - 7x$ **5.** $3a^3 - 6a^2$ **7.** $-5x^4 + 5x^3$ **9.** $-3x^5 + 7x^3$ **11.** $12x^3 - 6x^2$ **13.** $6x^2 - 12x$

15. $3x^2 + 4x$ **17.** $-x^3y + xy^3$ **19.** $2x^4 - 3x^2 + 2x$ **21.** $2a^3 + 3a^2 + 2a$ **23.** $3x^6 - 3x^4 - 2x^2$

25. $-6y^4 - 12y^3 + 14y^2$ **27.** $-2a^3 - 6a^2 + 8a$ **29.** $6y^4 - 3y^3 + 6y^2$ **31.** $x^3y - 3x^2y^2 + xy^3$

33. $x^3 + 4x^2 + 5x + 2$ **35.** $a^3 - 6a^2 + 13a - 12$ **37.** $-2b^3 + 7b^2 + 19b - 20$ **39.** $-6x^3 + 31x^2 - 41x + 10$

41. $x^3 - 3x^2 + 5x - 15$ **43.** $x^4 - 4x^3 - 3x^2 + 14x - 8$ **45.** $15y^3 - 16y^2 - 70y + 16$

47. $5a^4 - 20a^3 - 5a^2 + 22a - 8$ **49.** $y^4 + 4y^3 + y^2 - 5y + 2$ **51.** $x^2 + 4x + 3$ **53.** $a^2 + a - 12$ **55.** $y^2 - 5y - 24$

57. $y^2 - 10y + 21$ **59.** $2x^2 + 15x + 7$ **61.** $3x^2 + 11x - 4$ **63.** $4x^2 - 31x + 21$ **65.** $3y^2 - 2y - 16$

67. $9x^2 + 54x + 77$ **69.** $21a^2 - 83a + 80$ **71.** $6a^2 - 25ab + 14b^2$ **73.** $2a^2 - 11ab - 63b^2$

75. $100a^2 - 100ab + 21b^2$ **77.** $15x^2 + 56xy + 48y^2$ **79.** $14x^2 - 97xy - 60y^2$ **81.** $56x^2 - 61xy + 15y^2$

85. $y^2 - 25$ **87.** $4x^2 - 9$ **89.** $x^2 + 2x + 1$ **91.** $9a^2 - 30a + 25$ **93.** $9x^2 - 49$ **95.** $4a^2 + 4ab + b^2$

97. $16 - 9y^2$ **99.** $25x^2 + 20xy + 4y^2$ **101.** The area is $(10x^2 - 35x)$ ft^2. **103.** The area is $(4x^2 + 4x + 1)$ km^2.

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105. The area is $(18x^2 + 12x + 2)$ in^2. **107.** The area is $(4x^2 + 10x)$ m^2. **109.** The total area is $(3000 + 60w)$ yd^2.
111. $4ab$ **113.** $a^3 + 9a^2 + 27a + 27$ **115.a.** $7x^2 - 11x - 8$ **b.** $x^3 - 7x^2 - 7$

SECTION 6.4

1. $x + 8$ **3.** $x^2 + \frac{2}{x-3}$ **5.** $3x + 5 + \frac{3}{2x+1}$ **7.** $5x + 7 + \frac{2}{2x-1}$ **9.** $4x^2 + 6x + 9 + \frac{18}{2x-3}$

11. $3x^2 + 1 + \frac{1}{2x^2-5}$ **13.** $x^2 - 3x - 10$ **15.** $x^2 - 2x + 1 - \frac{1}{x-3}$ **17.** $2x^3 - 3x^2 + x - 4$

19. $2x + \frac{x+2}{x^2+2x-1}$ **21.** $x^2 + 4x + 6 + \frac{10x+8}{x^2-2x-1}$ **23.** $x^2 - 2x + 4 + \frac{-6x-7}{x^2+2x+3}$ **25.** $2x - 8$ **27.** $3x - 8$

29. $3x + 3 - \frac{1}{x-1}$ **31.** $2x^2 - 3x + 9$ **33.** $4x^2 + 8x + 15 + \frac{12}{x-2}$ **35.** $2x^2 - 3x + 7 - \frac{8}{x+4}$

37. $3x^3 + 2x^2 + 12x + 19 + \frac{33}{x-2}$ **39.** $3x^3 - x + 4 - \frac{2}{x+1}$ **41.** $2x^3 + 6x^2 + 17x + 51 + \frac{155}{x-3}$ **43.** $f(3) = 0$

45. $P(3) = 8$ **47.** $R(4) = 43$ **49.** $P(-2) = -39$ **51.** $Z(-3) = -60$ **53.** $Q(2) = 31$ **55.** $F(-3) = 178$
57. $P(5) = 122$ **59.** $R(-3) = 302$ **61.** $Q(2) = 0$ **63.** $R(-2) = -65$ **65.a.** $a^2 - ab + b^2$

b. $x^4 - x^3y + x^2y^2 - xy^3 + y^4$ **c.** $x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5$ **67.a.** $\frac{1}{2}x^2 + \frac{3}{4}x - 1$

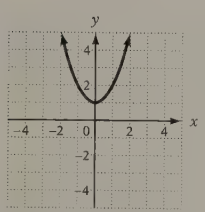
b. $\frac{2}{3}x^2 + \frac{7}{2}x + \frac{10}{3} + \frac{64}{6x-12}$ **c.** $2x^2 - x + 1 + \frac{3x-11}{x^2-x+1}$ **d.** $x^2 + 6x + 17 + \frac{51x+56}{x^2-2x-3}$

CHAPTER REVIEW

1. $-6a^3b^6$ [6.1A] **2.** $9x^4y^6$ [6.1A] **3.** $\frac{c^{10}}{2b^{17}}$ [6.1B] **4.** 0.00254 [6.1C]

5. -7 [6.2A] **6.** $4x^2 - 8xy + 5y^2$ [6.2B] **7.** $\frac{18x^5y^4}{z}$ [6.1B] **8.** $-54a^{13}b^5c^7$ [6.1A]

9.



[6.2A]

10.a. 3 **b.** 8 **c.** 5 [6.2A] **11.** 1 [6.4C] **12.** $-8x^3 - 14x^2 + 18x$ [6.3A]

13. $8a^3b^3 - 4a^2b^4 + 6ab^5$ [6.3A] **14.** $6y^3 + 17y^2 - 2y - 21$ [6.3B] **15.** $10a^2 + 31a - 63$ [6.3C]

16. 1.27×10^{-7} [6.1C] **17.** $25y^2 - 70y + 49$ [6.3D] **18.** $5x + 4 + \frac{6}{3x-2}$ [6.4A]

19. $4x^2 + 3x - 8 + \frac{50}{x+6}$ [6.4B] **20.** $x^3 + 4x^2 + 16x + 64 + \frac{252}{x-4}$ [6.4B] **21.** 8,100,000,000 [6.1C]

22. $-\frac{2a^3}{3b^3}$ [6.1B] **23.** $25a^2 - 4b^2$ [6.3D] **24.** 68 [6.4C] **25.** $21y^2 + 4y - 1$ [6.2B]

26. $12b^5 - 4b^4 - 6b^3 - 8b^2 + 5$ [6.3B] **27.** $a^2 - 49$ [6.3D] **28.** 7.65×10^{11} [6.1C]
29. The area of the Ping-Pong table is $(2w^2 - w)$ ft^2. [6.3E] **30.** The Great Galaxy of Andromeda is 1.291224×10^{19} mi from Earth. [6.1D] **31.** There are 6.048×10^5 s in one week. [6.1D] **32.** The area is $(9x^2 - 12x + 4)$ in^2. [6.3E]
33. The area is $(10x^2 - 29x - 21)$ cm^2. [6.3E]

CHAPTER TEST

1. $4x^3 - 6x^2$ [6.3A] **2.** -8 [6.4C] **3.** $-\frac{4}{x^6}$ [6.1B] **4.** $-6x^3y^6$ [6.1A] **5.** $x - 1 + \frac{2}{x+1}$ [6.4A/6.4B]

6. $x^3 - 7x^2 + 17x - 15$ [6.3B] **7.** $-8a^6b^3$ [6.1A] **8.** $\frac{9y^{10}}{x^{10}}$ [6.1B] **9.** $a^2 + 3ab - 10b^2$ [6.3B] **10.** -3 [6.2A]

11. $x + 7$ [6.4A/6.4B] **12.** $6y^4 - 9y^3 + 18y^2$ [6.3A] **13.** $-4x^4 + 8x^3 - 3x^2 - 14x + 21$ [6.3B]

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14. $16y^2 - 9$ [6.3D] **15.** a^4b^7 [6.1A] **16.** $8ab^4$ [6.1B] **17.** $\frac{2b^7}{a^{10}}$ [6.1B] **18.** $-5a^3 + 3a^2 - 4a + 3$ [6.2B]

19. $4x^2 - 20x + 25$ [6.3D] **20.** $x^2 - 5x + 10 - \frac{23}{x+3}$ [6.4A/6.4B] **21.** $10x^2 - 43xy + 28y^2$ [6.3C]

22. $3x^3 + 6x^2 - 8x + 3$ [6.2B] **23.** 3.02×10^{-9} [6.1C] **24.** The mass of the moon is 8.103×10^{19} tons. [6.1D]

25. The area of the circle is $(\pi x^2 - 10\pi x + 25\pi)$ m^2. [6.3E]

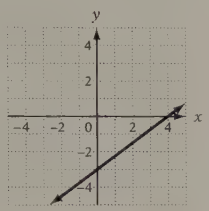
CUMULATIVE REVIEW

1. -3 and 3 [1.1A] **2.** -83 [1.1B] **3.** 6 [1.3A] **4.** $-\frac{5}{4}$ [1.4A] **5.** $-50\sqrt{3}$ [1.2F] **6.** The Inverse Property of Addition [1.4B] **7.** $-186x + 8$ [1.4D] **8.** $-\frac{1}{6}$ [2.2A] **9.** $-\frac{11}{4}$ [2.2B] **10.** -1 and $\frac{7}{3}$ [2.5A]

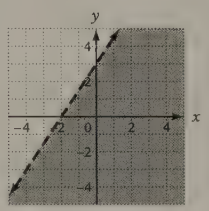
11. 18 [4.2A] **12.** Yes [4.2A] **13.** $-\frac{1}{6}$ [4.4A] **14.** $y = -\frac{3}{2}x + \frac{1}{2}$ [4.5A] **15.** $y = \frac{2}{3}x + \frac{16}{3}$ [4.6A]

16. $\left(-\frac{7}{5}, -\frac{8}{5}\right)$ [5.3B] **17.** $\left(-\frac{9}{7}, \frac{2}{7}, \frac{11}{7}\right)$ [5.2B] **18.** $5x - 3xy$ [1.4B] **19.** $4x^3 - 7x + 3$ [6.3B]

20. 5.01×10^{-6} [6.1C] **21.**

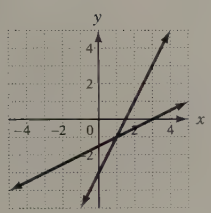


[4.3B] **22.**

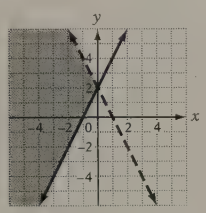


[4.7A]

23.



[5.1A] **24.**



[5.5A]

The solution is $(1, -1)$.

25. $\frac{b^5}{a^8}$ [6.1B] **26.** $\frac{y^2}{25x^6}$ [6.1B]

27. The two integers are 9 and 15. [2.2D] **28.** 40 oz of pure gold must be used. [2.3A] **29.** The cyclists are traveling at 5 mph and 7.5 mph. [2.3D] **30.** \$4500 must be invested at an annual simple interest rate of 10%. [2.3C]

31. The slope is 50. The slope represents the average speed in miles per hour. [4.4A] **32.** The length is 15 m. The width is 6 m. [3.2A] **33.** The area is $(4x^2 + 12x + 9)$ m^2. [6.3E]

Answers to Chapter 7 Selected Exercises

PREP TEST

1. $2 \cdot 3 \cdot 5$ [1.2D] **2.** $-12y + 15$ [1.4D] **3.** $-a + b$ [1.4D] **4.** $-3a + 3b$ [1.4D] **5.** 0 [2.1B]

6. $-\frac{1}{2}$ [2.2A] **7.** $x^2 - 2x - 24$ [6.3C] **8.** $6x^2 - 11x - 10$ [6.3C] **9.** x^3 [6.1B] **10.** $3x^3y$ [6.1B]

SECTION 7.1

1. $5(a + 1)$ **3.** $8(2 - a^2)$ **5.** $4(2x + 3)$ **7.** $6(5a - 1)$ **9.** $x(7x - 3)$ **11.** $a^2(3 + 5a^3)$ **13.** $y(14y + 11)$

15. $2x(x^3 - 2)$ **17.** $2x^2(5x^2 - 6)$ **19.** $4a^5(2a^3 - 1)$ **21.** $xy(xy - 1)$ **23.** $3xy(xy^3 - 2)$ **25.** $xy(x - y^2)$

27. $5y(y^2 - 4y + 2)$ **29.** $3y^2(y^2 - 3y - 2)$ **31.** $3y(y^2 - 3y + 8)$ **33.** $a^2(6a^3 - 3a - 2)$ **35.** $ab(2a - 5ab + 7b)$

37. $2b(2b^4 + 3b^2 - 6)$ **39.** $x^2(8y^2 - 4y + 1)$ **41.** $a^{2n}(2a^{3n} + 1)$ **43.** $y^{2n}(y^{2n} + 1)$ **45.** $b^5(b^n - 1)$

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47. $(a + z)(y + 7)$ **49.** $(a - b)(3r + s)$ **51.** $(m - 7)(t - 7)$ **53.** $(4a - b)(2y + 1)$ **55.** $(x + 2)(x + 2y)$
57. $(p - 2)(p - 3r)$ **59.** $(a + 6)(b - 4)$ **61.** $(2z - 1)(z + y)$ **63.** $(4v + 7)(2v - 3y)$ **65.** $(2x - 5)(x - 3y)$
67. $(y - 2)(3y - a)$ **69.** $(3x - y)(y + 1)$ **71.** $(3s + t)(t - 2)$ **73a.** 28 **b.** 496

SECTION 7.2

1. $(x + 1)(x + 2)$ **3.** $(x + 1)(x - 2)$ **5.** $(a + 4)(a - 3)$ **7.** $(a - 1)(a - 2)$ **9.** $(a + 2)(a - 1)$ **11.** $(b - 3)(b - 3)$
13. $(b + 8)(b - 1)$ **15.** $(y + 11)(y - 5)$ **17.** $(y - 2)(y - 3)$ **19.** $(z - 5)(z - 9)$ **21.** $(z + 8)(z - 20)$
23. $(p + 3)(p + 9)$ **25.** $(x + 10)(x + 10)$ **27.** $(b + 4)(b + 5)$ **29.** $(x + 3)(x - 14)$ **31.** $(b + 4)(b - 5)$
33. $(y + 3)(y - 17)$ **35.** $(p + 3)(p - 7)$ **37.** nonfactorable over the integers **39.** $(x - 5)(x - 15)$ **41.** $(x - 7)(x - 8)$
43. $(x + 8)(x - 7)$ **45.** $(a + 3)(a - 24)$ **47.** $(a - 3)(a - 12)$ **49.** $(z + 8)(z - 17)$ **51.** $(c + 9)(c - 10)$
53. $(z + 4)(z + 11)$ **55.** $(c + 2)(c + 17)$ **57.** $(x + 8)(x - 12)$ **59.** $(x - 8)(x - 14)$ **61.** $(b + 15)(b - 7)$
63. $(a + 3)(a - 12)$ **65.** $(b - 6)(b - 17)$ **67.** $(a + 3)(a + 24)$ **69.** $(x + 12)(x + 13)$ **71.** $(x + 6)(x - 16)$
73. $2(x + 1)(x + 2)$ **75.** $-(x + 2)(x - 9)$ **77.** $a(b + 5)(b - 3)$ **79.** $x(y - 2)(y - 3)$ **81.** $z(z - 3)(z - 4)$
83. $-3y(y - 2)(y - 3)$ **85.** $3(x + 4)(x - 3)$ **87.** $5(z + 4)(z - 7)$ **89.** $2a(a + 8)(a - 4)$ **91.** $(x - 2y)(x - 3y)$
93. $(a - 4b)(a - 5b)$ **95.** $(x + 4y)(x - 7y)$ **97.** nonfactorable over the integers **99.** $z^2(z - 5)(z - 7)$
101. $b^2(b - 10)(b - 12)$ **103.** $2y^2(y + 3)(y - 16)$ **105.** $-x^2(x + 8)(x - 1)$ **107.** $4y(x + 7)(x - 2)$
109. $c(c + 20)(c - 2)$ **111.** $-4x(x + 3)(x - 2)$ **113.** $(y + x)(y - 8x)$ **115.** $(y + 7z)(y - 3z)$ **117.** $(y - z)(y - 15z)$
119. $4y(x + 1)(x - 18)$ **121.** $4x(x + 8)(x - 5)$ **123.** $5z(z + 2)(z - 12)$ **125.** $5x(x + 2)(x + 4)$ **127.** $4(p + 8)(p - 15)$
129. $p^2(p + 8)(p - 7)$ **131.** $(a - 5b)(a - 5b)$ **133.** $(x + 10y)(x - 6y)$ **135.** $6x(x - 5)(x + 4)$ **137.** $y(x + 6)(x - 9)$
139. $k = -36, 36, -12, 12$ **141.** $k = 22, -22, 10, -10$ **143.** $k = 6, 10, 12$ **145.** $k = 6, 10, 12$ **147.** $k = 4, 6$

SECTION 7.3

1. $(x + 1)(2x + 1)$ **3.** $(y + 3)(2y + 1)$ **5.** $(a - 1)(2a - 1)$ **7.** $(b - 5)(2b - 1)$ **9.** $(x + 1)(2x - 1)$
11. $(x - 3)(2x + 1)$ **13.** $(t + 2)(2t - 5)$ **15.** $(p - 5)(3p - 1)$ **17.** $(3y - 1)(4y - 1)$ **19.** nonfactorable over the integers **21.** $(2t - 1)(3t - 4)$ **23.** $(x + 4)(8x + 1)$ **25.** nonfactorable over the integers **27.** $(3y + 1)(4y + 5)$
29. $(a + 7)(7a - 2)$ **31.** $(b - 4)(3b - 4)$ **33.** $(z - 14)(2z + 1)$ **35.** $(p + 8)(3p - 2)$ **37.** $2(x + 1)(2x + 1)$
39. $5(y - 1)(3y - 7)$ **41.** $x(x - 5)(2x - 1)$ **43.** $b(a - 4)(3a - 4)$ **45.** nonfactorable over the integers
47. $-3x(x - 3)(x + 4)$ **49.** $4(4y - 1)(5y - 1)$ **51.** $z(2z + 3)(4z + 1)$ **53.** $y(2x - 5)(3x + 2)$ **55.** $5(t + 2)(2t - 5)$
57. $p(p - 5)(3p - 1)$ **59.** $2(z + 4)(13z - 3)$ **61.** $2y(y - 4)(5y - 2)$ **63.** $yz(z + 2)(4z - 3)$ **65.** $3a(2a + 3)(7a - 3)$
67. $y(3x - 5y)(3x - 5y)$ **69.** $xy(3x - 4y)(3x - 4y)$ **71.** $(2x - 3)(3x - 4)$ **73.** $(b + 7)(5b - 2)$ **75.** $(3a + 8)(2a - 3)$
77. $(z + 2)(4z + 3)$ **79.** $(2p + 5)(11p - 2)$ **81.** $(y + 1)(8y + 9)$ **83.** $(6t - 5)(3t + 1)$ **85.** $(b + 12)(6b - 1)$
87. $(3x + 2)(3x + 2)$ **89.** $(2b - 3)(3b - 2)$ **91.** $(3b + 5)(11b - 7)$ **93.** $(3y - 4)(6y - 5)$ **95.** $(3a + 7)(5a - 3)$
97. $(2y - 5)(4y - 3)$ **99.** $(2z + 3)(4z - 5)$ **101.** nonfactorable over the integers **103.** $(2z - 5)(5z - 2)$
105. $(6z + 5)(6z + 7)$ **107.** $(x + y)(3x - 2y)$ **109.** $(a + 2b)(3a - b)$ **111.** $(y - 2z)(4y - 3z)$ **113.** $-(z - 7)(z + 4)$
115. $-(x - 1)(x + 8)$ **117.** $3(x + 5)(3x - 4)$ **119.** $4(2x - 3)(3x - 2)$ **121.** $a^2(5a + 2)(7a - 1)$ **123.** $5(b - 7)(3b - 2)$
125. $(x - 7y)(3x - 5y)$ **127.** $3(8y - 1)(9y + 1)$ **129.** $-(x - 1)(x + 21)$ **131.** $(5a + 7b)(3a - 2b)$
133. $-z(z + 11)(z - 3)$ **135.** $(3a + 2)(a + 3)$ **137.** $k = 7, -7, 5, -5$ **139.** $k = 7, -7, 5, -5$

SECTION 7.4

1. 4; $25x^6$; $100x^4y^4$ **3.** $4z^4$ **5.** $9a^2b^3$ **7.** $(x + 4)(x - 4)$ **9.** $(2x + 1)(2x - 1)$ **11.** $(4x + 11)(4x - 11)$
13. $(1 + 3a)(1 - 3a)$ **15.** $(xy + 10)(xy - 10)$ **17.** nonfactorable over the integers **19.** $(5 + ab)(5 - ab)$
21. $(a^n + 1)(a^n - 1)$ **23.** $(x - 6)^2$ **25.** $(b - 1)^2$ **27.** $(4x - 5)^2$ **29.** nonfactorable over the integers
31. nonfactorable over the integers **33.** $(x + 3y)^2$ **35.** $(5a - 4b)^2$ **37.** $(x^n + 3)^2$ **39.** $(x - 7)(x - 1)$
41. $(x - y + a + b)(x - y - a - b)$ **43.** 8; x^9; $27c^{15}d^{18}$ **45.** $2x^3$ **47.** $4a^2b^6$ **49.** $(x - 3)(x^2 + 3x + 9)$
51. $(2x - 1)(4x^2 + 2x + 1)$ **53.** $(x - y)(x^2 + xy + y^2)$ **55.** $(m + n)(m^2 - mn + n^2)$ **57.** $(4x + 1)(16x^2 - 4x + 1)$
59. $(3x - 2y)(9x^2 + 6xy + 4y^2)$ **61.** $(xy + 4)(x^2y^2 - 4xy + 16)$ **63.** nonfactorable over the integers
65. nonfactorable over the integers **67.** $(a - 2b)(a^2 - ab + b^2)$ **69.** $(x^{2n} + y^n)(x^{4n} - x^{2n}y^n + y^{2n})$

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71. $(x^n + 2)(x^{2n} - 2x^n + 4)$ **73.** $(xy - 3)(xy - 5)$ **75.** $(xy - 5)(xy - 12)$ **77.** $(x^2 - 3)(x^2 - 6)$
79. $(b^2 + 5)(b^2 - 18)$ **81.** $(x^2y^2 - 2)(x^2y^2 - 6)$ **83.** $(x^n + 1)(x^n + 2)$ **85.** $(3xy - 5)(xy - 3)$ **87.** $(2ab - 3)(3ab - 7)$
89. $(2x^2 - 15)(x^2 + 1)$ **91.** $(2x^n - 1)(x^n - 3)$ **93.** $(2a^n + 5)(3a^n + 2)$ **95.** $3(2x - 3)^2$ **97.** $a(3a - 1)(9a^2 + 3a + 1)$
99. $5(2x + 1)(2x - 1)$ **101.** $y^3(y + 11)(y - 5)$ **103.** $(4x^2 + 9)(2x + 3)(2x - 3)$ **105.** $2a(2 - a)(4 + 2a + a^2)$
107. $b^3(ab - 1)(a^2b^2 + ab + 1)$ **109.** $2x^2(2x - 5)^2$ **111.** $(x^2 + y^2)(x + y)(x - y)$ **113.** $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
115. nonfactorable over the integers **117.** $2a(2a - 1)(4a^2 + 2a + 1)$ **119.** $2b^2(3a + 5b)(4a - 9b)$ **121.** $(x - 2)^2(x + 2)$
123. $(x + y)(x - y)(2x + 1)(2x - 1)$ **125.** $x(x^n + 1)^2$ **127.** $b^n(3b - 2)(b + 2)$ **129.** The dimensions are $(4x + 3)$ m by $(4x + 3)$ m. Yes, x can equal 0. The possible values of x are $x > -\frac{3}{4}$.

SECTION 7.5

3. 3, 5 **5.** −8, 9 **7.** 0, −2 **9.** 0, −12 **11.** $0, \frac{7}{4}$ **13.** $0, -\frac{5}{2}$ **15.** 8, −3 **17.** 11, −11 **19.** $\frac{1}{4}, -\frac{1}{4}$
21. $\frac{7}{4}, -\frac{7}{4}$ **23.** 3, 5 **25.** 8, −9 **27.** $-\frac{2}{3}, -4$ **29.** $\frac{5}{3}, \frac{3}{2}$ **31.** 0, 5 **33.** 0, 4 **35.** −3, 8 **37.** −1, 8
39. $\frac{5}{3}, -2$ **41.** $\frac{6}{5}, 2$ **43.** 12, −1 **45.** −3, −5 **47.** 5, −4 **49.** 3, 5 **51.** 15, −1 **53.** −6, 3 **55.** 1, 3
57. −12, 5 **59.** $-\frac{3}{2}, -2$ **61.** The number is −3. **63.** The numbers are 3 and 5. **65.** The integers are 8 and 10.
67. The length is 38 cm, and the width is 10 cm. **69.** 18 consecutive natural numbers beginning with 1 will give a sum of 171. **71.** There are 10 teams in the league. **73.** The object will hit the ground 4 s later. **75.** The ball will be 64 ft above the ground 2 s later. **77.** The height of the triangle is 14 m. **79.** The width of the border is 1.5 ft.
81. The radius of the original circle is 3.81 in. **83.** The length is 20 in., and the width is 10 in. **85.** 1, 18

CHAPTER REVIEW

1. $5x(x^2 + 2x + 7)$ [7.1A] **2.** $3ab(4a + b)$ [7.1A] **3.** $7y^3(2y^6 - 7y^3 + 1)$ [7.1A] **4.** $(x - 3)(4x + 5)$ [7.1B]
5. $(2x + 5)(5x + 2y)$ [7.1B] **6.** $(3a - 5b)(7x + 2y)$ [7.1B] **7.** $(b - 3)(b - 10)$ [7.2A] **8.** $(c + 6)(c + 2)$ [7.2A]
9. $(y - 4)(y + 9)$ [7.2A] **10.** $3(a + 2)(a - 7)$ [7.2B] **11.** $4x(x - 6)(x + 1)$ [7.2B] **12.** $n^2(n + 1)(n - 3)$ [7.2B]
13. $(2x - 7)(3x - 4)$ [7.3A] **14.** $(6y - 1)(2y + 3)$ [7.3A] **15.** nonfactorable over the integers [7.3A]
16. $(3x - 2)(x - 5)$ [7.3B] **17.** $(2a + 5)(a - 12)$ [7.3B] **18.** $(6a - 5)(3a + 2)$ [7.3B]
19. $(xy + 3)(xy - 3)$ [7.4A] **20.** $(2x + 3y)^2$ [7.4A] **21.** $(x^n - 6)^2$ [7.4A] **22.** $(4a - 3b)(16a^2 + 12ab + 9b^2)$ [7.4B]
23. $(3x^2 + 2)(5x^2 - 3)$ [7.4C] **24.** $(7x^2y^2 + 3)(3x^2y^2 + 2)$ [7.4C] **25.** $3a^2(a^2 + 1)(a^2 - 6)$ [7.4D]
26. $\frac{1}{4}$ and −7 [7.5A] **27.** −3 and 7 [7.5A] **28.** The length is 100 yd. The width is 60 yd. [7.5B]
29. The width of the frame is $\frac{3}{2}$ in. or 1.5 in. or $1\frac{1}{2}$ in. [7.5B] **30.** The length of a side is 20 ft. [7.5B]

CHAPTER TEST

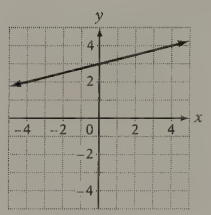
1. $(b + 6)(a - 3)$ [7.1B] **2.** $2y^2(y + 1)(y - 8)$ [7.2B] **3.** $4(x + 4)(2x - 3)$ [7.3B] **4.** $(2x + 1)(3x + 8)$ [7.3A]
5. $(a - 3)(a - 16)$ [7.2A] **6.** $2x(3x^2 - 4x + 5)$ [7.1A] **7.** $(x + 5)(x - 3)$ [7.2A] **8.** $\frac{1}{2}, -\frac{1}{2}$ [7.5A]
9. $5(x^2 - 9x - 3)$ [7.1A] **10.** $(p + 6)^2$ [7.4A] **11.** 3 and 5 [7.5A] **12.** $3(x + 2y)^2$ [7.4D]
13. $(3x - 2)(9x^2 + 6x + 4)$ [7.4B] **14.** $3y^2(2x + 1)(x + 1)$ [7.3B] **15.** $(2a^2 - 5)(3a^2 + 1)$ [7.4C]
16. $(x - 2)(a + b)$ [7.1B] **17.** $(p + 1)(x - 1)$ [7.1B] **18.** $3(a + 5)(a - 5)$ [7.4D] **19.** nonfactorable over the integers [7.3A] **20.** $(x + 3)(x - 12)$ [7.2A] **21.** $(2a - 3b)^2$ [7.4A] **22.** $(2x + 7y)(2x - 7y)$ [7.4A]
23. $\frac{3}{2}$ and −7 [7.5A] **24.** The length of the rectangle is 15 cm, and the width is 6 cm. [7.5B] **25.** The numbers are 3 and 7. [7.5B]

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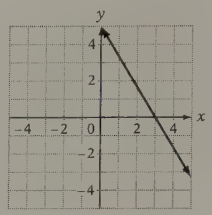
CUMULATIVE REVIEW

1. 7 [1.1C] **2.** 4 [1.3A] **3.** -7 [1.4A] **4.** $15x^2$ [1.4C] **5.** 12 [1.4D] **6.** $\frac{2}{3}$ [2.1C] **7.** $\frac{7}{4}$ [2.2B]

8. 3 [2.2C] **9.** 45 [2.1D] **10.** 1 [4.2A] **11.**



[4.3A] **12.**



[4.3B]

13. $y = \frac{2}{3}x + 6$ [4.5A] **14.** (1, 6) [5.1B] **15.** $(-1, -2)$ [5.2A] **16.** $9a^6b^4$ [6.1A] **17.** $x^3 - 3x^2 - 6x + 8$ [6.3B]

18. $4x + 8 + \frac{21}{2x - 3}$ [6.4A] **19.** $\frac{y^6}{x^8}$ [6.1B] **20.** $(a - b)(3 - x)$ [7.1B] **21.** $5xy^2(3 - 4y^2)$ [7.1A]

22. $(x - 7y)(x + 2y)$ [7.2A] **23.** $\frac{2}{3}$ and -7 [7.5A] **24.** $\frac{5}{2}$ and 4 [7.5A] **25.** The third angle measures 59°. [3.1C]

26. The width is 15 ft. [3.2A] **27.** The pieces are 4 ft long and 6 ft long. [2.2D] **28.** $6500 was invested at 11%. [2.3C] **29.** The distance to the resort is 168 mi. [2.3D] **30.** The length of the base of the triangle is 12 in. [7.5B]

Answers to Chapter 8 Selected Exercises

PREP TEST

1. 50 [1.2B] **2.** $-\frac{1}{6}$ [1.2D] **3.** $-\frac{3}{2}$ [1.2D] **4.** $\frac{1}{24}$ [1.2C] **5.** $\frac{5}{24}$ [1.2C] **6.** $\frac{1}{3}$ [1.4A] **7.** -2 [2.2C]

8. $\frac{10}{7}$ [2.2C] **9.** 110 mph, 130 mph [2.3D]

SECTION 8.1

3. $\frac{3}{4x}$ **5.** $\frac{1}{x + 3}$ **7.** -1 **9.** $\frac{2}{3y}$ **11.** $-\frac{3}{4x}$ **13.** $\frac{a}{b}$ **15.** $-\frac{2}{x}$ **17.** $\frac{y - 2}{y - 3}$ **19.** $\frac{x + 5}{x + 4}$ **21.** $\frac{x + 4}{x - 3}$

23. $-\frac{x + 2}{x + 5}$ **25.** $\frac{2(x + 2)}{x + 3}$ **27.** $\frac{2x - 1}{2x + 3}$ **29.** $-\frac{x + 7}{x + 6}$ **31.** $\frac{35ab^2}{24x^2y}$ **33.** $\frac{4x^3y^3}{3a^2}$ **35.** $\frac{3}{4}$ **37.** ab^2

39. $\frac{x^2(x - 1)}{y(x + 3)}$ **41.** $\frac{y(x - 1)}{x^2(x + 10)}$ **43.** $-ab^2$ **45.** $\frac{x + 5}{x + 4}$ **47.** 1 **49.** $-\frac{n - 10}{n - 7}$ **51.** $\frac{x(x + 2)}{2(x - 1)}$ **53.** $-\frac{x + 2}{x - 6}$

55. $\frac{x + 5}{x - 12}$ **57.** $\frac{7a^3y^2}{40bx}$ **59.** $\frac{4}{3}$ **61.** $\frac{3a}{2}$ **63.** $\frac{x^2(x + 4)}{y^2(x + 2)}$ **65.** $\frac{x(x - 2)}{y(x - 6)}$ **67.** $-\frac{3by}{ax}$ **69.** $\frac{(x - 3)(x + 6)}{(x + 7)(x - 6)}$

71. 1 **73.** $-\frac{x + 8}{x - 4}$ **75.** $\frac{2n + 1}{2n - 3}$ **79.** $\frac{n - 2}{n + 3}$ **81.** $\frac{8}{3y}$ **83.** 1

SECTION 8.2

1. $24x^3y^2$ **3.** $30x^4y^2$ **5.** $8x^2(x + 2)$ **7.** $6x^2y(x + 4)$ **9.** $36x(x + 2)^2$ **11.** $6(x + 1)^2$ **13.** $(x - 1)(x + 2)(x + 3)$

15. $(2x + 3)^2(x - 5)$ **17.** $(x - 1)(x - 2)$ **19.** $(x - 3)(x + 2)(x + 4)$ **21.** $(x + 4)(x + 1)(x - 7)$

23. $(x - 6)(x + 6)(x + 4)$ **25.** $(x - 10)(x - 8)(x + 3)$ **27.** $(3x - 2)(x - 3)(x + 2)$ **29.** $(x + 2)(x - 3)$

31. $(x - 5)(x + 1)$ **33.** $(x - 3)(x - 2)(x - 1)(x - 6)$ **35.** $\frac{5}{ab^2}$; $\frac{6b}{ab^2}$ **37.** $\frac{15y^2}{18x^2y}$; $\frac{14x}{18x^2y}$

39. $\frac{ay + 5a}{y^2(y + 5)}$; $\frac{6y}{y^2(y + 5)}$ **41.** $\frac{a^2y + 7a^2}{y(y + 7)^2}$; $\frac{ay}{y(y + 7)^2}$ **43.** $\frac{b}{y(y - 4)}$; $\frac{-b^2y}{y(y - 4)}$ **45.** $-\frac{3y - 21}{(y - 7)^2}$; $\frac{2}{(y - 7)^2}$

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47. $\frac{2y^2}{y^2(y-3)}$; $\frac{3}{y^2(y-3)}$ **49.** $\frac{x^3+4x^2}{(2x-1)(x+4)}$; $\frac{2x^2+x-1}{(2x-1)(x+4)}$ **51.** $\frac{3x^2+15x}{(x-5)(x+5)}$; $\frac{4}{(x-5)(x+5)}$

53. $\frac{x-3}{(3x-2)(x+2)}$; $\frac{6x-4}{(3x-2)(x+2)}$ **55.** $\frac{x^2-1}{(x+5)(x-3)(x+1)}$; $\frac{x^2-3x}{(x+5)(x-3)(x+1)}$ **57.** $\frac{11}{y^2}$ **59.** $-\frac{7}{x+4}$

61. $\frac{8x}{2x+3}$ **63.** $\frac{5x+7}{x-3}$ **65.** $\frac{2x-5}{x+9}$ **67.** $\frac{-3x-4}{2x+7}$ **69.** $\frac{1}{x+5}$ **71.** $\frac{1}{x-6}$ **73.** $\frac{3}{2y-1}$ **75.** $\frac{1}{x-5}$

79. $\frac{7b+5a}{ab}$ **81.** $\frac{11}{12a}$ **83.** $\frac{11}{12y}$ **85.** $\frac{120+7y}{20y^2}$ **87.** $\frac{21b+4a}{12ab}$ **89.** $\frac{14x+3}{12x}$ **91.** $\frac{8x-3}{6x}$ **93.** $\frac{7}{36}$

95. $\frac{-3x^2-11x-2}{3x^2}$ **97.** $\frac{6x^2+5x+10}{6x^2}$ **99.** $\frac{2x^2+1}{x}$ **101.** $\frac{4x+2}{x+1}$ **103.** $\frac{x^2-x+2}{x^2y}$

105. $\frac{16xy-12y+6x^2+3x}{12x^2y^2}$ **107.** $\frac{3xy-6y-2x^2-14x}{24x^2y}$ **109.** $\frac{9x+2}{(x-2)(x+3)}$ **111.** $\frac{2(x+23)}{(x-7)(x+3)}$

113. $\frac{2x^2-5x+1}{(x+1)(x-3)}$ **115.** $\frac{4x^2-34x+5}{(2x-1)(x-6)}$ **117.** $\frac{2a-5}{a-7}$ **119.** $\frac{4x+9}{(x+3)(x-3)}$ **121.** $\frac{-x+9}{(x+2)(x-3)}$

123. $\frac{8y}{(y+2)^2}$ **125.** $\frac{3x-y}{x-y}$ **127.** $\frac{11}{(a-3)(a-4)}$ **129.** $\frac{-x-1}{(x+2)(x-5)}$ **131.** $\frac{x+3}{x+5}$ **133.** $\frac{x-2}{x+4}$

135. $\frac{4(2x-1)}{(x+5)(x-2)}$ **137a.** $\frac{3}{y}+\frac{6}{x}$ **b.** $\frac{4}{b^2}+\frac{3}{ab}$ **c.** $\frac{1}{4mn}+\frac{1}{6m^2}$ **139a.** $\frac{19{,}200}{x}$ dollars **b.** $\frac{96{,}500}{x(x+5)}$ dollars

c. \$128

SECTION 8.3

1. $\frac{x}{x-3}$ **3.** $\frac{2}{3}$ **5.** $\frac{y+3}{y-4}$ **7.** $\frac{2(2x+13)}{5x+36}$ **9.** $\frac{x+2}{x+3}$ **11.** $\frac{x-6}{x+5}$ **13.** $-\frac{x-2}{x+1}$ **15.** $x-1$

17. $\frac{1}{2x-1}$ **19.** $\frac{x-3}{x+5}$ **21.** $\frac{x-7}{x-8}$ **23.** $\frac{2y-1}{2y+1}$ **25.** $\frac{x-2}{2x-5}$ **27.** $-\frac{x+1}{4x-3}$ **29.** $\frac{x+1}{2(5x-2)}$

31. $\frac{5}{3}$ **33.** $-\frac{1}{x-1}$ **35.** $\frac{y+4}{2(y-2)}$

SECTION 8.4

1. 3 **3.** 1 **5.** 9 **7.** 1 **9.** $\frac{1}{4}$ **11.** 1 **13.** -3 **15.** $\frac{1}{2}$ **17.** 8 **19.** 5 **21.** -1 **23.** 5

25. The equation has no solution. **27.** 2, 4 **29.** $-\frac{3}{2}$, 4 **31.** 3 **33.** 4 **37.** 9 **39.** 36 **41.** 10 **43.** 113

45. -2 **47.** 15 **49.** 4 **51.** 6.7 cm **53.** 2.9 m **55.** 22.5 ft **57.** 48 m^2 **59.** 6.25 cm **61.** 6 in.

63. 13 cm **65.** The height of the flagpole is 14.375 ft. **67.** The width of the ravine is 82.5 ft. **69.** 20,000 voters voted in favor of the amendment. **71.** There are 175 mi between the two cities. **73.** There are 160 ml of water in 280 ml of soft drink. **75.** It would take 18 min to print a document 45 pages long. **77.** The window should be 40 ft^2. **79.** The person's height is 67.5 in. **81.** The first person's share of the winnings is \$1.25 million. **83.** The basketball player made 210 shots.

SECTION 8.5

1. $h=\frac{2A}{b}$ **3.** $t=\frac{d}{r}$ **5.** $T=\frac{PV}{nR}$ **7.** $L=\frac{P-2W}{2}$ **9.** $b_1=\frac{2A-hb_2}{h}$ **11.** $h=\frac{3V}{A}$ **13.** $S=C-Rt$

15. $P=\frac{A}{1+rt}$ **17.** $w=\frac{A}{S+1}$ **19a.** $h=\frac{S-2\pi r^2}{2\pi r}$ **b.** The height is 5 in. **c.** The height is 4 in.

21a. $S=\frac{F+BV}{B}$ **b.** The required selling price is \$180. **c.** The required selling price is \$75.

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SECTION 8.6

1. It would take 2 h with both sprinklers working. **3.** It would take both skiploaders 3 h working together. **5.** It would take 30 h with both computers working. **7.** It would take 30 min with both air conditioners working. **9.** It would take the second pipeline 90 min to fill the tank. **11.** It would take the apprentice 15 h to construct the wall. **13.** It would take the second technician 3 h to complete the wiring. **15.** It would have taken one welder 40 h to complete the welds. **17.** It would take one machine 28 h working alone. **19.** The rate through the congested traffic is 20 mph. **21.** The rate of the jogger is 8 mph, and the rate of the cyclist is 20 mph. **23.** The rate of the jet is 360 mph. **25.** The rate of the runner is 8 mph. **27.** The rate of the car is 48 mph. **29.** The rate of the wind is 20 mph. **31.** The rate of the gulf current is 6 mph. **33.** The rate of the trucker for the first 330 mi was 55 mph. **35.** The less-experienced helper can complete the job in 30 h. **37.** The bus usually travels 60 mph.

SECTION 8.7

3. The profit is $80,000. **5.** The pressure is 6.75 lb/in^2. **7.** The object will fall 1600 ft. **9.** The ball will roll 54 ft in 3 s. **11.** At a rate of 65 mph, the time to travel between the two cities is 4.2 h. **13.** The pressure is 112.5 lb/in^2. **15.** The repulsive force is 80 lb. **17.** y is doubled. **19.** inversely **21.** inversely

CHAPTER REVIEW

1. $\frac{b^3y}{10ax}$ [8.1C] **2.** $\frac{7x+22}{60x}$ [8.2D] **3.** $\frac{x-2}{3x-10}$ [8.3A] **4.** $-\frac{x+6}{x+3}$ [8.1A] **5.** $\frac{2x^4}{3y^7}$ [8.1A]

6. 62 [8.4B] **7.** $\frac{(3y-2)^2}{(y-1)(y-2)}$ [8.1C] **8.** $\frac{by^3}{6ax^2}$ [8.1B] **9.** $\frac{x}{x-7}$ [8.3A]

10. $\frac{3x^2-x}{(2x+3)(6x-1)(3x-1)}, \frac{24x^3-4x^2}{(2x+3)(6x-1)(3x-1)}$ [8.2B] **11.** $a=\frac{T-2bc}{2b+2c}$ [8.5A] **12.** 2 [8.4A]

13. $c=\frac{100m}{i}$ [8.5A] **14.** The equation has no solution. [8.4A] **15.** $\frac{1}{x^2}$ [8.1C] **16.** $\frac{2y-3}{5y-7}$ [8.2D]

17. $\frac{1}{x+3}$ [8.2C] **18.** $(5x-3)(2x-1)(4x-1)$ [8.2A] **19.** $y=-\frac{4}{9}x+2$ [8.5A] **20.** $\frac{8x+5}{3x-4}$ [8.1B]

21. 5 [8.4A] **22.** $\frac{3x-1}{x-5}$ [8.2D] **23.** 10 [8.4B] **24.** 12 [8.4B] **25.** The perimeter of triangle ABC is 24 in. [8.4C] **26.** It would take 6 h to fill the pool using both hoses. [8.6A] **27.** The rate of the car is 45 mph. [8.6B] **28.** The rate of the wind is 20 mph. [8.6B] **29.** The pitcher's ERA is 1.35. [8.4D] **30.** The current is 2 amps. [8.7A]

CHAPTER TEST

1. $\frac{2x^3}{3y^3}$ [8.1A] **2.** $-\frac{x+5}{x+1}$ [8.1A] **3.** $\frac{x+1}{x^3(x-2)}$ [8.1B] **4.** $\frac{(x-5)(2x-1)}{(x+3)(2x+5)}$ [8.1B] **5.** $\frac{x+5}{x+4}$ [8.1C]

6. $3(2x-1)(x+1)$ [8.2A] **7.** $\frac{3(x+2)}{x(x+2)(x-2)}; \frac{x^2}{x(x+2)(x-2)}$ [8.2B] **8.** $\frac{2}{x+5}$ [8.2C]

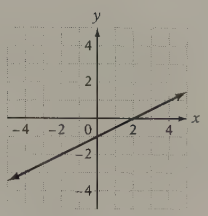
9. $\frac{5}{(2x-1)(3x+1)}$ [8.2D] **10.** $\frac{x^2-4x+5}{(x-2)(x+3)}$ [8.2D] **11.** $\frac{x-3}{x-2}$ [8.3A] **12.** 2 [8.4A] **13.** The equation has no solution. [8.4A] **14.** -1 [8.4B] **15.** The area is 64.8 m^2. [8.4C] **16.** $t=\frac{d-s}{r}$ [8.5A] **17.** 14 rolls of wallpaper are needed. [8.4D] **18.** It would take 10 min with both landscapers working. [8.6A] **19.** The rate of the cyclist is 10 mph. [8.6B] **20.** The resistance is 0.4 ohm. [8.7A]

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CUMULATIVE REVIEW

1. $\frac{31}{30}$ [1.3A] **2.** 21 [1.4A] **3.** $5x - 2y$ [1.4B] **4.** $-8x + 26$ [1.4D] **5.** $-\frac{9}{2}$ [2.2A] **6.** -12 [2.2C]

7. 10 [2.1D] **8.** $\{x|x \leq 8\}$ [2.4A] **9.** The volume is 200 ft^3. [3.3A] **10.**



[4.3B]

11. $\frac{3}{7}$ [4.2A] **12.** $y = \frac{3}{2}x + 2$ [4.6A] **13.** -28 [5.3A] **14.** a^3b^7 [6.1A] **15.** $\frac{a^5}{b^6}$ [6.1B]

16. 3.5×10^{-8} [6.1C] **17.** $-4a^4 + 6a^3 - 2a^2$ [6.3A] **18.** $a^2 + ab - 12b^2$ [6.3C] **19.** $x^2 + 2x + 4$ [6.4B]

20. $(y - 6)(y - 1)$ [7.2A] **21.** $(4x + 1)(3x - 1)$ [7.3A/7.3B] **22.** $a(2a - 3)(a + 5)$ [7.3A/7.3B]

23. $4(b + 5)(b - 5)$ [7.4D] **24.** -3 and $\frac{5}{2}$ [7.5A] **25.** $\frac{2x^3}{3y^5}$ [8.1A] **26.** $-\frac{x - 2}{x + 5}$ [8.1A] **27.** 1 [8.1C]

28. $\frac{3}{(2x - 1)(x + 1)}$ [8.2D] **29.** $\frac{x + 3}{x + 5}$ [8.3A] **30.** 4 [8.4A] **31.** The alloy contains 70% silver. [2.3B]

32. It would cost \$160. [8.4D] **33.** It would take 6 min to fill the tank. [8.6A]

Answers to Chapter 9 Selected Exercises

PREP TEST

1. 16 [1.1D] **2.** 32 [1.2E] **3.** 9 [1.2D] **4.** $\frac{1}{12}$ [1.2C] **5.** $-5x - 1$ [1.4D] **6.** $\frac{xy^5}{4}$ [6.1B]

7. $9x^2 - 12x + 4$ [6.3C] **8.** $-12x^2 + 14x + 10$ [6.3C] **9.** $36x^2 - 1$ [6.3D] **10.** $-1, 15$ [7.5A]

SECTION 9.1

1. 2 **3.** 27 **5.** $\frac{1}{9}$ **7.** 4 **9.** $(-25)^{5/2}$ is not a real number. **11.** $\frac{343}{125}$ **13.** x **15.** $y^{1/2}$ **17.** $x^{1/12}$ **19.** $a^{7/12}$

21. $\frac{1}{a}$ **23.** $\frac{1}{y}$ **25.** $y^{3/2}$ **27.** $\frac{1}{x}$ **29.** $\frac{1}{x^4}$ **31.** a **33.** $x^{3/10}$ **35.** a^3 **37.** $\frac{1}{x^{1/2}}$ **39.** $y^{1/9}$ **41.** x^4y

43. $x^6y^3z^9$ **45.** $\frac{x}{y^2}$ **47.** $\frac{x^{3/2}}{y^{1/4}}$ **49.** $\frac{x^2}{y^8}$ **51.** $\frac{1}{x^{11/12}}$ **53.** $\frac{1}{y^{5/2}}$ **55.** $\frac{1}{b^{7/8}}$ **57.** a^5b^{13} **59.** $\frac{m^2}{4n^{3/2}}$ **61.** $\frac{y^{17/2}}{x^3}$

63. $\frac{16b^2}{a^{1/3}}$ **65.** $y^2 - y$ **67.** $a - a^2$ **69.** x^{4n} **71.** $x^{3n/2}$ **73.** $y^{3n/2}$ **75.** x^{2n^2} **77.** $x^{2n}y^n$ **79.** $x^{4n}y^{2n}$

81. $\sqrt[4]{3}$ **83.** $\sqrt{a^3}$ **85.** $\sqrt{32t^5}$ **87.** $-2\sqrt[3]{x^2}$ **89.** $\sqrt[3]{a^4b^2}$ **91.** $\sqrt[5]{a^6b^{12}}$ **93.** $\sqrt[4]{(4x - 3)^3}$ **95.** $\frac{1}{\sqrt[3]{x^2}}$

97. $14^{1/2}$ **99.** $x^{1/3}$ **101.** $x^{4/3}$ **103.** $b^{3/5}$ **105.** $(2x^2)^{1/3}$ **107.** $-(3x^5)^{1/2}$ **109.** $3xy^{2/3}$ **111.** $(a^2 - 2)^{1/2}$ **113.** x^8

115. $-x^4$ **117.** xy^3 **119.** $-x^5y$ **121.** $4a^2b^6$ **123.** $\sqrt{-16x^4y^2}$ is not a real number. **125.** $3x^3$ **127.** $-4x^3y^4$

129. $-x^2y^3$ **131.** x^4y^2 **133.** $3xy^5$ **135.** $2ab^2$ **137a.** false; 2 **b.** true **c.** true **d.** false; $(a^n + b^n)^{1/n}$

e. false; $a + 2a^{1/2}b^{1/2} + b$ **f.** false; $a^{n/m}$

SECTION 9.2

1. $x^2yz^2\sqrt{yz}$ **3.** $2ab^4\sqrt{2a}$ **5.** $3xyz^2\sqrt{5yz}$ **7.** $\sqrt{-9x^3}$ is not a real number. **9.** $a^5b^2\sqrt[3]{ab^2}$ **11.** $-5y\sqrt[3]{x^2y}$

13. $abc^2\sqrt[3]{ab^2}$ **15.** $2x^2y\sqrt[4]{xy}$ **17.** $-6\sqrt{x}$ **19.** $-2\sqrt{2}$ **21.** $3\sqrt{2b} + 5\sqrt{3b}$ **23.** $-2xy\sqrt{2y}$

25. $6ab^2\sqrt{3ab} + 3ab\sqrt{3ab}$ **27.** $-\sqrt[3]{2}$ **29.** $8b\sqrt[3]{2b^2}$ **31.** $3a\sqrt[4]{2a}$ **33.** $17\sqrt{2} - 15\sqrt{5}$ **35.** $5b\sqrt{b}$

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37. $-8xy\sqrt{2x} + 2xy\sqrt{xy}$ **39.** $2y\sqrt[3]{2x}$ **41.** $-4ab\sqrt[4]{2b}$ **43.** 16 **45.** $2\sqrt[3]{4}$ **47.** $xy^3\sqrt{x}$ **49.** $8xy\sqrt{x}$
51. $2x^2y\sqrt[3]{2}$ **53.** $2ab\sqrt[4]{3a^2b}$ **55.** 6 **57.** $x - \sqrt{2x}$ **59.** $4x - 8\sqrt{x}$ **61.** $x - 6\sqrt{x} + 9$ **63.** $84 + 16\sqrt{5}$
65. $672x^2y^2$ **67.** $4a^3b^3\sqrt[3]{a}$ **69.** $-8\sqrt{5}$ **71.** $x - y^2$ **73.** $12x - y$ **77.** $y\sqrt{5y}$ **79.** $b\sqrt{13b}$ **81.** $\frac{\sqrt{2}}{2}$
83. $\frac{2\sqrt{3y}}{3y}$ **85.** $\frac{3\sqrt{3a}}{a}$ **87.** $\frac{\sqrt{2y}}{2}$ **89.** $\frac{5\sqrt[3]{3}}{3}$ **91.** $\frac{5\sqrt[3]{9y^2}}{3y}$ **93.** $\frac{b\sqrt{2a}}{2a^2}$ **95.** $\frac{\sqrt{15x}}{5x}$ **97.** $2 + 2\sqrt{2}$
99. $\frac{-12 - 4\sqrt{2}}{7}$ **101.** $-\frac{10 + 5\sqrt{7}}{3}$ **103.** $-\frac{7\sqrt{x} + 21}{x - 9}$ **105.** $-\sqrt{6} + 3 - 2\sqrt{2} + 2\sqrt{3}$ **107.** $-\frac{17 + 5\sqrt{5}}{4}$
109. $\frac{8a - 10\sqrt{ab} + 3b}{16a - 9b}$ **111.** $\frac{3 - 7\sqrt{y} + 2y}{1 - 4y}$ **113a.** false; $\sqrt[6]{432}$ **b.** true **c.** false; $\sqrt[3]{x^2}$ **d.** false; $\sqrt{x} + \sqrt{y}$
e. false; $\sqrt[2]{2} + \sqrt[3]{3}$ **f.** true **115.** $\sqrt[4]{a + b}$

SECTION 9.3

3. $2i$ **5.** $7i\sqrt{2}$ **7.** $3i\sqrt{3}$ **9.** $4 + 2i$ **11.** $2\sqrt{3} - 3i\sqrt{2}$ **13.** $4\sqrt{10} - 7i\sqrt{3}$ **15.** $8 - i$ **17.** $-8 + 4i$
19. $6 - 6i$ **21.** $19 - 7i\sqrt{2}$ **23.** $6\sqrt{2} - 3i\sqrt{2}$ **25.** 63 **27.** -4 **29.** $-3\sqrt{2}$ **31.** $-4 + 12i$ **33.** $-2 + 4i$
35. $17 - i$ **37.** $8 + 27i$ **39.** 1 **41.** 1 **43.** $-3i$ **45.** $\frac{3}{4} + \frac{1}{2}i$ **47.** $\frac{10}{13} - \frac{2}{13}i$ **49.** $\frac{4}{5} + \frac{2}{5}i$ **51.** $-i$
53. $-\frac{\sqrt{5}}{5} + \frac{2\sqrt{5}}{5}i$ **55.** $\frac{3}{10} - \frac{11}{10}i$ **57.** $\frac{6}{5} + \frac{7}{5}i$ **59a.** yes **b.** yes

SECTION 9.4

1. 25 **3.** 27 **5.** 48 **7.** -2 **9.** The equation has no solution. **11.** 9 **13.** -23 **15.** -16 **17.** $\frac{9}{4}$
19. 14 **21.** 7 **23.** 7 **25.** -122 **27.** 35 **29.** $-5, -2$ **31.** 45 **33.** 23 **35.** -4 **37.** 10 **39.** 1
41. 3 **43.** 21 **45.** $\frac{25}{72}$ **47.** 5 **49.** $-2, 4$ **51.** 6 **53.** 0, 1 **55.** $-2, 2$ **57.** $-2, 1$ **59.** 2, 11
61. The width is 6 ft. **63.** The bottom of the ladder is 10 ft from the wall. **65.** The object has fallen 2500 ft.
67. The periscope must be 8.64 ft above the water. **69.** The length of the pendulum is 4.67 ft. **71.** The distance is 180 m. **73.** The HDTV screen is 7.15 in. wider. **75a.** 2, -2; integer, rational number, real number
b. $i, -i$; imaginary **c.** 16; integer, rational number, real number **77.** $a = \sqrt{c^2 - b^2}$ or $a = -\sqrt{c^2 - b^2}$ **79.** $x = \sqrt{6}$

CHAPTER REVIEW

1. $20x^2y^2$ [9.1A] **2.** 7 [9.4A] **3.** $39 - 2i$ [9.3C] **4.** $7x^{2/3}y$ [9.1B] **5.** $6\sqrt{3} - 13$ [9.2C] **6.** -2 [9.4A]
7. $\frac{1}{x^5}$ [9.1A] **8.** $\frac{8\sqrt{3y}}{3y}$ [9.2D] **9.** $-2a^2b^4$ [9.1C] **10.** $2a^2b\sqrt{2b}$ [9.2B] **11.** $\frac{x\sqrt{x} - x\sqrt{2} + 2\sqrt{x} - 2\sqrt{2}}{x - 2}$ [9.2D]
12. $\frac{2}{3} - \frac{5}{3}i$ [9.3D] **13.** $3ab^3\sqrt{2a}$ [9.2A] **14.** $-4\sqrt{2} + 8i\sqrt{2}$ [9.3B] **15.** $5x^3y^3\sqrt[3]{2x^2y}$ [9.2B]
16. $4xy^2\sqrt[3]{x^2}$ [9.2C] **17.** $7 + 3i$ [9.3C] **18.** $3\sqrt[4]{x^3}$ [9.1B] **19.** $-2ab^2\sqrt[5]{2a^3b^2}$ [9.2A]
20. $-2 + 7i$ [9.3D] **21.** $-6\sqrt{2}$ [9.3C] **22.** 30 [9.4A] **23.** $3a^2b^3$ [9.1C] **24.** $5i\sqrt{2}$ [9.3A]
25. $-12 + 10i$ [9.3B] **26.** $31 - 10\sqrt{6}$ [9.2C] **27.** $6x^2\sqrt{3y}$ [9.2B] **28.** The amount of power generated is 120 watts. [9.4B] **29.** The distance required to reach a velocity of 88 ft/s is 242 ft. [9.4B] **30.** The bottom of the ladder is 6.63 ft from the building. [9.4B]

CHAPTER TEST

1. $\frac{1}{2}x^{3/4}$ [9.1B] **2.** $-2x^2y\sqrt[3]{2x}$ [9.2B] **3.** $3\sqrt[5]{y^2}$ [9.1B] **4.** $18 + 16i$ [9.3C] **5.** $4x + 4\sqrt{xy} + y$ [9.2C]
6. $r^{1/6}$ [9.1A] **7.** 4 [9.4A] **8.** $2xy^2$ [9.1C] **9.** $-4x\sqrt{3}$ [9.2C] **10.** $-3 + 2i$ [9.3B] **11.** $4x^2y^3\sqrt{2y}$ [9.2A]

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12. $14 + 10\sqrt{3}$ [9.2C] **13.** 4 [9.3B] **14.** 2 [9.2D] **15.** $8a\sqrt{2a}$ [9.2B] **16.** $2a - \sqrt{ab} - 15b$ [9.2C]

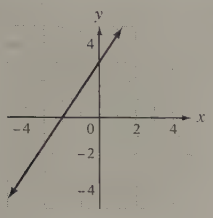
17. $\frac{64x^3}{y^6}$ [9.1A] **18.** $\frac{x + \sqrt{xy}}{x - y}$ [9.2D] **19.** $-\frac{4}{5} + \frac{7}{5}i$ [9.3D] **20.** -3 [9.4A] **21.** $\frac{b^3}{8a^6}$ [9.1A]

22. $3abc^2\sqrt[3]{ac}$ [9.2A] **23.** $\frac{4x^2}{y}$ [9.2D] **24.** -4 [9.3C] **25.** The object has fallen 576 ft. [9.4B]

CUMULATIVE REVIEW

1. 92 [1.3A] **2.** 56 [1.4A] **3.** $-10x + 1$ [1.4D] **4.** $\frac{3}{2}$ [2.2A] **5.** $\frac{2}{3}$ [2.2C] **6.** $\{x|x > 1\}$ [2.4A]

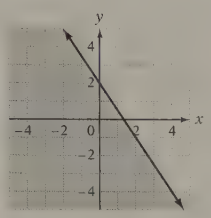
7. $\frac{1}{3}, \frac{7}{3}$ [2.5A] **8.** $\{x|-6 \le x \le 3\}$ [2.5B] **9.** The area is 187.5 cm². [3.2B] **10.** 3 [5.3A]

11. 

$m = \frac{3}{2}$

$b = 3$

[4.3B]

12. 

[4.7A]

13. $y = \frac{1}{3}x + \frac{7}{3}$ [4.5B]

14. $\left(\frac{17}{4}, \frac{9}{2}\right)$ [5.3B] **15.** $2x^2y^2$ [6.1B] **16.** $(9x + y)(9x - y)$ [7.4A] **17.** $x(x^2 + 3)(x + 1)(x - 1)$ [7.4D]

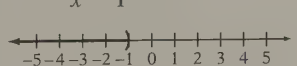
18. $C = R - nP$ [8.5A] **19.** $\frac{y^5}{x^4}$ [9.1A] **20.** $-x\sqrt{10x}$ [9.2B] **21.** $13 - 7\sqrt{3}$ [9.2C] **22.** $\sqrt{6} + \sqrt{2}$ [9.2D]

23. $-\frac{1}{5} + \frac{3}{5}i$ [9.3D] **24.** -20 [9.4A] **25.** The length of side DE is 27 m. [8.4C] **26.** \$4000 must be invested at 8.4%. [2.3C] **27.** The rate of the plane was 250 mph. [8.6B] **28.** It takes 1.25 s for light to travel to Earth from the moon. [6.1D] **29.** The slope is 0.08. The slope represents the simple interest rate on the investment. The interest rate is 8%. [4.4A] **30.** The periscope must be 32.7 ft above the water. [9.4B]

Answers to Chapter 10 Selected Exercises

PREP TEST

1. $3\sqrt{2}$ [1.2F] **2.** $3i$ [9.3A] **3.** $\frac{2x - 1}{x - 1}$ [8.2B] **4.** 8 [1.4A] **5.** yes [7.4A] **6.** $(2x - 1)^2$ [7.4A]

7. $(3x + 2)(3x - 2)$ [7.4A] **8.** 

[1.5C] **9.** $-3, 5$ [7.5A] **10.** 4 [8.4B]

SECTION 10.1

3. $2x^2 - 4x - 5 = 0$; $a = 2, b = -4, c = -5$ **5.** $4x^2 - 5x + 6 = 0$; $a = 4, b = -5, c = 6$ **7.** 0, 4 **9.** 5, −5

11. 3, −2 **13.** 3 **15.** 0, 2 **17.** 5, −2 **19.** 2, 5 **21.** $6, -\frac{3}{2}$ **23.** $2, \frac{1}{4}$ **25.** $\frac{1}{3}, -4$ **27.** $\frac{9}{2}, -\frac{2}{3}$

29. $\frac{1}{4}, -4$ **31.** 9, −2 **33.** $-2, -\frac{3}{4}$ **35.** 2, −5 **37.** $-4, -\frac{3}{2}$ **39.** $2b, 7b$ **41.** $7c, -c$ **43.** $-\frac{b}{2}, -b$

45. $4a, \frac{2a}{3}$ **47.** $-\frac{a}{3}, 3a$ **49.** $-\frac{3y}{2}, -\frac{y}{2}$ **51.** $-\frac{a}{2}, -\frac{4a}{3}$ **53.** $x^2 - 7x + 10 = 0$ **55.** $x^2 + 6x + 8 = 0$

57. $x^2 - 5x - 6 = 0$ **59.** $x^2 - 9 = 0$ **61.** $x^2 - 8x + 16 = 0$ **63.** $x^2 - 5x = 0$ **65.** $x^2 - 3x = 0$

67. $2x^2 - 7x + 3 = 0$ **69.** $4x^2 - 5x - 6 = 0$ **71.** $3x^2 + 11x + 10 = 0$ **73.** $9x^2 - 4 = 0$ **75.** $6x^2 - 5x + 1 = 0$

77. $10x^2 - 7x - 6 = 0$ **79.** $8x^2 + 6x + 1 = 0$ **81.** $50x^2 - 25x - 3 = 0$ **83.** 7, −7 **85.** $2i, -2i$ **87.** 2, −2

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89. $\frac{9}{2}, -\frac{9}{2}$ **91.** $7i, -7i$ **93.** $4\sqrt{3}, -4\sqrt{3}$ **95.** $5\sqrt{3}, -5\sqrt{3}$ **97.** $3i\sqrt{2}, -3i\sqrt{2}$ **99.** $7, -5$ **101.** $0, -6$

103. $-7, 3$ **105.** $1, 0$ **107.** $-5 + \sqrt{6}, -5 - \sqrt{6}$ **109.** $3 + 3i\sqrt{5}, 3 - 3i\sqrt{5}$ **111.** $\frac{2 - 9\sqrt{2}}{3}, \frac{2 + 9\sqrt{2}}{3}$

113. $x^2 - 2 = 0$ **115.** $x^2 - 18 = 0$ **117.** $\frac{3b}{a}, -\frac{3b}{a}$ **119.** $-\frac{1}{2}$

SECTION 10.2

3. $5, -1$ **5.** $-9, 1$ **7.** 3 **9.** $-2 + \sqrt{11}, -2 - \sqrt{11}$ **11.** $3 + \sqrt{2}, 3 - \sqrt{2}$ **13.** $1 + i, 1 - i$ **15.** $8, -3$

17. $4, -9$ **19.** $\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$ **21.** $\frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}$ **23.** $3 + \sqrt{13}, 3 - \sqrt{13}$ **25.** $5, 3$ **27.** $2 + 3i, 2 - 3i$

29. $-3 + 2i, -3 - 2i$ **31.** $1 + 3\sqrt{2}, 1 - 3\sqrt{2}$ **33.** $\frac{1 + \sqrt{17}}{2}, \frac{1 - \sqrt{17}}{2}$ **35.** $1 + 2i\sqrt{3}, 1 - 2i\sqrt{3}$

37. $\frac{1}{2} + i, \frac{1}{2} - i$ **39.** $\frac{1}{3} + \frac{1}{3}i, \frac{1}{3} - \frac{1}{3}i$ **41.** $\frac{2 + \sqrt{14}}{2}, \frac{2 - \sqrt{14}}{2}$ **43.** $1, -\frac{3}{2}$ **45.** $1 + \sqrt{5}, 1 - \sqrt{5}$ **47.** $\frac{1}{2}, 5$

49. $2 + \sqrt{5}, 2 - \sqrt{5}$ **51.** $1.236, -3.236$ **53.** $1.707, 0.293$ **55.** $0.309, -0.809$ **57.** $x = 2a, x = -a$

59. $x = 2a, x = -5a$

SECTION 10.3

3. $5, -2$ **5.** $4, -9$ **7.** $4 + 2\sqrt{22}, 4 - 2\sqrt{22}$ **9.** $3, -8$ **11.** $\frac{-5 + \sqrt{33}}{4}, \frac{-5 - \sqrt{33}}{4}$ **13.** $\frac{3}{2}, -\frac{1}{4}$

15. $7 + 3\sqrt{5}, 7 - 3\sqrt{5}$ **17.** $\frac{1 + \sqrt{3}}{2}, \frac{1 - \sqrt{3}}{2}$ **19.** $-1 + i, -1 - i$ **21.** $1 + 2i, 1 - 2i$ **23.** $2 + 3i, 2 - 3i$

25. $\frac{1 + \sqrt{11}}{2}, \frac{1 - \sqrt{11}}{2}$ **27.** $-\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i$ **29.** $\frac{3}{4} + \frac{3\sqrt{3}}{4}i, \frac{3}{4} - \frac{3\sqrt{3}}{4}i$ **31.** $7.606, 0.394$ **33.** $0.236, -4.236$

35. $1.851, -1.351$ **37.** two complex number solutions **39.** one real number solution **41.** two real number solutions **43.** $\{p \mid p < 9\}$ **45.** $\{p \mid p > 1\}$ **47.** i and $-2i$

SECTION 10.4

1. $3, -3, 2, -2$ **3.** $\sqrt{2}, -\sqrt{2}, 2, -2$ **5.** $1, 4$ **7.** 16 **9.** $2i, -2i, 1, -1$ **11.** $4i, -4i, 2, -2$ **13.** 16

15. $1, 512$ **17.** $\frac{2}{3}, -\frac{2}{3}, 1, -1$ **19.** 3 **21.** 9 **23.** $2, -1$ **25.** $0, 2$ **27.** $2, -\frac{1}{2}$ **29.** -2 **31.** 1 **33.** 1

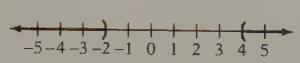
35. -3 **37.** $10, -1$ **39.** $-\frac{1}{2} + \frac{\sqrt{7}}{2}i, -\frac{1}{2} - \frac{\sqrt{7}}{2}i$ **41.** $1, -3$ **43.** $0, -1$ **45.** $\frac{1}{2}, -\frac{1}{3}$ **47.** $-\frac{2}{3}, 6$ **49.** $\frac{4}{3}, 3$

51. $-\frac{1}{4}, 3$ **53.** 4

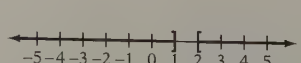
SECTION 10.5

1. The height is 3 cm; the base is 14 cm. **3.** The dimensions of Colorado are approximately 272 mi by 383 mi. **5.** The maximum safe speed is 33 mph. **7.** The time for the projectile to return to Earth is 12.5 s. **9.** The maximum speed is 72.5 km/h. **11.** The smaller pipe requires 12 min. The larger pipe requires 6 min. **13.** The rate of the wind is 108 mph. **15.** The rowing rate of the guide is 6 mph. **17.** The radius of the cone 1.5 in.

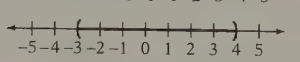
SECTION 10.6

3. 

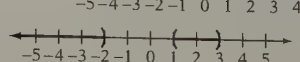
$\{x \mid x < -2 \text{ or } x > 4\}$

5. 

$\{x \mid x \le 1 \text{ or } x \ge 2\}$

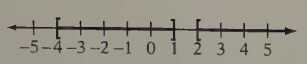
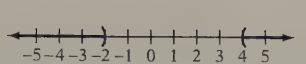
7. 

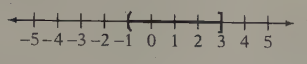
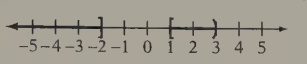
$\{x \mid -3 < x < 4\}$

9. 

$\{x \mid x < -2 \text{ or } 1 < x < 3\}$

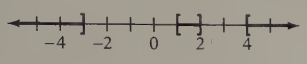
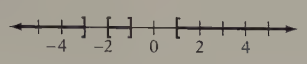
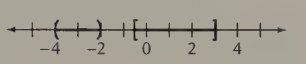
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11.  $\{x|-4 \le x \le 1 \text{ or } x \ge 2\}$ **13.**  $\{x|x < -2 \text{ or } x > 4\}$

15.  $\{x|-1 < x \le 3\}$ **17.**  $\{x|x \le -2 \text{ or } 1 \le x < 3\}$

19. $\{x|x > 4 \text{ or } x < -4\}$ **21.** $\{x|-3 \le x \le 12\}$ **23.** $\left\{x\middle|\frac{1}{2} < x < \frac{3}{2}\right\}$ **25.** $\left\{x|x < 1 \text{ or } x > \frac{5}{2}\right\}$

27. $\{x|x < -1 \text{ or } 1 < x \le 2\}$ **29.** $\left\{x\middle|\frac{1}{2} < x \le 1\right\}$ **31.** $\{x|2 < x \le 3\}$ **33.** $\{x|x > 5 \text{ or } -4 < x < -1\}$

35.  **37.**  **39.** 

CHAPTER REVIEW

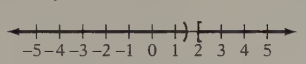
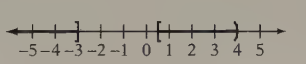
1. $0, \frac{3}{2}$ [10.1A] **2.** $\frac{c}{2}, -2c$ [10.1A] **3.** $4\sqrt{3}, -4\sqrt{3}$ [10.1C] **4.** $-\frac{1}{2} + 2i, -\frac{1}{2} - 2i$ [10.1C]

5. $-3, -1$ [10.2A] **6.** $\frac{7 + 2\sqrt{7}}{7}, \frac{7 - 2\sqrt{7}}{7}$ [10.2A] **7.** $\frac{3}{4}, \frac{4}{3}$ [10.3A] **8.** $\frac{1}{2} + \frac{\sqrt{31}}{2}i, \frac{1}{2} - \frac{\sqrt{31}}{2}i$ [10.3A]

9. $x^2 + 3x = 0$ [10.1B] **10.** $12x^2 - x - 6 = 0$ [10.1B] **11.** $1 + i\sqrt{7}, 1 - i\sqrt{7}$ [10.2A] **12.** $2i, -2i$ [10.2A]

13. $\frac{11 + \sqrt{73}}{6}, \frac{11 - \sqrt{73}}{6}$ [10.3A] **14.** two real number solutions [10.3A] **15.** $\left\{x\middle|-3 < x < \frac{5}{2}\right\}$ [10.6A]

16. $\left\{x|x \le -4 \text{ or } -\frac{3}{2} \le x \le 2\right\}$ [10.6A] **17.** 27, −64 [10.4A] **18.** $\frac{5}{4}$ [10.4B] **19.** $-1, 3$ [10.4C]

20. -1 [10.4C] **21.**  [10.6A] **22.**  [10.6A]

$\left\{x\middle|x < \frac{3}{2} \text{ or } x \ge 2\right\}$ $\left\{x|x \le -3 \text{ or } \frac{1}{2} \le x < 4\right\}$

23. 4 [10.4B] **24.** 5 [10.4B] **25.** $\frac{-3 + \sqrt{249}}{10}, \frac{-3 - \sqrt{249}}{10}$ [10.4C] **26.** $\frac{-11 + \sqrt{129}}{2}, \frac{-11 - \sqrt{129}}{2}$ [10.4C]

27. The length is 12 cm, and the width is 5 cm. [10.5A] **28.** The three integers are 2, 4, and 6 or −6, −4, and −2. [10.5A] **29.** Working alone, the new computer takes 12 min to print the payroll. [10.5A] **30.** The speed of the first car is 40 mph. The speed of the second car is 50 mph. [10.5A]

CHAPTER TEST

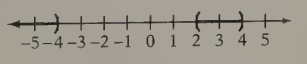
1. $\frac{2}{3}, -4$ [10.1A] **2.** $\frac{3}{2}, -\frac{2}{3}$ [10.1A] **3.** $x^2 - 9 = 0$ [10.1B] **4.** $2x^2 + 7x - 4 = 0$ [10.1B]

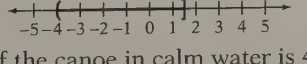
5. $2 + 2\sqrt{2}, 2 - 2\sqrt{2}$ [10.1C] **6.** $3 + \sqrt{11}, 3 - \sqrt{11}$ [10.2A] **7.** $\frac{3 + \sqrt{15}}{3}, \frac{3 - \sqrt{15}}{3}$ [10.2A]

8. $\frac{1 + \sqrt{3}}{2}, \frac{1 - \sqrt{3}}{2}$ [10.3A] **9.** $-2 + 2i\sqrt{2}, -2 - 2i\sqrt{2}$ [10.3A] **10.** two real number solutions [10.3A]

11. two complex number solutions [10.3A] **12.** $\frac{1}{4}$ [10.4A] **13.** $1, -1, \sqrt{3}, -\sqrt{3}$ [10.4A] **14.** 4 [10.4B]

15. The equation has no solution. [10.4B] **16.** 2, −9 [10.4C]

17. $\{x|x < -4 \text{ or } 2 < x < 4\}$  [10.6A]

18. $\left\{x\middle|-4 < x \le \frac{3}{2}\right\}$  [10.6A] **19.** The base is 15 ft. The height is 4 ft. [10.5A]

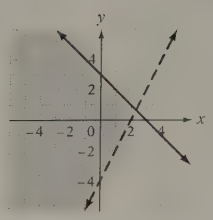
20. The rowing rate of the canoe in calm water is 4 mph. [10.5A]

CUMULATIVE REVIEW

1. 14 [1.4A] **2.** $\left\{x\middle|-2 < x < \frac{10}{3}\right\}$ [2.5B] **3.** The volume is 54π m^3. [3.3A] **4.** $-\frac{7}{3}$ [4.2A] **5.** $-\frac{3}{2}$ [4.4A]

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6. $\left(\frac{5}{2}, 0\right), (0, -3)$ [4.3B] **7.** $y = x + 1$ [4.6A] **8.** $(1, -1, 2)$ [5.2B] **9.**



[5.5A]

10. The height of triangle *DEF* is 16 cm. [8.4C] **11.** $x^2 - 3x - 4 - \frac{6}{3x - 4}$ [6.4A] **12.** $-3xy(x^2 - 2xy + 3y^2)$ [7.1A]

13. $(2x - 5)(3x + 4)$ [7.3A/7.3B] **14.** $\frac{x}{2}$ [8.1B] **15.** $-\frac{3}{2}$ and -1 [8.4A] **16.** $b = \frac{2S - an}{n}$ [8.5A]

17. $1 - a$ [9.1A] **18.** $-8 - 14i$ [9.3C] **19.** 0, 1 [9.4A] **20.** $2, -2, \sqrt{2}, -\sqrt{2}$ [10.4A] **21.** The lower limit is $9\frac{23}{64}$ in. The upper limit is $9\frac{25}{64}$ in. [2.5C] **22.** The area is $(x^2 + 6x - 16)$ ft². [6.3E] **23.** The slope is $-\frac{25{,}000}{3}$. The building decreases \$8333.33 in value each year. [4.4A] **24.** The ladder will reach 15 ft up on the building. [9.4B]
25. There are two complex number solutions. [10.3A]

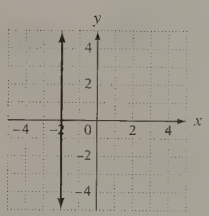
Answers to Chapter 11 Selected Exercises

PREP TEST

1. 1 [1.4A] **2.** -7 [1.4A] **3.** 30 [4.2A] **4.** $h^2 + 4h - 1$ [4.2A] **5.** $-\frac{2}{3}, 3$ [10.1A]

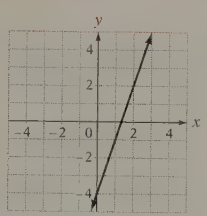
6. $2 - \sqrt{3}, 2 + \sqrt{3}$ [10.2A, 10.3A] **7.** $y = \frac{1}{2}x - 2$ [8.5A] **8.** domain: $\{-2, 3, 4, 6\}$; range: $\{4, 5, 6\}$; yes [4.2A]

9. [4.3B]

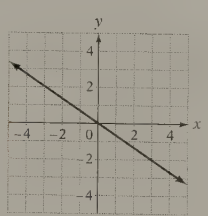


SECTION 11.1

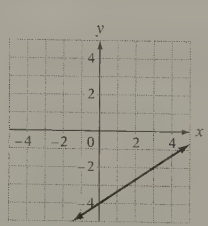
1.



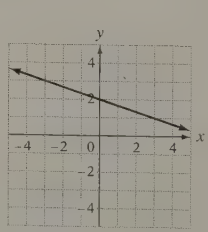
3.



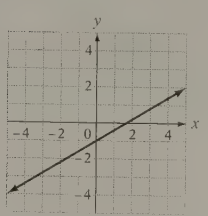
5.



7.



9.

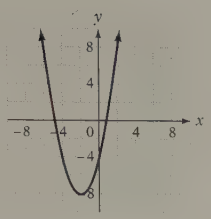


11. If the cost is \$150, the markup is \$37.50. **13.** The linear function is $y = -\frac{3}{5}x + 545$. If the room rate is \$100, 485 rooms will be rented. **15a.** III **b.** IV **c.** II **d.** I

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SECTION 11.2

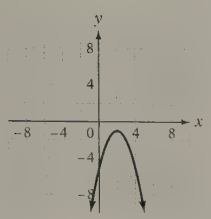
3. 8 **5.** $x = -4$ **7.**



vertex: $(-2, -8)$

axis of symmetry: $x = -2$

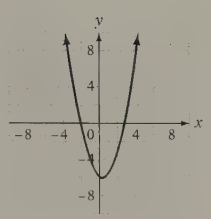
9.



vertex: $(2, -1)$

axis of symmetry: $x = 2$

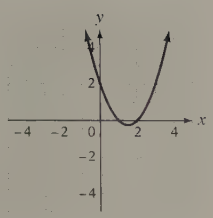
11.



vertex: $\left(\frac{1}{2}, -\frac{25}{4}\right)$

axis of symmetry: $x = \frac{1}{2}$

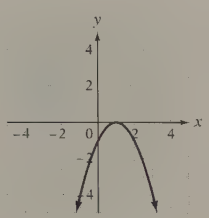
13.



vertex: $\left(\frac{3}{2}, -\frac{1}{4}\right)$

axis of symmetry: $x = \frac{3}{2}$

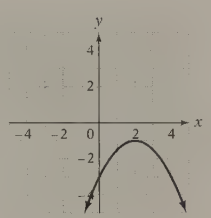
15.



vertex: $(1, 0)$

axis of symmetry: $x = 1$

17.



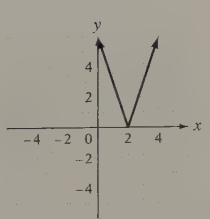
vertex: $(2, -1)$

axis of symmetry: $x = 2$

19. The domain is $\{x|x \in \text{real numbers}\}$. The range is $\{y|y \geq -5\}$. **21.** The domain is $\{x|x \in \text{real numbers}\}$. The range is $\{y|y \leq -1\}$. **23.** $(2, 0), (-2, 0)$ **25.** $(0, 0), (2, 0)$ **27.** $(2, 0), (-1, 0)$ **29.** $\left(-\frac{1}{2}, 0\right), (1, 0)$ **31.** $(-1 + \sqrt{2}, 0)$, $(-1 - \sqrt{2}, 0)$ **33.** The parabola has no x-intercepts. **35.** The parabola has no x-intercepts. **37.** The parabola has two x-intercepts. **39.** The parabola has one x-intercept. **41.** The parabola has no x-intercepts. **43.** The parabola has two x-intercepts. **45a.** maximum **b.** minimum **c.** maximum **47.** minimum: 2 **49.** maximum: -1 **51.** minimum: -2 **53.** maximum: -3 **55.** minimum: $-\frac{73}{8}$ **57.** b **59.** The minimum height of the cable above the bridge is 24.36 ft. **61.** The diver will be a maximum of 13.1 m above the water. **63.** The maximum height is 1.5 ft. **65.** A speed of 41 mph will yield the maximum fuel efficiency. The maximum fuel efficiency is 33.658 mi/gal. **67.** $-\frac{3}{2}$

SECTION 11.3

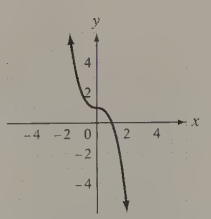
1. yes **3.** no **5.** yes **7.**



domain: $\{x|x \in \text{real numbers}\}$

range: $\{y|y \geq 0\}$

9.

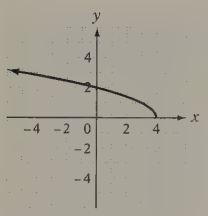


domain: $\{x|x \in \text{real numbers}\}$

range: $\{y|y \in \text{real numbers}\}$

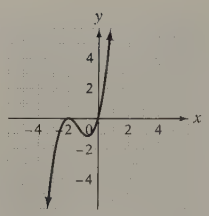
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11.



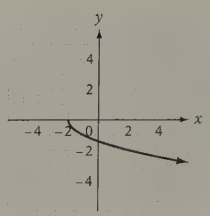
domain: $\{x|x \le 4\}$
range: $\{y|y \ge 0\}$

13.



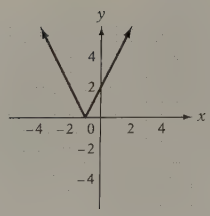
domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \in \text{real numbers}\}$

15.



domain: $\{x|x \ge -2\}$
range: $\{y|y \le 0\}$

17.



domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \ge 0\}$

19. $a = 18$ **21.** $f(2, 5) + g(2, 5) = 17$ **23.** $f(14) = 8$ **25.** $x = -3$

SECTION 11.4

1. 5 **3.** 1 **5.** 0 **7.** $-\frac{29}{4}$ **9.** $\frac{2}{3}$ **11.** 2 **13.** 39 **15.** -8 **17.** $-\frac{4}{5}$ **19.** -2 **21.** 7 **23.** -13

25. -29 **27.** $8x - 13$ **29.** 4 **31.** 3 **33.** $x + 4$ **35.** 5 **37.** 11 **39.** $3x^2 + 3x + 5$ **41.** -3 **43.** -27

45. $x^3 - 6x^2 + 12x - 8$ **47.** $h^2 + 6h$ **49.** $2 + h$ **51.** $h + 2a$ **53.** -1 **55.** -6 **57.** $6x - 13$

SECTION 11.5

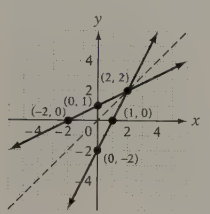
3. yes **5.** no **7.** yes **9.** no **11.** no **13.** no **17.** $\{(0, 1), (3, 2), (8, 3), (15, 4)\}$ **19.** no inverse

21. $\{(-2, 0), (5, -1), (3, 3), (6, -4)\}$ **23.** no inverse **25.** $f^{-1}(x) = \frac{1}{4}x + 2$ **27.** $f^{-1}(x) = \frac{1}{2}x - 2$

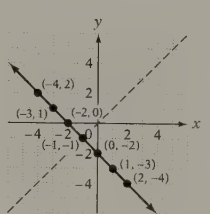
29. $f^{-1}(x) = 2x + 2$ **31.** $f^{-1}(x) = -\frac{1}{2}x + 1$ **33.** $f^{-1}(x) = \frac{3}{2}x - 6$ **35.** $f^{-1}(x) = -3x + 3$ **37.** $f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$

39. $f^{-1}(x) = \frac{1}{5}x + \frac{2}{5}$ **41.** $f^{-1}(x) = \frac{1}{6}x + \frac{1}{2}$ **43.** $\frac{5}{3}$ **45.** 3 **47.** yes; yes **49.** yes **51.** no **53.** yes **55.** yes

57.

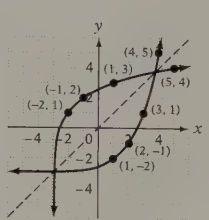


59.



The inverse is the same graph.

61.

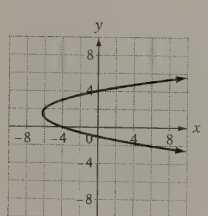


63. no **65.** 5 **67.** 0 **69.** -4

SECTION 11.6

1a. a vertical line **b.** opens up **3a.** a horizontal line **b.** opens right **5a.** a horizontal line **b.** opens left

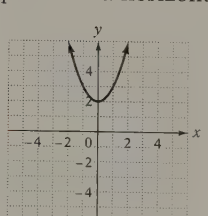
7.



vertex: $\left(-\frac{25}{4}, \frac{3}{2}\right)$

axis of symmetry: $y = \frac{3}{2}$

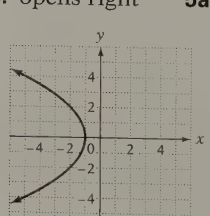
9.



vertex: (0, 2)

axis of symmetry: $x = 0$

11.

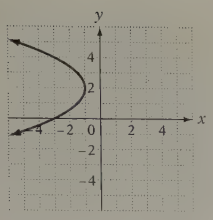


vertex: $(-1, 0)$

axis of symmetry: $y = 0$

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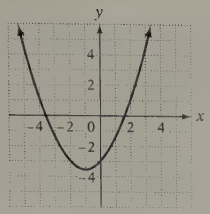
13.



vertex: $(-1, 2)$

axis of symmetry: $y = 2$

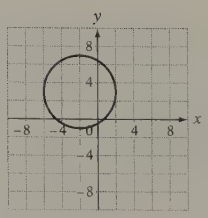
15.



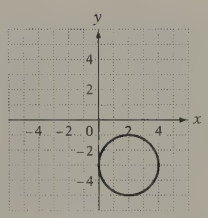
vertex: $\left(-1, -\frac{7}{2}\right)$

axis of symmetry: $x = -1$

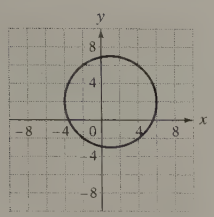
17.



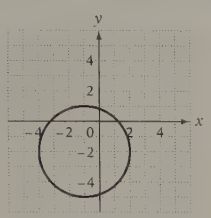
19.



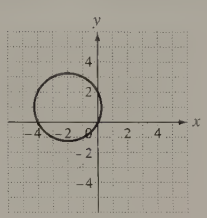
21.



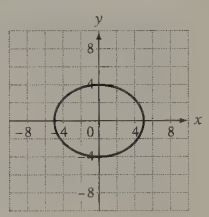
23. $(x + 1)^2 + (y + 2)^2 = 9$



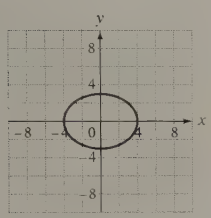
25. $(x + 2)^2 + (y - 1)^2 = 5$



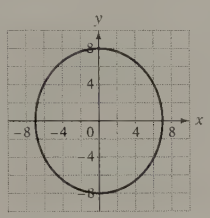
27.



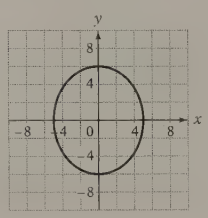
29.



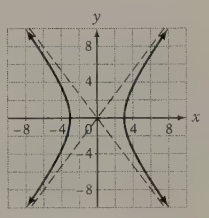
31.



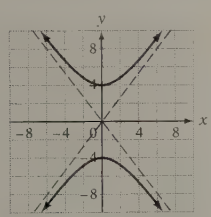
33.



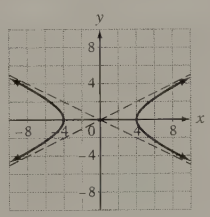
35.



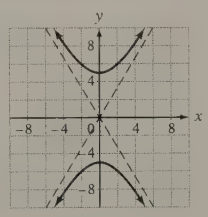
37.



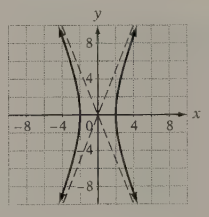
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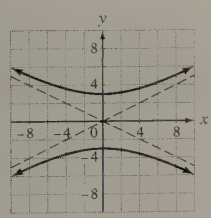
41.



43.

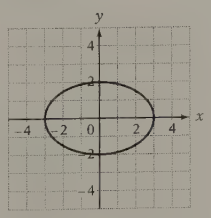


45.



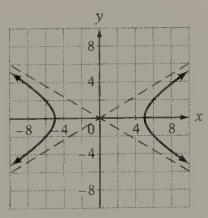
47. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

ellipse



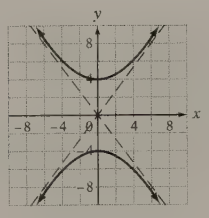
49. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

hyperbola



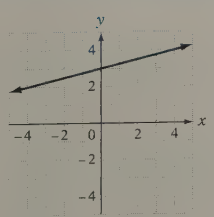
51. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

hyperbola



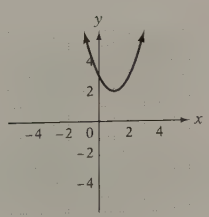
CHAPTER REVIEW

1.



[11.1A]

2.



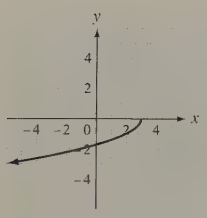
[11.2A]

3. There are no x-intercepts. [11.2B]

4. The maximum value of the function is 9. [11.2C]

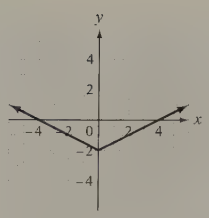
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5.



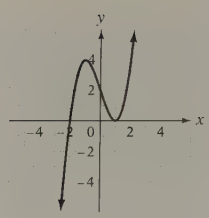
domain: $\{x|x \le 3\}$
range: $\{y|y \le 0\}$ [11.3A]

6.



domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \ge -2\}$ [11.3A]

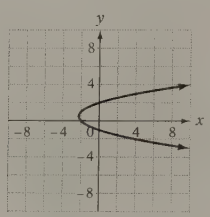
7.



domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \in \text{real numbers}\}$ [11.3A]

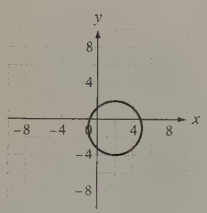
8. no [11.5A] **9.** -2 [11.4A] **10.** $-\frac{13}{2}$ [11.4A] **11.** 5 [11.4B] **12.** $2x^2 - 4x - 5$ [11.4B]

13. $\{(6, 2), (5, 3), (4, 4), (3, 5)\}$ [11.5B] **14.** $f^{-1}(x) = 4x + 16$ [11.5B] **15.**



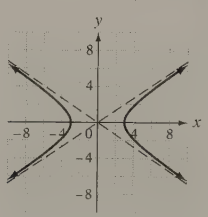
[11.6A]

16.



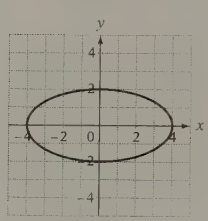
[11.6B]

17.



[11.6D]

18.

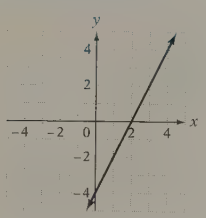


[11.6C]

19. $(x + 3)^2 + (y + 3)^2 = 16$ [11.6B] **20.** The dimensions are 50 cm by 50 cm. The area is 2500 cm^2. [11.2D]

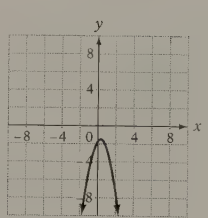
CHAPTER TEST

1.



[11.1A]

2.

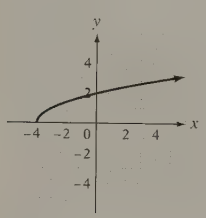


[11.2A]

3. (0, 0), (−3, 0) [11.2B]

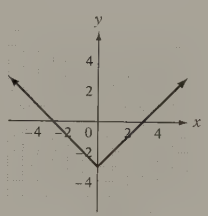
4. The maximum value of the function is 3. [11.2C]

5.



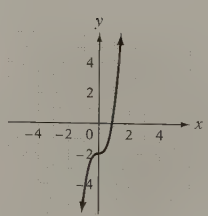
domain: $\{x|x \ge -4\}$
range: $\{y|y \ge 0\}$ [11.3A]

6.



domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \ge -3\}$ [11.3A]

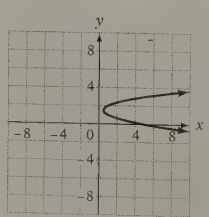
7.



domain: $\{x|x \in \text{real numbers}\}$
range: $\{y|y \in \text{real numbers}\}$ [11.3A]

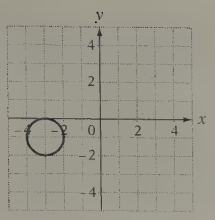
8. yes [11.5A] **9.** 7 [11.4A] **10.** −9 [11.4A] **11.** 70 [11.4A] **12.** $\frac{12}{7}$ [11.4A]

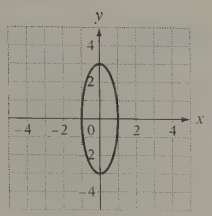
13. $6x^2 + 3x - 16$ [11.4B] **14.** $f^{-1}(x) = -\frac{1}{6}x + \frac{2}{3}$ [11.5B] **15.**

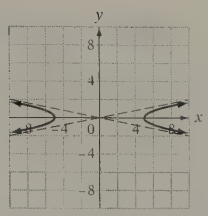


[11.6A]

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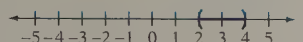
16.  [11.6B]

17.  [11.6C]

18.  [11.6D]

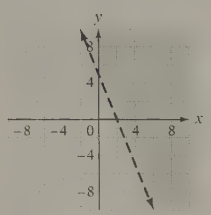
19. yes [11.5B] **20.** The rectangle will have a maximum area when the dimensions are 7 ft by 7 ft. [11.2D]

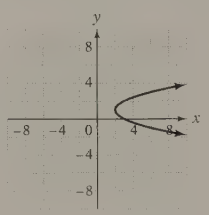
CUMULATIVE REVIEW

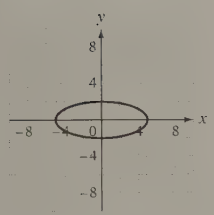
1.  [1.5C] **2.** $\frac{38}{53}$ [8.4A] **3.** -20 [4.2A] **4.** $y = -\frac{3}{2}x$ [4.5A] **5.** $y = x - 6$ [4.6A]

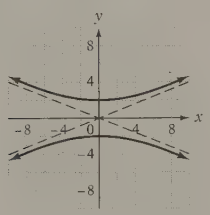
6. $\frac{x}{x+y}$ [8.1A] **7.** $\frac{x-5}{3x-2}$ [8.2D] **8.** $\frac{5}{2}$ [8.4A] **9.** $\frac{4a}{3b^{12}}$ [6.1B] **10.** $2x^{3/4}$ [9.1B] **11.** $3\sqrt{2} - 5i$ [9.3A]

12. $\frac{-1+\sqrt{7}}{2}, \frac{-1-\sqrt{7}}{2}$ [10.2A/10.3A] **13.** 6 [10.4B] **14.** $\{x \mid -4 < x \leq 3\}$ [10.6A] **15.** 0 [11.2C]

16. $f^{-1}(x) = \frac{1}{4}x - 2$ [11.5B] **17.**  [4.7A]

18.  [11.6A]

19.  [11.6C]

20.  [11.6D]

21. There were 82 adult tickets sold. [5.4B]

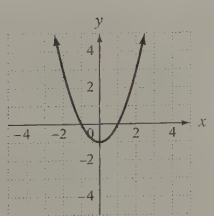
22. The rate of the motorcycle is 60 mph. [8.6B] **23.** The rate of the crew in calm water is 4.5 mph. [5.4A]

24. A gear with 60 teeth will make 18 revolutions per minute. [8.7A] **25.** The maximum product is 400. [11.2D]

Answers to Chapter 12 Selected Exercises

PREP TEST

1. $\frac{1}{9}$ [6.1B] **2.** 16 [6.1B] **3.** -3 [6.1B] **4.** 0; 108 [4.2A] **5.** -6 [2.2B] **6.** $-2, 8$ [10.1A]

7. 6326.60 [1.4A] **8.**  [11.2A]

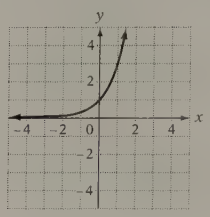
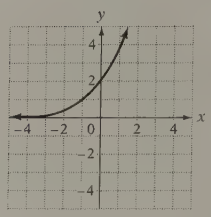
SECTION 12.1

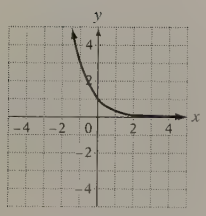
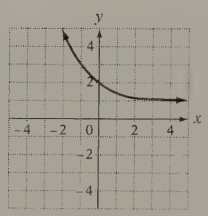
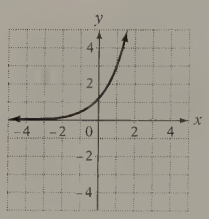
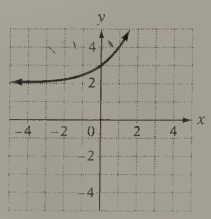
3. c **5a.** $f(2) = 9$ **b.** $f(0) = 1$ **c.** $f(-2) = \frac{1}{9}$ **7a.** $g(3) = 16$ **b.** $g(1) = 4$ **c.** $g(-3) = \frac{1}{4}$ **9a.** $P(0) = 1$

b. $P\left(\frac{3}{2}\right) = \frac{1}{8}$ **c.** $P(-2) = 16$ **11a.** $G(4) = 7.3891$ **b.** $G(-2) = 0.3679$ **c.** $G\left(\frac{1}{2}\right) = 1.2840$

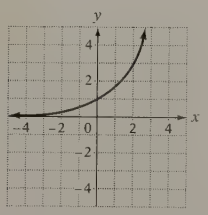
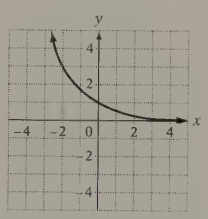
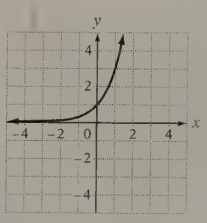
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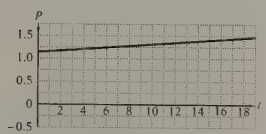
13a. $H(-1) = 54.5982$ **b.** $H(3) = 1$ **c.** $H(5) = 0.1353$ **15a.** $F(2) = 16$ **b.** $F(-2) = 16$ **c.** $F\left(\frac{3}{4}\right) = 1.4768$

17a. $f(-2) = 0.1353$ **b.** $f(2) = 0.1353$ **c.** $f(-3) = 0.0111$ **19.**  **21.** 

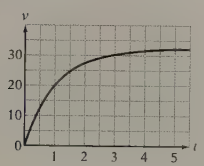
23.  **25.**  **27.**  **29.** 

31. b and d **33.** (0, 1) **35.** no x-intercept; y-intercept: (0, 1)

37.  **39.**  **41.**  **43.** e

45a. 

b. The point (9, 1.303) means that in the year 2002, the Chinese population was approximately 1.303 billion.

47a. 

b. The point (2, 27.7) means that after 2 s, the object is dropping at a speed of 27.7 ft/s.

SECTION 12.2

3. $\log_5 25 = 2$ **5.** $\log_4 \frac{1}{16} = -2$ **7.** $\log_{10} x = y$ **9.** $\log_a w = x$ **11.** $3^2 = 9$ **13.** $10^{-2} = 0.01$ **15.** $e^y = x$

17. $b^v = u$ **19.** $\log_3 81 = 4$ **21.** $\log_2 128 = 7$ **23.** $\log 100 = 2$ **25.** $\ln e^3 = 3$ **27.** $\log_8 1 = 0$

29. $\log_5 625 = 4$ **31.** 9 **33.** 64 **35.** $\frac{1}{7}$ **37.** 1 **39.** 316.23 **41.** 0.02 **43.** 7.39 **45.** 0.61

49. $\log_3 (x^3y^2)$ **51.** $\ln\left(\frac{x^4}{y^2}\right)$ **53.** $\log_7 x^3$ **55.** $\ln (x^3y^4)$ **57.** $\log_4 (x^2y^2)$ **59.** $\log_6 \sqrt{\frac{x}{y}}$ **61.** $\log_3\left(\frac{x^2z^2}{y}\right)$

63. $\ln\left(\frac{x}{y^2z}\right)$ **65.** $\log_4\left(\frac{s^2r^2}{t^4}\right)$ **67.** $\ln\left(\frac{x^3}{y^4z^4}\right)$ **69.** $\log_2\left(\frac{t^3v^2}{r^2}\right)$ **71.** $\log_4 \sqrt{\frac{x^3z}{y^2}}$ **73.** $\ln \sqrt{\frac{x}{y^3}}$ **75.** $\log_2\left(\frac{\sqrt{xz}}{\sqrt[3]{y^2}}\right)$

77. $\log_8 x + \log_8 z$ **79.** $5 \log_3 x$ **81.** $\log_b r - \log_b s$ **83.** $2 \log_3 x + 6 \log_3 y$ **85.** $3 \log_7 u - 4 \log_7 v$

87. $2 \log_2 r + 2 \log_2 s$ **89.** $2 \ln x + \ln y + \ln z$ **91.** $\log_5 x + 2 \log_5 y - 4 \log_5 z$ **93.** $2 \log_8 x - \log_8 y - 2 \log_8 z$

95. $\frac{3}{2}\log_4 x + \frac{1}{2}\log_4 y$ **97.** $\frac{3}{2}\log_7 x - \frac{1}{2}\log_7 y$ **99.** $\log_3 t - \frac{1}{2}\log_3 x$ **101.** 0.8451 **103.** −0.2218 **105.** 1.3863

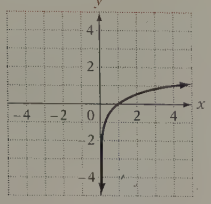
107. 1.0415 **109.** 0.8617 **111.** 2.1133 **113.** −0.6309 **115.** 0.2727 **117.** 1.6826 **119.** 1.9266

121. 0.6752 **123.** 2.6125 **125a.** false **b.** true **c.** false **d.** true **e.** false **f.** true

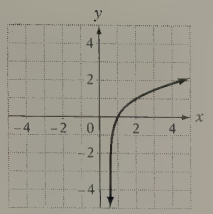
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SECTION 12.3

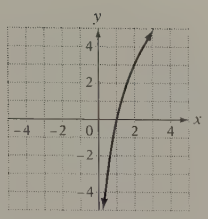
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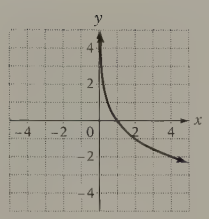
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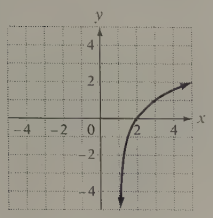
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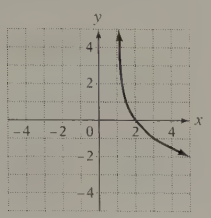
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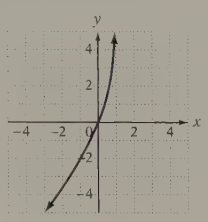
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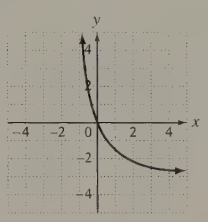
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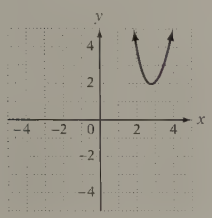
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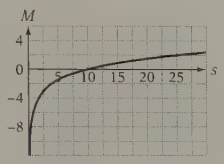
19.



21.



23a.



b. The point (25.1, 2) means that a star that is 25.1 parsecs from Earth has a distance modulus of 2.

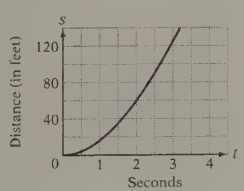
SECTION 12.4

3. $-\frac{1}{3}$ **5.** -3 **7.** 1 **9.** 6 **11.** $\frac{4}{7}$ **13.** $\frac{9}{8}$ **15.** 1.1133 **17.** 1.0986 **19.** 1.3222 **21.** -2.8074

23. 3.5850 **25.** 1.1309 **29.** $\frac{11}{2}$ **31.** $-4, 2$ **33.** $\frac{5}{3}$ **35.** $\frac{1}{2}$ **37.** $\frac{9}{2}$ **39.** 17.5327 **41.** 3 **43.** 2

45. The equation has no solution. **47.** -0.55 **49.** 1.38 **51.** The equation has no solution. **53.** 0.09

55a.



b. It will take the object approximately 2.64 s to fall 100 ft.

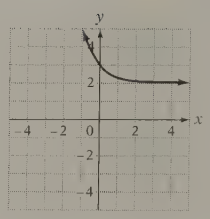
SECTION 12.5

1. The value of the investment after 2 years is \$1172. **3.** The investment will be worth \$15,000 in 18 years. **5a.** The technetium level will be 21.2 mg after 3 h. **b.** It will take 3.5 h for the technetium level to reach 20 mg. **7.** The half-life of promethium-147 is 2.5 years. **9a.** The approximate pressure at 40 km above Earth is 0.098 N/cm^2. **b.** The approximate pressure on Earth's surface is 10.13 N/cm^2. **c.** The atmospheric pressure decreases as you rise above Earth's surface. **11.** The pH of milk is 6.4. **13.** At a depth of 2.5 m, the percent of light will be 75% of the light at the surface of the pool. **15.** Normal conversation is 65 decibels. **17.** The average time of a major league baseball game increased 22.5 min during the years from 1981 to 1999. **19.** A thickness of 0.4 cm is needed. **21.** The Richter scale magnitude of this earthquake is 6.8. **23.** The intensity of this earthquake is 794,328,235I_0. **25.** The magnitude of the earthquake is 5.3. **27.** The magnitude of the earthquake is 5.6. **29.** The value of the investment after 5 years is \$3210.06.

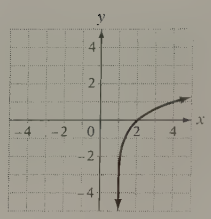
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CHAPTER REVIEW

1. 1 [12.1A] **2.** $5^2 = 25$ [12.2A] **3.**



[12.1B] **4.**



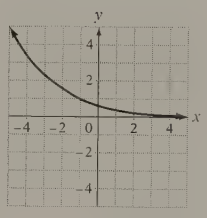
[12.3A]

5. $\frac{2}{5}\log_3 x + \frac{4}{5}\log_3 y$ [12.2B] **6.** $\log_3 \frac{x^2}{y^5}$ [12.2B] **7.** -12 [12.4A] **8.** $\frac{1}{4}$ [12.4B] **9.** 1.7251 [12.2C]

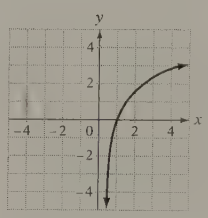
10. 32 [12.2A] **11.** 79 [12.4B] **12.** 1000 [12.2A] **13.** $\log_7 \sqrt[3]{xy^4}$ [12.2B] **14.** $\frac{5}{2}\log_8 x - \frac{3}{2}\log_8 y$ [12.2B]

15. $\log_2 32 = 5$ [12.2A] **16.** 0.4278 [12.2C] **17.** -0.535 [12.4A] **18.** $\frac{3}{2}$ [12.1A] **19.** $\frac{2}{3}$ [12.4B]

20. 2 [12.4B] **21.**



[12.1B] **22.**



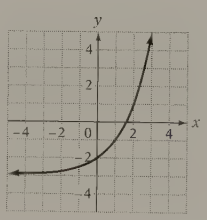
[12.3A]

23. The value of the investment after 2 years is \$4692. [12.5A] **24.** The magnitude of the earthquake is 7.6. [12.5A]

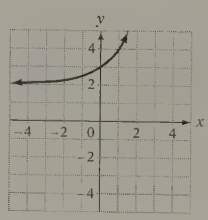
25. The half-life of the material is 27 days. [12.5A] **26.** Sound emitted from a busy street corner is 107 decibels. [12.5A]

CHAPTER TEST

1. 1 [12.1A] **2.** $\frac{1}{3}$ [12.1A] **3.**

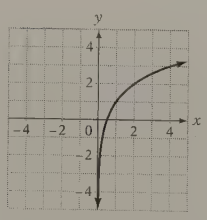


[12.1B] **4.**

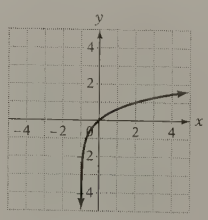


[12.1B]

5. 2 [12.2A] **6.** $\frac{1}{9}$ [12.2A] **7.**



[12.3A] **8.**



[12.3A]

9. $\frac{1}{2}\log_6 x + \frac{3}{2}\log_6 y$ [12.2B] **10.** $\log_3 \sqrt{\frac{x}{y}}$ [12.2B] **11.** $\ln x - \frac{1}{2}\ln z$ [12.2B] **12.** $\ln \frac{x^3}{y\sqrt{z}}$ [12.2B]

13. -2 [12.4A] **14.** -3 [12.4A] **15.** 2.5789 [12.4A] **16.** 6 [12.4B] **17.** 3 [12.4B]

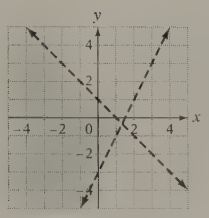
18. 1.3652 [12.2C] **19.** 2.6801 [12.2C] **20.** The half-life is 33 h. [12.5A]

CUMULATIVE REVIEW

1. $\frac{8}{7}$ [2.2C] **2.** $y = 2x - 6$ [4.6A] **3.** $(4x^n + 3)(x^n + 1)$ [7.3A/7.3B] **4.** $\frac{x-3}{x+3}$ [8.3A] **5.** $\frac{x\sqrt{y} + y\sqrt{x}}{x - y}$ [9.2D]

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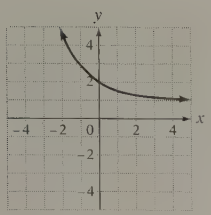
6. $2 + \sqrt{10}, 2 - \sqrt{10}$ [10.2A] **7.** The length of the side is 26 cm. [9.4B] **8.**



[5.5A]

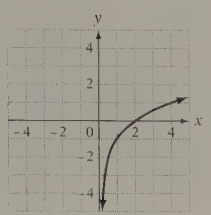
9. $(0, -1, 2)$ [5.2B] **10.** $-\dfrac{2x^2 - 17x + 13}{(x - 2)(2x - 3)}$ [8.2D] **11.** $\{x \mid -5 \le x \le 1\}$ [10.6A] **12.** $\{x \mid 1 \le x \le 4\}$ [2.5B]

13.



[12.1B]

14.



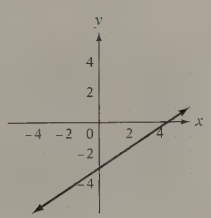
[12.3A]

15. The area is 28 yd^2. [3.2B]

16. 125 [12.2A] **17.** $\log_b \dfrac{x^3}{y^5}$ [12.2B] **18.** 1.7712 [12.2C] **19.** 2 [12.4A] **20.** $\dfrac{1}{2}$ [12.4B] **21.** The customer can write fewer than 50 checks. [2.4C] **22.** The mixture costs \$3.10 per pound. [2.3A] **23.** The rate of the wind is 25 mph. [5.4A] **24.** The force will stretch the spring 10.2 in. [8.7A] **25.** The redwood costs \$.40 per foot. The fir costs \$.25 per foot. [5.4B] **26.** The investment will double in 8 years. [12.5A]

Answers to Final Exam

1. -31 [1.3A] **2.** -1 [1.4A] **3.** $-10x + 33$ [1.4D] **4.** 8 [2.2A] **5.** $4, -\dfrac{2}{3}$ [2.5A] **6.** The volume is 268.1 ft^3. [3.3A] **7.**



[4.3B] **8.** $y = -3x + 7$ [4.5B] **9.** $y = -\dfrac{2}{3}x - \dfrac{1}{3}$ [4.6A]

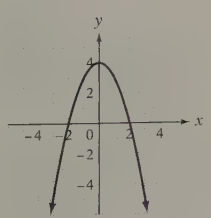
10. $6a^3 - 5a^2 + 10a$ [6.3A] **11.** $(2 - xy)(4 + 2xy + x^2y^2)$ [7.4B] **12.** $(x - y)(1 + x)(1 - x)$ [7.4D]

13. $x^2 - 2x - 3 - \dfrac{5}{2x - 3}$ [6.4A] **14.** $\dfrac{x(x - 1)}{2x - 5}$ [8.1C] **15.** $\dfrac{-10x}{(x + 2)(x - 3)}$ [8.2D] **16.** $\dfrac{x + 3}{x + 1}$ [8.3A]

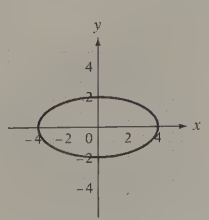
17. $-\dfrac{7}{4}$ [8.4A] **18.** $d = \dfrac{a_n - a_1}{n - 1}$ [8.5A] **19.** $\dfrac{y^4}{162x^3}$ [6.1B] **20.** $\dfrac{1}{64x^8y^5}$ [9.1A] **21.** $-2x^2y\sqrt{2y}$ [9.2B]

22. $\dfrac{x^2\sqrt{2y}}{2y^2}$ [9.2D] **23.** $\dfrac{6}{5} - \dfrac{3}{5}i$ [9.3D] **24.** $2x^2 - 3x - 2 = 0$ [10.1B] **25.** $\dfrac{3 + \sqrt{17}}{4}, \dfrac{3 - \sqrt{17}}{4}$ [10.3A]

26. $-8, 27$ [10.4A] **27.**



[11.2A] **28.**

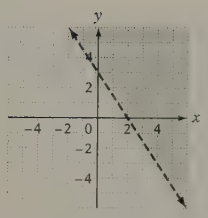


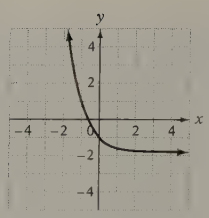
[11.6C]

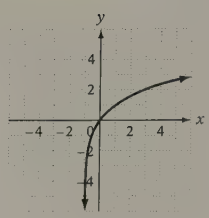
29. $-2, \dfrac{3}{2}$ [10.4C] **30.** $f^{-1}(x) = \dfrac{3}{2}x + 6$ [11.5B] **31.** $(3, 4)$ [5.2A] **32.** 10 [5.3A]

33. $\left\{x \mid x > \dfrac{3}{2}\right\}$ [2.4B] **34.** $\{x \mid -4 < x < -1\}$ [2.5B]

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35.  [4.7A]

36.  [12.1B]

37.  [12.3A]

38. $\log_2 \frac{a^2}{b^2}$ [12.2B] **39.** 6 [12.4B] **40.** The range of scores is $69 \le x \le 100$. [2.4C]

41. The cyclist rode 40 mi. [2.3D] **42.** There is \$8000 invested at 8.5% and \$4000 invested at 6.4%. [2.3C] **43.** The length is 20 ft. The width is 7 ft. [10.5A] **44.** An additional 200 shares are needed. [8.4B] **45.** The rate of the plane was 420 mph. [8.6B] **46.** The object has fallen 88 ft when the speed reaches 75 ft/s. [9.4B] **47.** The rate of the plane for the first 360 mi was 120 mph. [8.6B] **48.** The intensity is 200 foot-candles. [8.7A] **49.** The rate of the boat in calm water is 12.5 mph. The rate of the current is 2.5 mph. [5.4A] **50.** The value of the investment after 2 years is \$4785.65. [12.5A]

Answers to Chapter R Selected Exercises

SECTION R.1

1. 10 **3.** 21 **5.** -2 **7.** 5 **9.** -3 **11.** -1 **13.** 1 **15.** 18 **17.** -6 **19.** 2 **21.** -40 **23.** 2.01

25. $8x$ **27.** $3b$ **29.** $5a$ **31.** $11x$ **33.** $15x - 2y$ **35.** $9a$ **37.** $22y^2$ **39.** $\frac{1}{2}x$ **41.** $-20x$ **43.** $-24a$

45. x **47.** $7x$ **49.** $3y$ **51.** $-3a - 15$ **53.** $-16x - 48$ **55.** $-10y^2 + 5$ **57.** $18x^2 - 12xy - 6y^2$

59. $-8y - 7$ **61.** $5x - 75$ **63.** $-a + 56$

SECTION R.2

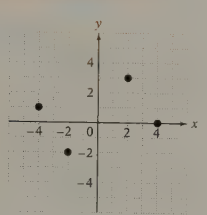
1. -12 **3.** -1 **5.** -8 **7.** -20 **9.** 8 **11.** -1 **13.** 1 **15.** $\frac{3}{4}$ **17.** 3 **19.** 3 **21.** 2 **23.** -1

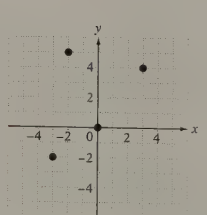
25. $\frac{1}{2}$ **27.** 5 **29.** $-\frac{1}{9}$ **31.** $\{x \mid x > 3\}$ **33.** $\{n \mid n \ge 2\}$ **35.** $\{x \mid x \le -3\}$ **37.** $\{n \mid n > 0\}$ **39.** $\{x \mid x > 4\}$

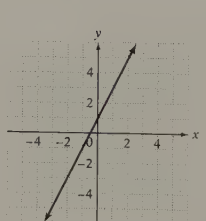
41. $\{x \mid x > -2\}$ **43.** $\{x \mid x > 3\}$ **45.** $\{x \mid x \ge 2\}$ **47.** $\{x \mid x < 2\}$ **49.** $\{x \mid x > 3\}$ **51.** $\{x \mid x \ge 2\}$ **53.** $\{x \mid x < 4\}$

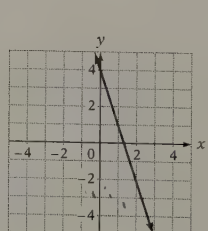
55. $\left\{x \middle| x > \frac{14}{11}\right\}$ **57.** $\left\{x \middle| x \le \frac{3}{8}\right\}$ **59.** $\left\{x \middle| x \ge -\frac{5}{4}\right\}$

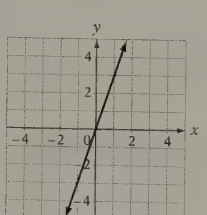
SECTION R.3

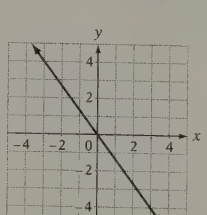
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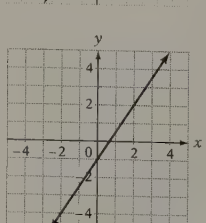
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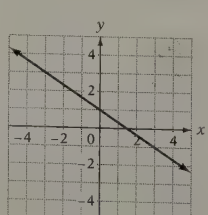
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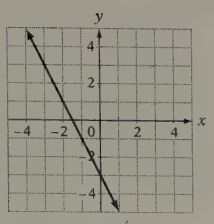
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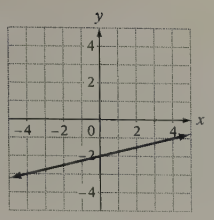
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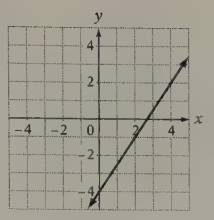
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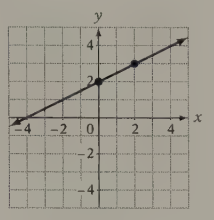
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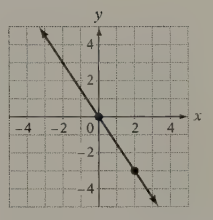
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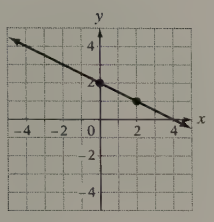
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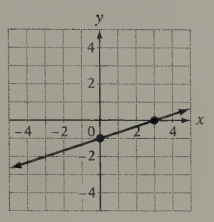
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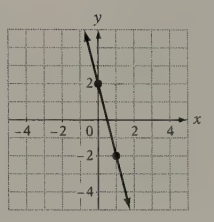
23. 3 **25.** $\frac{1}{4}$ **27.** $-\frac{3}{2}$

29. Undefined **31.** 0 **33.** 1 **35.** $m = \frac{5}{2}$, $b = (0, -4)$ **37.** 

39. 

41. 

43. 

45. 

47. 13; (−4, 13) **49.** 14; (−1, 14)

51. −1; (2, −1) **53.** −15; (−2, −15) **55.** $y = -3x - 1$ **57.** $y = -2x + 3$ **59.** $y = -\frac{3}{5}x$ **61.** $y = -\frac{2}{3}x - 4$

63. $y = -\frac{1}{2}x + 4$

SECTION R.4

1. z^8 **3.** x^{15} **5.** $x^{12}y^{18}$ **7.** a^6 **9.** $-m^9n^3$ **11.** $-8a^9b^3c^6$ **13.** mn^2 **15.** $-\frac{2a}{3}$ **17.** $-27m^2n^4p^3$

19. $54n^{14}$ **21.** $\frac{7xz}{8y^3}$ **23.** $-8x^{13}y^{14}$ **25.** $\frac{4}{x^7}$ **27.** $\frac{1}{d^{10}}$ **29.** $\frac{1}{x^5}$ **31.** $\frac{x^2}{3}$ **33.** $\frac{x^6}{y^{12}}$ **35.** $\frac{9}{x^2y^4}$ **37.** $\frac{2}{x^4}$

39. $\frac{1}{2x^3}$ **41.** $\frac{1}{2x^2y^6}$ **43.** $\frac{1}{x^6}$ **45.** $\frac{y^8}{x^4}$ **47.** $\frac{b^{10}}{4a^{10}}$ **49.** $7b^2 + b - 4$ **51.** $3a^2 - 3a + 17$ **53.** $-7x - 7$

55. $-2x^3 + x^2 + 2$ **57.** $x^3 + 2x^2 - 6x - 6$ **59.** $5x^3 + 10x^2 - x - 4$ **61.** $y^3 - y^2 + 6y - 6$ **63.** $4y^3 - 2y^2 + 2y - 4$

65. $12b^4 - 48b^3 - 24b$ **67.** $9b^5 - 9b^3 + 24b$ **69.** $-2x^4y + 6x^3y^2 - 4x^2y^3$ **71.** $x^3 + 4x^2 + 5x + 2$

73. $a^3 - 6a^2 + 13a - 12$ **75.** $-2b^3 + 7b^2 + 19b - 20$ **77.** $x^4 - 4x^3 - 3x^2 + 14x - 8$ **79.** $y^4 + 4y^3 + y^2 - 5y + 2$

81. $a^2 + a - 12$ **83.** $y^2 - 10y + 21$ **85.** $2x^2 + 15x + 7$ **87.** $3x^2 + 11x - 4$ **89.** $4x^2 - 31x + 21$

91. $3y^2 - 2y - 16$ **93.** $21a^2 - 83a + 80$ **95.** $2x^2 + 3xy + y^2$ **97.** $3x^2 - 10xy + 8y^2$ **99.** $10a^2 + 14ab - 12b^2$

101. $16x^2 - 49$ **103.** $x + 2$ **105.** $2y - 7$ **107.** $x - 2 + \frac{8}{x + 2}$ **109.** $3y - 5 + \frac{20}{2y + 4}$ **111.** $b - 5 - \frac{24}{b - 3}$

113. $3x + 17 + \frac{64}{x - 4}$ **115.** $5y + 3 + \frac{1}{2y + 3}$ **117.** $x^2 - 5x + 2$ **119.** $x^2 + 5$ **121.** $y(12y - 5)$

123. $5xyz(2xz + 3y^2)$ **125.** $5(x^2 - 3x + 7)$ **127.** $3y^2(y^2 - 3y - 2)$ **129.** $x^2y^2(x^2y^2 - 3xy + 6)$

131. $8x^2y(2 - xy^3 - 6y)$ **133.** $(x + 2)(x - 1)$ **135.** $(a + 4)(a - 3)$ **137.** $(a - 1)(a - 2)$ **139.** $(b + 8)(b - 1)$

141. $(z + 5)(z - 9)$ **143.** $(z - 5)(z - 9)$ **145.** $(b + 4)(b + 5)$ **147.** Nonfactorable **149.** $(x - 7)(x - 8)$

151. $(y + 3)(2y + 1)$ **153.** $(a - 1)(3a - 1)$ **155.** $(x - 3)(2x + 1)$ **157.** $(2t + 1)(5t + 3)$ **159.** $(2z - 1)(5z + 4)$

161. Nonfactorable **163.** $(t + 2)(2t - 5)$ **165.** $(3y + 1)(4y + 5)$ **167.** $(a - 5)(11a + 1)$ **169.** $(2b - 3)(3b - 2)$

171. $3(x + 2)(x + 3)$ **173.** $a(b + 8)(b - 1)$ **175.** $2y^2(y + 3)(y - 16)$ **177.** $x(x - 5)(2x - 1)$ **179.** $5(t + 2)(2t - 5)$

181. $p(2p + 1)(3p + 1)$

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Glossary

abscissa The first number of an ordered pair; it measures a horizontal distance and is also called the first coordinate of an ordered pair. (Sec. 4.1)

absolute value of a number The distance of the number from zero on the number line. (Sec. 1.1, 2.5)

absolute-value equation An equation containing the absolute-value symbol. (Sec. 2.5)

acute angle An angle whose measure is between 0° and 90°. (Sec. 3.1)

acute triangle A triangle that has three acute angles. (Sec. 3.2)

addition method An algebraic method of finding an exact solution of a system of linear equations. (Sec. 5.2)

additive inverses Numbers that are the same distance from zero on the number line but lie on different sides of zero; also called opposites. (Sec. 1.1, 1.4)

adjacent angles Two angles that share a common side. (Sec. 3.1)

alternate exterior angles Two nonadjacent angles that are on opposite sides of the transversal and outside the parallel lines. (Sec. 3.1)

alternate interior angles Two nonadjacent angles that are on opposite sides of the transversal and between the parallel lines. (Sec. 3.1)

analytic geometry Geometry in which a coordinate system is used to study relationships between variables. (Sec. 4.1)

angle An angle is formed when two rays start at the same point; it is measured in degrees. (Sec. 3.1)

antilogarithm If $\log_b M = N$, then the antilogarithm, base b, of N is M. (Sec. 12.2)

area A measure of the amount of surface in a region. (Sec. 3.2)

asymptotes The two straight lines that a hyperbola "approaches." (Sec. 11.6)

axes The two number lines that form a rectangular coordinate system; also called coordinate axes. (Sec. 4.1)

axis of symmetry of a parabola A line of symmetry that passes through the vertex of the parabola and is parallel to the y-axis for an equation of the form $y = ax^2 + bx + c$ or parallel to the x-axis for an equation of the form $x = ay^2 + by + c$. (Sec. 11.2, 11.6)

base In an exponential expression, the number that is taken as a factor as many times as indicated by the exponent. (Sec. 1.2)

basic percent equation Percent times base equals amount. (Sec. 2.1)

binomial A polynomial of two terms. (Sec. 6.2)

center of a circle The central point that is equidistant from all the points that make up a circle. (Sec. 11.6)

center of an ellipse The intersection of the two axes of symmetry of the ellipse. (Sec. 11.6)

characteristic The integer part of a common logarithm. (Sec. 12.2)

circle A plane figure in which all points are the same distance from point O, which is called the center of the circle. In a rectangular coordinate system, the set of all points (x, y) in the plane that are a fixed distance from a given point (h, k) called the center. (Sec. 3.2, 11.6)

circumference The distance around a circle. (Sec. 3.2)

clearing denominators Removing denominators from an equation that contains fractions by multiplying each side of the equation by the LCM of the denominators. (Sec. 8.4)

coefficient The number part of a variable term. (Sec. 1.4)

cofactor of an element of a matrix $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is its column number. (Sec. 5.3)

combined variation A variation in which two or more types of variation occur at the same time. (Sec. 8.7)

combining like terms Using the Distributive Property to add the coefficients of like variable terms; adding like terms of a variable expression. (Sec. 1.4)

common logarithms Logarithms to the base 10. (Sec. 12.2)

complementary angles Two angles whose sum is 90°. (Sec. 3.1)

completing the square Adding to a binomial the constant term that makes it a perfect-square trinomial. (Sec. 10.2)

complex fraction A fraction whose numerator or denominator contains one or more fractions. (Sec. 8.3)

complex number A number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. (Sec. 9.3)

composition of functions The operation on two functions f and g denoted by $f \circ g$. The value of the composition of f and g is given by $(f \circ g)(x) = f[g(x)]$. (Sec. 11.4)

compound inequality Two inequalities joined with a connective word such as "and" or "or." (Sec. 2.4)

compound interest Interest that is computed not only on the original principal but also on the interest already earned. (Sec. 12.5)

conic section A curve that can be constructed from the intersection of a plane and a right circular cone. The four conic sections are the parabola, hyperbola, ellipse, and circle. (Sec. 11.6)

conjugates Binomial expressions that differ only in the sign of a term. The expressions $a + b$ and $a - b$ are conjugates. (Sec. 9.2)

consecutive even integers Even integers that follow one another in order. (Sec. 7.5)

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consecutive integers Integers that follow one another in order. (Sec. 7.5)

consecutive odd integers Odd integers that follow one another in order. (Sec. 7.5)

constant of proportionality k in a variation equation; also called the constant of variation. (Sec. 8.7)

constant of variation k in a variation equation; also called the constant of proportionality. (Sec. 8.7)

constant term A term that includes no variable part; also called a constant. (Sec. 1.4)

coordinate axes The two number lines that form a rectangular coordinate system; also called axes. (Sec. 4.1)

coordinates of a point The numbers in the ordered pair that is associated with the point. (Sec. 4.1)

corresponding angles Two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. (Sec. 3.1)

cube A rectangular solid in which all six faces are squares. (Sec. 3.3)

cube root of a perfect cube One of the three equal factors of the perfect cube. (Sec. 7.4)

cubic function A third-degree polynomial function. (Sec. 6.2)

decimal notation Notation in which a number consists of a whole-number part, a decimal point, and a decimal part. (Sec. 1.2)

degree Unit used to measure angles; one complete revolution is 360°. (Sec. 3.1)

degree of a monomial The sum of the exponents of the variables. (Sec. 6.1)

degree of a polynomial The greatest of the degrees of any of its terms. (Sec. 6.2)

dependent system of equations A system of equations whose graphs coincide. (Sec. 5.1)

dependent variable In a function, the variable whose value depends on the value of another variable known as the independent variable. (Sec. 4.2)

descending order The terms of a polynomial in one variable are arranged in descending order when the exponents of the variable decrease from left to right. (Sec. 6.2)

determinant A number associated with a square matrix. (Sec. 5.3)

diameter of a circle A line segment with endpoints on the circle and going through the center. (Sec. 3.2)

diameter of a sphere A line segment with endpoints on the sphere and going through the center. (Sec. 3.3)

direct variation A special function that can be expressed as the equation $y = kx$, where k is a constant called the constant of variation or the constant of proportionality. (Sec. 8.7)

discriminant For an equation of the form $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the discriminant. (Sec. 10.3)

domain The set of the first coordinates of all the ordered pairs of a relation. (Sec. 4.1)

double root When a quadratic equation has two solutions that are the same number, the solution is called a double root of the equation. (Sec. 10.1)

element of a matrix A number in a matrix. (Sec. 5.3)

elements of a set The objects in the set. (Sec. 1.5)

ellipse An oval shape that is one of the conic sections. (Sec. 11.6)

empty set The set that contains no elements; also called the null set. (Sec. 1.5)

equation A statement of the equality of two mathematical expressions. (Sec. 2.1)

equilateral triangle A triangle that has three sides of equal length; the three angles are also of equal measure. (Sec. 3.2)

equivalent equations Equations that have the same solution. (Sec. 2.1)

evaluating a function Replacing x in $f(x)$ with some value and then simplifying the numerical expression that results. (Sec. 4.1)

evaluating a variable expression Replacing each variable by its value and then simplifying the resulting numerical expression. (Sec. 1.4)

even integer An integer that is divisible by 2. (Sec. 7.5)

expanding by cofactors A technique for finding the value of a 3×3 or larger determinant. (Sec. 5.3)

exponent In an exponential expression, the raised number that indicates how many times the factor, or base, occurs in the multiplication. (Sec. 1.2)

exponential equation An equation in which the variable occurs in the exponent. (Sec. 12.4)

exponential form The expression 2^6 is in exponential form. Compare *factored form.* (Sec. 1.2)

exponential function The exponential function with base b is defined by $f(x) = b^x$, where b is a positive real number not equal to one. (Sec. 12.1)

exterior angle An angle adjacent to an interior angle of a triangle. (Sec. 3.1)

extraneous solution When each side of an equation is raised to an even power, the resulting equation may have a solution that is not a solution of the original equation. Such a solution is called an extraneous solution. (Sec. 9.4)

factor In multiplication, a number being multiplied. (Sec. 1.1)

factored form The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in factored form. Compare *exponential form.* (Sec. 1.2)

factoring a polynomial Writing the polynomial as a product of other polynomials. (Sec. 7.1)

factoring a trinomial Expressing the trinomial as the product of two binomials. (Sec. 7.2)

first-degree equation An equation in which all variables have an exponent of 1. (Sec. 4.3)

FOIL A method of finding the product of two binomials. The letters stand for First, Outer, Inner, and Last. (Sec. 6.3)

formula A literal equation that states rules about measurement. (Sec. 8.5)

function A relation in which no two ordered pairs that have the same first coordinate have different second coordinates. (Sec. 4.2)

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functional notation A function designated by $f(x)$, which is the value of the function at x. (Sec. 4.2)

geometric solid A figure in space. (Sec. 3.3)

graph of a real number A heavy dot placed directly above the number on the number line. (Sec. 1.1)

graph of an equation in two variables A graph of the ordered-pair solutions of the equation. (Sec. 4.3)

graph of an ordered pair The dot drawn at the coordinates of the point in the plane. (Sec. 4.1)

graphing a point in the plane Placing a dot at the location given by the ordered pair; also called plotting a point in the plane. (Sec. 4.1)

greater than A number that lies to the right of another number on the number line is said to be greater than that number. (Sec. 1.1)

greatest common factor The greatest common factor (GCF) of two or more integers is the greatest integer that is a factor of all the integers. The greatest common factor of two or more monomials is the product of the GCF of the coefficients and the common variable factors. (Sec. 7.1)

half-plane The solution set of a linear inequality in two variables. (Sec. 4.7)

hyperbola A conic section formed by the intersection of a cone and a plane perpendicular to the base of the cone. (Sec. 11.6)

hypotenuse In a right triangle, the side opposite the 90° angle. (Sec. 9.4)

imaginary number A number of the form ai, where a is a real number and $i = \sqrt{-1}$. (Sec. 9.3)

imaginary part of a complex number For the complex number $a + bi$, b is the imaginary part. (Sec. 9.3)

inconsistent system of equations A system of equations that has no solution. (Sec. 5.1)

independent system of equations A system of equations whose graphs intersect at only one point. (Sec. 5.1)

independent variable In a function, the variable that varies independently and whose value determines the value of the dependent variable. (Sec. 4.2)

index In the expression $\sqrt[n]{a}$, n is the index of the radical. (Sec. 9.1)

inequality An expression that contains the symbol $>$, $<$, $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to). (Sec. 1.5)

integers The numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$. (Sec. 1.1)

interior angles The angles within the region enclosed by a triangle. (Sec. 3.1)

intersecting lines Lines that cross at a point in the plane. (Sec. 3.1)

intersection of two sets The set that contains all elements that are common to both of the sets. (Sec. 1.5)

inverse of a function The set of ordered pairs formed by reversing the coordinates of each ordered pair of the function. (Sec. 11.5)

inverse variation A function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. (Sec. 8.7)

irrational number The decimal representation of an irrational number never terminates or repeats and can only be approximated. (Sec. 1.2, 9.2)

isosceles triangle A triangle that has two sides of equal length; the angles opposite the equal sides are of equal measure. (Sec. 3.2)

joint variation A variation in which a variable varies directly as the product of two or more variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. (Sec. 8.7)

leading coefficient In a polynomial, the coefficient of the variable with the largest exponent. (Sec. 6.2)

least common denominator The smallest number that is a multiple of each denominator in question. (Sec. 1.2)

least common multiple (LCM) The LCM of two or more numbers is the smallest number that is a multiple of each of those numbers. (Sec. 1.2)

least common multiple of two polynomials The simplest polynomial of least degree that contains the factors of each polynomial. (Sec. 8.2)

leg In a right triangle, one of the two sides that are not opposite the 90° angle. (Sec. 9.4)

less than A number that lies to the left of another number on the number line is said to be less than that number. (Sec. 1.1)

like terms Terms of a variable expression that have the same variable part. Having no variable part, constant terms are like terms. (Sec. 1.4)

line A line extends indefinitely in two directions in a plane; it has no width. (Sec. 3.1)

linear equation in three variables An equation of the form $Ax + By + Cz = D$, where A, B, and C are coefficients of the variables and D is a constant. (Sec. 5.2)

linear equation in two variables An equation of the form $y = mx + b$, where m is the coefficient of x and b is a constant; also called a linear function. (Sec. 4.3)

linear function An equation of the form $y = mx + b$, where m is the coefficient of x and b is a constant; also called a linear equation in two variables. (Sec. 4.3, 6.2, 11.1)

linear inequality in two variables An inequality of the form $y > mx + b$ or $Ax + By > C$. (The symbol $>$ could be replaced by $\geq$, $<$, or $\leq$.) (Sec. 4.7)

line segment Part of a line; it has two endpoints. (Sec. 3.1)

literal equation An equation that contains more than one variable. (Sec. 8.5)

logarithm For b greater than zero and not equal to 1, the statement $y = \log_b x$ (the logarithm of x to the base b) is equivalent to $x = b^y$. (Sec. 12.2)

mantissa The decimal part of a common logarithm. (Sec. 12.2)

matrix A rectangular array of numbers. (Sec. 5.3)

minor of an element The minor of an element in a 3×3 determinant is the 2×2 determinant obtained

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by eliminating the row and column that contain that element. (Sec. 5.4)

monomial A number, a variable, or a product of a number and variables; a polynomial of one term. (Sec. 6.1, 6.2)

multiplicative inverse The multiplicative inverse of a nonzero real number a is $\frac{1}{a}$; also called the reciprocal. (Sec. 1.4)

natural exponential function The function defined by $f(x) = e^x$, where $e \approx 2.71828$. (Sec. 12.1)

natural logarithm When e (the base of the natural exponential function) is used as the base of a logarithm, the logarithm is referred to as the natural logarithm and is abbreviated $\ln x$. (Sec. 12.2)

natural numbers The numbers 1, 2, 3, . . . ; also called the positive integers. (Sec. 1.1)

negative integers The numbers . . . , −3, −2, −1. (Sec. 1.1)

negative slope The slope of a line that slants downward to the right. (Sec. 4.4)

nonfactorable over the integers A polynomial is nonfactorable over the integers if it does not factor using only integers. (Sec. 7.2)

n*th root of *a A number b such that $b^n = a$. The nth root of a can be written $a^{1/n}$ or $\sqrt[n]{a}$. (Sec. 9.1)

null set The set that contains no elements; also called the empty set. (Sec. 1.5)

numerical coefficient The number part of a variable term. When the numerical coefficient is 1 or −1, the 1 is usually not written. (Sec. 1.4)

obtuse angle An angle whose measure is between 90° and 180°. (Sec. 3.1)

obtuse triangle A triangle that has one obtuse angle. (Sec. 3.2)

odd integer An integer that is not divisible by 2. (Sec. 7.5)

one-to-one function In a one-to-one function, given any y, there is only one x that can be paired with the given y. (Sec. 11.5)

opposites Numbers that are the same distance from zero on the number line but lie on different sides of zero; also called additive inverses. (Sec. 1.1)

order $m \times n$ A matrix of m rows and n columns is of order $m \times n$. (Sec. 5.3)

Order of Operations Agreement A set of rules that tell us in what order to perform the operations that occur in a numerical expression. (Sec. 1.3)

ordered pair A pair of numbers expressed in the form (a, b) and used to locate a point in the plane determined by a rectangular coordinate system. (Sec. 4.1)

ordinate The second number of an ordered pair; it measures a vertical distance and is also called the second coordinate of an ordered pair. (Sec. 4.1)

origin The point of intersection of the two number lines that form a rectangular coordinate system. (Sec. 4.1)

parabola The graph of a quadratic function is called a parabola. (Sec. 11.2)

parallel lines Lines that never meet; the distance between them is always the same. In a rectangular coordinate system, parallel lines have the same slope and thus do not intersect. (Sec. 3.1, 4.6)

parallelogram A quadrilateral that has opposite sides equal and parallel. (Sec. 3.2)

percent Parts of 100. (Sec. 1.2)

perfect cube The product of the same three factors. (Sec. 7.4)

perfect square The product of a term and itself. (Sec. 1.2, 7.4)

perimeter The distance around a plane figure. (Sec. 3.2)

perpendicular lines Intersecting lines that form right angles. The slopes of perpendicular lines are negative reciprocals of each other. (Sec. 3.1, 4.6)

plane A flat surface. (Sec. 3.1)

plane figure A figure that lies totally in a plane. (Sec. 3.1)

plotting a point in the plane Placing a dot at the location given by the ordered pair; also called graphing a point in the plane. (Sec. 3.1)

point-slope formula The equation $y - y_1 = m(x - x_1)$, where m is the slope of a line and (x_1, y_1) is a point on the line. (Sec. 4.5)

polygon A closed figure determined by three or more line segments that lie in a plane. (Sec. 3.2)

polynomial A variable expression in which the terms are monomials. (Sec. 6.2)

positive integers The numbers 1, 2, 3, . . . ; also called the natural numbers. (Sec. 1.1)

positive slope The slope of a line that slants upward to the right. (Sec. 4.4)

prime polynomial A polynomial that is nonfactorable over the integers. (Sec. 7.2)

principal square root The positive square root of a number. (Sec. 1.2, 9.1)

product In multiplication, the result of multiplying two numbers. (Sec. 1.1)

proportion An equation that states the equality of two ratios or rates. (Sec. 8.4)

Pythagorean Theorem The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. (Sec. 9.4)

quadrant One of the four regions into which a rectangular coordinate system divides the plane. (Sec. 4.1)

quadratic equation An equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$; also called a second-degree equation. (Sec. 7.5, 10.1)

quadratic formula A general formula, derived by applying the method of completing the square to the standard form of a quadratic equation, used to solve quadratic equations. (Sec. 10.3)

quadratic function A function that can be expressed by the equation $f(x) = ax^2 + bx + c$, where a is not equal to zero. (Sec. 6.2, 11.2)

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quadratic inequality An inequality that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where a is not equal to zero. The symbols $\leq$ and $\geq$ can also be used. (Sec. 10.6)

quadratic trinomial A trinomial of the form $ax^2 + bx + c$, where a and b are nonzero coefficients and c is a nonzero constant. (Sec. 7.4)

quadrilateral A four-sided closed figure. (Sec. 3.2)

radical equation An equation that contains a variable expression in a radicand. (Sec. 9.4)

radical sign The symbol $\sqrt{\ }$, which is used to indicate the positive, or principal, square root of a number. (Sec. 1.2, 9.1)

radicand In a radical expression, the expression under the radical sign. (Sec. 1.2, 9.1)

radius of a circle A line segment going from the center to a point on the circle. The fixed distance, from the center of a circle, of all points that make up the circle. (Sec. 3.2, 11.6)

radius of a sphere A line segment going from the center to a point on the sphere. (Sec. 3.3)

range The set of the second coordinates of all the ordered pairs of a relation. (Sec. 4.2)

rate The quotient of two quantities that have different units. (Sec. 8.4)

rate of work That part of a task that is completed in one unit of time. (Sec. 8.6)

ratio The quotient of two quantities that have the same unit. (Sec. 8.4)

rational expression A fraction in which the numerator or denominator is a polynomial. (Sec. 8.1)

rational number A number of the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero. (Sec. 1.2)

rationalizing the denominator The procedure used to remove a radical from the denominator of a fraction. (Sec. 9.2)

ray A ray starts at a point and extends indefinitely in one direction. (Sec. 3.1)

real numbers The rational numbers and the irrational numbers taken together. (Sec. 1.2)

real part of a complex number For the complex number $a + bi$, a is the real part. (Sec. 9.3)

reciprocal The reciprocal of a nonzero real number a is $\frac{1}{a}$; also called the multiplicative inverse. (Sec. 1.4)

reciprocal of a rational expression The rational expression with the numerator and denominator interchanged. (Sec. 8.1)

rectangle A parallelogram that has four right angles. (Sec. 3.2)

rectangular coordinate system A coordinate system formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. (Sec. 4.1)

rectangular solid A solid in which all six faces are rectangles. (Sec. 3.3)

regular polygon A polygon in which each side has the same length and each angle has the same measure. (Sec. 3.2)

relation A set of ordered pairs. (Sec. 4.2)

repeating decimal A decimal formed when dividing the numerator of its fractional counterpart by the denominator results in a decimal part wherein one or more digits repeat infinitely. (Sec. 1.2)

right angle A 90° angle. (Sec. 3.1)

right triangle A triangle that contains one right angle. (Sec. 3.1)

roster method A method of designating a set by enclosing a list of its elements in braces. (Sec. 1.5)

scatter diagram A graph of collected data as points in a coordinate system. (Sec. 4.5)

scientific notation Notation in which a number is expressed as the product of a number between 1 and 10 and a power of 10. (Sec. 6.1)

second-degree equation An equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$; also called a quadratic equation. (Sec. 10.1)

set A collection of objects. (Sec. 1.5)

set-builder notation A method of designating a set that makes use of a variable and a certain property that only elements of that set possess. (Sec. 1.5)

similar objects Similar objects have the same shape but not necessarily the same size. (Sec. 8.4)

simplest form of a rational expression A rational expression is in simplest form when the numerator and denominator have no common factors. (Sec. 8.1)

slope A measure of the slant, or tilt, of a line. The symbol for slope is m. (Sec. 4.4)

slope-intercept form of a straight line The equation $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. (Sec. 4.4)

solution of a system of equations in three variables An ordered triple that is a solution of each equation of the system. (Sec. 5.2)

solution of a system of equations in two variables An ordered pair that is a solution of each equation of the system. (Sec. 5.1)

solution of an equation A number that, when substituted for the variable, results in a true equation. (Sec. 2.1)

solution of an equation in three variables An ordered triple (x, y, z) whose coordinates make the equation a true statement. (Sec. 5.2)

solution of an equation in two variables An ordered pair whose coordinates make the equation a true statement. (Sec. 4.1)

solution set of a system of inequalities The intersection of the solution sets of the individual inequalities. (Sec. 5.5)

solution set of an inequality A set of numbers, each element of which, when substituted for the variable, results in a true inequality. (Sec. 2.4)

solving an equation Finding a solution of the equation. (Sec. 2.1)

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sphere A solid in which all points are the same distance from point O, which is called the center of the sphere. (Sec. 3.3)

square A rectangle that has four equal sides. (Sec. 3.2)

square root A square root of a positive number x is a number a for which $a^2 = x$. (Sec. 1.2)

square matrix A matrix that has the same number of rows as columns. (Sec. 5.3)

square root of a perfect square One of the two equal factors of the perfect square. (Sec. 7.4)

standard form of a quadratic equation A quadratic equation is in standard form when the polynomial is in descending order and equal to zero. (Sec. 7.5, 10.1)

substitution method An algebraic method of finding an exact solution of a system of linear equations. (Sec. 5.1)

straight angle A 180° angle. (Sec. 3.1)

sum In addition, the total of two or more numbers. (Sec. 1.1)

supplementary angles Two angles whose sum is 180°. (Sec. 3.1)

synthetic division A shorter method of dividing a polynomial by a binomial of the form $x - a$. This method uses only the coefficients of the variable terms. (Sec. 6.4)

system of equations Two or more equations considered together. (Sec. 5.1)

system of inequalities Two or more inequalities considered together. (Sec. 5.5)

terminating decimal A decimal formed when dividing the numerator of its fractional counterpart by the denominator results in a remainder of zero. (Sec. 1.2)

terms of a variable expression The addends of the expression. (Sec. 1.4)

tolerance of a component The acceptable amount by which the component may vary from a given measurement. (Sec. 2.5)

transversal A line intersecting two other lines at two different points. (Sec. 3.1)

triangle A three-sided closed figure. (Sec. 3.1)

trinomial A polynomial of three terms. (Sec. 6.2)

undefined slope The slope of a vertical line is undefined. (Sec. 4.4)

uniform motion The motion of an object whose speed and direction do not change. (Sec. 2.3, 8.6)

union of two sets The set that contains all elements that belong to either of the sets. (Sec. 1.5)

value of a function The value of the dependent variable for a given value of the independent variable. (Sec. 4.2)

value mixture problem A problem that involves combining two ingredients that have different prices into a single blend. (Sec. 2.3)

variable A letter of the alphabet used to stand for a number that is unknown or that can change. (Sec. 1.1)

variable expression An expression that contains one or more variables. (Sec. 1.4)

variable part In a variable term, the variable or variables and their exponents. (Sec. 1.4)

variable term A term composed of a numerical coefficient and a variable part. When the numerical coefficient is 1 or -1, the 1 is usually not written. (Sec. 1.4)

vertex The common endpoint of two rays that form an angle. (Sec. 3.1)

vertex of a parabola The point on the parabola with the smallest y-coordinate or the largest y-coordinate. (Sec. 11.1)

vertical angles Two angles that are on opposite sides of the intersection of two lines. (Sec. 3.1)

volume A measure of the amount of space inside a closed surface. (Sec. 3.3)

***x*-coordinate** The abscissa in an xy-coordinate system. (Sec. 4.1)

***x*-intercept** The point at which a graph crosses the x-axis. (Sec. 4.3)

***y*-coordinate** The ordinate in an xy-coordinate system. (Sec. 4.1)

***y*-intercept** The point at which a graph crosses the y-axis. (Sec. 4.3)

zero slope The slope of a horizontal line. (Sec. 4.4)

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Table of Measurement Abbreviations

U.S. Customary System

Length		Capacity		Weight		Area		Rate	
in.	inches	oz	ounces	oz	ounces	in^2	square inches	ft/s	feet per second
ft	feet	c	cups	lb	pounds	ft^2	square feet	mph	miles per hour
yd	yards	qt	quarts			yd^2	square yards		
mi	miles	gal	gallons			mi^2	square miles		

Metric System

Length		Capacity		Weight		Area		Rate	
mm	millimeters	ml	milliliters	mg	milligrams	cm^2	square centimeters	m/s	meters per second
cm	centimeters	cl	centiliters	cg	centigrams	m^2	square meters	km/s	kilometers per second
m	meters	L	liters	g	grams	km^2	square kilometers	km/h	kilometers per hour
km	kilometers	kl	kiloliters	kg	kilograms				

Time

h hours min minutes s seconds

Table of Equations and Geometric Formulas

Perimeter

Triangle: $P = a + b + c$

Rectangle: $P = 2L + 2W$

Square: $P = 4s$

Circle: $C = \pi d$ or $C = 2\pi r$

Volume

Rectangular solid: $V = LWH$

Cube: $V = s^3$

Sphere: $V = \frac{4}{3}\pi r^3$

Right circular cylinder: $V = \pi r^2 h$

Right circular cone: $V = \frac{1}{3}\pi r^2 h$

Regular pyramid: $V = \frac{1}{3}s^2 h$

Triangles

$\angle A + \angle B + \angle C = 180°$

Slope-Intercept Form of a Straight Line

$y = mx + b$, where m = slope and $b = y$-intercept

Slope of a Line

$m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$

Area

Triangle: $A = \frac{1}{2}bh$

Rectangle: $A = LW$

Square: $A = s^2$

Circle: $A = \pi r^2$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Surface Area

Rectangular solid: $S = 2LW + 2LH + 2WH$

Cube: $S = 6s^2$

Sphere: $S = 4\pi r^2$

Right circular cylinder: $S = 2\pi r^2 + 2\pi rh$

Right circular cone: $S = \pi r^2 + \pi rl$

Regular pyramid: $S = s^2 + 2sl$

Pythagorean Theorem

$c^2 = a^2 + b^2$

Point-Slope Formula

$y - y_1 = m(x - x_1)$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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MATH STUDY SKILLS WORKBOOK

Your Guide to Reducing Test Anxiety and Improving Study Strategies

Paul D. Nolting, Ph.D.
Learning Specialist

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Preface

Many students have difficulty in mathematics; in fact, math is considered one of the most difficult subjects in college. Although other students may not find it as difficult, it still takes them many homework hours to understand it. Most students do not know that 25 percent of their math grade is based on affective characteristics, such as math study skills. However, most students are never taught math study skills. In any case, these students need to pass their math course to graduate.

What kind of assistance do most students want? Students want tips and procedures they can use to help them improve their math grades. The math study suggestions, however, have to be based on research and be statistically proven to improve student learning and grades. ***Math Study Skills Workbook*** is based on ***Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills*** (1997), which is the only math study skills text that can boast statistical evidence demonstrating an improvement in students' ability to learn math and make better grades. Learning and using these study skills techniques will improve your grades.

Math Study Skills Workbook is designed to supplement your math course or study skills course or to be used as part of a math anxiety workshop. The workbook is designed for independent work or to be supplemented by lectures. To gain the most benefit, the workbook needs to be completed by midterm.

This workbook is designed to enhance learning by teaching math learning skills in small "chunks." After each chapter section, you are required to recall the most important information by writing it down. The writing exercises are especially designed for you to personalize learning techniques, such as the ten steps to better test taking. Each chapter review is designed to reinforce your learning and to

give you the opportunity to immediately select and use the best learning strategy from that chapter. Lecture Notes pages have been added, where possible, to allow you to include your notes in this workbook.

> **Remember:** To get the most benefit, you must complete this workbook and be using these learning techniques by midterm.

Chapter 1

What You Need to Know to Study Math

Mathematics (math) courses are not like other courses. Because they are different, they require different study procedures. Passing most of your other courses requires only that you read, understand, and recall the subject material. To pass math, however, an extra step is required: You must use the information you have learned to correctly solve math problems.

Learning general study skills can help you pass most of your courses. But special math study skills are needed to help you learn more and get better grades in math. In this chapter, you will find out

- why learning math is different from learning other subjects,
- what the differences are between high school and college math, and
- why your first math test is very important.

Why learning math is different from learning other subjects

In a math course, you must be able to do three things:

1. *Understand* the material
2. *Process* the material
3. *Apply* what you have learned to correctly solve a problem.

Of these three tasks, applying what you have learned to correctly solve a problem is the hardest.

Examples: Political science courses require that you learn about politics and public service. But your instructor will not make you run for governor to pass the course. Psychology courses require you to understand the concepts of different psychology theories. But you will not have to help a patient overcome depression to pass the course. In math, however, you must be able to correctly solve problems to pass the course.

Sequential Learning Pattern

Another reason learning math is different from learning other subjects is that it follows a *sequential learning pattern*, which simply means that the material learned on one day is used the next day and the next day, and so forth. This building-block approach to learning math is the reason it is difficult to catch up when you fall behind. *All* building blocks must be included to be successful in learning math.

You can compare learning math to building a house. Like a house, which must be built foundation first, walls second, and roof last, math must be learned in a specific order. Just as you cannot build a house roof first, you cannot learn to solve complex problems without first learning to solve simple ones.

Example: In a history class, if you study Chapters 1 and 2, do not understand Chapter 3, and end up studying and having a test on Chapter 4, you *could* pass. Understanding Chapter 4 in history is not totally based on comprehending Chapters 1, 2, and 3. To succeed in math, however, each previous chapter has to be completely understood before you can continue to the next chapter.

Sequential learning affects your ability to study for math tests, as well. If you study Chapter 1 and understand it, study Chapter 2 and understand it, and study Chapter 3 and *do not* understand it, then when you study for a test on Chapter 4, you are not going to understand it either, and you probably will not do well on the test.

> **Remember:** To learn the new math material for the test on Chapter 5, you must first go back and learn the material in Chapter 4. This means you will have to go back and learn Chapter 4 while learning Chapter 5. (The best of us can fall behind under these circumstances.) However, if you do not understand the material in Chapter 4, you will not understand the material in Chapter 5 either, and you will fail the test on Chapter 5. This is why the sequential learning of math concepts if so important.

The sequential learning pattern is also affected by

- your previous math course grade,
- your math placement tests scores, and
- the time elapsed since your last math course.

Sequential learning is influenced by how much math knowledge you have at the beginning of your course. Students who forgot or never acquired the necessary skills from their previous math course will have difficulty with their current math course. If you do not remember what you learned in your last math course, you will have to relearn the math concepts from the previous course as well as the new material for the current course. In most other courses, such as the humanities, previous course knowledge is not required. However, in math you must remember what the last course taught you so that you are prepared for the current course. Measuring previous course knowledge will be explained in Chapter 2, "How to Discover Your Math-Learning Strengths and Weaknesses."

Sequential learning is also affected by your math placement test scores. If you barely scored high enough to be placed into a math course, then you will have math-learning gaps. Learning problems will occur when new math material is based on one of your learning gaps. The age of the placement test score also affects sequential learning. Placement test scores are designed to measure your *current* math knowledge and are to be used immediately.

Sequential learning is interrupted if math courses are taken irregularly. Math courses are designed to be taken one after another. By taking math courses each semester, without semester breaks in between courses, you are less likely to forget the concepts required for the next course.

Math as a Foreign Language

Another helpful technique for studying math is to consider it a foreign language. Looking at math as a foreign language can improve your study procedures. If you do not practice a foreign language, what happens? You forget it. If you do not practice math, what happens? You are likely to forget it, too. Students who excel in a foreign language must practice it *at least* every other day. The same study skills apply to math, because it is considered a foreign language.

Like a foreign language, math has unfamiliar vocabulary words or terms that must be put in sentences called *equations*. Understanding and solving a math equation is similar to speaking and understanding a sentence in a foreign language.

Example: Math sentences use symbols (which stand for spoken words), such as

= (for which you *say*, "equal"),
- (for which you *say*, "less"), and
a (for which you *say*, "unknown").

Learning *how* to speak math as a language is the key to math success. Currently, most universities consider computer and statistics (a form of math) courses as foreign languages. Some universities have now gone so far as to actually classify math as a foreign language.

Math as a Skill Subject

Math is a *skill subject*, which means you have to practice actively the skills involved to master it. Learning math is similar to learning to play a sport, learning to play a musical instrument, or learning auto mechanics skills. You can listen and watch your coach or instructor all day, but unless you *practice* those skills yourself, you will not learn.

Examples: In basketball, the way to improve your free throw is to *see and understand* the correct shooting form and then *practice* the shots yourself. Practicing the shots improves your free-throwing percentage. However, if you simply listen to your coach describe the correct form and see him demonstrate it, but you do not practice the correct form yourself, you will not increase your shooting percentage.

Suppose you want to play the piano, and you hired the best available piano instructor. You sit next to your instructor on the piano bench and watch the instructor demonstrate beginning piano-playing techniques. You see and understand how to place your hand on the keys and play. But what does it take to learn to play the piano? You have to place your hands on the keys and *practice*.

continued

Math works the same way. You can go to class, listen to your instructor, watch the instructor demonstrate skills, and understand everything that is said (and feel that you are quite capable of solving the problems). However, if you leave the class *and do not practice* – by working and successfully solving the problems – you will not learn math.

Many of your other courses can be learned by methods other than practicing. In social studies, for example, information can be learned by listening to your instructor, taking good notes, and participating in class discussions. Many students mistakenly believe that math can be learned the same way.

Remember: Math is different. If you want to learn math, *you must practice.* Practice not only means doing your homework but also means spending the time it takes to understand the reasons for each step in each problem.

A Bad Math "Attitude"

Students' attitudes about learning math are different from their attitudes about learning their other subjects. Many students who have had bad past experiences with math do not like math and have a bad attitude about learning it. In fact, some students actually *hate* math, even though these same students have positive experiences with their other subjects and look forward to going to class.

Society, as a whole, reinforces students' negative attitudes about math. It has become socially acceptable not to do well in math. This negative attitude has become evident even in popular comic strips, such as *Peanuts*. The underlying message is that math should be feared and hated and that it is all right not to learn math.

This "popular" attitude toward math may reinforce your belief that it is all right to fail math. Such a belief is constantly being reinforced by others.

The bad math attitude is not a major problem,

however. Many students who hate math pass it anyway, just as many students who hate history still pass it. The major problem concerning the bad math attitude is how you use this attitude. If a bad math attitude leads to poor class attendance, poor concentration, and poor study skills, then you have a bad math attitude *problem*.

Remember: Passing math is your goal, regardless of your attitude.



1. How does a sequential learning pattern affect math learning?

Give two examples:

Example 1:

Example 2:

2. List two examples of how learning math is similar to learning a foreign language.

Example 1:

Example 2:

3. How is math similar to a skills subject?

4. List three ways a bad math attitude could affect your mathematics learning.

Way 1:

Way 2:

Way 3:

The differences between high school and college math

Math, as a college-level course, is almost two to three times as difficult as high school-level math courses. There are many reasons for the increased difficulty: course class time allowance, the amount of material covered in a course, the length of a course, and the college grading system.

The first important difference between high school and college math courses is the length of time devoted to instruction each week. Most college math instruction, for the fall and spring semesters, has been cut to three hours per week; high school math instruction is provided five hours per week. Additionally, college courses cover twice the material in the same time frame as do high school courses. What is learned in one year of high school math is learned in one semester (four months) of college math.

Simply put, in college math courses you are receiving less instructional time per week and covering twice the ground per course as you were in high school math courses. The responsibility for learning in college is the student's. As a result, most of your learning (and *practicing*) will have to occur outside of the college classroom.

College Summer Semester Versus Fall or Spring Semester and the Difference Between Night and Day

College math courses taught during summer semesters are more difficult than those taught during fall or spring. Further, math taught during night courses is more difficult than math taught during day courses.

Students attending a six-week summer math session must learn the information – and master the skills – two and a half times as fast as students attending regular, full-semester math sessions. Though you receive the same amount of instructional classroom time, there is less time to understand and *practice the skills* between class sessions.

Summer classes are usually two hours per day, four days per week (nighttime summer classes are four hours per night, two nights per week).

> **Example:** If you do not understand the lecture on Monday, then you have only Monday night to learn the material before progressing to more difficult material on Tuesday. During a night course, you have to learn and understand the material before the break; after the break, you will move on to the more difficult material – *that night.*

Because math is a sequential learning experience, where every building block must be understood before proceeding to the next block, you can quickly fall behind, and you may never catch up. In fact, some students become lost during the first half of a math lecture and never understand the rest of the lecture (this can happen during just one session of night class). This is called "kamikaze" math because most students do not survive summer courses.

If you *must* take a summer math course, take a 10- or 12-week *daytime* session so that you will have more time to process the material between classes.

Course Grading System

The grading system for math is different in college than in high school.

> **Example:** While in high school, if you make a D or borderline D/F, the teacher more than likely will give you a D, and you may continue to the next course. However, in some college math courses, a D is not considered a passing grade, or if a D is made, the course will not count toward graduation.

College instructors are more likely to give the grade of N (no grade), W (withdrawal from class), or F for barely knowing the material. This is because the instructor knows that you will be unable to pass the next course if you barely know the current one.

Most colleges require students to pass two col-

lege-level algebra courses to graduate. In most high schools, you may graduate by passing one to three math courses. In some college-degree programs, you may even have to take four math courses and make a C in all of them to graduate.

The grading systems for math courses are very precise compared to the grading systems for humanities courses.

> **Example:** In a math course, if you have a 79 percent average and you need 80 percent to get a B, you will get a C in the course. But if you make a 79 percent in English class, you may be able to talk to your instructor and do extra credit work to earn a B.

Because math is an exact science and not as subjective as English, do not expect your math instructor to let you do extra work to earn a better grade. In college, there usually is not a grade given for "daily work," as is often offered in high school. In fact, *your test scores may be the only grades that will count toward your final grade.* Therefore, you should not assume that you will be able to "make up" for a bad test score.

The Ordering of College Math Courses

College math courses should be taken, *in order,* from the fall semester to the spring semester. If at all possible, avoid taking math courses from the spring to fall semesters. There is less time between the fall and spring semester for you to forget the information. During the summer break, you are more likely to forget important concepts required for the next course and therefore experience greater difficulty.



1. Compare the amount of college class time to high school class time.

_____

_____

_____

_____

2. How do college summer math courses differ from courses offered in the spring or fall?

_____

_____

_____

_____

_____

_____

3. How can the order of taking math courses affect your learning?

_____

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_____

Why your first math test is very important

Making a high grade on the first major math test is more important than making a high grade on the first major test in other subjects. The first major math test is the easiest and, most often, the one the student is least prepared for.

Students often feel that the first major math test is mainly a review and that they can make a B or C without much study. These students are overlooking an excellent opportunity to make an A on the easiest major math test of the semester. (Do not forget that this test counts the same as the more difficult remaining math tests.)

At the end of the semester, these students sometimes do not pass the math course or do not make an A because of their first major test grade. In other words, the first math test score was not high enough to "pull up" a low test score on one of the remaining major tests.

Studying hard for the first major math test and obtaining an A offers you several advantages:

- A high score on the first test can compensate for a low score on a more difficult fourth or fifth math test. All major tests have equal value in the final grade calculations.

- A high score on the first test can provide assurance that you have learned the basic math skills required to pass the course. This means you will not have to spend time relearning the misunderstood material covered on the first major test while learning new material for the next test.

- A high score on the first test can motivate you to do well. Improved motivation can cause you to increase your math study time, which will allow you to master the material.

- A high score on the first test can improve your confidence for higher test scores. With more confidence, you are more likely to work harder on the difficult math homework assignments, which will increase your chances of doing well in the course.



1. Why is your first math test supposed to be the easiest?

_____
_____
_____
_____
_____
_____

2. Give three reasons why your first math test is so important.

Reason 1: _____
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Reason 2: _____
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Reason 3: _____
_____
_____
_____
_____
_____
_____

Lecture Notes



Chapter 1 Review



1. In a math course, you must be able to do three things. You must _____ the material, _____ the material, and _____ what you have learned to solve the problem.

2. Math requires _____ _____, which means one concept builds on the next concept.

3. Placement test scores are designed to measure your _____ math knowledge and are to be used _____.

4. Like a foreign language, math has unfamiliar vocabulary words or terms that must be put in sentences called _____.

5. Learning how to speak math as a _____ _____ is one key to math success.

6. Keeping a _____ attitude about math will help you study more efficiently.

7. College math courses are _____ to _____ times as difficult as high school math courses.

8. Math course are even more difficult than other courses because a grade of _____ or better is usually required to take the next course.

9. The math grading is exact; in many cases, you cannot do _____ _____ to improve your grade.

10. College courses should be taken _____ _____, from the fall semester to the spring semester.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
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_____

How can you immediately use it?

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Chapter 2

How to Discover Your Math-Learning Strengths and Weaknesses

Math-learning strengths and weaknesses affect students' grades. You need to understand your strengths and weaknesses to improve your math-learning skills.

Just as a mechanic does a diagnostic test on a car before repairing it, you need to do your own testing to learn what you need to improve upon. You do not want the mechanic to charge you for something that does not need repairing, nor do you want to work on learning areas that do not need improvement. You want to identify learning areas you need to improve or better understand.

In this chapter, you will learn

- how what you know about math affects your grades,
- how quality of math instruction affects your grades,
- how affective student characteristics affect your math grades,
- how to determine your learning style,
- how to develop a math-learning profile of your strengths and weaknesses, and
- how to improve your math knowledge.

Areas of math strengths and weaknesses include math knowledge, level of test anxiety, study skills, study attitudes, motivation, and test-taking skills. Before we start identifying your math strengths and weaknesses, you need to understand what contributes to math academic success.

Dr. Benjamin Bloom, a famous researcher in the field of educational learning, discovered that your IQ (intelligence) and your cognitive entry skills account for 50 percent of your course grade (see Figure 1 on the next page). Quality of instruction represents 25 percent of your course grade, while affective student characteristics reflect the remaining 25 percent of your grade.

- *Intelligence* may be considered, for our purpose, how fast you can learn or relearn math concepts.

- *Cognitive entry skills* refer to how much math you knew before entering your current math course.

- *Quality of instruction* is concerned with the effectiveness of math instructors when presenting material to students in the classroom and math lab. This effectiveness depends on the course textbook, curriculum, teaching style, extra teaching aids (videos, audiocassettes), and other assistance.

- *Affective student characteristics* are characteristics you possess that affect your course grades – excluding how much math you knew before entering your math course. Some of these affective characteristics are anxiety, study skills, study attitudes, self-concept, motivation, and test-taking skills.

How what you know about math affects your grades

Poor math knowledge can cause low grades. A student placed in a math course that requires a more extensive math background than the student possesses will probably fail that course. Without the correct math background, you will fall behind and never catch up.

Placement Tests and Previous Course Grades

The math you need to know to enroll in a particular math course can be measured by a placement test (ACT, SAT) or by the grade received in the prerequisite math course. Some students are incorrectly placed in math courses by placement tests.

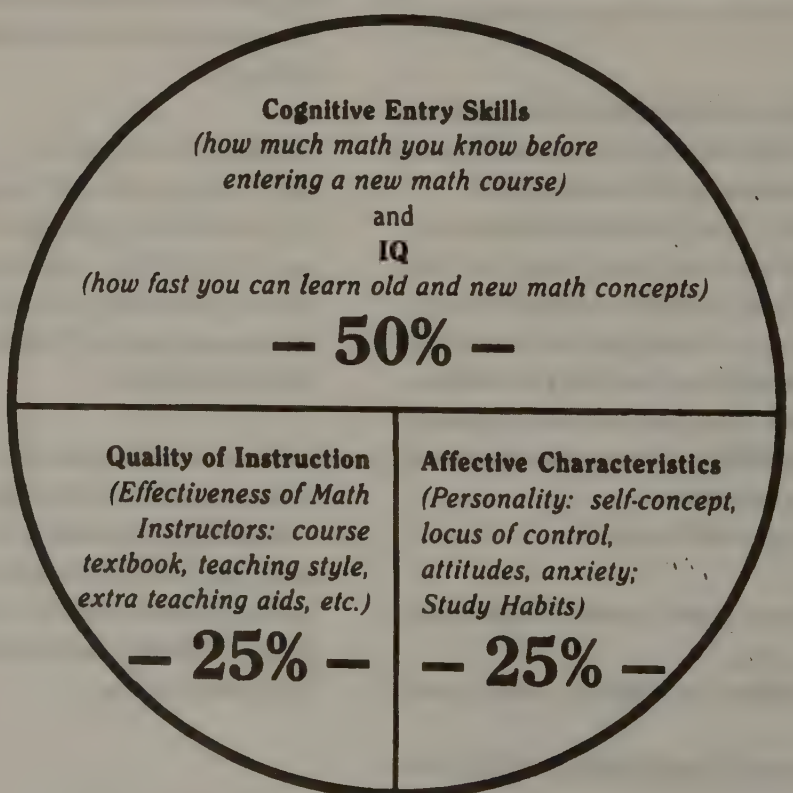
If, by the second class meeting, everything looks like Greek and you do not understand what is being explained, move to a lower-level course. In the lower-level math course you will have a better chance to understand the material and to pass the course. A false evaluation of math ability and knowledge can only lead to frustration, anxiety, and failure.

Requests by Students for Higher Placement

Some students try to encourage their instructors to move them to a higher-level math course because they believe they have received an inaccurate placement score. Many students try to avoid non-credit math courses, while other students do not want to repeat a course that they have previously failed. These moves can also lead to failure.

Some older students imagine that their math skills are just as good as when they completed their last math course, which was five to ten years ago. If they have not been practicing their math skills, they

Figure 1 – Variables Contributing to Student Academic Achievement



are just fooling themselves. Still other students believe they do not need the math skills obtained in a prerequisite math course to pass the next course. This is also incorrect thinking. Research indicates that students who were placed correctly in their prerequisite math course, and who subsequently failed it, will not pass the next math course.

What My Research Shows

I have conducted research on thousands of students who have either convinced their instructors to place them in higher-level math courses or have placed themselves in higher-level math courses. The results? These students failed their math courses many times before realizing they did not possess the prerequisite math knowledge needed to pass the course. Students who, without good reason, talk their instructors into moving them up a course level are setting themselves up to fail.

To be successful in a math course, you must have the appropriate math knowledge. If you think you may have difficulty passing a higher-level math course, you probably do not have an adequate math background. Even if you do pass the math course with a D or C, research indicates that you will most likely fail the next higher math course.

It is better to be conservative and pass a lower-level math course with an A or B instead of making a C or D in a higher-level math course and failing the next course at a higher level.

This is evident when many students repeat a higher-level math course up to five times before repeating the lower-level math course that was barely passed. After repeating the lower-level math course with an A or B, these students passed their higher-level math course.

How quality of math instruction affects your grades

Quality of instruction accounts for 25 percent of your grade. Quality of instruction includes such things as classroom atmosphere, instructor's teaching style, lab instruction, and textbook content and format. All of these "quality" factors can affect your ability to learn in the math classroom.

Interestingly enough, probably the most important "quality" variable is the compatibility of an instructor's teaching style with your learning style. You need to discover your learning style and compare it to the instructional style. Noncompatibility can be best solved by finding an instructor who better matches your learning style. However, if you cannot find an instructor to match your learning style, improving your math study skills and using the math lab or learning resource center (LRC) can compensate for most of the mismatch.

Use of the math lab or learning resource center (LRC) can dramatically improve the quality of instruction. With today's new technologies, students are able to select their best learning aids. These learning aids could be videotapes, CD-ROMs, computer programs, math study skills computer programs, and math texts.

The quality of tutors is also a major factor in the math lab or learning resource center. A low student-to-tutor ratio and trained tutors are essential for good tutorial instruction. Otherwise, the result is just a math study hall with a few helpers.

The math textbook should be up to date with good examples and a solutions manual. This increases the amount of learning from the textbook compared to older, poorly designed texts.

The curriculum design affects the sequence of math courses, which could cause learning problems. Some math courses have gaps between them that cause learning problems for all students.

How affective student characteristics affect your math grades

Affective student characteristics account for about 25 percent of your grade. These affective characteristics include math study skills, test anxiety, motivation, locus of control, learning style, and other variables that affect your personal ability to learn math.

Most students do not have this 25 percent of the grade in their favor. In fact, most students have never been taught *any* study skills, let alone how to study math, specifically. Students also do not know their best learning style, which means they may study ineffectively by using their least effective learning style. However, it is not your fault that you were not taught math study skills or made aware of your learning style.

By working to improve your affective characteristics, you will reap the benefits by learning math more effectively and receiving higher grades. Thousands of students have improved their math affective characteristics and thereby their grades by using this workbook.



1. List and explain the three variables that contribute to your academic success.

   Variable 1: _____

   Variable 2: _____

   Variable 3: _____

2. How does your placement test or previous math course grade affect your math learning?

3. How does the quality of math instructors affect your grade?

4. How do your affective characteristics affect your grade?

How to determine your learning style

Research has shown that matching a student's best learning style with the instructional style improves learning. Research has also shown that students who understand their learning style can improve their learning effectiveness. A learning disadvantage will occur for students who do not know or do not understand their learning style. Students should talk to their instructor or counselor about taking one or more learning style inventories.

Taking Stock of Your Learning Style

There are different types of learning styles assessments. One type of learning style assessment focuses on learning modalities, whereas other assessments focus on cognitive or environmental learning styles. We will focus on learning modalities and cognitive learning styles.

Learning modalities
(using your senses)

If it is available to you, take a learning style inventory that measures *learning modalities*. Learning modalities focus on the best way your brain receives information – that is, learning *visually* (seeing), *auditorially* (hearing), or *kinesthetically* (touching).

If possible, take an inventory that offers math-learning modalities, such as *Learning Styles Inventory* (Brown and Cooper, 1978). Other learning style inventories can measure learning modalities, but they focus on English- or reading-learning modes. Sometimes students' math-learning modes may be different from their language-learning modes.

If you cannot locate a modality-type inventory, then you may want to do a self-assessment on your math-learning modality. Do you learn math best by seeing it, by hearing it, or by having hands-on learning experiences? Hands-on learning means you need to touch or feel things to best learn about them. This is not the most accurate way to determine your learning modality; however, you may use this informal assessment until you have an opportunity to take a more formal modality learning style inventory.

Cognitive learning styles
(processing what you sense)

The second type of learning style inventory focuses on *cognitive learning styles*. Cognitive learning styles are based on how you process math information once it is received though a learning modality. In other words, once you have heard, seen, or felt the information, how does your brain process it? Based on McCarthy (1981), you can process the information four different ways: innovatively, analytically, through common sense, or dynamically. Results of either the *Kolb Learning Styles Inventory* (1985) or the *McCarthy Inventory* (1981) will determine your cognitive learning style.

If you do not have the opportunity to take a cognitive learning style inventory, you may want to do a self-assessment of your cognitive learning style. Which of the following best describes you?

Innovative learners solve problems or make decisions by personally relating to them and using their feelings. They learn best by listening and discussing their ideas with others while searching for the meaning. They are students with the good ideas, and they are innovative. They usually major in social sciences and the psychological helping professions.

Analytic learners look for facts and ask experts for advice when solving problems. They learn by thinking about the concepts and solving problems by using logic. They have more interest in ideas and concepts than in people. They generally major in math and the hard sciences; they enjoy doing research.

Common sense learners solve problems by knowing how things work. They learn by testing concepts in a practical way and make decisions by applying theories. They want to know practical ways to use theories and concepts. They major in applied sciences, health fields, engineering, and technologies.

Dynamic learners solve problems by looking at hidden possibilities and processing information concretely. They learn best by independent self-instruction and trial-and-error practice. They are flexible and will take risks. They major in business and sales occupations.

Learning modalities and cognitive learning styles are neither good nor bad. Learning modalities and cognitive styles are concerned with how you best take in information and process it. Different subject areas are best understood through certain learning modalities and cognitive styles.

Most math instructors are visual/abstract learners, and they tend to teach the way they best learn. Visual/abstract learners teach through a visual mode, and they have students process information abstractly. The second most frequent learning modality/style of math instructors is visual/common sense.

> **Remember:** It is important to recognize that students who do not have a math-learning modality/style can still learn math. A mismatch of modality/learning styles between student and instructor can be compensated by good study skills.

– Section Review –

1. My learning modality is _____ because I learn math best by _____
   _____
   _____
   _____

2. My cognitive learning style is _____ because I process math information by _____
   _____
   _____
   _____
   _____

How to develop a math-learning profile of your strengths and weaknesses

Students have learning strengths and weaknesses that affect math learning and their course grade. They must identify and understand these strengths and weaknesses to know which strengths support them and which weaknesses to improve upon.

> **Examples:** Having good math study skills is a *positive* math-learning characteristic, whereas having high test anxiety is a *negative* math-learning characteristic.

Understanding Math-Learning Profiles

By developing a math-learning profile, you will be able to identify your learning strengths and weaknesses.

Figure 2, on the facing page, allows you to rate yourself as having excellent, good, or poor math-learning skills. It helps you measure your previous math study skills, test anxiety, math knowledge, learning styles, and math attitude. These measurements are based on both the results of certain surveys and your previous math background. Doing all or some of these personal assessments will provide you with a good learning profile, and they may predict your success in math.

Now complete question 1 of the Math-Learning Profile Sheet (Figure 2). Check excellent or good if you were taught math study skills or poor if you were not taught math study skills.

Measuring Your Math Anxiety Level

The second part of the assessment measures your math anxiety level. You need to determine if you have high, medium, or low test anxiety. If you never get anxious on a math test, then select low anxiety. If you sometimes get anxious on a math test, select medium. If you always get anxious on a math test, select high.

Figure 2
Math-Learning Profile Sheet

Student _____ Math Course _____ Math Instructor _____

1. **My math study skills are**

   Excellent _____ Good _____ Poor _____
   (5 points) (3 points) (1 point)

2. **My math test anxiety level is**

   High _____ Medium _____ Low _____
   (1 point) (3 points) (5 points)

3. **Math knowledge (choose either placement score or last course grade):**

   (Placement score)
   High _____ Middle _____ Low _____
   (5 points) (3 points) (1 point)

   OR

   (Last course and grade) _____
   A ____ B ____ C, D, F, W, N, I ____
   (5 points) (3 points) (1 point)

4. **Math attitude:**

   Good _____ Neutral _____ Negative _____
   (5 points) (3 points) (1 point)

5. **Years since last math course:**

   1 Year ____ 2-3 Years ____ 4 or more years ____
   (5 points) (3 points) (1 point)

6. **Previous study skills course or training:**

   Course ____ Study-skills training ____ None ____
   (5 points) (3 points) (1 point)

7. **Previous/current required reading course:**

   No required college-level reading course _____
   (5 points)
   First developmental course _____
   (3 points)
   Second developmental course _____
   (1 point)

8. **How much control do you have over your math learning?**

   A lot _____ Some _____ Very little _____
   (5 points) (3 points) (1 point)

9. **My learning modality is**

   Visual _____ Auditory _____ Kinesthetic _____
   (5 points) (3 points) (1 point)

10. **My cognitive learning style is**

    Analytic _____
    (5 points)
    Commonsense _____
    (3 points)
    Innovative/dynamic _____
    (1 point)

TOTAL POINTS: _____

SCORING

45+ points — an excellent math-learning profile

40-45 points — a good math-learning profile

40 or below — a poor math-learning profile

Complete question 2 of the Math-Learning Profile Sheet.

Measuring Previous Math Knowledge

The third part of the assessment measures your previous math knowledge. Your previous math knowledge is an excellent predictor of future math success. In fact, the best predictor of math success is your previous math grade (if taken within one year of your current course).

The measurement of your math knowledge is based on your previous math grade or your math placement test. If your college does not require a placement test, contact the chairperson of the math department to help you with your course placement. Selecting a math course yourself is like playing Russian roulette with five bullets in the chamber. Without help, you will probably commit mathematical suicide.

Visit your counselor or instructor to discuss your math placement test score. Ask your counselor or instructor how your placement score relates to the scores for the math course below and above your course.

Find out if your score barely placed you into your course, if your score was in the middle, or if your score almost put you into the next course. Mark one of these three outcomes in question 3. If your placement score barely put you into your current math course, you will have difficulty learning the math required in that course.

A barely passing placement score means that you are missing the math knowledge you require to be successful, and you have a poor chance for passing the course. A middle placement score indicates that you probably have most of the math knowledge required for the course; however, you still may have difficulty passing the course. A high placement score means you have most of the math knowledge needed to pass and may have a good chance of passing the course. (Still, if you have poor math study skills and high test anxiety, you may not pass the course.)

Students who have low placement test scores need to improve their math knowledge as soon as possible. Students with middle placement scores should also consider improving their math knowledge.

You need to improve your math skills within the first few weeks of the semester. If you wait too long, you will become hopelessly lost. Suggestions for improving your math knowledge are included in a later section in this chapter.

If you are taking your second math course, use your previous math course grade to determine your math knowledge. Your previous course grade is the best predictor of success in your next math course. (However, this is only true if your last math course was taken within the last year.)

Record the name and grade of your previous math course in question 3. Students who made an A or B in their previous math course have a good chance of passing their next math course. However, students who made a C in their previous math course have a poor chance of passing their next math course. Students who made below a C or withdrew from the previous math course have virtually no chance of passing their next math course.

Determining Your Math Attitude

Math attitude can play a major part in being successful in your course. Some students who have a poor attitude toward math may not attend class as often as they should or may procrastinate in doing their homework. Other students avoid math until they *have* to take it. Mark the type of math attitude you have in question 4 of the Math-Learning Profile Sheet.

The Amount of Time Since Your Last Math Course

Another indicator of math success can be the length of time since your last math course. The longer the time since your last math course, the less likely you will be to pass the current course. The exception to this rule is if you were practicing math while you were not taking math courses. Math is very

easily forgotten if not practiced. Indicate in question 5 of the Math-Learning Profile Sheet the number of years since your last math course.

Previous Study Skills Courses or Training

General study skills training that does not focus on math may help some students in time management, reading techniques, learning styles, and overcoming procrastination. If you had a course in study skills or some study skills training, mark that area in question 6 of the Math-Learning Profile Sheet. If you had no study skills training, indicate that on your profile sheet.

Previous/Current Required Reading Courses

College reading skills are also needed for success in math courses. This is especially true when you are trying to solve story or word problems.

At community colleges and some universities, there are three levels of reading courses. Usually, the first two levels are noncredit reading courses, and the third level is for college credit.

If you are enrolled in the lowest reading level, you will have difficulty reading and understanding the math text. Students who are enrolled in the second level may also have difficulty reading the text. Students who are not required to take a reading class or who are reading at grade level may still have some difficulty reading the math text.

> **Remember:** Math texts are not written like English or history texts. Even students with college-level reading skills may experience difficulty understanding their math text.

If you do not know your reading level, ask to take a reading test. If in doubt about your reading level, take a reading class. Indicate the type of reading skills you have in question 7 of the Math-Learning Profile Sheet.

Your Locus of Control

"Locus of control" is a concept that describes how much control you feel you have over both your life and your course grades. Some students feel they have a lot of control over their life and learning, while other students feel they have very little control over their life or grades. Indicate in question 8 of the Math-Learning Profile Sheet how much control you feel you have over your math learning and grades.

Your Best Learning Modality

Learning modalities are best described by your preference in how you receive lecture information. As discussed, some of these modalities match your math instructor and some do not. Most math instructors teach through a visual mode, with the distant second modality being auditory. Mark your best learning modality in question 9 of the Math-Learning Profile Sheet.

Your Cognitive Learning Style

Cognitive learning styles are best described as how information is processed once it is received by your brain. Most math instructors process information *analytically*, and they therefore teach math the same way. Using *common sense* is the second most popular way math instructors cognitively process information.

If you do not process information analytically or through common sense, then you may be at a learning disadvantage. However, improved math study skills can compensate for a mismatch in learning modes/styles. Mark your cognitive learning style in question 10 of the Math-Learning Profile Sheet.

Evaluating Your Findings

Your profile sheet is an informal way to mea-

sure your math strengths, weaknesses, and learning modalities/styles. Any area with 5 points is a strength, and an area with 1 point is a weakness. Review your Math-Learning Profile Sheet and locate your strengths, weaknesses, or mismatches.

Learning modalities/styles are counted the same way but are considered matches and mismatches instead of strengths and weaknesses. Total up your score to determine if you have an overall strong or weak math-learning profile.

Some of the weak areas can be compensated for, while other areas, such as learning styles, can be made more effective. Strategies for improving math knowledge and finding the best instructor will be discussed in the last part of this chapter. Other profile weaknesses will be improved by using the suggestions in this workbook.



1. Complete the Math-Learning Profile Sheet if it has not been completed.

2. See your instructor about improving your weaknesses.

How to improve your math knowledge

Instructors always operate on the premise that you finished your previous math course just last week; they do not wait for you to catch up on current material. It does not matter if your previous math course was a month ago or five years ago. Instructors expect you to know the previous course material – period.

Review Your Previous Math Course Material and Tests

There are several ways to improve your math knowledge. Review your previous math course material before attending your present math course. Look closely at your final exam to determine your weak areas. Work on your weak areas as soon as possible so they can become building blocks (instead of stumbling blocks) for your current course.

If it has been some time since your last math course, visit the math lab or learning resource center to locate review material. Ask the instructor if there are any computer programs that will assess your math skills to determine your strengths and weaknesses for your course. Review math videotapes on the math course below your level. Also review any computer software designed for the previous math course.

Another way to enhance learning is to review the previous math course text by taking all of the chapter review tests. If you score above 80 percent on one chapter review test, move on to the next chapter. A score below 80 percent means you need to work on that chapter before moving on to the next chapter. Get a tutor to help you with those chapters if you have trouble. Make sure you review all the chapters required in the previous course as soon as possible. If you wait more than two weeks to conclude this exercise, it may be too late to catch up (while learning new material at the same time).

Employ a Tutor

One last way to improve your cognitive entry skills is to employ a private tutor. If you have a history of not doing well in math courses, you may need to start tutorial sessions *the same week class begins.* This will give the tutor a better chance of helping you regain those old math skills.

You still need to work hard to relearn old math skills while continuing to learn the new material. If you wait four to five weeks to employ a tutor, it will probably be too late to catch up and do well or even pass the course.

Remember: Tutorial sessions work best when the sessions begin during the first two weeks of a math course.

Schedule Math Courses "Back to Back"

Another way to maintain appropriate math knowledge is to take your math courses "back to back." It is better to take math courses every semester – even if you do not like math – so that you can maintain sequential (linear) learning.

I have known students who have made B's or C's in a math class and who then waited six months to a year to take the next math course. Inevitably, many failed. These students did not complete any preparatory math work before the math course and were lost after the second chapter. This is similar to having one semester of Spanish, not speaking it for a year, then visiting Spain and not understanding what is being said.

The only exception to taking math courses "back to back" is taking a six-week "kamikaze" math course (an ultracondensed version of a regular course), which should be avoided.

If you are one of the unfortunate many who are currently failing a math course, you need to ask yourself, "Am I currently learning any math or just becoming more confused?" If you are learning some math, stay in the course. If you are getting more confused, withdraw from the course. Improve your math knowledge prior to enrolling in a math course during the next semester.

Example: You have withdrawn from a math course after midterm due to low grades. Instead of waiting until next semester, attend a math lab or seek a tutor and learn Chapters 1, 2, and 3 *to perfection*. Also use this time to improve your math study skills. You will enter the same math course next semester with excellent math knowledge and study skills. In fact, you can make an A on the first test and complete the course with a high grade. Does this sound farfetched? It may, but I know hundreds of students who have used this learning procedure and passed their math course instead of failing it again and again.

Finding Your Best Instructor

Finding an instructor who best matches your learning style can be a difficult task. Your learning style is important; your learning style is how you best acquire information.

Example: Auditory learners do better when hearing the information over and over again instead of carefully reading the information. If an auditory learner is taught by a visual-style instructor who prefers that students read materials on their own and who prefers working problems instead of describing them, the mismatch could cause the student to do worse than if the student were taught by an auditory instructor.

Most students are placed in their first math course by an academic advisor. Usually, academic advisors know who are the most popular and least popular math instructors. However, advisors can be reluctant to discuss teacher popularity. And unfortunately, students may want the counselor to devise a course schedule based on the student's time limits instead of teacher selection.

To learn who are the best math instructors, ask the academic advisor which math instructor's classes fill up first. This does not place the academic advisor in the position of making a value judgment; neither does it guarantee the best instructor. But it will increase the odds in your favor.

Another way to acquire a good math instructor is to ask your friends about their current and previous math instructors. However, if another student says that an instructor is excellent, make sure your learning style matches your friend's learning style. Ask you friend, "Exactly what makes the instructor so good?" Then compare the answer with how you learn best. If you have a different learning style than your friend, look for another instructor, or ask another friend whose learning style more closely matches your own.

To obtain the most from an instructor, discover your best learning style and match it to the instructor's teaching style. Most learning centers or student personnel offices will have counselors who

can measure and explain your learning style. Interview or observe the instructor while the instructor is teaching. This process is time consuming, but it is well worth the effort!

Once you have found your best instructor, *do not* change. Remain with the same instructor for every math class whenever possible.

Remember: The first step in becoming a better math student is knowing your learning strengths and weaknesses. Now you can focus on what you need to improve.

– Section Review –

1. Give four examples of how you can review math course materials.

Example 1: _____

Example 2: _____

Example 3: _____

Example 4: _____

2. What is the advantage of taking math courses back to back?

3. Give two examples of how to find your best math instructor.

Example 1: _____

Example 2: _____



Chapter 2 Review



1. _____ _____ _____ refers to how much math you knew before entering your current math course.

2. _____ _____ _____ are the learning skills you possess that affect your grades.

3. Students who talk their instructors into moving up a course level usually _____ that course.

4. The _____ of tutors is a major part of the math lab or learning resource center.

5. Affective student characteristics account for about _____ percent of your grade.

6. _____ learners look for facts and ask experts for their advice.

7. Hands-on learners are called _____ learners.

8. Developing your math-learning profile will help you identify your _____ and _____ to help you improve your learning.

9. Students having previous difficulty in mathematics should start tutorial sessions the _____ week class begins.

10. Once you find your best math instructor _____ _____ change instructors for your next math course.

What is the most important information you learned from this chapter?

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

How can you immediately use it?

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Chapter 3

How to Reduce Math Test Anxiety

Math test anxiety is a common problem for many college students, especially for students who are in developmental courses. Unfortunately, it is also common for students to experience test anxiety *only* in math and not in their other courses.

Mild test anxiety can be a motivational factor, but high test anxiety can cause major problems both in learning and in taking tests. For many students, reducing test anxiety is the key to becoming successful in math. Such students need to learn the causes of test anxiety and how to reduce the test anxiety that affects their learning and grades.

Several techniques have proved helpful in reducing both math anxiety and math test anxiety. However, reducing math test anxiety does not guarantee good math grades. It must be coupled with effective study skills and a desire to do well in math.

In this chapter, you will learn

- how to recognize test anxiety,
- the causes of test anxiety,
- the different types of test anxiety, and
- how to reduce test anxiety.

How to recognize test anxiety

Test anxiety has existed for as long as tests have been used to evaluate student performance. Because it is so common and because it has survived the test of time, test anxiety has been carefully studied over the last 50 years. Pioneering studies indicate that test anxiety generally leads to low test scores.

At the University of South Florida (Tampa), Dr. Charles Spielberger investigated the relationship between test anxiety and intellectual ability. The study results suggested that anxiety coupled with high ability can improve academic performance; but anxiety coupled with low or average ability can interfere with academic performance. That is:

Anxiety + high ability = improvement
Anxiety + low or average ability = no improvement

Example: Students with average ability and low test anxiety had better performance and higher grades than did students with average ability and high test anxiety. However, there are students who make good grades, take calculus, and still have test anxiety.

Test anxiety is a *learned* response; a person is not born with it. An environmental situation brings about test anxiety. The good news is that because it is a learned response, it can be *unlearned*. Test anxiety is a special kind of general stress. General stress is considered "strained exertion," which can lead to physical and psychological problems.

Defining Test Anxiety

There are several definitions of test anxiety. One definition states, "Test anxiety is a conditioned emotional habit to either a single terrifying experience, recurring experience of high anxiety, or a continuous condition of anxiety" (Wolpe, 1958).

Another definition of test anxiety relates to the educational system. The educational system develops evaluations that measure one's mental performance, and this creates test anxiety. This definition suggests that test anxiety is the *anticipation* of some realistic or nonrealistic situational threat (Cattell, 1966). The "test" can be a research paper, an oral report, work at the chalkboard, a multiple-choice exam, a written essay, or a math test.

Math test anxiety is a relatively new concept in education. *Ms.* magazine published "Math Anxiety: Why Is a Smart Girl Like You Counting on Your Fingers?" (Tobias, 1976) and coined the phrase *math anxiety*. During the 1970s, other educators began referring to *mathophobia* and *mathemaphobia* as a possible cause for children's unwillingness to learn math. Additional studies on the graduate level discovered that math anxiety was common among adults as well as children. Previously, educators had regarded math anxiety as a skill deficiency rather than a state of mind.

There are several other definitions of math anxiety:

> Math anxiety is the extreme reaction to a very negative attitude toward math. (There is a strong relationship between low math confidence and high math test anxiety; this could be the same concept.) (Fox, 1977)

> Math anxiety is the feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of math problems during school tests. (Richardson and Suinn, 1972)

> Math anxiety is a state of panic, helplessness, paralysis and mental disorganization that occurs in some students when required to solve math problems. This discomfort varies in intensity and is the outcome of numerous previous situations. (Tobias, 1986)

One of my students once described math test anxiety as "being in a burning house with no way out." No matter how you define it, math test anxiety is real, and it affects millions of students.

Why Math Tests Create Anxiety

Math anxiety can be divided into two separate anxieties: *math test* anxiety and *numerical* anxiety. Math test anxiety involves anticipation, completion, and feedback of math tests. Numerical anxiety refers to everyday situations requiring working with numbers.

It has been shown that math anxiety exists among many students who usually do not suffer from other tensions. Counselors at a major university reported that one-third of the students who enrolled in behavior therapy programs offered through counseling centers had problems with math anxiety (Suinn, 1988).

Educators know that math anxiety is common among college students and is more prevalent in women than in men. They also know that math anxiety frequently occurs in students with a poor high school math background. These students were found to have the greatest amount of anxiety.

Approximately half of the students in college prep math courses (designed for students with inadequate high school math background or low placement scores) could be considered to have math anxiety. However, math anxiety also occurs in students in high-level math courses, such as college algebra and calculus.

Educators investigating the relationship between anxiety and math have indicated that anxiety contributes to poor grades in math. They also found that simply *reducing* math test anxiety does not guarantee higher math grades. Students often have other problems that affect their math grades, such as poor study skills, poor test-taking skills, or poor class attendance.

– Section Review –

1. What is your personal definition of test anxiety?

_____
_____
_____
_____
_____
_____

2. What type of student does math anxiety affect most?

_____
_____
_____
_____
_____
_____

The causes of test anxiety

The causes of test anxiety can be different for each student, but they can be explained by seven basic concepts (described on the next page).

The most common situation students have reported as a known cause of their math test anxiety is their elementary school math experiences. When asked, many students indicated that they were made fun of when trying to solve math problems at the chalkboard. When they could not solve the problem, the teacher and/or students would call them "stupid."

Teacher and peer embarrassment and humiliation become the conditioning experience that causes some students' test anxiety. Over the years, this test anxiety is reinforced and still exists. In fact, many math-anxious students – now 30 and 40 years old – *still* have extreme fear about working math problems on the board. Some students said that they absolutely refuse to go to the board.

Math Autobiography

The math autobiography is a way to find out where some of your fears about math began. In the math autobiography, you write down some of your positive and negative math experiences. A better understanding of your math history can lead to more control over math. Use the answers to the following questions to help you write your own math autobiography.

My first positive math experience was when _____
_____
_____
_____.

My first negative math experience was when _____
_____
_____
_____.

One of my best math teachers helped me in math by
_____
_____
_____.

One math teacher didn't help me learn math because he/she _____
_____
_____
_____.

I did well in math until _____
_____
_____
_____
_____. OR

I am still doing well in math because _____
_____
_____
_____.

When I hear the word *math* I first think of _____
_____
_____
_____

Because _____
_____
_____
_____.

Math is only for people who _____
_____
_____
_____.

Doing math successfully means I can _____
_____
_____
_____.

For me, math is like _____
_____
_____
_____.

After answering these questions, you now have a better understanding of how math affects you. Write a three- to five-page math autobiography about your math history from elementary school to your current math course. Write down how you felt about math and how you feel about math now. Your conclusion should include what you want to do NOW about your math success. Turn the autobiography in to your instructor.

Remember: For some students, just writing about their previous math history helps them.

– Section Review –

1. List and describe five causes of test anxiety.

   Cause 1: _____

The Causes of Test Anxiety

Test anxiety can be a learned behavior resulting from the expectations of parents, teachers, or other significant people in the student's life.

Test anxiety can be caused by the association between grades and a student's personal worth.

Test anxiety develops from fear of alienating parents, family, or friends due to poor grades.

Test anxiety can stem from a feeling of lack of control and an inability to change one's life situation.

Test anxiety can be caused by a student's being embarrassed by the teacher or other students when trying to do math problems.

Test anxiety can be caused by timed tests and the fear of not finishing the test, even if one can do all the problems.

Test anxiety can be caused by being put in math courses above one's level of competence.

What are the cause(s) of *your* test anxiety?

Cause 2: _____

Cause 3: _____

Cause 4: _____

Cause 5: _____

2. Put into your own words your cause(s) of test anxiety. If you don't have test anxiety, list what you think is the major cause of test anxiety.

The different types of test anxiety

The two basic types of test anxiety are emotional (educators call this *somatic*) and worry (educators call this *cognitive*). Students with high test anxiety have *both* emotional and worry anxiety.

Signs of emotional anxiety are upset stomach, nausea, sweaty palms, pain in the neck, stiff shoulders, high blood pressure, rapid shallow breathing, rapid heartbeat, or general feelings of nervousness. As anxiety increases, these feelings intensify. Some students even run to the bathroom to throw up or have diarrhea.

Even though these *feelings* are caused by anxiety, the physical response is real. These feelings and physical inconveniences can affect your concentration and your testing speed, and they can cause you to completely "draw a blank."

Worry anxiety causes the student to think about failing the test. These negative thoughts can happen either before or during the test. This negative "self-talk" causes students to focus on their anxiety instead of recalling math concepts.

The effects of test anxiety range from a "mental block" on a test to avoiding homework. One of the most common side effects of test anxiety is getting the test and immediately forgetting information that you know. Some students describe this event as having a "mental block," "going blank," or finding that the test looks like Greek.

After five or ten minutes into the test, some of these students can refocus on the test and start working the problems. They have, however, lost valuable time. For other students, anxiety persists throughout the test and they cannot recall the needed math information. It is only after they walk out the door that they can remember how to work the problems.

Sometimes math anxiety does not cause students to "go blank" but slows down their mental processing speed. This means it takes longer to recall formulas and concepts and to work problems. The result is frustration and loss of time, leading to *more* anxiety. Since,

The 12 Myths About Test Anxiety

Students are born with test anxiety.

Test anxiety is a mental illness.

Test anxiety cannot be reduced.

Any level of test anxiety is bad.

All students who are not prepared have test anxiety.

Students with test anxiety cannot learn math.

Students who are well prepared will not have test anxiety.

Very intelligent students and students taking high-level courses, such as calculus, do not have test anxiety.

Attending class and doing homework should reduce all test anxiety.

Being told to relax during a test will make a student relaxed.

Doing nothing about test anxiety will make it go away.

Reducing test anxiety will guarantee better grades.

in most cases, math tests are *speed* tests (those in which you have a certain amount of time to complete the test), you may not have enough time to work all the problems or to check the answers if you have mentally slowed down. The result is a lower test score, because even though you know the material, you do not complete all of the questions before test time runs out.

Not using all of the time allotted for the test is another problem caused by test anxiety. Students know that they should use all of the test time to check their answers. In fact, math is one of the few subjects in which you can check test problems to find out if your work is correct. However, most students do not use all of the test time, and this results in lower test scores. Why does this happen?

Students with high test anxiety do not want to stay in the classroom. This is especially true of students whose test anxiety increases as the test progresses. The test anxiety gets so bad that they would rather leave early and receive a lower grade than stay in that "burning house."

Students have another reason for leaving the test early: the fear of what the instructor and other students will think about them for being the last one to hand in the test. These students refuse to be the last ones to finish the test because they think that the instructor or other students will think they are dumb. This is middle-school thinking, but the feelings are still real – no matter what the age of the student. These students do not realize that some students who turn in their tests first fail, whereas many students who turn in their tests last make A's and B's.

Another effect of test anxiety relates to completing homework assignments. Students who have high test anxiety may have difficulty starting or completing their math homework. Doing homework reminds some students of their learning problems in math. More specifically, it reminds them of their previous math failures, which causes further anxiety. This anxiety can lead to total homework avoidance or "approach-avoidance" behavior.

Total homework avoidance is called procrastination. The very thought of doing their homework

causes these students anxiety, which causes them to put off tackling their homework. This makes them feel better for a short amount of time – *until test day.*

Example: Some students begin their homework and work some problems successfully. They then get stuck on an problem that causes them anxiety, so they take a break. During their break the anxiety disappears until they start doing their homework again. Doing their homework causes more anxiety, which leads to another break. The breaks become more frequent. Finally, the student ends up taking one long break and not doing the homework. Quitting, to them, means *no more anxiety* until the next homework assignment.

The effects of math test anxiety can be different for each student. Students can have several of the mentioned characteristics that can interfere with math learning and test taking. However, there are certain myths about math that each student needs to know. Review The 12 Myths About Test Anxiety on the facing page to see which ones you believe. If you have test anxiety, which of the mentioned characteristics are true for you?



1. List and describe the two basic types of test anxiety.

   Type 1: _____

   _____

   _____

   _____

   Type 2: _____

   _____

   _____

   _____

2. List two reasons students leave the test room early instead of checking their answers.

   Reason One: _____

   _____

   _____

   _____

   Reason Two: _____

   _____

   _____

   _____

3. List 6 of the 12 myths about test anxiety that you most believed.

Myth 1: _____

_____

_____

Myth 2: _____

_____

_____

Myth 3: _____

_____

_____

Myth 4: _____

_____

_____

Myth 5: _____

_____

_____

Myth 6: _____

_____

_____

How to reduce test anxiety

To reduce math test anxiety, you need to understand both the relaxation response and how negative self-talk undermines your abilities.

Relaxation Techniques

The relaxation response is any technique or procedure that helps you to become relaxed. It will take the place of an anxiety response. Someone simply telling you to relax or even telling yourself to relax, however, does little to reduce your test anxiety. There are both short-term and long-term relaxation response techniques that help control emotional (somatic) math test anxiety. These techniques will also help reduce worry (cognitive) anxiety. Effective *short-term* techniques include the tensing and differential relaxation method, the palming method, and deep breathing.

Short-Term Relaxation Techniques

The Tensing and Differential Relaxation Method

The tensing and differential relaxation method helps you relax by tensing and relaxing your muscles all at once. Follow these procedures while you are sitting at your desk before taking a test:

1. Put your feet flat on the floor.
2. With your hands, grab underneath the chair.
3. Push down with your feet and pull up on your chair at the same time for about five seconds.
4. Relax for five to ten seconds.
5. Repeat the procedure two or three times.
6. Relax all your muscles except the ones that are actually used to take the test.

The Palming Method

The palming method is a visualization procedure used to reduce test anxiety. While you are at your desk before or during a test, follow these procedures:

1. Close and cover your eyes using the center of the palms of your hands.
2. Prevent your hands from touching your eyes by resting the lower parts of your palms on your cheekbones and placing your fingers on your forehead. Your eyeballs must not be touched, rubbed, or handled in any way.
3. Think of some real or imaginary relaxing scene. Mentally visualize this scene. Picture the scene as if you were actually there, looking through your own eyes.
4. Visualize this relaxing scene for one to two minutes.

Practice visualizing this scene several days before taking a test and the effectiveness of this relaxation procedure will improve.

Deep Breathing

Deep breathing is another short-term relaxation technique that can help you relax. Proper breathing is a way to reduce stress and decrease test anxiety. When breathing properly, enough oxygen gets into your bloodstream to nourish your body and mind. A lack of oxygen in your blood contributes to an anxiety state that makes it more difficult to react to stress. Proper deep breathing can help you control your test anxiety.

Deep breathing can replace the rapid, shallow breathing that sometimes accompanies test anxiety, or it can prevent test anxiety. Here are the steps to deep breathing:

1. Sit straight up in your chair in a good posture position.
2. Slowly inhale through your nose.
3. As you inhale, first fill the lower section of your lungs and work your way up to the upper part of your lungs.
4. Hold your breath for a few seconds.
5. Exhale slowly through your mouth.
6. Wait a few seconds and repeat the cycle.

You need to practice deep breathing exercises before using them during a test.

Side one of the audiocassette *How to Reduce Test Anxiety* (Nolting, 1986) further explains test anxiety and discusses these and other short-term relaxation response techniques. Short-term relaxation techniques can be learned quickly but are not as successful as the long-term relaxation technique. Short-term techniques are intended to be used while learning the long-term technique.

Long-Term Relaxation Techniques

The cue-controlled relaxation response technique is the best long-term relaxation technique. It is presented on side two of the audiocassette *How To Reduce Test Anxiety* (Nolting, 1986). Cue-controlled relaxation means you can induce your own relaxation based on repeating certain cue words to yourself. In essence, you are taught to relax and then silently repeat cue words, such as "I am relaxed."

After enough practice, you can relax during math tests. The cue-controlled relaxation technique has worked with thousands of students. For a better understanding of test anxiety and how to reduce it, listen to *How to Reduce Test Anxiety* (Nolting, 1986).

Negative Self-Talk

Negative self-talk is a form of worry (cognitive) anxiety. This type of worrying can interfere with your test preparation and can keep you from concentrating on the test. Worrying can motivate you to study, but too much worrying may prevent you from studying at all.

Negative self-talk is defined as the negative statements you tell yourself before and during tests. Negative self-talk causes students to lose confidence and to give up on tests. Further, it can give you an inappropriate excuse for failing math and cause you to give up on learning math.

Students need to change their negative self-talk to positive self-talk without making unrealistic statements.

Positive self-statements can improve your studying and test preparation. During tests, positive self-talk can build confidence and decrease your test anxiety. These positive statements (see examples on the next page) can help reduce your test anxiety and improve your grades. Some more examples of positive self-statements are on the cassette *How to Reduce Test Anxiety* (Nolting, 1986). Before the test, make up some positive statements to tell yourself.

Thought-Stopping Techniques

Some students have difficulty stopping their negative self-talk. These students cannot just tell themselves to eliminate those thoughts. These students need to use a thought-stopping technique to overcome their worry and become relaxed.

Thought stopping involves focusing on the unwanted thoughts and, after a few seconds, suddenly stopping those thoughts by emptying your mind. Using the command "Stop!" or a loud noise like clapping your hands can effectively interrupt the negative self-talk. In a homework situation, you may be able to use a loud noise to stop your thoughts, but don't use it during a test.

To stop your thoughts in the classroom or during a test, silently shout to yourself "Stop!" or "Stop

thinking about that." After your silent shout, either relax yourself or repeat one of your positive self-talk statements. You may have to shout to yourself several times during a test or while doing homework to control negative self-talk. After every shout, use a different relaxation technique/scene or positive self-talk statement.

Examples of Negative Self-Talk:

"No matter what I do, I will not pass this course."
"I failed this course last semester, and I will fail it again."
"I am no good at math, so why should I try?"
"I cannot do it; I cannot do the problems, and I am going to fail this test."
"I have forgotten how to do the problems, and I am going to fail."
"I am going to fail this test and never graduate."
"If I can't pass this test, I am too dumb to learn math and will flunk out."

Examples of Positive Self-Talk:

"I failed the course last semester, but I can now use my math study skills to pass this course."
"I went blank on the last test, but I now know how to reduce my test anxiety."
"I know that my poor math skills are due to poor study skills, not my own ability, and since I am working on my study skills, my math skills will improve."
"I know that with hard work, I will pass math."
"I prepared for this test and will do the best I can. I will reduce my test anxiety and use the best test-taking procedures. I expect some problems will be difficult, but I will not get discouraged."
"I am solving problems and feel good about myself. I am not going to worry about that difficult problem; I am going to work the problems that I can do. I am going to use all the test time and check for careless errors. Even if I do not get the grade I want on this test, it is not the end of the world."

Thought stopping works because it interrupts the worry response before it can cause high negative emotions. During that interruption, you can replace the negative self-talk with positive statements or relaxation. However, students with high worry anxiety should practice this technique three days to one week before taking a test. Contact your counselor if you have additional questions about the thought-stopping techniques.

– Section Review –

1. Describe your best short-term relaxation technique.

2. You can use the palming method by closing your eyes and visualizing a scene without putting your hands to your face. Describe a very relaxing scene that you could visualize. Make sure to include some sounds and visual images in your scene.

3. Practice your relaxation scene for three to five minutes for the next five days. List the times and dates you practiced your scene.

Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____

4. How does negative self-talk cause you to have text anxiety?

5. Make up three positive self-talk statements that are not listed in this text.

Statement 1: _____

Statement 2: _____

Statement 3: _____

6. Describe the thought-stopping technique in your own words.

7. What words will you use as your silent shout?

8. What will you do after your silent shout?

9. How does the thought-stopping technique work?

Lecture Notes



Chapter 3 Review



1. Reducing math test anxiety _____ _____ guarantee good math grades.

2. Test anxiety is a _____ response; a person is not _____ with it.

3. Math test anxiety involves _____, _____,and _____ of math tests.

4. One cause of test anxiety is that a student goes to the board to work a problem but is called _____ when he or she cannot work the problem.

5. The two basic types of test anxiety are _____and _____.

6. The effects of test anxiety range from a "_____ _____" on a test to _____ homework.

7. The _____and _____ _____ method helps you relax by tensing and relaxing your muscles all at once..

8. The palming method is a _____ procedure used to reduce text anxiety.

9. Negative self-talk is a form of _____anxiety.

10. Thought stopping works because it interrupts the _____ response before it can cause _____emotions.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
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Chapter 4

How to Improve Your Listening and Note-Taking Skills

Listening and note-taking skills in a math class are very important, since most students either do not read the math text or have difficulty understanding it. In most of your other classes, if you do not understand the lecture, you can read the text and get almost all the information. In math class, however, the instructor can usually explain the textbook better than the students can read and understand it.

Students who do not have good listening skills or note-taking skills will be at a disadvantage in learning math. Most math understanding takes place in the classroom. Students must learn how to take advantage of learning math in the classroom by becoming effective listeners, calculator users, and notetakers.

In this chapter you will learn

- how to become an effective listener,
- how to become a good notetaker,
- when to take notes,
- the seven steps to math note-taking, and
- how to rework your notes.

How to become an effective listener

Becoming an effective listener is the foundation for good note-taking. You can become an effective listener using a set of skills that you can learn and practice. To become an effective listener, you must prepare yourself both physically and mentally.

Sitting in the Golden Triangle

The physical preparation for becoming an effective listener involves *where you sit* in the classroom. Sit in the best area to obtain high grades, that is, in "the golden triangle of success." The golden triangle of success begins with seats in the front row facing the instructor's desk (see Figure 3 on the next page).

Students seated in this area (especially in the front row) directly face the teacher and are most likely to pay attention to the lecture. This is a great seating location for visual learners. There is also less tendency for them to be distracted by activities outside the classroom or by students making noise within the classroom.

The middle seat in the back row is another point in the golden triangle for students to sit, especially those who are auditory (hearing) learners. You can hear the instructor better because the instructor's voice is projected to that point. This means that there is less chance of misunderstanding the instructor, and you can hear well enough to ask appropriate questions.

By sitting in the golden triangle of success, you can force yourself to pay more attention during class and be less distracted by other students. This is

very important for math students because math instructors usually go over a point once and continue on to the next point. If you miss that point in the lesson, then you could be lost for the remainder of the class.

Warming Up for Math Class

The mental preparation for note-taking involves "warming up" before class begins and becoming an active listener. Just as an athlete must warm up before a game begins, you must warm up before taking notes. Warm up by

- reviewing the previous day's notes,
- reviewing the reading material,
- reviewing the homework,
- preparing questions, and
- working one or two unassigned homework problems.

This mental warm up before the lecture allows you to refresh your memory and prepare pertinent questions, making it easier to learn the new lecture material.

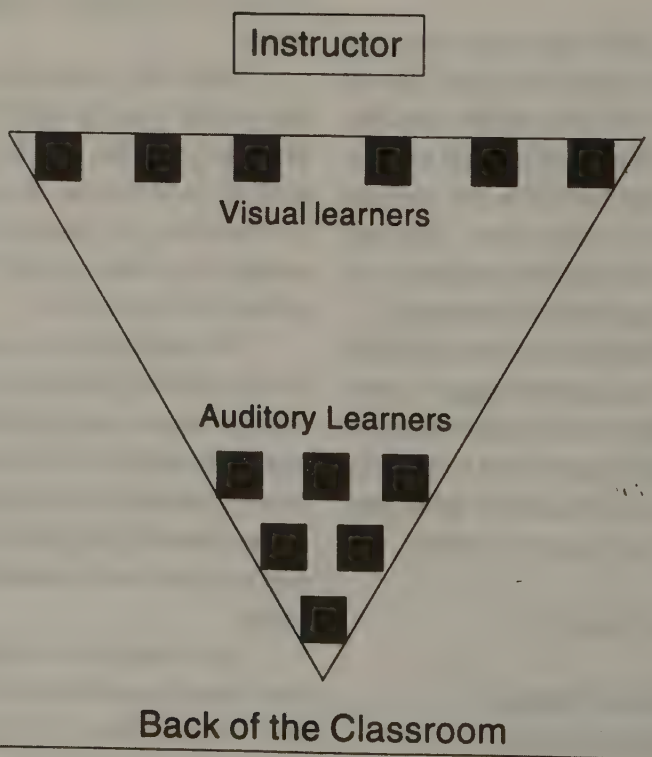
How to Become an Active Listener

Becoming an active listener is the second part of the mental preparation for note-taking.

Do not anticipate what the instructor is going to say or immediately judge the instructor's information before the point is made. This will distract you from learning the information.

Figure 3

The Golden Triangle of Success



Examples: Watch the speaker, listen for main ideas, and nod your head or say to yourself, "I understand," when agreeing with the instructor.

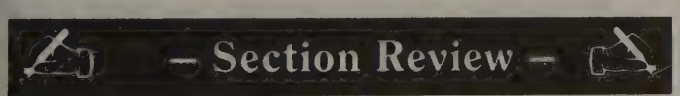
Expend energy looking for interesting topics in the lecture. When the instructor discusses information that you need to know, immediately repeat it to yourself to begin the learning process.

You can practice this exercise by viewing math videotapes and repeating important information. This is an especially good learning technique for auditory learners.

Remember: Class time is an intense study period that should not be wasted.

Listening and Learning

Some students think that listening to the instructor and taking notes is a waste of valuable time. Students too often sit in class and use only a fraction of their learning ability. Class time should be considered a valuable study period where you can listen, take notes, and learn at the same time. One way to do this is by memorizing important facts when the instructor is talking about material you already know. Another technique is to repeat back to yourself the important concepts right after the instructor says them in class. Using class time to learn math is an efficient learning system.



1. Explain how sitting in the golden triangle of success makes you a better listener.

2. List five ways you can warm up before math class begins.

Way 1: _____

Way 2: _____

Way 3: _____

Way 4: _____

Way 5: _____

3. Select your best warm-up process and try it before your next math class. What was your warm-up process, and how did it help you?

4. How can you listen and learn at the same time?

How to become a good notetaker

Becoming a good notetaker requires two basic strategies. One strategy is to be specific in detail. In other words, *copy* the problems down, step by step. The second strategy is to *understand* the general principles, general concepts, and general ideas.

Copying from the Board

While taking math notes, you need to copy each and every step of the problem even though you may already know every step of the problem. While in the classroom, you might understand each step, but a week later you might not remember how to do the problem unless all the steps were written down. In addition, as you write down each step, you are memorizing it. Make sure to copy every step for each problem written on the board.

There will be times when you will get lost while listening to the lecture. Nevertheless, you should keep taking notes, even though you do not understand the problem. This will provide you with a reference point for further study. Put a question mark (?) by those steps that you do not understand; then, after class, review the steps you did not understand with the instructor, your tutor, or another student.

Taking Notes

The goal of note-taking is to take the least amount of notes and get the greatest amount of information on your paper. This could be the opposite of what most instructors have told you. Some instructors tell you to take down everything. This is not necessarily a good note-taking system, since it is very difficult to take precise, specific notes while trying to understand the instructor.

What you need to develop is a note-taking system in which you write the least amount possible and get the most information down while still understanding what the instructor is saying.

Develop an Abbreviation List

To reduce the amount of written notes, an abbreviation system is needed. An abbreviation system is your own system of reducing long words to shorter versions that you still can understand. By writing less, you can listen more and have a better understanding of the material.

> **Example:** When the instructor starts explaining the commutative property, you need to write "commutative property" out the first time. After that, use "COM." You should develop abbreviations for all the most commonly used words in math.

Figure 4 on the next page provides a list of abbreviations. Add your own abbreviations to this list. By using abbreviations as much as possible, you can obtain the same meaning from your notes and have more time to listen to the instructor.

When to take notes

To become a better notetaker, you must know when to take notes and when not to take notes. The instructor will give cues that indicate what material is important. These cues include

- presenting usual facts or ideas,
- writing on the board,
- summarizing,
- pausing,
- repeating statements,
- enumerating, such as "1, 2, 3" or "A, B, C,"
- working several examples of the same type of problem on the chalkboard,

Figure 4
Abbreviations

E.G. (for example)

CF. (compare, remember in context)

N.B. (note well, this is important)

∴ (therefore)

(because)

⊃ (implies, it follows from this)

> (greater than)

< (less than)

= (equals, is the same)

≠ (does not equal, is not the same)

( ) (parentheses in the margin, around a sentence or group of sentences, indicates an important idea)

? (used to indicate you do not understand the material)

0 (a circle around a word may indicate that you are not familiar with it; look it up)

E (marks important materials likely to be used in an exam)

1, 2, 3, 4 (to indicate a series of facts)

D (shows disagreement with statement or passage)

REF (reference)

et al. (and others)

bk (book)

p (page)

etc. (and so forth)

V (see)

VS (see above)

SC (namely)

SQ (the following)

Com. (commutative)

Dis. (distributive)

APA (associative property of addition)

AI (additive inverse)

IPM (identity property of multiplication)

- saying, "This is a tricky problem." Most students will miss it (for example, 5/0 is "undefined" instead of "zero").
- saying, "This is the most difficult step in the problem,"
- indicating that certain types of problems will be on the test, such as coin or age problems, and
- explaining bold-face words.

You must learn the cues your instructor gives to indicate important material. If you are in doubt about the importance of the class material, do not hesitate to ask the instructor about its importance.

While taking notes, you may become confused about math material. At that point, take as many notes as possible, and do not give up on note-taking.

As you take notes on confusing problem steps, leave extra space; then go back and fill in information that clarifies your misunderstanding of the steps in question. Ask your tutor or instructor for help with the uncompleted problem steps, and write down the reasons for each step in the space provided.

Another procedure to save time while taking notes is to stop writing complete sentences. Write your main thoughts in phrases. Phrases are easier to jot down and easier to memorize.

– Section Review –

1. Note-taking requires two basic strategies:

   _____and
   _____.

2. List five abbreviations (and their meanings) that you use in your math class (one abbreviation and meaning per line).

   _____
   _____
   _____
   _____

3. List five cues that your instructor gives that indicate what material is important (one cue per line).

The seven steps to math note-taking

The key to effective note-taking is to record the fewest words while retaining the greatest information. As you know, it is very difficult to record notes and at the same time fully understand the instructor. The seven steps to math note-taking were developed to decrease the amount of note-taking while at the same time improving math learning.

The seven steps to math note-taking consists of three major areas. The first area, steps 1 through 3, focuses on *recording* your notes. Steps 4 through 6 focus on *checking* yourself to see how much information is retained. This is done by recalling key words and concepts and putting a check mark by misunderstood information. Recalling information is one of the best learning techniques. Step 7 focuses on *understanding* key words and concepts that are frequently used.

Note-Taking Memory Cues

One of the best math note-taking methods is demonstrated in Figure 5 (Modified Three-Column Note-Taking Sample) on the facing page. To use this effective note-taking system, you need to record a few memory cues as reminders. Label the top space between the notebook ring and the red line "Key Words." Label the other side of the red line "Examples." Next, label "Explanations/ Rules" about 4 inches from the red line. Draw a vertical line between the "Examples" and "Explanations/Rules" sections. Record the same information on the next ten pages. After using this system for ten pages, you may not need to label each page.

Follow these steps to improve your note-taking:

Step 1 Record each problem step in the "Examples" section.

Step 2 Record the reasons for each step in the "Explanations/Rules" section by using

- abbreviations,
- short phrases, not sentences, and
- key words, properties, principles, or formulas.

Step 3 Record key words and concepts in the left 2-inch margin either during or immediately after lecture by reworking your notes.

Step 4 Cover up the "Examples" and "Explanations/ Rules" sections, and recite out loud the meaning of the key words or concepts.

Step 5 Place a check mark by the key words and concepts that you *did not* know.

Step 6 Review the information that you checked until it is understood.

Step 7 Develop a math glossary for difficult-to-remember key words and concepts.

A Math Glossary

The third area of focus in the seven steps to math note-taking is devoted to developing a math glossary. The math glossary is created to define a math vocabulary in your own words. Since math is consid-

Figure 5
Modified Three-Column Note-Taking Sample

Key Words	Examples	Explanations/Rules
Natural numbers	1, 2, 3, 4, 5, . . .	You can count them.
Whole numbers	0, 1, 2, 3, 4, . . .	Natural numbers and zero
Integers	. . . -2, -1, 0, 1, 2, . . .	Negative numbers and whole numbers
$-n$	$-n$ If $n = s$, then $-n = -s$ $-(-10) = 10$ $-(-15)(-15) = -15$ Opposite of $-x$ is x	Opposite of any number Count the number of signs; even means +, odd means -
Rational	Rational numbers are fractions (¼, ½, ¾).	A/B, $B/0$ is rational Division by 0 undefined
Numerator Denominator	numerator/denominator N/D	Numerator on top Denominator on bottom (D = Down)
Terminating decimal	41.5, 45.0, 7, 1576, 0.39, are terminating decimals	Decimal numbers Some are rational numbers, some are not rational numbers.
Repeating decimals	0.333, 0.51515151 = 0.51	A repeating decimal = rational number
Pi, irrational number	$\pi = 3.14159$ but does not repeat and is not a rational number. It is irrational.	Irrational numbers do not repeat.

Figure 5
Modified Three-Column Note-Taking System
(Blank)

Key Words	Examples	Explanations/Rules

ered a foreign language, understanding the math vocabulary becomes the key to comprehending math. Creating a glossary for each chapter of your textbook will help you understand math.

Your glossary should include all words printed in bold face in the text and any words you do not understand. If you cannot explain the math vocabulary in your own words, ask your instructor or tutor for help. You may want to use the last pages in your notebook to develop a math glossary for each chapter in your textbook. Review your math glossary before each test.



1. List and describe the seven steps to note-taking.

   Step 1: _____
   _____
   _____
   Step 2: _____
   _____
   _____
   Step 3: _____
   _____
   _____
   Step 4: _____
   _____
   _____
   Step 5: _____
   _____
   _____
   Step 6: _____
   _____
   _____
   Step 7: _____
   _____
   _____

2. Make several copies of the facing page (the modified three-column note-taking system, and take notes using this system in your next math class. Or develop the modified three-column note-taking system in your own notebook.

3. List two benefits of a math glossary.

   Benefit 1: _____
   _____
   _____
   Benefit 2: _____
   _____
   _____

4. From your current math chapter, list and define five words that you can put in your math glossary.

   Word 1: _____
   _____
   Word 2: _____
   _____
   Word 3: _____
   _____
   Word 4: _____
   _____
   Word 5: _____
   _____

How to rework your notes

The note-taking system does not stop when you leave the classroom. As soon as possible after class, rework your notes. You can rework the notes between classes or as soon as you get home. By reworking your notes as soon as possible, you can decrease the amount of forgetting. This is an excellent procedure to transfer math information from short-term memory to long-term memory.

Remember: Most forgetting occurs right after learning the material. You need to rework the notes as soon as possible. Waiting means that you probably will not understand what was written.

Here are important steps in reworking your notes:

Step 1 *Rewrite the material you cannot read or will not be able to understand a few weeks later.* If you do not rework your notes, you will be frustrated when studying for a test if you come across notes you cannot read. Another benefit of rewriting the notes is that you immediately learn the new material. Waiting means it will take more time to learn the material.

Step 2 *Fill in the gaps.* Most of the time, when you are listening to the lecture, you cannot write down everything. It is almost impossible to write down everything, even if you know shorthand. Locate the portions of your notes that are incomplete. Fill in the concepts that were left out. In the future, skip two or three lines in your notebook page for anticipated lecture gaps.

Step 3 *Add additional key words and ideas in the left-hand column.* These key words or ideas were the ones not recorded during the lecture.

> **Example:** You did not know that you should add the *opposite* of 18 to solve a particular problem, and you incorrectly added 18. Put additional important key words and ideas (such as "opposite" and "negative of") in the notes; these are the words that will improve your understanding of math.

Step 4 *Make a problem log on those problems that the teacher worked in class.* The problem log is a separate section of your notebook that contains a listing of the problems (without explanations – just problems) that your teacher worked in class. If your teacher chose those problems to work in class, you can bet that they are considered important. The problems in this log can be used as a practice test for the next exam. Your regular class notes will not only contain the solutions but also all the steps involved in arriving at those solutions and can be used as a reference when you take your practice test.

Step 5 *Make a calculator handbook and put in it your keystroke sequences.* The calculator handbook can be a spiral-bound set of note cards or a separate section of your notebook that holds only calculator-related information. Your handbook should also include an explanation of when that particular set of keystrokes is to be used.

Step 6 *Reflection and synthesis.* Once you have finished going over your notes, review the major points in your mind. Combine your new notes with your previous knowledge to have a better understanding of what you have learned today.

Use a Tape Recorder

If you have a math class during which you cannot get all the information down while listening to the lecture, ask your instructor about using a tape recorder. To ensure success, the tape recorder must have a tape counter and must be voice activated.

The tape counter displays a number indicating the amount of tape to which you have listened. When you find you are in an area of confusing information, write the beginning and ending tape counter number in the left margin of your notes. When reviewing your notes, the tape count number will be a reference point for obtaining information to work the problem. You can also reduce the time it takes to listen to the tape by using the pause button to stop the recording of unnecessary material.

Ask Questions

To obtain the most from a lecture, you must ask questions in class. By asking questions, you improve your understanding of the material and decrease your homework time. By *not* asking questions, you create for yourself unnecessary confusion during the remainder of the class period. Also, it is much easier to ask questions in class about potential homework problems than it is to spend hours trying to figure out the problems on your own at a later time.

If you are shy about asking questions in class, write down the questions and read them to your instructor. If the instructor seems confused about the questions, tell him or her that you will discuss the problem after class. To encourage yourself to ask questions, remember that

- you have paid for the instructor's help,
- five other students probably have the same question,
- the instructor needs feedback on his or her teaching to help the class learn the material, and
- there is no such thing as a "stupid" question.

Record Each Problem Step

The final suggestion in note-taking is to record each step of every problem written or verbally explained. By recording each problem step, you begin *overlearning* how to work the problems. This will increase your problem-solving speed during future tests. If you get stuck on the homework, you will also have complete examples to review.

The major reason for recording every step of a problem is to understand how to do the problems while the instructor is explaining them instead of trying to remember unwritten steps. Although it may seem time consuming, it pays off during homework and test time.



1. List and describe the six steps to reworking your notes.

   Step 1: _____
   _____
   _____
   Step 2: _____
   _____
   _____
   Step 3: _____
   _____
   _____
   Step 4: _____
   _____
   _____
   Step 5: _____
   _____
   _____
   Step 6: _____
   _____
   _____

2. How can using a tape recorder in class improve your learning?

   _____
   _____
   _____
   _____

3. List three reasons for asking questions in class.

   Reason 1: _____
   _____
   _____
   Reason 2: _____
   _____
   _____

Reason 3: _____

_____

_____

4. What is the major reason for recording each problem step?

_____

_____

_____

_____

Lecture Notes

Name: _____ Date: _____



Chapter 4 Review



1. Sitting in the golden triangle of success can help you pay more _____during class.

2. Class time is an intense _____ _____ that should not be wasted.

3. Auditory learners should review the math videotape and _____back the important information.

4. The goal of note-taking is to take the _____amount of notes and get the _____amount of information.

5. Using abbreviations can help you write _____ and listen _____.

6. Your _____ should include any words you don't know and their explanations.

7. Rework your notes to transfer information from _____-_____ memory to _____-_____memory.

8. A _____ _____is a separate section in your notebook that contains problems that your teacher worked in class.

9. Asking _____ in class will improve your understanding and decrease your homework time.

10. _____how to work problems will increase your problem-solving speed and memory.

What is the most important information you learned from this chapter?

_____
_____
_____
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How can you immediately use it?

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Chapter 5

How to Improve Your Reading, Homework, and Study Techniques

Reading a math textbook is more difficult than reading other textbooks. Math textbooks are written differently than your English or social science textbooks. Math textbooks contain condensed material and therefore take longer to read.

Mathematicians can reduce a page of writing to one paragraph using math formulas and symbols. To make sure you understood that same information, an English instructor would take that original page of writing and expand it into two pages. Mathematicians pride themselves on how little they can write and still cover the concept. This is one reason it may take you two to three times as long to read your math text as it would any other text.

Remember: Reading your math text will take longer than reading your other texts.

Math students are expected to know how to do their homework; however, most math students do not have a homework system. Most students begin their homework by going directly to the problems and trying to work them. When they get stuck, they usually quit. This is not a good homework system. A good homework system will improve your homework success and math learning at the same time.

In this chapter, you will learn

- how to read a math textbook,
- how to do your homework,
- how to solve word problems,
- how to work with a study buddy.

How to read a math textbook

The way you read a math textbook is different from the traditional way students are taught to read textbooks in high school or college. Students are taught to read quickly or skim the material. If you do not understand a word, you are supposed to keep on reading. Instructors of other courses want students to continue to read so they can pick up the unknown words and their meanings from context.

This reading technique may work with your other classes, but using it in your math course will be totally confusing. By skipping some major concept words or bold-face words, you will not understand the math textbook or be able to do the homework. Reading a math textbook takes more time and concentration than reading your other textbooks.

If you have a reading problem, it would be wise to take a developmental reading course before taking math. This is especially true with reformed math courses, where reading and writing are emphasized.

Reform math classes deal more with word problems than do traditional math courses. If you cannot take the developmental reading course before taking math, then take it during the same semester as the math course.

Eight Steps to Understanding Reading Materials

There are several appropriate steps in reading a math textbook:

Step 1 *Skim the assigned reading material.* Skim the material to get the general idea about the major topics. Read the chapter introduction and each section summary. You do not want to learn the material at this time; you simply want to get an overview of the assignment. Then think about similar math topics that you already know.

Example: Skimming will allow you to see if problems presented in one chapter section are further explained in later chapter sections.

Step 2 *As you skim the chapter, circle (using pencil) the new words that you do not understand.* If you do not understand these new words after reading the assignment, then ask the instructor for help. Skimming the reading assignments should take only five to ten minutes.

Step 3 *Put all your concentration into reading.* While reading the textbook, highlight the material that is important to you. However, do not highlight more than 50 percent of a page because the material is not being narrowed down enough for future study. Especially highlight the material that is also discussed in the lecture. Material discussed in both the textbook and lecture usually appears on the test. The purpose of highlighting is to emphasize the important material for future study. Do not skip reading assignments.

Remember: Reading a math textbook is very difficult. It might take you half an hour to read and understand just *one* page.

Step 4 *When you get to the examples, go through each step.* If the example skips any steps, make sure you write down each one of those skipped steps in the textbook for better understanding. Later on, when you go back and review, the steps are already filled in. You will understand how each step was completed. Also, by filling in the extra steps, you are starting to overlearn the material for better recall on future tests.

Step 5 *Mark the concepts and words that you do not know.* Maybe you marked them the first time while skimming. If you understand them now, erase the marks. If you do not understand the words or concepts, then reread the page or look them up in the glossary. Try not to read any further until you understand all the words and concepts.

Step 6 *If you do not clearly understand some words or concepts, add these words to the note-taking glossary in the back of your notebook.* Your glossary will contain the bold-face words that you do not understand. If you have difficulty understanding the bold-face words, ask the instructor for a better explanation. You should know all the words and concepts in your notebook's glossary before taking the test.

Step 7 *If you do not understand the material, follow these eight points, one after the other, until you do understand the material:*

Point 1 – Go back to the previous page and reread the information to maintain a train of thought.

Point 2 – Read ahead to the next page to discover if any additional information better explains the misunderstood material.

Point 3 – Locate and review any diagrams, examples, or rules that explain the misunderstood material.

Point 4 – Read the misunderstood paragraph(s) several times aloud to better understand their meaning.

Point 5 – Refer to your math notes for a better explanation of the misunderstood material.

Point 6 – Refer to another math textbook, computer software program, or videotape that expands the explanation of the misunderstood material.

Point 7 – Define exactly what you do not understand and call your study buddy for help.

Point 8 – Contact your math tutor or math instructor for help in understanding the material.

Step 8 *Reflect on what you have read.* Combine what you already know with the new information that you just read. Think about how this new information enhances your math knowledge. Prepare questions for your instructor on the confusing information. Ask those questions at the next class meeting.

By using this reading technique, you have narrowed down the important material to be learned. You have skimmed the textbook to get an overview of the assignment. You have carefully read the material and highlighted the important parts. You then added to your note-taking glossary unknown words or concepts.

Remember: The highlighted material should be reviewed before doing the homework problems, and the glossary has to be learned 100 percent before taking the test.

How Reading Ahead Can Help

Reading ahead is another way to improve learning. If you read ahead, do not expect to understand everything. Read ahead two or three sections and put question marks (in pencil) by the material you do not understand.

When the instructor starts discussing that material, have your questions prepared and take good notes. Also, if the lecture is about to end, ask the instructor to explain the confusing material in the textbook. Reading ahead will take more time and effort, but it will better prepare you for the lectures.

How to Establish Study Period Goals

Before beginning your homework, establish goals for the study period. Do not just do the homework problems.

Ask yourself this question: "What am I going to do tonight to become more successful in math?"

By setting up short-term homework goals and reaching them, you will feel more confident about math. This also improves your self-esteem and helps you become a more internally motivated student. Set up homework tasks that you can complete. Be realistic.

Study period goals are set up either on a time-line basis or an item-line basis. Studying on a time-line basis is studying math for a certain amount of time.

Example: You may want to study math for an hour, then switch to another subject. You will study on a time-line basis.

Studying on an item-line basis means you will study your math until you have completed a certain number of homework problems.

Example: You might set a goal to study math until you have completed all the odd-numbered problems in the chapter review. The odd-numbered problems are the most important problems to work. These, in most texts, are answered in the answer section in the back of the book. Such problems provide the opportunity to recheck your work if you do not get the answer correct. Once you have completed these problems, do the even-numbered problems.

No matter what homework system you use, remember this important rule: Always finish a homework session by understanding a concept or doing a homework problem correctly.

Do not end a homework session with a problem you cannot complete. You will lose confidence, since all you will think about is the last problem you could not solve instead of the 50 problems you correctly solved. If you did quit on a problem you could not solve, return and rework problems you have done correctly.

Remember: Do not end your study period with a problem you could not complete.



1. List and describe the eight steps to reading a math textbook.

Step 1: _____

_____

_____

Step 2: _____

_____

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Step 3: _____

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Step 4: _____

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Step 5: _____

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Step 6: _____

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Step 7: _____

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Step 8: _____

_____

_____

2. After trying each of the eight steps to reading your math text, make up, list, and try your own condensed version.

Step 1: _____

_____

Step 2: _____

_____

Step 3: _____

_____

Step 4: _____

_____

Step 5: _____

_____

Step 6: _____

_____

Step 7: _____

_____

Step 8: _____

_____

3. List two reasons why reading ahead can improve your learning.

Reason 1: _____

_____

_____

Reason 2: _____

_____

_____

4. Describe the two types of study period goals. Which one do you use?

Type 1: _____

_____

_____

Type 2: _____

_____

_____

How to do your homework

Doing your homework can be frustrating or rewarding. Most students jump right into their homework, become frustrated, and stop studying. These students usually go directly to the math problems and start working them without any preparation. When they get stuck on one problem, they flip to the back of the text for the answer. Then they either try to work the problem backward to understand the problem steps or they just copy down the answer.

Other students go to the solution guide and just copy the steps. After getting stuck several times, these students will inevitably quit doing their homework assignment. Their homework becomes a frustrating experience, and they may even quit doing their math homework altogether.

10 Steps to Doing Your Homework

To improve your homework success and learning, refer to these ten steps:

Step 1 *Review the textbook material that relates to the homework.* A proper review will increase the chances of successfully completing your homework. If you get stuck on a problem, you will have a better chance of remembering the location of similar problems. If you do not review prior to doing your homework, you could get stuck and not know where to find help in the textbook.

Remember: To be successful in learning the material and in completing homework assignments, you must first review your textbook.

Step 2 *Review your lecture notes that relate to the homework.* If you could not understand the explanation in the textbook on how to complete the homework assignment, then review your notes.

Remember: Reviewing your notes will give you a better idea about how to complete your homework assignment.

Step 3 *Do your homework as neatly as possible.* Doing your homework neatly has several benefits. When approaching your instructor about problems with your homework, he or she will be able to understand your previous attempts to solve the problem. The instructor will easily locate the mistakes and show you how to correct the steps without having to decipher your handwriting. Another benefit is that when you review for midterm or final exams, you can quickly relearn the homework material without having to decipher your own writing.

Remember: Neatly prepared homework can help you now and in the future.

Step 4 *When doing your homework, write down every step of the problem.* Even if you can do the step in your head, write it down anyway. This will increase the amount of homework time, but you are overlearning how to solve problems, which improves your memory. Doing every step is an easy way to memorize and understand the material. Another advantage is that when you rework the problems you did wrong, it is easy to review each step to find the mistake.

Remember: In the long run, doing every step of the homework will save you time and frustration.

Step 5 *Understand the reasons for each problem step and check your answers.* Do not get into the bad habit of memorizing how to do problems without knowing the reasons for each step. Many students are smart enough to memorize procedures required to complete a set of homework problems. However, when similar home-

work problems are presented on a test, the student cannot solve the problems. To avoid this dilemma, keep reminding yourself about the rules, laws, or properties used to solve problems.

Example: *Problem:* $2(a + 5) = 0$. What property allows you to change the equation to $2a + 10 = 0$? *Answer:* The distributive property.

Once you know the correct reason for going from one step to another in solving a math problem, you can solve any problem requiring that property. Students who simply memorize how to do problems instead of understanding the reasons for correctly working the steps will eventually fail their math course.

How to Check Your Answers

Checking your homework answers should be part of your homework duties. Checking your answers can improve your learning and help you prepare for tests.

Check the answers of the problems for which solutions are not given. This may be the even-numbered or odd-numbered problems or the problems not answered in the solutions manual.

First, check your answer by estimating the correct answer.

Example: If you are multiplying 2.234 by 5.102, Remember that 2 times 5 is 10. The answer should by a little over 10.

You can also check your answers by substituting the answer back into the equation or doing the opposite function required to answer the question. The more answers you check, the faster you will become. This is very important because increasing your answer-checking speed can help you catch more careless errors on future tests.

Step 6 *If you do not understand how to do a problem, refer to the following points:*

Point 1 – Review the textbook material that relates to the problem.

Point 2 – Review the lecture notes that relate to the problem.

Point 3 – Review any similar problems, diagrams, examples, or rules that explain the misunderstood material.

Point 4 – Refer to another math textbook, solutions guide, math computer program software, or videotape to obtain a better understanding of the material.

Point 5 – Call your study buddy.

Point 6 – Skip the problem and contact your tutor or math instructor as soon as possible for help.

Step 7 *Always finish your homework by successfully completing problems.* Even if you get stuck, go back and successfully complete previous problems before quitting. You want to end your homework assignment with feelings of success.

Step 8 *After finishing your homework assignment, recall to yourself or write down the most important learned concepts.* Recalling this information will increase your ability to learn these new concepts. Additional information about step 8 will be presented later in this chapter.

Step 9 *Make up note cards containing hard-to-remember problems or concepts.* Note cards are an excellent way to review material for a test. More information on the use of note cards as learning tools is presented later in this chapter.

Step 10 *Do not fall behind.* As mentioned in Chapter 1, math is a sequential learning process. If you get behind, it is difficult to catch up because each topic builds on the next. It would

be like going to Spanish class without learning the last set of vocabulary words. The teacher would be talking to you using the new vocabulary, but you would not understand what was being said.

Do Not Fall Behind

To keep up with your homework, it is necessary to complete the homework every school day and even on weekends. Doing your homework one-half hour each day for two days in a row is better than one hour every other day.

If you have to get behind in one of your courses, *make sure it is not math.* Fall behind in a course that does not have a sequential learning process, such as psychology or history. After using the ten steps to doing your homework, you may be able to combine two steps into one. Find your best combination of homework steps and use them.

Remember: Getting behind in math homework is the fastest way to fail the course.



1. List the ten steps to doing your math homework.

Step 1: _____
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Step 2: _____
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Step 3: _____
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Step 4: _____
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Step 5: _____
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Step 6: _____
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Step 7: _____
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Step 8: _____
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Step 9: _____
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Step 10: _____
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2. After trying the ten steps to doing your math homework, make up, list, and try your own condensed version.

Step 1: _____
_____
Step 2: _____
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Step 3: _____
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Step 4: _____
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Step 5: _____
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Step 6: _____
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Step 7: _____
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Step 8: _____
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Step 9: _____
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Step 10: _____
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continued on the next page

3. How will reviewing your notes or math text help you complete your homework more successfully?

_____
_____
_____
_____

4. List two ways you can check your homework answers.

Way 1: _____
_____
_____
_____

Way 2: _____
_____
_____
_____

5. What happens to students who fall behind in their homework?

_____
_____
_____
_____
_____
_____

How to solve word problems

The most difficult homework assignment for most math students is working story/word problems. Solving word problems requires excellent reading comprehension and translating skills.

Students often have difficulty substituting algebraic symbols and equations for English terms. But once an equation is written, it is usually easily solved. To help you solve word problems, follow these ten steps:

Step 1 *Read the problem three times.* Read the problem quickly the first time as a scanning procedure. As you are reading the problem the second time, answer these three questions:

1. *What is the problem asking me?* (Usually at the end of the problem)

2. *What is the problem telling me that is useful?* (Cross out unneeded information.)

3. *What is the problem implying?* (Usually something you have been told to remember)

Read the problem a third time to check that you fully understand its meaning.

Step 2 *Draw a simple picture of the problem to make it more real to you* (e.g., a circle with an arrow can represent travel in any form – by train, by boat, by plane, by car, or by foot).

Step 3 *Make a table of information and leave a blank space for the information you are not told.*

Step 4 *Use as few unknowns in your table as possible.* If you can represent all the unknown information in terms of a single letter, do so! When using more than one unknown, use a letter that reminds you of that unknown. Then write down what your unknowns represent. This eliminates the problem of assigning the right answer to the wrong unknown. Remember that you have to create as many separate equations as you have unknowns.

Step 5 *Translate the English terms into an algebraic equation using the list of terms in Figures 6 and 7 on page 69.* Remember, the English terms are sometimes stated in a different order than the algebraic terms.

Figure 6

Translating English Terms into Algebraic Symbols

Sum	+
Add	+
In addition	+
More than	+
Increased	+
In excess	+
Greater	+
Decreased by	-
Less than	-
Subtract	-
Difference	-
Diminished	-
Reduce	-
Remainder	-
Times as much	x
Percent of	x
Product	x
Interest on	x
Per	/
Divide	/
Quotient	/
Quantity	( )
Is	=
Was	=
Equal	=
Will be	=
Results	=
Greater than	>
Greater than or equal to	≥
Less than	<
Less than or equal to	≤

Figure 7

Translating English Words into Algebraic Expressions

English Words	Algebraic Expressions
Ten more than x	$x + 10$
A number added to 5	$5 + x$
A number increased by 13	$x + 13$
5 less than 10	$10 - 5$
A number decreased by 7	$x - 7$
Difference between x and 3	$x - 3$
Difference between 3 and x	$3 - x$
Twice a number	$2x$
Ten percent of x	$0.10x$
Ten times x	$10x$
Quotient of x and 3	$x/3$
Quotient of 3 and x	$3/x$
Five is three more than a number	$5 = x + 3$
The product of 2 and a number is 10	$2x = 10$
One-half a number is 10	$x/2 = 10$
Five times the sum of x and 2	$5(x + 2)$
Seven is greater than x	$7 > x$
Five times the difference of a number and 4	$5(x - 4)$
Ten subtracted from 10 times a number is that number plus 5	$10x - 10 = x + 5$
The sum of $5x$ and 10 is equal to the product of x and 15	$5x + 10 = 15x$
The sum of two consecutive integers	$(x) + (x + 1)$
The sum of two consecutive even integers	$(x) + (x + 2)$
The sum of two consecutive odd integers	$(x) + (x + 2)$

Step 6 *Immediately retranslate the equation, as you now have it, back into English.* The translation will not sound like a normal English phrase, but the meaning should be the same as the original problem. If the meaning is not the same, the equation is incorrect and needs to be rewritten. Rewrite the equation until it means the same as the English phrase.

Step 7 *Review the equation to see if it is similar to equations from your homework and if it makes sense.* Some formulas dealing with specific word problems may need to be rewritten. Distance problems, for example, may need to be written solving for each of the other variables in the formula. Distance = rate x time; therefore, time = distance/rate, and rate = distance/time. Usually, a distance problem will identify the specific variable to be solved.

Step 8 *Solve the equation using the rules of algebra.*

Remember: Whatever is done to one side of the equation must be done to the other side of the equation. The unknown must end up on one side of the equation, by itself. If you have more than one unknown, then use the substitution or elimination method to solve the equations.

Step 9 *Look at your answer to see if it makes common sense.*

Example: If tax was added to an item, it should cost more; if a discount was applied to an item, it should cost less. Is there more than one answer? Does your answer match the original question? Does your answer have the correct units?

Step 10 *Put your answer back into the original equation to see if it is correct.* If one side of the equation equals the other side of the equation, then you have the correct answer. If you do not have the correct answer, go back to step 5.

The most difficult part of solving word problems is translating part of a sentence into algebraic symbols and then into algebraic expressions. Review Figure 6 (Translating English Terms into Algebraic Symbols) and Figure 7 (Translating English Words into Algebraic Expressions), both on the previous page.

How to Recall What You Have Learned

After completing your homework problems, a good visual learning technique is to make note cards. Note cards are 3" by 5" index cards on which you place information that is difficult to learn or material you think will be on the test.

On the front of the note cards write a math problem or information that you need to know. Color-code the important information in red or blue. On the back of the note card, write how to work the problem or explain important information.

Example: If you are having difficulty remembering the rules for multiplying positive and negative numbers, you would write some examples on the front of the note card with the answers on the back.

Make note cards on important information you might forget. Every time you have five spare minutes, pull out your note cards and review them. You can glance at the front of the card, repeat to yourself the answer, and check yourself with the back of the card. If you are correct and know the information on a card, do not put it back in the deck. Mix up the cards you do not know and pick another card to test yourself on the information. Keep doing this until there are no cards left that you do not know.

If you are an auditory learner, then use the tape recorder like the note cards. Record the important information as you would on the front of the note card. Then leave a blank space on the recording before recording the answer. Play the tape back. When you hear the silence, put the tape on pause. Then say the answer out loud to yourself. Take the

tape player off pause and see if you were correct.

Review What You Have Learned

After finishing your homework, close the textbook and try to remember what you have learned. Ask yourself these questions: "What major concepts did I learn tonight?" "What test questions might the instructor ask on this material?"

Recall for about three to four minutes the major points of the assignment, especially the areas you had difficulty understanding. Write down questions for the instructor or tutor. Since most forgetting occurs right after learning the material, this short review will help you retain the new material.



1. List the ten steps to solving word problems.

Step 1: _____

_____

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Step 2: _____

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Step 3: _____

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Step 4: _____

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Step 5: _____

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Step 6: _____

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Step 7: _____

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Step 8: _____

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Step 9: _____

_____

_____

Step 10: _____

_____

_____

2. What is your best way to recall what you have learned?

_____

_____

_____

_____

3. How does reviewing right after doing your homework help you remember?

_____

_____

_____

_____

How to work with a study buddy

You need to have a study buddy when you miss class or when you do your homework. A study buddy is a friend or classmate who is taking the same course. You can find a study buddy by talking to your classmates or making friends in the math lab.

Try to find a study buddy who knows more about math than you do. Tell the class instructor that you are trying to find a study buddy and ask which students make the best grades. Meet with your study buddy several times a week to work on problems and to discuss math. If you miss class, get the notes from

your study buddy so you will not get behind.

Call your study buddy when you get stuck on your homework. You can solve math problems over the phone. Do not sit for half an hour or an hour trying to work one problem; that will destroy your confidence, waste valuable time, and possibly alienate your study buddy. Think how much you could have learned by trying the problem for 15 minutes and then calling your study buddy for help. Spend, at the maximum, 15 minutes on one problem before going on to the next problem or calling your study buddy.

Remember: A study buddy can improve your learning while helping you complete the homework assignment. Just do not overuse your study buddy or expect that person to do your homework for you.



1. How can you select your study buddy?

2. How can a study buddy help you learn math?

The benefits of study breaks

Psychologists have discovered that learning decreases if you do not take study breaks. Therefore, take a break after studying math for 45 minutes to one hour.

If you have studied for only 15 or 20 minutes and feel you are not retaining the information or your mind is wandering, take a break. If you continue to force yourself to study, you will not learn the material. After taking a break, return to studying.

If you still cannot study after taking a break, review your purpose for studying and your educational goals. Think about what is required to graduate. It will probably come down to the fact that you will have to pass math. Think about how studying math today will help you pass the next test; this will increase your chances of passing the course and of graduating.

Write on an index card three positive statements about yourself and three positive statements about studying. Look at this index card every time you have a study problem. Use every opportunity available to reinforce your study habits.



1. How can study breaks improve your learning?

Name: _____ Date: _____



Chapter 5 Review



1. Reading one page of a math text might take you _____ _____ _____.

2. Checking your homework answers can improve your _____ and help prepare you for a _____.

Matching

(for students in pre-algebra class)

3. _____	Product	A. +
4. _____	Increased	B. -
5. _____	Sum	C. x
6. _____	Per	D. /
7. _____	Product	E. =
8. _____	Is	
9. _____	Reduced	
10. _____	Remainder	

(for students taking elementary algebra or higher-level math course)

3. _____	Difference between x and 3	A. $10x$
4. _____	Quotient of x and 3	B. $x/3$
5. _____	Five times the sum of x and 2	C. $5(x-4)$
		D. $5(x+2)$
6. _____	A number added to 5	E. $5=x+3$
7. _____	Five less than 10	F. $5+x$
8. _____	Ten times x	G. 10 -5
9. _____	Five is three more than a number	H. $x-3$
10. _____	Five times the difference of a number and four	

11. Make sure you finish your homework by working problems you _____ do.

12. Using _____ _____ is a visual way to recall what you learned in math.

13. Using a _____ _____ is an auditory way to recall what you learned in math.

14. Most _____ occurs right after learning; a short review will help you _____ the new material.

15. A _____ _____ can improve your learning while helping you complete the homework assignment.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
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_____
_____

Chapter 6

How to Remember What You Have Learned

To understand the learning process, you must understand how your memory works. You learn by conditioning and thinking. But memorization is different from learning. For memorization, your brain must perform several tasks, including receiving the information, storing the information, and recalling the information.

By understanding how your memory works, you will be better able to learn at which point your memory is failing you. Most students usually experience memory trouble between the time their brain receives the information and the time the information is stored.

There are many techniques for learning information that can help you receive and store information without losing it in the process. Some of these techniques may be more successful than others, depending on how you learn best.

In this chapter, you will discover

- how you learn,
- how short-term memory affects what you remember,
- how working memory affects what you remember,
- how long-term memory/reasoning affects what you remember,
- how to use memory techniques,
- how to develop practice tests, and
- how to use number sense.

How you learn

Educators tell us that learning is the process of "achieving competency." More simply put, it is how you become good at something. The three ways of learning are by conditioning, by thinking, and by a combination of conditioning and thinking.

Learning by Conditioning and Thinking

Conditioning is learning things with a maximum of physical and emotional reaction and a minimum of thinking.

Examples: Repeating the word *pi* to yourself and practicing where the symbol is found on a calculator are two forms of conditioned learning. You are learning using your voice and your eye-hand coordination (physical activities), and you are doing very little thinking.

Thinking is defined as learning with a maximum of thought and a minimum of emotional and physical reaction.

Example: Learning about *pi* by thinking is different from learning about it by conditioning. To learn "pi" by thinking, you would have to do the calculations necessary to result in the numeric value that the word *pi* represents. You are learning *using your mind* (thought activities), and you are using very little emotional or physical energy to learn *pi* in this way.

The most successful way to learn is to combine thinking and conditioning. The best learning combination is to learn by thinking first and conditioning second.

Learning by thinking means that you learn by

- *observing*,
- *processing*, and
- *understanding* the information.

How Your Memory Works

Memory is different from learning; it requires reception, storage, and retrieval of information. You *receive* information through your five senses (*sensory input*): what you see, feel, hear, smell, and taste.

Examples: In math class, you will use your sense of *vision* both to watch the instructor demonstrate problems on the chalkboard and to read printed materials. You will use your sense of *hearing* to listen to the instructor and other students discuss the problems. Your sense of *touch* will be used to operate your calculator and to appreciate geometric shapes. In chemistry and other classes, however, you may additionally use your senses of *smell* and *taste* to identify substances.

The sensory register briefly holds an exact image or sound of each sensory experience until it can be processed. If the information is not processed immediately, it is forgotten. The sensory register helps us go from one situation to the next without cluttering up our minds with trivial information.

Processing the information involves placing it in either short-term memory, working memory, or long-term memory.

How short-term memory affects what you remember

Information that passes through the sensory register is stored in short-term memory. Remembering something for a short time is not hard to do for most students. By conscious effort, you can remember math laws, facts, and formulas received by the sensory register (your five senses) and put them in short-term memory. You can recognize them and register them in your mind as something to remember for a short time.

Example: When you are studying math, you can tell yourself the distributive property is illustrated by $a(b + c) = ab + ac$. By deliberately telling yourself to remember that fact (by using conditioning – repeating or writing it again and again), you can remember it, at least for a while, because you have put it in short-term memory.

Psychologists have found that short-term memory cannot hold an unlimited amount of information. You may be able to use short-term memory to remember one phone number or a few formulas, but not five phone numbers or ten formulas.

Items placed into short-term memory usually fade fast, as the name suggests.

Examples: Looking up a telephone number in the directory, remembering it long enough to dial, then forgetting it immediately; learning the name of a person at a large party or in a class but forgetting it completely within a few seconds: cramming for a test and forgetting most of it before taking the test.

Short-term memory involves the ability to recall information immediately after it is given (without any interruptions). It is useful in helping you to concentrate on a few concepts at a time, but it is not the best way to learn. This is especially true of students who have attention problems or problems with

short-term memory.

Students with short-term memory problems forget a math step as soon as the instructor explains it. To remember more facts or ideas and keep them in memory a longer period of time – especially at test time – use of a better system than short-term memory is required.

Changes in the way math is taught include the idea that conditioning (the most often used teaching form) leads only to short-term memory. Conditioning is often used for short-term recall of concept skills. The information learned in this way can only be stored for longer periods of time by thinking. Thinking takes place when applying the skills to working the problems.

How working memory affects what you remember

Working memory (or long-term retrieval) is that part of the brain that can work on problems for a longer period of time than can short-term memory. Working memory, then, offers an increase in the amount of *time* that information is held in memory. (An increase in the *volume* of information that can be held requires long-term memory.)

Working memory involves the ability to recall information after learning has been consistently interrupted over a period of several minutes. Students with working memory problems may listen to a math lecture and understand each step as it is explained. When the instructor goes back to a previous step discussed several minutes prior, however, the student has difficulty explaining or remembering the reasons for the steps. These students have difficulty remembering series steps long enough to understand the concept.

Working memory can be compared to a mental work space or an internal chalkboard. Just like a chalkboard, working memory has limited space, which can cause a "bottleneck" in learning.

Working memory uses the information (such as multiplication tables) recalled from long-term memory. Overlearning math facts – by storing them in long-term memory – can free up more working memory to solve problems. Working memory is like how much RAM you have in your computer.

> **Example:** In solving 26 x 32, you would put the intermediate products 52 (*from 2* x *26*) and 780 (*from 30* x *26* – remember, 3 is in the 10s place, so make it 30) into working memory and add them together. The more automatic the multiplication, the less working memory you use. If you cannot remember your multiplication, you use up working memory trying to solve the multiplication problem. This leaves you with less working memory to solve the resulting addition problem.

How long-term memory/reasoning affects what you remember

Long-term memory is a storehouse of material that is retained for long periods of time. Long-term memory is recalled into working memory to solve problems. It is *not* a matter of trying harder and harder to remember more and more unrelated facts or ideas; it is a matter of organizing your short-term memories and working memories into meaningful information.

Reasoning is thinking about memories, comprehending their meanings, and understanding their concepts. Without rehearsing the information, it will not be processed into long-term memory/reasoning.

The main problem students face is converting learned material from short-term memory or working memory into long-term memory – and understanding it. To place information into long-term memory, you need to understand it and effectively concentrate on it.

> **Remember:** Securing math information into long-term memory is not accomplished by just doing the homework – you must also understand it.

The Role of "Memory Output" in Testing

Memory output is what educators call a retrieving process. It is necessary for verbal or written examinations. It is the method by which you put information stored in your long-term memory onto paper or into words. Memory output can be blocked by three things:

1. insufficient processing of information into long-term memory,
2. test anxiety, or
3. poor test-taking skills.

If you did not completely place all of the information you learned into long-term memory, you may not be able to give complete answers on tests.

Test anxiety can decrease your ability to recall important information or it can cause you to block out information totally. Ways to decrease test anxiety were discussed in Chapter 3. Students who work on their test-taking skills can improve their memory output. Techniques to improve both your memory output and your test-taking skills will be explained in Chapter 7.

Understanding the Stages of Memory

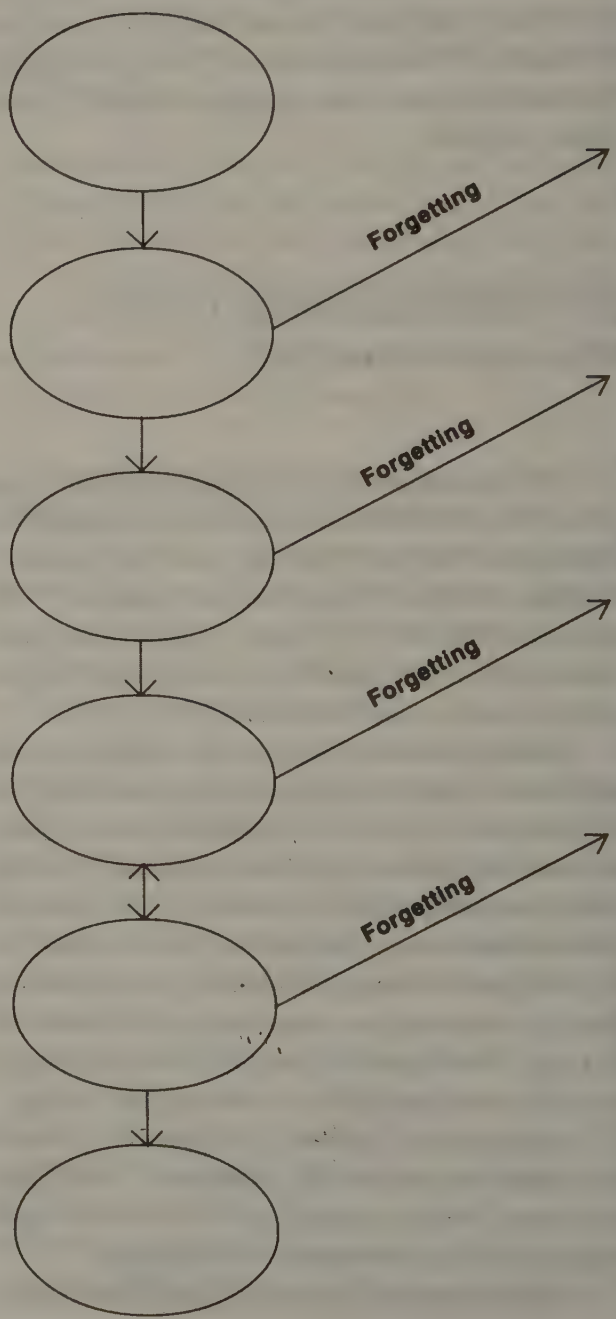
Understanding the stages of memory will help you answer this common question about learning math: "How can I understand the procedures to solve a math problem one day and forget how to solve a similar problem two days later?"

There are three good answers to this question. First, after initially learning how to solve the problem, you did not *rehearse* the solving process enough for it to enter your long-term memory. Second, you did get the information into long-term memory, but the information was not *reviewed* frequently enough and was therefore forgotten. Third, you memorized how to work the problem but did not *understand* the concept.

Remember: Locating where your memory breaks down and compensating for those weaknesses will improve your math learning.

– Section Review –

1. Label the ovals below with stages of memory and list the reasons for forgetting under the slanted arrows.



2. Describe the stages of memory by explaining and giving examples of the following terms:

Sensory input _____

_____

_____

Example _____

_____

Sensory register _____

_____

_____

Example _____

_____

Short-term memory _____

_____

_____

Example _____

_____

Working memory _____

_____

_____

Example _____

_____

Long-term memory _____

_____

_____

Example _____

_____

Memory output _____

_____

_____

Example _____

_____

3. What are three conditions that can block your memory output?

Condition 1: _____

_____

_____

Condition 2: _____

_____

_____

Condition 3: _____

_____

_____

4. Review the stages of memory and list the stages at which your memory breaks down. For each stage of breakdown, say how you can prevent it.

Stage _____

_____

_____

Prevention _____

_____

_____

_____

Stage _____

_____

_____

Prevention _____

_____

_____

_____

Stage _____

_____

_____

Prevention _____

_____

_____

_____

How to use memory techniques

There are many different techniques that can help you store information in your long-term memory. Having a positive attitude about studying, using your best learning sense(s), decreasing distractions, and other techniques can improve your long-term memory.

Have a Good Study/Math Attitude

Having a positive attitude about studying will help you concentrate and improve your retention. This means you need to have at least a neutral math attitude (you neither like nor dislike it), and you should reserve the right to actually learn to *like* math. View studying as an opportunity to learn rather than as an unpleasant task. Tell yourself that you *can* learn the material and that *learning it will help you pass the course and graduate.*

Use Your Best "Learning Sense"

Using your best *learning sense* (what educators call your "predominant learning modality") can improve how well you learn and enhance the transfer of knowledge into long-term memory/reasoning. Your learning senses are vision, hearing, touching, and so on. Ask yourself if you learn best by watching (vision), listening (hearing), or touching (feeling).

Visual (Watching) Learner

Knowing that you are a *visual math learner* can help you select the memory technique that will work best for you. Repeatedly reading and writing math materials being studied is the best way for a visual learner to study.

A visual way to decrease distractions is using the "my mind is full" concept. Imagine that your mind is completely filled with thoughts of learning math, and other distracting thoughts cannot enter. Your mind has one-way input and output, which only responds to thinking about math when you are doing homework or studying.

Auditory (Hearing) Learner

If you are an *auditory learner* (one who learns best by hearing the information), then learning formulas may be best accomplished by repeating them back to yourself or recording them on a tape recorder and listening to them. Reading out loud is one of the best auditory ways to get important information into long-term memory. Saying facts and ideas out loud improves your ability to think and remember. If you cannot recite out loud, recite the material to yourself, emphasizing the key words.

An auditory way to improve your concentration is by becoming aware of your distractions and telling yourself, out loud, to concentrate. If you are in a location where talking out loud will cause a disturbance, mouth the words "start concentrating" as you say them in your mind. Your concentration periods should increase.

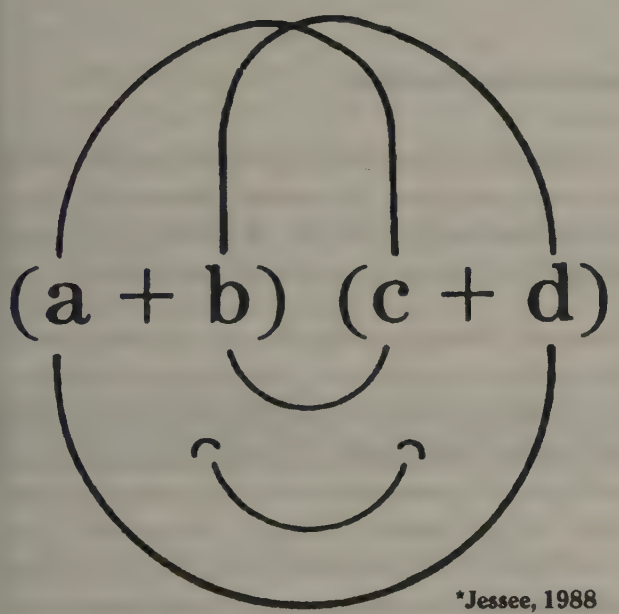
Tactile/Concrete (Touching) Learner

A tactile/concrete learner needs to feel and touch the material to learn it. Unfortunately, this learning sense is not used by most math instructors. As a result, students who depend heavily on feeling and touching for learning will have the most difficulty developing effective learning techniques. For example, if you wanted to learn the FOIL method (see Figure 8, on the facing page, and the acronym example on page 83), you would take your fingers and trace the "face" to remember the steps.

A tactile/concrete way to improve your study concentration is by counting the number of distractions for each study session. Place a sheet of paper by your book when doing homework. When you catch yourself not concentrating, put the letter C on the sheet of paper. This will remind you to concentrate and get back to work. After each study period, count up the number of C's and watch the number decrease.

Figure 8
The FOIL Method

F (a) (c)
O (a) (d)
I (b) (c)
L (b) (d)



*Jessee, 1988

FOIL is used to remember the procedure to *multiply two binomials.* The letters in FOIL stand for **F**irst, **O**utside, **I**nside, and **L**ast. To use FOIL, multiply

— the **F**irst terms, (*a*) (*c*),
— the **O**utside terms, (*a*) (*d*),
— the **I**nside terms, (*b*) (*c*),
— the **L**ast terms, (*b*) (*d*).

To learn FOIL, trace your finger along the FOIL route.

Using Multiple Senses

If you have difficulty learning material from one sense, you might want to try learning through two or three senses. Involving two or more senses in learning improves your learning and remembering.

If your primary sense is visual and your secondary sense is auditory, you may want to write down equations while saying them out loud to yourself. Writing and reciting the material at the same time combines visual and auditory and some tactile/concrete styles of learning and is therefore an excellent way to improve learning

Studying with a pen or highlighter is a visual as well as a tactile/concrete way to improve your concentration. Placing the pen or highlighter in your hand and using it will force you to concentrate more on what you are reading. After you correctly write and recite the material back to yourself one time, do it five or ten more times to overlearn it.

Be a Selective Learner

Being selective in your math learning will improve your memory. Prioritize the materials you are studying; decide which facts you need to know and which ones you can ignore. Narrow down information into laws and principles that can be generalized. Learn the laws and principles 100 percent.

Example: If you have been given a list of math principles and laws to learn for a test, put each one on an index card. As you go through them, create two piles: a "I already know this" pile and a "I don't know this" pile. Then study *only* the "I don't know this" pile. Study the "I don't know this" pile until it is completely memorized and understood.

Become an Organizer

Organizing your math material into idea/fact clusters or groups will help you learn and memorize it. Grouping similar material in a problem log or

calculator log is an example of categorizing information. Do not learn isolated facts; always try to connect them to other similar material.

Use Visual Imagery

Using mental pictures or diagrams to help you learn the material is especially helpful for the visual learners and those who are right-hemisphere dominant (who tend to learn best by visual and spatial methods). Mental pictures and actual diagrams involve 100 percent of your brain power. Picture the steps to solve difficult math problems in your mind.

Example: Use the FOIL method (see Figure 8, page 81) to visually learn how to multiply binomials. Memorize the face until you can sketch it from memory. If you need to use it during a test, you can then sketch the face onto your scratch paper and refer to it.

Make Associations

Association learning can help you remember better. Find a link between new facts and some well-established old facts and study them together. The recalling of old facts will help you remember the new facts and strengthen a mental connection between the two. Make up your own associations to remember math properties and laws.

Remember: The more ridiculous the association, the more likely you are to remember it.

Examples: When learning the *commutative property*, remember that the word *commutative* sounds like the word *community*. A community is made up of different types of people who could be labeled *as* an a group and a b group. However, in a community of a people and b people, it does not matter if we count the a people first or the b people first; we still have the same total number of people in the community. Thus, $a + b = b + a$.

When learning the *distributive law of multiplication over addition*, such as $a(b + c)$, remember that "distributive" sounds like "distributor," which is associated with giving out a product. The distributor a is giving its products to b and c.

Use Mnemonic Devices

The use of mnemonic devices is another way to help you remember. Mnemonic devices are easily remembered words, phrases, or rhymes associated with difficult-to-remember principles or facts.

Example: Many students become confused with the *order of operations*. These students mix up the order of the steps in solving a problem, such as dividing before adding the numbers in the parentheses. A mnemonic device to remember the order of operations is "Please Excuse My Dear Aunt Sally." The first letter in each of the words represents the math function to be completed from the first to the last. Thus, the order of operations is Parentheses (***Please***), Exponents (***Excuse***), Multiplication (***My***), Division (***Dear***), Addition (***Aunt***), and Subtraction (***Sally***). Remember to multiply and/or divide whatever comes first, from left to right. Also, add or subtract whatever comes first, from left to right.

Use Acronyms

Acronyms are another memory device to help you learn math. Acronyms are word forms created from the first letters of a series of words.

Example: FOIL is one of the most common math acronyms. FOIL is used to remember the procedure for multiplying two binomials. Each letter in the word *FOIL* represents a math operation. **FOIL** stands for **F**irst, **O**utside, **I**nside, and **L**ast, as it applies to multiplying two binomials such as $(2x + 3)(x + 7)$. The **F**irst quantities are $2x$ (in the first expression) and x (in the second expression). The **O**utside quantities are $2x$ (in the first expression) and 7 (in the second expression). The **I**nside quantities are 3 (in the first expression) and x (in the second expression). The **L**ast quantities are 3 (in the first expression) and 7 (in the second expression). This results in **F** $(2x)(x)$ + **O** $(2x)(7)$ + **I** $(3)(x)$ + **L** $(3)(7)$.

Do the multiplication to get $2x^2 + 14x + 3x + 21$, which adds up to $2x^2 + 17x + 21$. See Figure 8 (The FOIL Method), page 81.

– Section Review –

1. How can visual learners improve their memory?

2. How can auditory learners improve their memory?

3. How can tactile/concrete learners improve their memory?

4. How can you use multiple senses to improve your memory?

5. Give an example of being a selective learner in your math class.

6. From your current math lessons, make up and explain one association remembering device that is not in this workbook.

7. For your next major math test, make up and explain one mnemonic device that is not in this workbook.

8. For your next major math test, make up and explain one acronym that is not in this workbook.

How to develop practice tests

Developing a practice test is one of the best ways to evaluate your memory and math skills before taking a real test. You want to find out what you do not know *before* the real test instead of *during* the test. Practice tests should be as real as possible and should include time constraints.

You can create a practice test by reworking all the problems since your last test that you have recorded in your problem log. Another practice test can be developed using every other problem in the textbook chapter tests. Further, you can use the solutions manual to generate other problems with which to test yourself. You can also use old exams from the previous semester. Check to see if the math lab/LRC or library has tests on file from previous semesters, or ask your instructor for other tests. For some students, a better way to prepare for a test is the group method.

Example: Hold a group study session several days before the test. Have each student prepare a test with ten questions. On the back of the test, have the answers listed, worked out step by step. Have each member of the study group exchange his or her test with another member of the group. Once all the tests have been completed, have the author of each test discuss with the group the procedures used to solve those problems.

If group work improves your learning, you may want to hold a group study session at least once a week. Make sure the individual or group test is completed at least three days before the real test.

Completing practice math tests will help you increase your testing skills. It will also reveal your test problem weaknesses in enough time for you to learn how to solve the problem before the real test. If you have difficulty with any of the problems during class or after taking the practice test, be sure to see your tutor or instructor.

After taking the practice test(s), you should know what parts you do not understand (and need to study) and what is likely to be on the test. Put this valuable information on one sheet of paper. This information needs to be understood and memorized. It may include formulas, rules, or steps to solving a problem.

Use the learning strategies discussed in this chapter to remember this information. A good example of how this information should look is what students might call a mental "cheat sheet." Obviously, you cannot use the written form of this sheet during the real test.

If you cannot take a practice test, put down on your mental cheat sheet the valuable information you will need for the test. Work to understand and memorize your mental cheat sheet. Chapter 7, "How to Improve Your Math Test-Taking Skills," will discuss how to use the information on the mental cheat sheet – *without cheating.*



1. List three different ways you can make up a practice test.

   Way 1: _____
   _____
   _____
   _____

   Way 2: _____
   _____
   _____
   _____

   Way 3: _____
   _____
   _____
   _____

2. List ten different types of problems you would put

on your practice test for the next exam.

Problem 1: _____

Problem 2: _____

Problem 3: _____

Problem 4. _____

Problem 5: _____

Problem 6: _____

Problem 7: _____

Problem 8: _____

Problem 9: _____

Problem 10: _____

3. Give these ten problems to your study buddy and get ten problems from him or her. Compare your answers with those of your study buddy.

   How many did you get right? _____
   How can you correct the wrong answers? _____

How to use number sense

When taking your practice tests, you should use "number sense," or estimations, to make sure your answer is reasonable. Number sense is like common sense but it applies to math. Number sense is the ability to see if your answer makes sense without using algorithms. (Algorithms are the sequential math steps used to solve problems.) These following examples demonstrate solving a math problem (from a national math test given to high school students) using algorithms and number sense.

Example: Solve 3.04 x 5.3. Students use algorithms to solve this problem by multiplying the number 3.04 by 5.3, in sequence. Seventy-two percent of the students answered the problem correctly using algorithms.

Example: Estimate the product 3.04 x 5.3, and choose from the following answers.

A)	1.6	C) 160
B)	16	D) 1600

Only 15 percent of the students chose B, which is the correct answer. Twenty-eight percent of the students chose A. Using *estimating* to solve the answer, a whopping 85 percent of the students got the problem wrong.

These students were incorrectly using their "mental chalkboard" instead of using number sense. In using number sense to answer, you would multiply the numbers to the left of the decimal in each number to get an estimate of the answer. To estimate the answer, then, you would multiply 3 (the number to the left of the decimal in 3.04) by 5 (the number to the left of the decimal in 5.3) and expect the answer to be a little larger than 15.

It appears that the students' procedural processing (the use of algorithms) was good, but when asked to solve a non-routine problem using estimating (which is easier than using algorithms), the results were disappointing.

Another example of using number sense, or estimating, is in "rounding off."

Taking the time to estimate the answer to a math problem is a good way to check your answer. Another way to check your answer is to see if it is reasonable. Many students forget this important step and get the answer wrong. This is especially true of word or story problems.

Example: Solve 48 + 48 by rounding off. Rounding off means mentally changing the number (up or down) to make it more manageable to you, without using algorithms. By rounding off, 48 becomes 50 (easier to work with). 50 + 50 = 100. If the choices for answers were 104, 100, 98 and 96, you would then subtract 4 from the 100 (since each number was rounded up by 2) and you would get 96.

Also, when dealing with an equation, make sure to put the answer back into the equation to see if one side of the equation equals the other. If the two sides are not equal, you have the wrong answer. If you have extra time left over after you have completed a test, you should check answers using this method.

Examples: When solving a rate-and-distance problem, use your common sense to realize that one car cannot go 500 miles per hour to catch the other car. However, the car can go 50 miles per hour.

The same common sense rule applies to age-word problems, where the age of a person cannot be 150 years, but it can be 15 years.

Further, in solving equations, x is *usually* a number that is less than 20. When you solve a problem for x and get 50, then, this isn't reasonable, and you should recheck your calculations.

Example: In solving the equation $x + 3 = 9$, you calculated that $x = 5$. To check your answer, substitute x with 5 and see if the problem works out correctly. 5 + 3 does not equal 9, so you know you have made a mistake and need to recalculate the problem. The correct answer, by the way, is $x = 6$.

Remember: Number sense is a way to get more math problems correct by estimating your answer to determine if it is reasonable.

– Section Review –

1. Give an explanation of number sense. _____

2. How does number sense compare to common sense? _____

3. Give an example of a recent math problem you missed due to poor number sense. _____

4. List two ways you can improve your number sense.

   First Way _____

   Second Way _____

Name: _____ Date: _____



Chapter 6 Review



1. _____ is learning with a maximum of physical and _____ reaction and a minimum of thinking.

2. _____ is defined as learning with a maximum of thought and a minimum of emotional and _____ reaction.

3. The way you receive information is through your five senses, which are _____, _____, _____, _____, and _____.

4. The _____ _____ briefly holds an exact image or sound of each sensory experience until it can be processed or forgotten.

5. Short-term memory involves the ability to _____ information _____ after it is given without any interruptions.

6. Working memory involves the ability to recall information after learning has been _____ over a period of several _____.

7. Working memory can also be considered your internal _____ that recalls information from _____ _____.

8. The main problem most students have is converting learned material into _____ _____ and _____ it.

9. Repeatedly reading and writing math material is one of the best ways for a _____ learner to study.

10. Reading _____ _____ is one of the best auditory ways to learn material.

11. A _____/_____ learner needs to feel and touch material to learn it.

12. Being a selective learner means _____ the material to study and learning the laws and principles _____ percent.

13. Mnemonic devices are easy-to-remember _____, _____, or _____ associated with difficult-to-remember principles or facts.

14. _____ are word forms created from the first letters of a series of words.

15. The reason to develop a practice test is to find out what you do not know _____ the test instead of _____ the test.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
_____
_____
_____
_____

Chapter 7

How to Improve Your Math Test-Taking Skills

Taking a math test is different from taking tests in other subjects. Math tests require not only that you to recall the information, but that you apply the information. Multiple-choice tests, for example, usually test you on recall, and if you do not know the answer, you can guess.

Math tests build on each other, whereas history tests often do not test you on previous material. Most math tests are speed tests, where the faster your are, the better grade you can receive; most social science tests are designed for everyone to finish.

Math test preparation and test-taking skills are different from preparation and skills needed for other tests. You need to have a test-taking plan and a test analysis plan to demonstrate your total knowledge on math tests. Students with these plans make better grades compared to students without them. Math instructors want to measure your math knowledge, not your poor test-taking skills.

In this chapter, you will learn

- why attending class and doing your homework may not be enough to pass,
- the general pretest rules,
- the ten steps to better test taking,
- the six types of test-taking errors, and
- how to prepare for the final exam.

Why attending class and doing your homework may not be enough to pass

Most students and some instructors believe that attending class and doing all the homework ensures an A or B on tests. This is far from true. Doing all the homework and getting the correct answers is very different in many ways from taking tests:

1. While doing homework, there is little anxiety. A test situation is just the opposite.

2. You are not under a time constraint while doing your homework; you may have to complete a test in 55 minutes or less.

3. If you get stuck on a homework problem, your textbook and notes are there to assist you. This is not true for most tests.

4. Once you learn how to do several problems in a homework assignment, the rest are similar. In a test, the problems are in random order.

5. In doing homework, you have the answers to at least half the problems in the back of the text and answers to all the problems in the solutions guide. This is not true for tests.

6. While doing homework, you have time to figure out how to correctly use your calculator.

During the test, you can waste valuable time figuring out how to use your calculator.

7. When doing homework, you can call your study buddy or ask the tutor for help, something you cannot do on the test.

Do not develop a false sense of security by believing you can make an A or B by doing just your homework. Tests measure more than just your math knowledge.

The general pretest rules

General rules are important when taking any type of test:

1. *Get a good night's sleep before taking a test.* This is true for the ACT, the SAT, and your math tests. If you imagine you are going to cram all night and perform well on your test with three to four hours of sleep, you are wrong. It would be better to get seven or eight hours of sleep and be fresh enough to use your memory to recall information needed to answer the questions.

2. *Start studying for the test at least three days ahead of time.* Make sure you take a practice test to find out, before the test, what you do not know. Review your problem log and work the problems. Review the concept errors you made on the last test. (How to identify and correct your concept errors will be discussed later in this chapter.) Meet with the instructor or tutor for help on those questions you cannot solve.

3. *Review only already learned material the night before a test.*

4. *Make sure you know all the information on your mental cheat sheet.* Review your notebook and glossary to make sure you understand the concepts. Work a few problems and recall the information on your mental cheat sheet right before you go to bed. Go directly to bed; do not watch television, listen to the radio, or party. While you are asleep, your mind will work on and remember the last thing you did before going to sleep.

5. *Get up in the morning at your usual time and review your notes and problem log.* Do not do any new problems, but make sure your calculator is working.



1. List five reasons why only attending class and doing your homework may not be enough to pass your math course.

Reason 1: _____
_____
_____

Reason 2: _____
_____
_____

Reason 3: _____
_____
_____

Reason 4: _____
_____
_____

Reason 5: _____
_____
_____

2. List and explain three general pretest rules that best apply to you.

Rule 1: _____

_____

_____

Rule 2: _____

_____

_____

Rule 3: _____

_____

_____

The ten steps to better test taking

Once you begin a test, follow the ten steps to better test taking:

Step 1 *Use a memory data dump.* Upon receiving your test, turn it over and write down the information that you put on your mental cheat sheet. Your mental cheat sheet has now turned into a mental list, and writing down this information is not cheating. Do not put your name on the test, do not skim it, just turn it over and write down those facts, figures, and formulas from your mental cheat sheet or other information you might not remember during the test. This is called your *first memory data dump.* The data dump provides memory cues for test questions.

Example: It might take you a while to remember how to do a coin problem. However, if you had immediately turned your test over and written down different ways of solving coin problems, it would be easier to solve the coin problem.

Step 2 *Preview the test.* Put your name on the test and start previewing. Previewing the test requires you to look through the entire test to find different types of problems and their point values. Put a mark by the questions that you can do without thinking. These are the questions that you will solve first.

Step 3 *Do a second memory data dump.* The secondary data dump is for writing down material that was jarred from your memory while previewing the test. Write this information on the back of the test.

Step 4 *Develop a test progress schedule.* When you begin setting up a test schedule, determine the point value for each question. Some test questions might be worth more points than others.

In some tests, word problems are worth 5 points and other questions might be worth 2 or 3 points. You must decide the best way to get the most points in the least amount of time. This might mean working the questions worth 2 or 3 points first and leaving the more difficult word problems for last.

Decide how many problems should be completed halfway though the test. You should have more than half the problems completed by that time.

Step 5 *Answer the easiest problems first.* Solve, in order, the problems you marked while previewing the test. Then review the answers to see if they make sense. Start working through the test as fast as you can while being accurate. Answers should be reasonable.

Example: The answer to a problem asking you to find the area of a rectangle cannot be negative, and the answer to a land-rate-distance problem cannot be 1000 miles per hour.

Clearly write down each step to get partial credit. Even if you end up missing the problem,

you might get some credit. In most math tests, the easier problems are near the beginning of the first page; you need to answer them efficiently and quickly. This will give you both more time for the harder problems and time to review.

Step 6 *Skip difficult problems.* If you find a problem that you do not know how to work, read it twice and automatically skip it. Reading it twice will help you understand the problem and put it into your working memory. While you are solving other problems, your mind is still working on that problem. Difficult problems could be the type of problem you have never seen before or a problem in which you get stuck on the second or third step. In either case, skip the problem and go on to the next one.

Step 7 *Review the skipped problems.* When working the skipped problems, think how you have solved other, similar problems as a cue to solving the skipped ones. Also try to remember how the instructor solved that type of problem on the board.

While reviewing skipped problems, or at any other time, you may have the "Aha!" response. The "Aha!" response is your remembering how to do a skipped problem. Do not wait to finish your current problem. Go to the problem on which you had the "Aha!" and finish that problem. If you wait to finish your current problem, your "Aha!" response could turn into an "Oh, no!" response.

Step 8 *Guess at the remaining problems.* Do as much work as you can on each problem, even if it is just writing down the first step. If you cannot write down the first step, rewrite the problem. Sometimes rewriting the problem can jar your memory enough to do the first step or the entire problem. If you leave the problem blank, you will get a zero. Do not waste too much time on guessing or trying to work the problems you cannot do.

Step 9 *Review the test.* Look for careless errors or other errors you may have made. Students usually lose 2 to 5 test points on errors that could have been caught in review. Do not talk yourself out of an answer just because it may not look right. This often happens when an answer does not come out even. It is possible for the answer to be a fraction or decimal.

Remember: Answers in math do not have "dress codes." Research reveals that the odds of changing a right answer to a wrong answer are greater than the odds of changing a wrong answer to a right one.

Step 10 *Use all the allowed test time.* Review each problem by substituting the answer back into the equation or doing the opposite function required to answer the question. If you cannot check the problem by the two ways mentioned, rework the problem on a separate sheet of paper and compare the answers. Do not leave the test room unless you have reviewed each problem two times or until the bell rings.

Remember: There is no prize for handing your test in first, and students who turn their papers in last do make A's.

Stapling your scratch paper to the math test when handing it in has several advantages:

- If you miscopied the answer from the scratch paper, you will probably get credit for the answer.
- If you get the answer incorrect due to a careless error, your work on the scratch paper could give you a few points.
- If you do get the problem wrong, it will be easier to locate the errors when the instructor reviews the test. This will prevent you from making the same mistakes on the next math test.

Remember: Handing in your scratch paper may get you extra points or improve your next test score.

– Section Review –

1. List and explain the ten steps to test taking.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

Step 9: _____

Step 10: _____

2. After trying the ten steps to test taking, develop your own personalized test-taking steps.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

Additional Steps _____

The six types of test-taking errors

To improve future test scores, you must conduct a test analysis of previous tests. In analyzing your tests, you should look for the following kinds of errors:

1. misread-directions errors
2. careless errors
3. concept errors
4. application errors
5. test-taking errors
6. study errors

Students who conduct math test analyses will improve their total test scores.

Misread-directions errors occur when you skip directions or misunderstand directions, but do the problem anyway.

Examples: Suppose you have this type of problem to solve: $(x + 1)(x + 1)$. Some students will try to solve for x, but the problem only calls for multiplication. You would solve for x only if you have an equation such as $(x + 1)(x + 1) = 0$.
Another common mistake is not reading the directions before doing several word problems or statistical problems. All too often, when a test is returned, you find only three out of the five problems had to be completed. Even if you did get all five of them correct, it cost you valuable time that could have been used obtaining additional test points.

To avoid misread-directions errors, read all the directions. If you do not understand them, ask the instructor for clarification.

Careless errors are mistakes that you can catch automatically upon reviewing the test. Both good and poor math students make careless errors. Such errors can cost a student a higher letter grade on a test.

Examples: *Dropping the sign*: $-3(2x) = 6x$, instead of $-6x$, which is the correct answer.
Not simplifying your answer: Leaving $(3x - 12)/3$ as your answer instead of simplifying it to $x - 4$.
Adding fractions: $1/2 + 1/3 = 2/5$, instead of $5/6$, which is the correct answer.
Word problems: $x = 15$ instead of "The student had 15 tickets."

However, many students want all their errors to be careless errors. This means that the students *did* know the math, but simply made silly mistakes. In such cases, I ask the student to solve the problem immediately, while I watch.

If the student can solve the problem or point out his or her mistake in a few seconds, it is a careless error. If the student cannot solve the problem immediately, it is not a careless error and is probably a concept error.

When working with students who make careless errors, I ask them two questions: First, "How many points did you lose due to careless errors?" Then I follow with, "How much time was left in the class period when you handed in your test?" Students who lose test points to careless errors are giving away points if they hand in their test papers before the test period ends.

To reduce careless errors, you must realize the types of careless errors made and recognize them when reviewing your test. If you cannot solve the missed problem immediately, it is not a careless error. If your major error is not simplifying the answer, review each answer as if it were a new problem and try to reduce it.

Concept errors are mistakes made when you do not understand the properties or principles required to work the problem. Concept errors, if not corrected, will follow you from test to test, causing a loss of test points.

Examples: Some common concept errors are not knowing:

$(-)(-)x = x$, *not* $-x$

$-1(2) > x(-1) = 2 < x$, *not* $2 > x$

$5/0$ is undefined, *not* 0

$(a + x) / x$ is *not* reduced to a

the order of operations

Concept errors must be corrected to improve your next math test score. Students who have numerous concept test errors will fail the next test and the course if concepts are not understood. Just going back to rework the concept error problems is not good enough. You must go back to your textbook or notes and learn why you missed those types of problems, not just the one problem itself.

The best way to learn how to work those types of problems is to set up a concept problem error page in the back of your notebook. Label the first page "Test One Concept Errors." Write down all your concept errors and how to solve the problems. Then, work five more problems that use the same concept. Now, in your own words, write the concepts that you are using to solve these problems.

If you cannot write the concept in your own words, you do not understand it. Get assistance from your instructor if you need help finding similar problems using the same concept or cannot understand the concept. Do this for every test.

Application errors occur when you know the concept but cannot apply it to the problem. Application errors usually are found in word problems, deducing formulas (such as the quadratic equation), and graphing. Even some better students become frustrated with application errors; they understand the material but cannot apply it to the problem.

To reduce application errors, you must predict the type of application problems that will be on the test. You must then think through and practice solving those types of problems using the concepts.

Example: If you must derive the quadratic formula, you should practice doing that backward and forward while telling yourself the concept used to move from one step to the next.

Application errors are common with word problems. After completing the word problem, reread the question to make sure you have applied the answer to the intended question. Application errors can be avoided with appropriate practice and insight.

Test-taking errors apply to the specific way you take tests. Some students consistently make the same types of test-taking errors. Through recognition, these bad test-taking habits can be replaced by good test-taking habits. The result will be higher test scores. The list that follows includes the test-taking errors that can cause you to lose many points on an exam:

1. *Missing more questions in the first third, second third, or last third of a test* is considered a test-taking error.

   Missing more questions in the first third of a test can be due to carelessness when doing easy problems or due to test anxiety.

   Missing questions in the last part of the test can be due to the fact that the last problems are more difficult than the earlier questions or due to increasing your test speed to finish the test.

   If you consistently miss more questions in a certain part of the test, use your remaining test time to review that section of the test first. This means you may review the last part of your test first.

2. *Not completing a problem to its last step* is another test-taking error. If you have this bad habit, review the last step of the test problem first, before doing an in-depth test review.

3. *Changing test answers from correct ones to incorrect ones* is a problem for some students. Find out if you are a good or bad answer-changer

by comparing the number of answers changed to correct and to incorrect answers. If you are a bad answer-changer, write on your test, "Don't change answers." Change answers only if you can prove to yourself or the instructor that the changed answer is correct.

4. *Getting stuck on one problem and spending too much time on it* is another test-taking error. You need to set a time limit on each problem before moving to the next problem. Working too long on a problem without success will increase your test anxiety and waste valuable time that could be used in solving other problems or in reviewing your test.

5. *Rushing through the easiest part of the test and making careless errors* is a common test-taking error for the better student. If you have the bad habit of getting more points taken off for the easy problems than for the hard problems, first review the easy problems, and later review the hard problems.

6. *Miscopying an answer from your scratch work to the test* is an uncommon test-taking error, but it does cost some students points. To avoid these kinds of errors, systematically compare your last problem step on scratch paper with the answer written on the test. In addition, always hand in your scratch work with your test.

7. *Leaving answers blank* will get you zero points. If you look at a problem and cannot figure out how to solve it, do not leave it blank. Write down some information about the problem, rewrite the problem, or try to do at least the first step.

   Remember: Writing down the first step of a problem is the key to solving the problem and obtaining partial credit.

8. *Solving only the first step of a two-step problem* causes problems for some students. These students get so excited when answering the first step of the problem that they forget about the second part. This is especially true on two-step word problems. To correct this test-taking error, write "two" in the margin of the problem. That will remind you that their are two steps or two answers to this problem.

9. *Not understanding all the functions of your calculator* can cause major testing problems. Some students only barely learn how to use the critical calculator functions. They then forget or have to relearn how to use their calculator, which costs test points and test time. Do not wait to learn how to use your calculator on the test. Overlearn the use of your calculator *before* the test.

10. *Leaving the test early without checking all your answers is a costly habit.* Do not worry about the first person who finishes the test and leaves. Many students start to get nervous when students start to leave after finishing the test. This can lead to test anxiety, mental blocks, and loss of recall.

    According to research, the first students finishing the test do not always get the best grades. It sometimes is the exact opposite. Ignore the exiting students, and always use the full time allowed.

Make sure you follow the ten steps to better test taking (see page 91). Review your test-taking procedures for discrepancies in following the ten steps to better test taking. Deviating from these proven ten steps will cost you points.

Study errors, the last type of mistake to look for in test analysis, occur when you study the wrong type of material or do not spend enough time on pertinent material. Review your test to find out if you missed problems because you did not practice that type of problem or because you did practice it but forgot how to do it during the test. Study errors will take some time to track down. But correcting study errors will help you on future tests.

Most students, after analyzing one or several tests, will recognize at least one major, common test-taking error. Understanding the effects of this test-taking error should change your study techniques or test-taking strategy.

Example: If there are seven minutes left in the test, should you review for careless errors or try to answer those two problems you could not totally solve? This is a trick question. The real question is, Do you miss more points due to careless errors or concept errors, or are the missed points about even? The answer to this question should determine how you will spend the last minutes of the test. If you miss more points due to careless errors or miss about the same number of points due to careless/concept errors, review for careless errors.

Careless errors are easier to correct than concept errors. However, if you make very few or no careless errors, you should be working on those last two problems to get the greatest number of test points. Knowing your test-taking errors can add more points to your test by changing your test-taking procedure.



1. List the six types of test-taking errors.

   Error 1: _____
   _____
   _____

   Error 2: _____
   _____
   _____

   Error 3: _____
   _____
   _____

   Error 4: _____
   _____
   _____

   Error 5: _____
   _____
   _____

   Error 6: _____
   _____
   _____

2. Which of these errors have you made in the past?

   _____
   _____
   _____
   _____
   _____
   _____

3. How can you avoid these types of errors on the next test?

   _____
   _____
   _____
   _____
   _____
   _____

4. Review your last test to list the type of errors, the points lost for each error, and examples of the errors. (Remember: Careless errors are problems you can solve immediately.)

   Error 1: _____
   Points Lost _____
   Error Example _____
   _____
   _____

Error 2: _____
Points Lost _____
Error Example _____
_____
_____

Error 3: _____
Points Lost _____
Error Example _____
_____
_____

Error 4: _____
Points Lost _____
Error Example _____
_____
_____

Error 5: _____
Points Lost _____
Error Example _____
_____
_____

Error 6: _____
Points Lost _____
Error Example _____
_____
_____

5. List any additional concept error problems that were not included in question 2.

Concept Error Problems _____
_____
_____
_____

Using the pages in the back of your math notebook, rework the concept error problems and do five math problems just like them. For each concept error, write why you could not solve the problem on the test and what new information you learned to solve the problem. Do this for every major test. Give an example of one of your concept error(s) and the reasons you can now solve it.

Concept Error Problem(s) _____
_____
_____
_____

Reasons I Can Now Solve It _____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How to prepare for the final exam

The first day of class is when you start preparing for the final exam. Look at the syllabus or ask the instructor if the final exam is cumulative. A cumulative exam covers everything from the first chapter to the last chapter. Most math final exams are cumulative.

The second question you should ask is if the final exam is a departmental exam or if it is made up by your instructor. In most cases, departmental exams are more difficult and need a little different preparation. If you have a departmental final, you need to ask for last year's test and ask other students what their instructors say will be on the test.

The third question is, How much will the final exam count? Does it carry the same weight as a regular test or, as in some cases, will it count a third of your grade? If the latter is true, the final exam will usually make a letter grade difference on your final grade. The final exam could also determine if you

pass or fail the course. Knowing this information before the final exam will help you prepare.

Preparing for the final exam is similar to preparing for each chapter test. You must create a pretest to discover what you have forgotten. You can use questions from the textbook chapter tests or questions from your study group.

Review the concept errors that you recorded in the back of your notebook labeled "Test One," "Test Two," and so on. Review your problem log for questions you consistently miss. Even review material that you knew for the first and second test but which was not used on any other tests. Students forget how to work some of these problems.

Make sure to use the ten steps to better test taking (see page 91), and the information gained from your test analysis. Use all the time on the final exam, because you could improve your final grade by a full letter if you make an A on the final.



1. What are the three qustions you need to ask about your final exam?

   Question 1: _____
   _____
   _____

   Question 2: _____
   _____
   _____

   Question 3: _____
   _____
   _____

Lecture Notes

Name: _____ Date: _____



Chapter 7 Review



1. You should start studying for a test at least _____ days ahead of time.

2. You should be reviewing only already-_____ material the night before the test.

3. When previewing the test, put a _____ by the questions you can solve without thinking, and do those problems _____.

4. When taking a test, you must decide the best way to get the _____ points in the _____ amount of time.

5. Halfway through the test, you should have _____ than half of the problems completed.

6. If you cannot write down the first step of a problem, you should _____ the problem to help you remember how to solve it.

7. Do not leave the test room until you have reviewed each problem _____ times or until the bell rings.

8. In reviewing the test, if you cannot solve the problem immediately (within a few seconds), it is not a _____ error and is probably a _____ error.

9. Students who have numerous _____ errors will fail the next test and the course if the _____ are not understood.

10. The _____ day of class is when you start preparing for the final exam.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

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About the Author

Over the past 15 years, Learning Specialist Dr. Paul Nolting has helped thousands of students improve their math learning and obtain better grades. Dr. Nolting is a national expert in assessing math learning problems – from study skills to learning disabilities – and developing effective learning strategies and testing accommodations.

Dr. Nolting is also a nationally recognized consultant and trainer of math study skills and of learning and testing accommodations for students with learning disabilities. He has conducted national training grant workshops on math learning for the Association on Higher Education and Disabilities and for a two-year University of Wyoming 1991 Training Grant of Basic Skills.

Dr. Nolting has conducted numerous national conference workshops on math learning for the National Developmental Education Association, the National Council of Educational Opportunity Association, and the American Mathematical Association of Two-Year Colleges. He is a consultant for the American College Test (ACT) and the Texas Higher Education Coordinating Board. He is also a consultant for Houghton Mifflin faculty development programs, conducting workshops on math study skills for math faculty. His text *Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills* is used throughout the United States, Canada, and the world as the definitive text for math study skills.

Dr. Nolting has consulted with numerous universities and colleges. Some of the universities with which Dr. Nolting has consulted are the University of Massachusetts, the University of Colorado-Boulder, Texas Tech. University, Black Hills State University, Tennessee Tech. University, and the University of Connecticut.

Some of the colleges with which he has consulted are San Antonio College, St. Louis Community College, J. Sargeant Reynolds College, Montgomery College, Broward Community College, Miami-Dade Community College, Northeast State Technical Community College, Landmark College, Denver Community College, Valencia Community College, and Pensacola Junior College. Dr. Nolting has consulted with over 75 colleges, universities, and high schools over the last 15 years.

Dr. Nolting holds a Ph.D. in education from the University of South Florida. His Ph.D. dissertation was "The Effects of Counseling and Study Skills Training on Mathematics Academic Achievement." He is an adjunct instructor for the University of South Florida, teaching assessment and appraisal courses at the graduate level.

His book *Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills* was selected Book of the Year by the National Association of Independent Publishers. "The strength of the book is the way the writer leads a reluctant student through a course from choosing a teacher to preparing for the final examination," says *Mathematics Teacher*, a publication of the National Council of Teachers of Mathematics. *Winning at Math* and Dr. Nolting's other titles are available at http://www.academicsuccess.com.

His two audiocassettes, *How to Reduce Test Anxiety* and *How to Ace Tests*, were also winners of awards in the National Association of Independent Publishers competition. "Dr. Nolting," says *Publisher's Report*, "is an innovative and outstanding educator and learning specialist."

A key speaker at numerous regional and national education conferences and conventions, Dr. Nolting has been widely acclaimed for his ability to communicate with faculty and students on the subject of improving math learning.

Books and Materials by Academic Success Press, Inc., and Dr. Paul Nolting

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Ordering Information

For ordering information on *Winning at Math* and other books and learning materials listed on the previous pages, call toll free: 1-800-247-6553 or visit www.academicsuccess.com. Credit card, checks, and purchase orders are accepted. Shipping charges are $4.00 for the first item and $2.00 for any additional items. Academic Success Press can be contacted at PO Box 25002, Box 132, Bradenton, FL 34206. For a free catalog, call (941) 359-2819.

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Table of Properties

Properties of Real Numbers

Commutative Property of Addition
If a and b are real numbers, then $a + b = b + a$.

Associative Property of Addition
If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

Addition Property of Zero
If a is a real number, then $a + 0 = 0 + a = a$.

Inverse Property of Addition
If a is a real number, then $a + (-a) = (-a) + a = 0$.

Multiplication Property of One
If a is a real number, then $a \cdot 1 = 1 \cdot a = a$.

Commutative Property of Multiplication
If a and b are real numbers, then $a \cdot b = b \cdot a$.

Associative Property of Multiplication
If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Multiplication Property of Zero
If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Inverse Property of Multiplication
If a is a real number and $a \neq 0$, then $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Distributive Property
If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

Properties of Exponents

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

If m and n are integers, then $(x^m)^n = x^{mn}$.

If x is a real number and $x \neq 0$, then $x^0 = 1$.

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If m, n, and p are integers, then $(x^m \cdot y^n)^p = x^{mp}y^{np}$.

If n is a positive integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Properties of Equations

Addition Property of Equations
If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations
If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.

Properties of Inequalities

Addition Property of Inequalities
If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Properties of Radical Expressions

If a and b are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

A radical expression is in simplest form when:
- the radicand has no perfect-square factors
- no radicals appear in the denominator
- no fractions appear in the radicand



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